Lithium depletion in late-type dwarfs

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ABSTRACT
Using a self-consistent dynamic theory of non-local convection, the atmospheric lithium-abundance depletion values of a series of stellar evolutionary models with $M = 0.725–1.5 \, M_\odot$ are calculated, and the results show that the general observed properties of lithium abundance in lower-main-sequence stars can be reproduced by overshooting mixing and gravitational settling. After a careful study of the mechanisms of lithium depletion in stars, it is concluded that overshooting mixing and microdiffusion induced by gravitational settling and radiative acceleration are the two primary depletion mechanisms: for warm stars with $M \geq 1.1 \, M_\odot$ (or $T_e \geq 6100 \, \text{K}$) microdiffusion dominates, while it is the other way round for cooler objects with $M \leq 1 \, M_\odot$ (or $T_e \leq 5800 \, \text{K}$).

Key words: convection – stars: abundances – stars: evolution – stars: late-type – open clusters and associations: general.

1 INTRODUCTION
Lithium abundance is very important for the theory of nuclear synthesis during the big bang, as is well known. In addition, lithium and beryllium are both very fragile elements which can easily be destroyed due to $(p, \alpha)$ reactions. This property of lithium and beryllium makes them ideal tracers with which to measure the depth of stellar surface convection zones during the course of evolution.

Since the original observational detection of lithium abundance in solar-type stars by Herbig (1965), rich observational data have been accumulated. Observations of the lithium abundance of Galactic open clusters of various ages and metallicities provide very conclusive constraints on the depletion mechanisms of such fragile metals. The overall observational properties of stellar atmospheric lithium abundance can be summarized as follows.

(i) For stars with the same effective temperature, lithium abundance decreases as age increases. For young clusters such as the Pleiades and α Per, main-sequence (MS) stars with $T_e > 5500 \, \text{K}$ do not show obvious depletion (Duncan & Jones 1983; Soderblom et al. 1993; Balachandran, Lambert & Stauffer 1988). This means that for stars earlier than spectral type G5, lithium depletion happens at the post-main-sequence stage instead of during the gravitational contraction period of the pre-MS stage. However, for type K or later stars just arriving on the MS at the same age, or perhaps even still contracting, lithium depletion (at least a large fraction) in such low-temperature stars ($T_e < 5000 \, \text{K}$) must very likely happen in the pre-MS stage during contraction.

(ii) For dwarf stars later than type G0 in a given cluster, lithium abundance tends to be smaller when stellar effective temperature becomes lower. This trend follows the increase of the depth of the stellar surface convection zone with lower temperature. This is by no means a coincidence, and there must be a connection between convection and lithium depletion in late-type stars.

(iii) For warm stars with $T_e \sim 6000 \, \text{K}$, lithium abundance has only a weak dependence on $T_e$ (corresponding to the so-called lithium-abundance plateau). The plateau of lithium abundance becomes lower with increasing cluster age.

(iv) For clusters with the the age of the Hyades or older, there exists a lithium-abundance gap in the mid-F stars (Boesgaard & Trippico 1986). Obviously, the mechanism making such a gap is not the same depletion mechanism as for late-type stars (nuclear burning in the deep interior), rather it is a stellar surface phenomenon. Very likely, it is due to diffusion by radiative acceleration and gravitational settling (Richer & Michaud 1993; Michaud & Beaudet 1995).

(v) For some clusters, lithium-abundance dispersion for stars at the same temperature was observed. This was once believed to be conclusive observational evidence for rotationally induced mixing. After careful analysis of the observational data of lithium abundance of the Pleiades member stars, King, Krishnamurthi & Pinsonneault (2000) showed that the dispersion in lithium equivalent widths at fixed colour in cool single Pleiades stars is at least partially caused by stellar-atmosphere effects rather than being completely explained by genuine
abundance differences. Based on both photometry and spectroscopy data for ω Per and Pleiades member stars, Xiong & Deng (2005a,b, 2006) made a thorough analysis of the effects of inhomogeneous reddening, stellar spots and stellar surface activity on stellar lithium-abundance measurements, and concluded that a large part, if not at all, of the dispersion does not represent true lithium-abundance variation; rather, it is only an apparent scatter due to observational effects. It was also generally believed that there should be intrinsic variations in atmospheric lithium abundance going from intermediate-age clusters such as the Hyades and Coma clusters (Soderblom et al. 1989; Rebolo & Beckman 1989) to relatively old clusters such as NGC 752 (Hobbs & Pilachowsky 1986a,b) and M67 (García López, Rebolo & Beckman 1988). However, more recent high-resolution spectroscopy observations show that NGC 752 (Sestito, Randich & Pallavicini 2004) and NGC 188 (Randich, Sestito & Pallavicini 2003) are both characterized by a tight Li versus $T_e$ distribution for solar-type stars, with no evidence for a lithium spread as large as the one observed in M67. M67 is likely an exception. The dispersion of lithium abundance in cluster stars is not general. Is there any intrinsic variation of lithium abundance among cluster member stars? Further studies based on a larger sample should eventually provide a clue to this puzzle.

