Alpha-Decay Studies of Rf, Sg, and Hs Isotopes within the Multi-Channel Cluster Model

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Besides \(\alpha\)-decay energies and half-lives, more information on nuclear structure properties can be obtained from fine structure observed in \(\alpha\) decay. The multi-channel cluster model (MCCM) for well-deformed \(\alpha\) emitters has very recently been proposed for a precise description of the \(\alpha\)-decay fine structure. In this paper, based on the newly proposed \(Q_{\alpha}\) formula and the predicted rotational energy for superheavy nuclei, we present a theoretical prediction of the fine structure in the \(\alpha\) decay of deformed Rf, Sg, and Hs isotopes within the MCCM. The branching ratios to various daughter states and total \(\alpha\)-decay half-lives are evaluated by the five-channels microscopic calculations. Any adjustable parameter is not introduced in our calculations. This is, to our knowledge, the first coupled-channel study of the \(\alpha\)-decay fine structure in superheavy nuclei. It is expected that the present predictions would be useful for ongoing or future experiments on structure researches of superheavy nuclei.

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§1. Introduction

Alpha decay is one of the most important decay channels of unstable nuclei. The experimental measured quantities are \(\alpha\)-decay energies \(Q_{\alpha}\) and half-lives \(T_{1/2}\). The decay energies pose a tough test for nuclear mass models and provide information on the excitation energy of final daughter states. The half-lives provide information on the stability of nuclides. In particular, the location of nuclear shell closures in the superheavy mass region can be extracted by analyzing systematic trends in \(\alpha\)-decay energies and half-lives. For example, the experimental data show that there exists the enhanced nuclear stability at \(N = 162\), confirming the predicted \(N = 162\) neutron shell closure.\(^1\) Besides, an experimental combination of \(\alpha\) decay and \(\gamma\) emission can greatly help the spectroscopic study of neutron-deficient nuclei.\(^2,3\) Identification and knowledge of new synthesized elements and nuclides wholly or mainly resort to observing \(\alpha\)-decay chains.\(^4,5,6\)

From the theoretical standpoint, thanks to the intensive interest in superheavy nuclei, lots of efforts have been devoted to pursue a precise interpretation of the \(\alpha\)-decay half-lives of superheavy nuclei, such as the cluster model,\(^7,8\) generalized liquid drop model (GLDM),\(^9\) density-dependent cluster model (DDCM),\(^10\) super asymmetric fission model (SAFM),\(^11\) unified model for \(\alpha\) decay and \(\alpha\) capture (UMADAC),\(^12\) Coulomb and proximity potential model (CPPM),\(^13\) density-dependent M3Y (DDM3Y) effective interaction,\(^14\) and so on. Information on nuclear deformed shell closures in this region can be achieved through such \(\alpha\)-decay studies.
On the other hand, the theoretical predictions would guide ongoing or forthcoming experiments on synthesis of new superheavy elements and nuclides. However, these studies are insufficient to obtain more information on nuclear structure properties of superheavy nuclei such as rotational bands and nuclear deformations, because all these calculations adopt the assumption of favored $\alpha$ transitions (i.e., the angular momentum carried by the $\alpha$ particle $\ell = 0$) which usually occur for spherical even-even $\alpha$ emitters. In fact, most of the known superheavy nuclei are expected to exhibit large deformations. For deformed even-even nuclei, $\alpha$ decay can proceed from the ground state of the parent nucleus to various members of the ground-state rotational band in the daughter nucleus. This is called fine structure observed in $\alpha$-decay. Just owing to the complexity of the deformed system, information on nuclear deformations and rotational bands could be extracted by analyzing the $\alpha$-decay fine structure. The coupled-channels analysis of Delion et al.\textsuperscript{15} indicated that the fine structure in $\alpha$ decay is closely associated with the deformation parameters of daughter nuclei. The generalized density-dependent cluster model (GDDCM)\textsuperscript{16} suggested that in addition to nuclear deformations the excitation spectrum of daughter nuclei also plays an important role in describing the fine structure. And further studies of the multi-channel cluster model (MCCM)\textsuperscript{17} showed that the $\alpha$ transition to high-spin states is an important and sensitive tool to probe the energy spectrum and deformation of daughter nuclei.

