this non-equilibrium state. These are admittedly the results he requires for his estimates of how the rate of expansion depends on the magnitude of the initial "stagnation." At the same time the investigation does not explain how the universe contrives initially to pass from the equilibrium state \( p = p_0, R = R_1 \), to the non-equilibrium state \( p = 0, R = R_1 \). It tells us what will happen after the equilibrium has been disturbed in a particular way, but provides no mechanism which will produce such a disturbance.

We conclude by stressing again the two conditions which, in our view, must be observed in an approach to this subject (cf. § 1 above):

(a) The initial state must be some equilibrium state (in our case an Einstein universe).

(b) There must be some "condition of permanency," such as the constancy of total proper mass, to ensure that we are dealing throughout all changes with the same universe.

5. Einstein's Universe.—One reason for retaining the \( \lambda \)-term in the gravitational field equations was that without it they gave a negative pressure for a positive uniform density of matter. Einstein * has lately pointed out that this conclusion holds only if the field is restricted to be static. If this restriction is removed he shows that there can be a positive uniform density with zero pressure for zero value of \( \lambda \). He proposes, therefore, to drop the \( \lambda \)-term from the field equations.

The point that concerns us here is that, if we take his resulting equation for the radius of space, \( R \), then \( d^2R/dt^2 \) is never zero, i.e. there is no equilibrium state. On this view our original problem ceases to exist. We have no longer to find an agency for disturbing an equilibrium state, since no such state can exist.

It would be out of place, however, to attempt here a full examination of Einstein's proposal. Our general conclusion is that our original problem is still unsolved, but that this new suggestion may destroy any physical interest in solving it.


1. The following notes on methods of reduction of the observations of Eros secured around the time of the opposition of 1931 February 17, and on the form of publication, have been prepared in order to secure uniformity in the published data with a view to facilitating the subsequent co-ordination of results from different observatories, and to draw attention to a few points of importance which may be overlooked. At the recent opposition, the photographic observations preponderate largely over visual observations; these notes are therefore concerned mainly with the reduction of photographic observations.

2. Corrections for Differential Refraction.—In observations obtained at large zenith distances, it is necessary to apply corrections for the second order terms in differential refraction. It should be noted that such terms change sign with the parallax, and therefore enter with full effect into the derivation of the solar parallax from the combination of observations secured at large eastern and western hour angles.

The correct formulæ to use in the reduction of photographic plates are

\[\Delta x = - k(2X^2 + Y(1 + 2Z^2)Y + Z(1 + Y^2)Z),\]
\[\Delta y = - k(Y + X^2)X + Z(1 + Y^2)Y + Y(2 + Y^2)Y^2],\]

where

\[x, y\] denote the measured co-ordinates of a star;
\[X, Y\] denote the co-ordinates of the zenith on the plate.

\(X, Y\) are given by

\[X = \sin \omega \tan \zeta, \quad Y = \cos \omega \tan \zeta,\]

\(\omega\) being the parallactic angle and \(\zeta\) the zenith distance. In the above formulæ, \(\Delta x, \Delta y\) are corrections applicable to the measured co-ordinates to obtain the co-ordinates free from the effects of the second-order terms. These corrections should be applied before making the solutions for the ordinary six constant reductions.

The application of these squared terms is facilitated by the diagrams given by Hinks.*

It should be noted that Kapteyn † and Baillaud ‡ derived expressions for the second-order terms in differential refraction in rectangular co-ordinates, which are equivalent to one another, but differ from the above expressions. These formulæ are incorrect, although the expressions for differential refraction in distance and position angle, from which they were deduced, are correct. The error in the formulæ given by Kapteyn and Baillaud arises as follows: if \(S_1, S_2\) denote the true positions of two stars, \(S_1\) being at the centre of the plate, and the arc \(S_1S_2\) is denoted by \(\sigma\), the projection of \(S_1S_2\) on the plate is \(\tan \sigma\); if \(S_1', S_2'\) are the positions of \(S_1, S_2\) as affected by refraction and if \(S_1'S_2' = \sigma + a\sigma\), it is assumed by Kapteyn and Baillaud that the projection of \(S_1'S_2'\) on the plate is \(\tan(\sigma + a\sigma)\). This is not correct to the second order.

Attention is drawn to this, because incorrect formulæ were used in the reduction of some of the observations obtained at the 1901 opposition.