The depletion of lithium in the Sun is still not fully understood. The so-called standard model of the Sun is based on local convection theory, therefore mixing mechanisms other than convection have not been taken into account. The temperature at the bottom of the solar convective zone is merely about $2.3 \times 10^6$ K, lower than the $2.5 \times 10^6$ K needed to start vigorous burning (at a rate proportional to $T^{22}$) of lithium. Thus, it is straightforward to see that there will be no depletion at all if there is no other mechanism of mixing. Over past 20 yr, a number of mechanisms have been proposed including mass-loss (Weymann & Sears 1965; Hobbs, Iben & Pilachowsky 1989; Schramm, Steigman & Dearborn 1990; Swenson & Faulkner 1992), wave-driven mixing (García López & Spruit 1991; Montalban & Schatzman 1996), rotationally induced mixing (Pinsonneault, Kawaler & Demarque 1990; Charbonnel, Vauclair & Zahn 1992; Chaboyer, Demarque & Pinsonneault 1995; Pinsonneault, Deliyannis & Demarque 1992; Pinsonneault et al. 1999), diffusion and turbulent mixing (Michaud 1986; Richer & Michaud 1993; Michaud & Beaudet 1995; Michaud et al. 2004; Richard, Michaud & Richer 2005) and convective overshooting mixing (Straus, Blake & Schramm 1976; Xiong, Kondo & Unno 1991; Xiong & Deng 2002, 2007; Deng & Xiong 2008). Unfortunately, none of these ideas can, by itself, explain perfectly the complete observed nature of stellar atmospheric lithium-abundance depletion mentioned above.

The study of rotating stellar evolution by the Yale group is no doubt very important and attractive, not only because it provides a clue to the general observational properties of stellar angular velocity evolution in low-mass stars (Pinsonneault et al. 1989, 1990; Krishnamurthi et al. 1997), but also because it can reproduce, given certain assumptions and parameters, the overall observed lithium-abundance patterns for low-mass stars (Pinsonneault et al. 1999). Owing to clearly defined physics, such a theory can be easily understood, making it a widely accepted lithium-depletion theory. However, one should be aware of the downside as pointed out by Richard et al. (2005) when reviewing rotationally induced mixing: ‘Prescriptions for the initial angular momentum distribution, angular momentum loss, and the turbulent transport of angular momentum and of Li are required. Given the current level of understanding of turbulent transport processes, different arbitrary parameters are used for angular momentum and particle transport, even though efforts are made to link the two. Various prescriptions were studied but, given input uncertainties, obtaining credible models is a formidable task. … Since little is known about either initial angular momentum, turbulent transport of angular momentum, or turbulent transport of Li, one should perhaps not be too surprised that these studies have not been very conclusive’. In the authors’ view, the depletion of stellar lithium abundance should not be a consequence of a unique physical process; instead it ought to be a combination of many. For warm stars with $T_e > 6000$ K, the diffusion process is very important. Otherwise one has no clues pointing toward Ap, Am, Fm and the lithium gap. For cool stars, however, convective overshooting plays a role or even becomes dominant for depletion. All these processes are excluded in the rotationally induced mixing theory, and arbitrary use of parameters makes its application quite uncertain. Nevertheless, rotationally induced mixing theory, in a certain sense, can still be a dimension-type theory.

The diffusion process can be reliably understood in theory. At the present level of physics, many important atomic diffusion coefficients can be accurately calculated. For warm stars having a shallow convection zone, the diffusion time-scale is short, therefore the diffusion process becomes dominant, but not unique, in separating elements in the atmosphere. In order to slow down element separation caused by the microdiffusion process, a turbulent diffusion coefficient $D_T$ in the radiative zone below the convective zone is needed. $D_T$ should be large enough that the overwhelming effect of element separation can be compensated for; at the same time, it should also vanish quickly enough towards the stellar interior that the elements are not completely mixed. Therefore, two different parametrizations of turbulent diffusion coefficients have been proposed (Richard et al. 2005):

$$D_T = 400D_{He}(T_0) \left[ \frac{\rho}{\rho(T_0)} \right]^{-3},$$

or

$$D_T = 7500 \left[ \frac{\rho}{\rho_{bcz}} \right]^{-3},$$

where $D_{He}(T_0)$ is the diffusion coefficient of He atoms at temperature $T_0$ and $\rho_{bcz}$ is the density at the bottom of the convective zone. The second expression clearly emulates convective overshooting mixing, while the first equation does the same for rotationally induced mixing or other turbulent mixing (including that of convective overshooting).