Recently, progress has been made in the theoretical studies of superheavy nuclei. Based on the macroscopic-microscopic plus cranking models, the experimentally known rotational bands in even-even U to No isotopes were well reproduced, and the theoretical predictions of rotational energies $E_2$ were made for the No to Cn isotopes.\textsuperscript{18} The $Q_\alpha$ formula for superheavy nuclei was proposed by Dong and Ren,\textsuperscript{19} where the macroscopic terms are based on the well-known Weizsäcker-Bethe mass formula and the shell corrections are simulated by the Mexican hat wavelet functions. Additionally, the predictive power of the MCCM has been checked for a wide range of deformed $\alpha$ emitters including the newly measured neutron-rich Pt isotopes,\textsuperscript{17} where the five-channels microscopic calculations well reproduced the available experimental data concerning $\alpha$-decay half-lives and fine structures. Encouraged by these successes, in this work we intend to perform a detailed study of the fine structure in the $\alpha$ decay of Rf to Hs isotopes within the MCCM. These $\alpha$ emitters are at the gateway to the superheavy mass region, and their enhanced stability is closely related with deformed shell closures, similar to predicted superheavy elements related with spherical shell closures. They are perfect research objects as tentative researches on superheavy nuclei.

§2. Multi-channel cluster model for deformed $\alpha$ emitters

The $\alpha$-decay process can be divided into two distinct parts: the $\alpha$-clustering of four valence nucleons at the nuclear surface, followed by the tunneling of the formed $\alpha$ cluster through the potential barrier. The former concerns the nuclear structure part and the latter concerns the nuclear dynamic part. From these two aspects, we present the theoretical basis in the following, which are necessary to understand
\(\alpha\) transitions from ground states of the parent nucleus to various members of the ground-state rotational band in the daughter system.

2.1. Nuclear dynamic part of \(\alpha\) decay

The cluster radial function representing the relative radial motion of the \(\alpha\) particle with respect to the daughter nucleus satisfies the following set of coupled-channel equations: \(^{15}, 16\)

\[
\left[ -\frac{\hbar^2}{2\mu} \left( \frac{d^2}{dr^2} - \frac{\ell_\alpha (\ell_\alpha + 1)}{r^2} \right) - (Q_0 - E_I) \right] u_\alpha(r) + \sum_{\alpha'} V_{\alpha,\alpha'}(r) u_{\alpha'}(r) = 0. \tag{2.1}
\]

In this equation, \(\alpha \equiv (n\ell I)\) labels the channel quantum numbers, \(Q_0\) is the \(Q_\alpha\) value for the decay to the ground state, \(E_I\) is the excitation energy of the daughter state \(I\), and \(V_{\alpha,\alpha'}(r)\) denotes the matrix element of the interaction \(V(r)\) taken between channels \(\alpha\) and \(\alpha'\). Here, the channel quantum numbers are determined from the Wildermuth rule: \(^{20}\) \(G = 2n + \ell\), which connects shell-model-type states with cluster-model-type states. The simple shell model suggests \(G = 22\) for the superheavy mass region. The interaction matrix elements are obtained by the multipole expansion of the potential \(V(r) = V_N(r) + V_C(r)\). After considering the dynamic effect of the core nucleus, one obtains \(^{21}, 22\)

\[
V_{\alpha,\alpha'}(r) = \sum_\lambda v_\lambda(r) \frac{(-1)^\lambda}{4\pi} (2\lambda + 1) \sqrt{(2\ell' + 1)(2I' + 1)} \times \langle \ell' \lambda 00 | \ell 0 \rangle \langle I' \lambda K 0 | I K \rangle W(\ell' \lambda JJ; \ell I'), \tag{2.2}
\]

where \(\langle ab\beta | c\gamma \rangle\) is the Clebsch-Gordan coefficient and \(W(ab\beta; ef)\) the Racah coefficient. This potential matrix contains most of the nuclear physics, including the dynamics of the core and the \(\alpha\) cluster.

When numerically solving the coupled equation (2.1), we adjust the depth of the nuclear potential \(V_0\) to make all channels simultaneously reproduce the \((Q_0 - E_I)\) values. Then, the channel wave function is achieved by matching it with outgoing Coulomb-Hankel waves at large distance \(R\). Ultimately, one can express the partial width of the channel \(\ell I\) in the following form \(^{15} - 17\)

\[
\Gamma_{\ell I} = \frac{\hbar^2 k_I}{\mu} \frac{|u_{n\ell I}(R)|^2}{G_\ell(k_I R)^2 + F_\ell(k_I R)^2}, \tag{2.3}
\]

where \(R\) denotes large distances beyond the range of the nuclear potential and beyond the distance where the Coulomb potential can be regarded as spherically symmetric.