The principal refraction terms depending upon the change of the constant of refraction with zenith distance and upon the square of the constant of refraction are linear, and are automatically absorbed into the orientation and scale constants.

3. Mean of Several Exposures.—In general, each plate contains more than one exposure on Eros. It is important to examine within what limits it is legitimate to assume that the parallax and position corresponding

* M.N., 63, 138–147, 1903 January.
‡ Ibid., 3, 19, 1902 (vide p. 48).
to the mean time of several exposures can be assumed to be equal respectively to the mean values of the parallax and position for the several exposures.

The limits for the parallax are very narrow. The parallax in right ascension is

\[ \pi_a = \pi_0 \cos \phi \sec \delta \sin h = \pi \sin h \text{ (say)}, \]

where \( \pi_0 \) is the horizontal parallax and \( h \) is the hour angle. Suppose the hour angle at the mean time of several exposures is \( h \), and \( t_1, t_2 \ldots t_n \) are the mid-times of several exposures, measured from the mean time, so that \( \Sigma t = 0 \).

The parallax at the mean time is \( \pi \sin h \).

The mean of the parallax values for the several exposures is

\[ \pi \sin h - \frac{1}{2} \pi \sin h \cdot \frac{\Sigma t^2}{n} \]

The parallax at the mean time is thus greater than the mean of the true values by

\[ \delta \pi_a = \frac{1}{2} \pi \sin h \cdot \frac{\Sigma t^2}{n} \]

\[ = \frac{1}{2} \pi_0 \cdot \frac{\Sigma t^2}{n} \]

If the time is expressed in minutes,

\[ \frac{\delta \pi_a}{\pi_a} = \frac{1}{2n} \cdot \left( \frac{\pi}{720} \right)^2 \cdot \Sigma t^2. \]

The deduced value of the parallax will be systematically too small. If there are four exposures on the plate, each of \( 3^m \) duration, with intervals of \( 1^m \) between the exposures, \( \delta \pi_a/\pi_a = 0.015/8 \). The deduced value of the solar parallax will be too small by \( 0.0016 \). If there are three exposures of \( 3^m \) duration, with intervals of \( 1^m \), the deduced value will be too small by \( 0.0009 \).

These quantities are not negligible compared with the probable errors to be expected from the programme. Therefore it is necessary in the derivation of the solar parallax from observations at east and west hour angles to reduce each exposure separately.

In the same way, in the case of observations taken near the meridian, it is not in general legitimate to assume that the parallax in declination corresponding to the mean time of several exposures is equal to the mean parallax for the several exposures.

If a single exposure extends over a sufficient time, the mean parallax for the exposure will not be equal to the parallax at the mid-time of the exposure. But as the exposures on Eros were for the most part short, no error is in general to be feared on this account. In any case of doubt it is easy to ascertain what error is involved.

The limits within which it may be assumed that the position corresponding to the mean time of several exposures is equal to the mean position
for the several exposures are considerably wider than in the case of the parallax. The maximum second difference for intervals of one day is about 5" in right ascension and 115" in declination. If the time-interval is less than about 30 minutes, the mean position may be assumed equal to the position at the mean time. But since this is not the case for the parallactic displacement, it is generally necessary to reduce each exposure independently.

4. Measurement and Reduction of Plates.—The reduction of the measurements on the plates follows the methods customary for astrographic plates, for which various formulæ are available. As some of the co-operating observatories are not familiar with astrographic reductions, suitable formulæ are summarised below.

If \( A, D \) are the right ascension and declination of the centre of the plate; \( \alpha, \delta \) the right ascension and declination of a star whose rectangular co-ordinates are \( \xi, \eta \), we have:

\[
\xi = \sin (\alpha - A) \sec (\theta - D) \sin \theta \cot \delta
= \tan (\alpha - A) \sec (\theta - D) \cos \theta,
\eta = \tan (\theta - D),
\]

where

\[
\tan \theta = \sec (\alpha - A) \tan \delta.
\]

Thus

\[
\theta = \delta + \tan^2 \frac{\alpha - A}{2} \sin 2\delta \quad \text{(approximately)}
\]

gives \( \theta \), and \( \xi, \eta \) are determined from the above formulæ when \( A \) and \( D \) are known. The standard co-ordinates of the reference stars can thus be computed. The measured co-ordinates, corrected for the second-order terms in differential refraction, are equated to the computed co-ordinates by linear formulæ, and the constants of these formulæ derived. The standard co-ordinates of Eros can thus be derived.