Following hydrodynamic theory, hydrodynamic simulations or the theory of stellar evolution, there must be a convective overshooting zone under (and/or above) a local convectively unstable zone. The existence of convective overshooting has also been confirmed by
observational tests of stellar evolutionary models. For the Sun, the bottom of the convective zone is located at \( T_e \sim 2.3 \times 10^6 \) K, setting it very close to the value of \( T_e \sim 2.5 \times 10^6 \) K required by vigorous nuclear burning of lithium. Therefore, for solar-like late-type stars, convective overshooting mixing is non-negligible, and may even dominate lithium depletion.

We have developed a non-local statistical theory of convection in chemical inhomogeneous stars (Xiong 1981), which was also applied to deal with convective overshooting mixing of elements during the course of stellar evolution (Xiong 1986), and lithium depletion in the Sun and solar-type stars (Xiong & Deng 2002, 2007). Our non-local convection theory is derived from the equations of hydrodynamics and the theory of turbulence, and can be used to describe accurately the dynamic behaviour of turbulent convection in stars. The observed properties of solar lithium abundance can be reproduced well in our calculations, with the following two restrictions (when diffusion is not considered).

(i) For hot stars with \( T_e \geq 6000 \) K, there is almost no depletion at all within their lifetimes. This is of course not a surprise. As already discussed in the above text, depletion of lithium in hot stars is not due to turbulent mixing bringing the element to high-temperature interiors, being completely different from the case for cooler stars with \( T_e \leq 5800 \) K. Instead, it is caused by the diffusion process in stellar atmospheres, which tends to separate elements.

(ii) For cool stars with \( T_e \leq 5800 \) K, the depletion due to overshooting mixing is slightly overestimated compared with observations. The reason for such overestimation is now understood: we assumed quasi-isotropy for turbulent convection in our previous work. Turbulent convection, in fact, is non-isotropic. Following the anisotropic non-local convection theory developed recently (Deng, Xiong & Chan 2006), turbulent motion in the convective (unstable) zone is radial-dominant, while in the convective overshooting zone the motion tends to be horizontal-dominant. Such an anisotropy of turbulent convection can explain the granular velocity field in the solar atmosphere (Keil & Canfield 1978; Nesis & Mattig 1989; Komm, Mattig & Nesis 1991), and is confirmed by hydrodynamical simulations of convection (Deng et al. 2006). Considering the anisotropy of turbulent convection, convective overshooting mixing may have a lower efficiency compared with isotropic convection theory. In the calculations of solar lithium depletion, the theoretical lithium abundance agrees with observations (Deng et al. 2006) if the non-local convection model is constrained to have the depth of the convective zone given by helioseismology.

Microdiffusion and convective overshooting mixing are perhaps the most reliable mixing theories to date. The primary goal of this work is to investigate the possibility of reproducing the main observational properties of lithium depletions in solar-type and late-type stars using a combination of overshooting mixing and microdiffusion. As the monochromatic absorption coefficients are not available, this work will only consider gravitational settling of microdiffusion, while radiative acceleration will be neglected for the following reasons. For MS stars with \( T_e \leq 6500 \) K (or \( M < 1.4M_\odot \)), convection is deep enough. At the bottom of the convective zone \( \text{radiative acceleration} < \text{gravity} \) (Richer & Michaud 1993), therefore radiative acceleration is negligible compared with gravitational settling. For this reason, this work is limited to discussions on lithium depletion in low-mass stars (\( M < 1.4M_\odot \)). Section 2 gives the basic equations and computation method. The numerical results are presented in Section 3 together with discussions on depletion mechanisms. Comparison between our theoretical results and the observed lithium abundance in Galactic clusters is described in Section 4. Our conclusions and discussion are found in the final section.

2 THE FUNDAMENTAL EQUATIONS

Within the convective and overshooting zones, convective mixing is very efficient, therefore elements are almost completely mixed. Gravitational settling functions only in the radiative zone below the convective overshooting zone. For MS stars with \( M < 1.2M_\odot \), the temperature at the bottom of the convective overshooting zone is greater than \( 10^6 \) K where H, He and Li are almost all ionized, therefore the diffusion process can be considered as the way in which the tracing particle Li\(^{++} \) diffuses in the mixture of H\(^+ \), He\(^{++} \) and electrons. Considering both convection and diffusion, the fractional mass conservation equation for lithium can be written as (Xiong 1981)

\[
\frac{DC}{Dt} = \frac{1}{\mu_0} \left[ \frac{4\pi r^2 \rho (U + J)}{\mu_0} \right] = -q, \tag{1}
\]

where \( C \) is the abundance of lithium by mass, \( \rho \) is the gas density, \( C_q \) is the burning rate of lithium, \( r \) is the radius and \( M_i \) is the mass within radius \( r \). All terms with a bar at top are the mean value. \( U \) is the statistical mean of the radial component of turbulent velocity \( w_i^r \) and the abundance fluctuation \( C' \) (turbulent velocity and lithium-abundance correlation), therefore the convective flux of lithium is \( U = w_i^r C' \).