2.2. Nuclear structure part of \(\alpha\) decay

The structure part of \(\alpha\) decay concerns one basic open problem for nuclear structure theory, that is, a unified description of mean-field-type structure and cluster structure. \(^{23}\) Microscopic calculations of the \(\alpha\)-preformation factor \(P_\alpha\) are generally complicated and difficult due to large configuration spaces and insufficient knowledge of the clusterization process. \(^{24}, 25\) However, based on the available experimental
facts\textsuperscript{26} and theoretical analysis,\textsuperscript{27,28} one can simulate this process in a straightforward and consistent manner using a constant $P_\alpha$ factor combined with the hypothesis of the Boltzmann distribution (BD) for daughter states, $\rho(E_I) = \exp(-cE_I)$.\textsuperscript{29} For additional details on this simplification, see Refs. 16, 17). The previous systematic calculations of 35 deformed $\alpha$ emitters\textsuperscript{17} have shown that the experimental branching ratios to various states and total $\alpha$-decay half-lives can be well reproduced with the same factor $P_\alpha = 0.36$ and the same BD $c = 2.38 \text{ MeV}^{-1}$.

§3. Results and discussion

In the $\alpha$-decay calculations, the important input quantities are the decay energy $Q_0$, excitation energies of daughter states, and nuclear deformations of daughter nuclei. At present, the experimental data of these quantities are not available in most cases. So the theoretical values are used instead. The $Q_0$ values are evaluated using the newly proposed $Q_\alpha$ formula.\textsuperscript{19} This formula well reproduced the experimental $Q_\alpha$ values of 170 nuclei with $Z = 90–118$ and $N \geq 140$ with the standard deviation of 0.226 MeV. This gives us confidence in the $Q_\alpha$ predictions and then $\alpha$-decay calculations. The low-lying excitation spectrum $E_I$ of daughter nuclei is established using the predicted $E_2$ energies of Ref. 18) together with the perfect-rotor approximation $E_I = \kappa I(I + 1)$. The deformation $\beta_2$ and $\beta_4$ parameters of daughter nuclei are taken from the theoretical values of Möller et al.\textsuperscript{30} Although there exist deviations between the theoretical predictions and future experimental measurements for these input quantities, the theoretical calculations could give a reasonable description of the structure evolution in superheavy nuclei. Therefore, they remains useful as a first resort to understand the $\alpha$-decay fine structure in the superheavy mass region.

The five-channels microscopic calculations are performed for the $\alpha$ decay of deformed Rf, Sg, and Hs isotopes. Table I displays the calculated branching ratios (BR) to various daughter states and total $\alpha$-decay half-lives. The first column is the $\alpha$ emitter in question. The second and third columns separately denote the theoretical $Q_0$ values and the predicted $E_2$ energies. Columns 4–7 give the calculated BR to $0^+$, $2^+$, $4^+$, and $6^+$ daughter states, respectively. The theoretical $\alpha$-decay half-lives are listed in the last column. First, there is clear correlation between the calculated BR to excited $2^+$ states and the predicted $E_2$ energy. In general, the larger the $E_2$ energy, the smaller the BR to excited $2^+$ states. This is quite consistent with the conclusions of Refs. 16 and 17). One can see that there are minimum values of the $E_2$ energy at the parent neutron number $N = 154$ and $N = 164$ corresponding to the $N = 152$ and $N = 162$ closed shell closures in the daughter nuclei. Consequently, at these points the BR to excited $2^+$ states exhibit the relatively large values with respect to the neighboring $\alpha$ emitters. Second, there exists an abrupt change of the BR to $0^+$, $2^+$, and $4^+$ daughter states at $A = 268$. As one can see, the BR to ground $0^+$ states at $A = 268$ are clearly larger than before $A = 268$ while the BR to excited $2^+$ and $4^+$ states are smaller. This may be induced by the relatively smaller deformations of the daughter nuclei with the mass number $A_d \geq 264$.\textsuperscript{30} This unexpected property is worth further investigations. Along with the enhancement of experimental sensitivity and the development of experimental technologies, experimental structure studies of
superheavy nuclei have been performed in some famous laboratories such as JYFL, Argonne, GSI, Dubna, JAERI, GANIL, and Berkeley. For example, the direct mass measurements were performed for $^{252-254}$No using Penning traps at GSI. The decay spectroscopy of $^{253}$No and its daughter $^{249}$Fm was established at Dubna. It would be the most welcome to compare the present results with future experimental measurements, and this comparison will provide a reference for us to improve $\alpha$-decay studies in the superheavy mass region.

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