Knowing \( \xi, \eta \), we can derive \( \alpha, \delta \) by the formulæ

\[
\tan (\alpha - A) = \frac{\xi \sec D}{1 - \eta \tan D},
\]

\[
\delta = D + \eta - \frac{1}{2} \xi^2 \tan (D + \eta) - \frac{\eta^3}{3} + \text{fourth power and higher terms.}
\]

Or it is possible to use the special tables given by Loewy in Circular 10 of the Astrographic Conference. These tables were computed primarily for the reduction of the Eros observations at the opposition of 1901. They extend to declination 55° 20' and therefore may be used also for the reduction of the present series of observations.

With these tables, \( x_0, y_0 \) denote the measured co-ordinates relative to the centre of the plate (corrected for orientation and scale). Then

\[
x = x_0 - ax_0 \quad \text{(Table I gives} \ ax_0, \ \text{arg.} \ x_0, \ y_0); \]
\[
y = y_0 - ay_0 \quad \text{(Table II gives} \ ay_0, \ \text{arg.} \ y_0).\]
If \( A, D \) denote the right ascension and declination of the centre of the plate, \( a, \delta \) the right ascension and declination of a star:—

\[
(a - A) \cos \delta = x + t_a \quad \text{(Table III gives } t_a, \text{ arg. } x_a \text{ and } \delta) ;
\]
\[
D + y = \delta + t_a \quad \text{(Table IV gives } t_a, \text{ arg. } D + y \text{ and } x) .
\]

\( t_a \) changes sign with change in the sign of \( x \).
\( t_a \) is independent of the sign of \( x \).
\( t_a \) changes unaltered with change in sign of \( \delta \).
\( t_a \) changes sign with \( \delta \); \( i.e. \) for negative declinations, \( t_a \) is to be treated as negative.

Loewy’s tables are not ideally arranged but their use will save a considerable amount of computing.

In view of the fact that the observations will be used for the dual purpose of deriving the solar parallax and also the mass of the Moon, it is desirable that all photographic plates obtained should be measured both in right ascension and in declination. For the plates taken at large east and west hour angles, the measures in declination are of little value for the derivation of the solar parallax; similarly, for the co-operative programmes between northern and southern observatories, the measures in right ascension are of little value for the derivation of the solar parallax. The measures in both co-ordinates will serve, however, to give a value of the mass of the Moon. The maximum value of the lunar equation was about 2°25′ in right ascension and 23″ in declination, so that the declination measures will contribute appreciably to the weight of the derived value of the mass of the Moon.

5. **Comparison Stars.**—The primary reference stars, contained in the three lists published by Professor Kopff, will serve as the comparison stars in the measurement of plates of astrophographic or shorter focal length. In order to eliminate as far as possible systematic errors arising from differential atmospheric dispersion, it is advisable that stars of extreme colour or spectral type (say, types B–Ao, K5 and M) should be excluded. The spectral types of these stars have been determined at Harvard, and their colour indices have been determined both by Seares, Sitterly and Miss Joyner ∗ at Mount Wilson and by Ross and Zug † at Yerkes. There is very little information available as to the colour of the secondary comparison stars, though it appears probable that the mean spectral type of these stars is not likely to differ very much from the colour of Eros.

For the measurement of photographic plates which cover a field of less than 2° x 2°, comparison stars should be selected as far as possible from the list of secondary reference stars which has been published by Professor Schorr.‡ Observers are recommended to select from this list suitable reference stars for each plate obtained, before measurement of the plates is commenced, and to prepare a list of the secondary reference stars for which positions will be required. The path of Eros has been photographed at the

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* *Ap. J., 72, 311, 1930.*
† *Ast. Nach., 239, 289, 1930.*
‡ *Ibid., 239, 321, 1930; 240, 65, 1930; 242, 249, 1931.*
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Bergedorf, Greenwich, Leipzig, Lick and Cape Observatories. If each co-operating observatory will forward, with as little delay as possible, to each of these observatories lists of secondary comparison stars for which positions are required, the work of measurement of the path plates will be lightened, as comparison stars for which positions are not required by any co-operating observatory need not be measured.