\( J \) is the microdiffusion flux of lithium expressed as

\[
J = -D \left( \frac{\partial \ln C}{\partial r} \right) + \frac{4\mu_0}{1 + \Phi} \left( \frac{g}{R T} + k_T \frac{\partial \ln T}{\partial r} \right), \tag{2}
\]

where \( T \) is temperature, \( R \) is the gas constant, \( D \) and \( k_T \) are respectively the diffusion and thermal diffusion coefficients, \( g = GM_i/r^2 \) is gravitational acceleration, and \( \mu_0 \) and \( \Phi \) are respectively the mean molecular weight and ionization degree, defined as

\[
\mu_0 = [X/A_H + (1 - X)/A_H]^{-1} = [X + (1 - X)/4]^{-1}.
\]

\[
\Phi = \mu_0 \left[ \frac{\Phi_H X/A_H + \Phi_{He} (1 - X)/A_{He}}{X} \right] \approx X + (1 - X)/2.
\]

When calculating \( \mu_0 \) and \( \Phi \), only hydrogen (X) and helium (\( Y = 1 - X \)) are considered and complete ionization is assumed. This is a natural approximation to make, which should not cause any sizable errors while providing us with great convenience. Equation (2) is
derived from the book of Chapman & Cowling (1970) and the work of Richer & Michaud (1993) under the above simplification hypothesis. Equation (2) was derived under the approximation of no separation between H and He, and would be modified if He settling were included. For details of the equation manipulations and the working equations for the calculations of envelope structure in non-local convection theories, please refer to our previous work (Xiong 1981, 1986). A brief review is given here.

The complete set of equations for the current work is composed of equations (1) and (2) and the following formulae:

\[
\frac{\partial r^3}{\partial M_r} = \frac{3}{4\pi \rho},
\]

\[
\frac{\partial}{\partial M_r} \left( \bar{P} + \bar{\rho} x^2 \right) + \frac{1}{r} \frac{\partial}{\partial M_r} \left( \bar{\rho} r^3 \mathcal{H}^{11} \right) = - \frac{GM_r}{4\pi r^2},
\]

\[
L_t + L_c + L_s = L_0.
\]

\[
\frac{\partial T^4}{\partial M_r} = - \frac{3\kappa_L}{16\pi^2 c r^4},
\]

\[
\frac{\partial x^2}{\partial t} - \frac{\partial}{\partial M_r} \left( \frac{Q}{\partial M_r} \right) + \frac{2}{3} \frac{GM_r}{r^2} \left( 1.56 \frac{\bar{\rho} x R_0}{c_1 P_r} + \rho U - BV \right) = 0,
\]

\[
\frac{\partial \mathcal{H}^{11}}{\partial t} - \frac{\partial}{\partial M_r} \left( \frac{Q}{\partial M_r} \right) - 4GM_r \frac{B V}{3r^2} + \frac{2}{3} \frac{1.56(1 + c_s)}{c_1 P_r} GM_r \rho x + \left[ \frac{2}{3} \frac{1.56}{c_1 P_r} (x + x_c) \right] \mathcal{H}^{11} = 0,
\]

\[
\frac{\partial Z}{\partial t} = \frac{1}{\rho \bar{C}_p} \frac{\partial}{\partial M_r} \left( \frac{\rho \bar{C}_p^2}{\partial M_r} \right) + 8\pi r^2 \bar{\rho} V \left( \frac{\partial \ln \bar{T}}{\partial M_r} - \nabla_{\text{ad}} \frac{\partial \ln \bar{P}}{\partial M_r} \right) + 1.56 \frac{GM_r R_0 \bar{\rho}}{c_1 r^3 P} (x + x_c) Z = 0,
\]

\[
\frac{\partial V}{\partial t} = \frac{1}{\bar{C}_p} \frac{\partial}{\partial M_r} \left( \bar{C}_p \frac{\partial V}{\partial M_r} \right) + 4\pi r^2 \bar{\rho} x^2 \left( \frac{\partial \ln \bar{T}}{\partial M_r} - \nabla_{\text{ad}} \frac{\partial \ln \bar{P}}{\partial M_r} \right) + \frac{GM_r}{r^2} \left[ \frac{x^2}{(x^2 + \mathcal{H}^{11})^{3/2}} + \frac{1}{2} (x + x_c) \right] \frac{\bar{\rho} V R_0}{c_1 P_r} + \rho U - BV \frac{W}{Z} = 0,
\]

\[
\frac{\partial Y}{\partial t} = \frac{\partial}{\partial M_r} \left( \frac{Q}{\partial M_r} \right) + 8\pi r^2 \bar{\rho} U \frac{\partial \bar{C}}{\partial M_r} + 1.56 \frac{GM_r R_0 \bar{\rho}}{c_1 r^3 P} x Y = 0,
\]

\[
\frac{\partial U}{\partial M_r} - \frac{\partial}{\partial M_r} \left( \frac{Q}{\partial M_r} \right) + 4\pi r^2 \bar{\rho} x^2 \frac{\partial \bar{C}}{\partial M_r} + 0.78 \frac{GM_r R_0 \bar{\rho}}{c_1 r^3 P} \left[ 1 + \frac{x}{(x^2 + \mathcal{H}^{11})^{3/2}} \right] x U = 0,
\]

\[
\frac{\partial W}{\partial t} - \frac{1}{\bar{C}_p} \frac{\partial}{\partial M_r} \left( \bar{C}_p \frac{\partial W}{\partial M_r} \right) + 4\pi r^2 \bar{\rho} V \frac{\partial \bar{C}}{\partial M_r} + 0.78 \frac{GM_r R_0 \bar{\rho}}{c_1 r^3 P} (x + x_c) \frac{W}{2} = 0,
\]

where \( P(=P_e + P_i) \) is the gas pressure (including radiative pressure \( P_i \)), \( C_p \) is specific heat at constant pressure, while

\[
\rho C = \left( \frac{\partial \rho}{\partial \bar{C}} \right)_{P,T}.
\]
\[ B = -\left( \frac{\partial \ln \bar{\rho}}{\partial \ln \bar{T}} \right)_{p,c}. \]

\(x^2, \mathcal{H}^{ij}, Y, Z, U, V\) and \(W\) are respectively the auto- and cross-correlations of turbulent velocity, lithium abundance and temperature, defined as

\[ \overline{w^i w^j} = g^{ij} x^2 + \mathcal{H}^{ij}, \]

\[ Y = C^2, \]

\[ Z = \left( \frac{T}{\bar{T}} \right)^2, \]

\[ U = w_i C, \]

\[ V = w_i T, \]

\[ W = C T, \]

\(g^{ij}\) is the metric tensor of the coordinate system, and therefore \(x^2\) and \(\mathcal{H}^{ij}\) are respectively the isotropic and anisotropic components of turbulent velocity correlation. \(w_i'\) is the r.m.s of the radial turbulent velocity. \(L_0\) is the total luminosity of the star, while \(L_0, L_c\) and \(L_e\) are respectively the corresponding radiative, convective thermal and turbulent kinetic energy components,

\[ L_c = 4\pi r^2 \bar{\rho} \bar{T} V, \]

\[ L_t = -\frac{3}{2} Q \frac{\partial x^2}{\partial \bar{M}_r}, \]

where

\[ Q = \frac{4\sqrt{3}/\pi c_2 r^3 \bar{P} (x^2 + \mathcal{H}^{11})^{1/2}}{GM R_0}. \]

\(c_1\) and \(c_2\) are the two convection parameters in our theory, while \(c_1\) is a parameter related to the anisotropy of turbulent convective motion. In the convectively unstable zone, the radial component \(w_i\) and the horizontal one \(w_0\) have a ratio of \(w_r^2/w_0^2 = (3 + c_3)/2c_1\). It follows from observations of the solar granular velocity field (Keil & Canfield 1978; Nesis & Mattig 1989; Komm et al. 1991) and hydrodynamic simulations (Deng et al. 2006) that \(c_3 \approx 3\) in the unstable zone, and \(w_r^2/w_0^2 \approx 1\); this corresponds to the most unstable linear convection mode (Unno 1961). \(c_1\) is a convection parameter related to turbulent viscous dissipation. Based on the theory of turbulence, the viscous dissipation of turbulent kinetic energy \(\epsilon \propto x^3/l_c\), where \(l_c\) is the scalelength of the turbulent energy-containing eddies (Hinze 1975). In our calculations, \(l_c = c_1 H r / R_0 = c r^3 P / C M R_0 R_0\) (Xiong 1981). The parameter \(c_1\) actually determines the efficiency of convective energy transport, being similar to the mixing-length parameter in the mixing-length theory. However, the two are distinctly different and have different origins. \(c_2\) is a convective parameter related to turbulent diffusion. In our non-local convection theory, the third-order correlations represent diffusion of non-local convection. For instance, \(w_i r w_j w^j/2\) is the turbulent kinetic energy flux and we used a gradient-type diffusion approximation to express the third-order correlations,

\[ \overline{w_i w_j w^j} = -w_k' \Lambda \nabla_k w_i' w_0'. \]

\(w_k' = (x^2 + \mathcal{H}_k^1)^{1/2}\) is the turbulent velocity component in direction \(k\), and \(\Lambda\) is the diffusion length of turbulence, which is further assumed to be proportional to the local pressure scaleheight \(H_P\), i.e.

\[ \Lambda = c_2 H_P r / R_0 = c_2 r^3 P / G M R_0 r. \]