In the case of micrometric observations, the star or stars nearest to Eros had in general to be used. Many such stars are not contained in Professor Schorr’s list of secondary comparison stars. Micrometric observers are requested to forward lists of comparison stars for which positions are required to the observatories which have obtained series of path plates; where such stars are not contained in Professor Schorr’s list, the approximate right ascension and declination should be given.

The final reductions of the photographic plates obtained with long-focus instruments and of the micrometric observations will not be possible until the various measures of the path plates have been completed and collated.

6. Form of Publication of Results.—The primary reference star places, as given in the catalogue prepared by Professor Kopff, form the basis to which the positions both of secondary reference stars and of Eros itself will be referred. The star places in the primary reference star catalogue are given for the mean equinox of 1930.0.

The definitive ephemeris of Eros, with which the observed places of Eros will be compared, will also be referred to the mean equinox of 1930.0 and will be barycentric, i.e. referred to the centre of gravity of the Earth-Moon system. The lunar equation will be given separately.

The position of the planet, derived from the measurements on the photographic plates, is thus referred directly to the mean equator and equinox of 1930.0. Precession and nutation do not enter into consideration, being allowed for automatically on the reduction. The circular aberration terms are also automatically removed from the place of the planet in the process of reduction. The aberration terms depending upon the eccentricity of the Earth’s orbit are not removed, however, since the mean places of the stars contain these terms as usual.

If, therefore, we add the circular aberration terms to the place of the planet, derived from the photographs, the resulting place will contain the whole aberration due to the motion of the Earth’s centre. The correction for parallax is also applied to reduce to geocentric place.

The whole aberration (circular + eccentricity terms) is equivalent to the alteration in the place of the planet during the light-time. The usual process of antedating the time of observation by the light-time will thus give the true geocentric position corresponding to the antedated time. It should be noted that the reduction to geocentric place (parallax correction) should be computed for the actual time of observation and not for the antedated time. The diurnal aberration terms are automatically removed in the reduction of the plate, being common to stars and planet.

The final ephemeris with which the observations are compared will be barycentric, as stated above. The observed place of the planet is therefore
to be corrected first to geocentric place (correction for diurnal parallax) and then to barycentric place (correction for lunar equation computed for the actual time of observation, not for the antedated time), and the circular aberration terms are to be added. The position so obtained is to be compared with the antedated position of the planet derived from the barycentric ephemeris. Proceeding in this way, the small aberration term produced by the motion of the Earth about the centre of gravity of the Earth and Moon (lunar aberration) is eliminated. If we antedate in a geocentric ephemeris, we allow both for the aberration due to the motion of the Earth’s centre and also for the lunar aberration. We are therefore allowing for too much, as the lunar aberration is automatically eliminated in the reduction of the plate.

For the computation of the diurnal parallax, the basic value, 8°·800, of the solar parallax should in all cases be used.

It is desired that, in the publication of results, both the G.M.T. (or Universal Time or Weltzeit) and the local sidereal time of the observations should be given. Errors not infrequently enter into the reduction from one to the other; if both are given, there is a check on such errors as well as on typographical errors. The time-interval or intervals of the observation period or periods corresponding to each derived position of the planet should also be given, in order that it can be tested whether it is permissible to use the parallactic factor for the mean time or whether terms of the second order must be taken into account.

If the reductions are carried through to the final stage of deriving the $\Delta \alpha$, $\Delta \delta$ differences (observed minus ephemeris), the following data should be given for each derived position: G.M.T. and local S.T. of observation; time-interval of observation; observed position of planet; circular aberration; parallax; lunar equation; deduced barycentric place; ephemeris position (for antedated time); observed minus ephemeris positions. If preferred, to avoid the delay which may result if the publication is withheld until the definitive ephemeris becomes available, the computation of the parallax, lunar equation and ephemeris position can be omitted.

It is desirable that details of the method of observation should be stated, e.g. whether Eros or a star was guided upon, in one or both co-ordinates; whether or not the magnitude of Eros was reduced to equality with that of the comparison stars; whether the method of interrupted exposures was used; etc.

In the case of micrometric observations, if preliminary publication is decided upon, it will be best if the observer gives merely the values of $\Delta \alpha$ and $\Delta \delta$, reduced to 1930-0, with an identification of the comparison star used for each observation. For these data, approximate star places suffice.