In fact, we are not choosing different parameters for different problems. Instead, we keep using the same set of parameters \((c_1, c_2\) and \(c_3)\) for all work, given they realize specific observational constraints for given problems. For instance, they are adjusted in such a way that the solar convective-zone depth \((r_c/R_0 \sim 0.715)\) defined by helioseismology is reproduced and that the solar surface Li abundance is the same as observed (see Fig. 8 later); also to give the observed turbulent velocity field and anisotropy in the solar atmosphere as functions of depth compatible with hydrodynamic simulations (Deng et al. 2006) and to reproduce the observed instability strips of variable stars, especially the red edges (Xiong & Deng 2001, 2007). The three adjustable parameters \((c_1, c_2\) and \(c_3)\) are in fact not completely independent. Tuning any one of them will make the depth of the convective zone different. \(c_3\) is more independent compared with the other two, which describe the isotropy of turbulent convection in stars. From the observations of the turbulent velocity field in the solar atmosphere and hydrodynamic simulations, \(c_3 \approx 3\) seems to be the optimal choice (Deng et al. 2006). Both \(c_1\) and \(c_2\) are linked to the characteristic wavenumbers of the turbulent power spectrum, and both are of order unity. The ratio \(c_2/c_1\) is more or less fixed in all cases. Following convective overshooting, and considering the depth of the solar convective zone and Li abundance in the solar atmosphere, we choose \(c_2/c_1 = 0.5\). As a result, the only really adjustable parameter is \(c_1\). Nevertheless, the adjustment of \(c_1\) is not at all random. For solar-type stars, the choice of \(c_1\) has to be constrained by the depth of the convective zone and the results of helioseismology. The selection of convection parameters in the current...
work follows from all the above arguments. Specifically, $c_1 = 0.83$, $c_2/c_1 = 0.5$ and $c_3 = 3$. Our recent work on variable stars (Xiong & Deng 2007) used this very set of parameters.

In the expressions for $l_0$ and $\Lambda$, an extra term $r/R_0$ shows up ($R_0$ is stellar radius). This is used to keep both $l_0$ and $\Lambda$ finite when $H_\beta$ tends to infinity at $r \rightarrow 0$. The depth of the stellar-surface convective zone is determined mainly by the superadiabatic zone atop the unstable zone, where $r \approx R_0$. Therefore such a modification factor will not have any sizable influence on stellar structure and the means of calibration of the parameters. Equations (1), (2) and (3)–(13) are a complete set of equations for anisotropic non-local convection theory. The first four equations of the set (equations 3–6) are the normal stellar-structure equations, with certain differences. The turbulent Reynolds stress is now included in the equation of momentum conservation (in our case static equilibrium), equation (4), where $\rho \mathbf{v}^2$ is the isotropic turbulent pressure and $\rho \mathbf{H}_\beta$ is the anisotropic component. In the energy equation, equation (5), both thermal convection ($H_\beta$) and turbulent kinetic energy flux ($L_u$) are included. Equation (6) is the radiation transfer equation under the radiative diffusion approximation. Equations (7)–(13) are the dynamic equations of turbulent correlations. This complete set of equations is derived from hydrodynamics and the theory of turbulence. For the original derivation of the equations, details can be found in an earlier work (Xiong 1981). Of course, the original equations were under the assumption of isotropy. A more recent version taking into account anisotropy can be found in Deng et al. (2006).

Equations (1)–(13) are a set of partial differential equations, where the first four plus equation (6) are first-order and equation (5) has been decreed to be algebraic. The remaining equations are, however, second-order ones. To solve the set of equations, we need to set up 19 boundary conditions defined by physics at both the surface and the centre of the stars. At the surface, we have

$$r = R_0,$$

$$P = P_0 = 2P_\odot = \frac{2}{3} a T_\odot^{4},$$

$$T = T_\odot = 2^{1/4} T_\odot,$$

$$\frac{\partial x}{\partial M_t} = - \beta_+ \frac{GM_\odot x}{4\pi R_0^4 P_0},$$

$$\frac{\partial H^{11}}{\partial M_t} = - \beta_+ \frac{GM_\odot H^{11}}{4\pi R_0^4 P_0},$$

$$\frac{\partial Z}{\partial M_t} = - (2\beta_+ - \nabla) \frac{GM_\odot Z}{4\pi R_0^4 P_0},$$

$$\frac{\partial V}{\partial M_t} = - (3\beta_+ - \nabla) \frac{GM_\odot V}{4\pi R_0^4 P_0},$$

$$\frac{\partial Y}{\partial M_t} = - (2\beta_+ - \nabla) \frac{GM_\odot Y}{4\pi R_0^4 P_0},$$

$$\frac{\partial U}{\partial M_t} = - (3\beta_+ - \nabla) \frac{GM_\odot U}{4\pi R_0^4 P_0},$$

$$\frac{\partial W}{\partial M_t} = - (2\beta_+ - \nabla) \frac{GM_\odot W}{4\pi R_0^4 P_0},$$

$$4\pi R_0^2 \rho J = 0,$$

$$4\pi R_0^2 \rho U = 0,$$

For the current purpose of calculating lithium depletion, the bottom boundary is set at a depth deeper than $5 \times 10^6$ K, in all cases lower than the convective zone. Therefore the bottom boundary conditions (at $M_t = M_0$) are

$$\frac{\partial x}{\partial M_t} = - \beta_- \frac{GM_\odot x}{4\pi r^4 P_0},$$

$$\frac{\partial H^{11}}{\partial M_t} = - \beta_- \frac{GM_\odot H^{11}}{4\pi r^4 P_0},$$

$$\frac{\partial Z}{\partial M_t} = - (2\beta_- - \nabla) \frac{GM_\odot Z}{4\pi r^4 P_0},$$

$$\frac{\partial V}{\partial M_t} = - (3\beta_- - \nabla) \frac{GM_\odot V}{4\pi r^4 P_0},$$

$$\frac{\partial Y}{\partial M_t} = - (2\beta_- - \nabla) \frac{GM_\odot Y}{4\pi r^4 P_0},$$

$$\frac{\partial U}{\partial M_t} = - (3\beta_- - \nabla) \frac{GM_\odot U}{4\pi r^4 P_0},$$

$$\frac{\partial W}{\partial M_t} = - (2\beta_- - \nabla) \frac{GM_\odot W}{4\pi r^4 P_0}.$$
\[ \frac{\partial U}{\partial M_i} = -(3\beta_\ast - \nabla) \frac{GM_\ast U}{4\pi r^3 P}, \]
\[ \frac{\partial W}{\partial M_i} = -(2\beta_\ast - \nabla) \frac{GM_\ast W}{4\pi r^3 P}, \]

where

\[ \beta_\ast = -\frac{\nabla - 0.3}{2.4} \pm \sqrt{\left(\frac{\nabla - 0.3}{2.4}\right)^2 + \frac{0.5 - \nabla}{6} + \frac{2\sqrt{3}\eta_\ast + 1}{3\sqrt{3}\gamma_1\gamma_2}}. \]

For the surface boundary-condition equation (14), a testing value of \( R_0 \) is used in real-time calculations, and we perform the integration up to an optical depth of \( \tau = 2/3 \). After a few iterations, a restricting condition of the following form is achieved:

\[ 4\pi R^2_{ph} \sigma T^4 = L_0, \]

where \( R_{ph} \) is the radius of the photosphere (\( \tau = 2/3 \)). Equations (16) and (34) are actually from the grey-atmosphere model under the Eddington approximation. The boundary-condition equation (15) means that the gas pressure equals the radiative pressure at the surface boundary. In a large dynamic range, the value \( P_0 \) does not affect the envelope structure. The convection boundary-condition equations (17)–(23) and equations (26)–(32) are derived by the properties of the asymptotic solution in the overshooting zone (Unno, Kondo & Xiong 1985; Xiong 1989).

Equations (1)–(13) form a complete set of equations for chemically inhomogeneous envelope structure, which when supplemented with boundary-condition equations (14)–(33) can be used to calculate the non-local envelope structure and temporal evolution of lithium abundance.

Our calculations start with the ZAMS for a given stellar mass, therefore the lithium abundance in the whole star is homogeneous,

\[ C = C_0. \]

Calculation of stellar evolution under non-local treatment of convection is far more difficult and complicated than the conventional model. The only success achieved so far is the early MS evolution of massive stars (Xiong 1986). For lithium-abundance depletion happening only at the envelope of stars, a full-scale non-local convection for stellar evolution is not needed. Lithium is an extremely under-abundant element, the existence and variation of which can slightly affect the equilibrium state of a star. Therefore, the calculations of stellar atmospheric lithium-abundance depletion can be realized in the following three steps.

(i) Computing the stellar evolutionary models using Padova code (Bressan et al. 1993) and defining the evolutionary tracks of equilibrium models.

(ii) Along the evolutionary tracks \( (L, T_e) \) given in the first step, a series of envelope models for non-local convection can be calculated. When building up the envelope models, lithium is not considered. As such, we just need eight equations (3)–(10) after neglecting all quantities related to lithium abundance \( (Y, U, W \text{ and } C) \), and the independent variants become only eight \( (r, P, T, L_x, x, Z, V \text{ and } H^{11}) \). For boundary conditions, with the same considerations, the number is now 11, namely equations (14)–(20) and equations (26)–(29). Such a set of equations is ready for the calculation of non-local convection envelope models. The well known Henyey scheme (Henyey, Forbes & Gould 1964) for stellar modelling is applied. The initial model for iteration is the so-called quasi-non-local convection model, which is made by minor revision to the local convection envelope model: in the local convectively unstable zone, all convection quantities \( (x, V \text{ and } V) \) take their corresponding local model values, while in the overshooting zone, \( x, V \text{ and } V \) are derived by asymptotic analytical expansion, and the four quantities \( (r, P, T \text{ and } L_x) \) take their local model values directly. Starting from the quasi-local convection models built in such a way, and following Henyey iterations, non-local convection envelope models can be calculated eventually.

(iii) Making a series of non-local convection envelope models in the above way, and applying equations (1), (2), (11)–(13), the boundary-condition equations (21)–(25) and (30)–(32), and the initial-condition equation (35), we are ready to calculate temporal lithium depletion. At this point, \( r, P, T, L_x, x, H^{11}, Z \text{ and } V \) are all known for each evolutionary step, and all that needs to be computed for evolution is the following five quantities \( (C, J, Y, U \text{ and } W) \).

3 NUMERICAL RESULTS AND LITHIUM-DEPLETION MECHANISM

3.1 Lithium depletion in stellar atmospheres

Using the equations and the computation scheme outlined in Section 2, lithium depletion in the atmospheres of stars with masses from 0.725–1.5 M\(_{\odot}\) and solar composition \( (X = 0.70, Z = 0.02) \) is calculated. Fig. 1 presents the lithium abundance as a function of time for a few models with different masses. Lithium abundance tends to decrease exponentially with time in the plots. In the models in shown in Fig. 1(a), only convective overshooting mixing is taken into account; gravitational settling is omitted. For models with mass larger than 1.1 M\(_{\odot}\), the convective zones are too shallow so that there is almost no change in lithium abundance during evolution. For lower stellar mass, the convective zone becomes deeper and therefore lithium depletion tends to be faster. In Fig. 1(b), the solid lines show models with...
Figure 1. Evolution of lithium abundance with time for stars with different mass. Stellar mass is marked on each track. The solid lines in panel (b) are the models with both convective overshooting mixing and gravitational settling considered, while the dotted lines in both panels are the model with only convective overshooting mixing. The location of the Sun is marked by ♂.

Figure 2. The e-folding time of lithium depletion as a function of (a) stellar mass and (b) effective temperature. The dotted lines are models with only convective overshooting mixing considered, while the solid lines are with both convective overshooting mixing and gravitational settling.

both convective overshooting mixing and gravitational settling taken into account. The model behaviour in Fig. 1(b) is obviously different from that in Fig. 1(a): for masses greater than $1.1 \, M_\odot$, the depletion speed increases with stellar mass. Apparently this is due to gravitational settling. For stars with $M \leq 1.0 \, M_\odot$, however, lithium depletion is slightly faster than for those in Fig. 1(a) when only overshooting mixing is considered. For these stars, overshooting mixing is the main reason for lithium depletion. This will be addressed in detail in Section 3.2.

The solid lines in Fig. 2(a) display the e-folding time $\tau_d$ of lithium depletion as a function of stellar mass. During the stellar evolution process, both effective temperature and luminosity of a star are changing with time. Therefore $\tau_d$ is also a function of time during evolution; this is why the temporal changes of lithium abundance in Fig. 1 are not straight lines. It should be borne in mind that the $\tau_d$–M relations shown in Fig. 2(a) are, of course, a mean relation, as is the case for the $\tau_d$–$T_e$ relation shown in Fig. 2(b).

Fig. 3 shows the theoretical curves of lithium abundance as a function of effective temperature for open clusters of various ages. Solar metallicity is assumed. Fig. 3(a) has only overshooting mixing considered. For stars with $T_e \geq 6100$ K, there is almost no depletion due to the fact that these hot stars all have a too-shallow convective zone at the surface; overshooting mixing can scarcely cause any depletion within the lifetimes of such stars. The depth of the convective zone deepens for lower effective temperature stars, overshooting mixing becomes stronger and therefore lithium depletion increases.
Lithium depletion in late-type dwarfs

The solid lines in Fig. 3(b) show the isochrones of lithium depletion with both overshooting mixing and gravitational settling included. The dotted lines are the ones without gravitational settling. It follows from Fig. 3(b) that, for lower temperature stars with \( T_e \leq 5800\) K, the depletion is similar in both cases. The reason is that for such cool stars overshooting mixing dominates the process. In contrast to the cases shown in Fig. 3(a), for warm stars with \( T_e \geq 5800\) K, the lithium abundance in stellar atmospheres drops down quickly with time, nicely reproducing the cooler edge of the Li gap of F-type stars (Boesgaard & Tramped 1986) and the observed lowering with increasing age of the lithium-abundance plateau.

3.2 The depletion mechanism of lithium

Comparing the dotted (without gravitational settling) and solid lines (including gravitational settling) in Figs 1–3, it is clear that, for warm stars with \( M \geq 1.1 M_\odot\) (or \( T_e \geq 6100\) K), gravitational settling is the dominant mechanism for lithium depletion in stellar atmospheres, while for cool stars with \( T_e \leq 5800\) K (or \( M \leq 1.0 M_\odot\)), overshooting mixing comes in turn to dominate. Details on the depletion mechanisms will be given in the following.

Figs 4(a)–(d) show the lithium profile in stars at three different times respectively, for four stars with various masses. The dashed and dash–dotted lines in the plots express respectively the lower boundaries of the convectively unstable and overshooting zones. It follows from Figs 4(a) and (b) that, when ignoring gravitational settling (dotted lines), the profile is a straight horizontal line up to the burning region of lithium, conserving the initial abundance; when gravitational settling is taken into account (solid lines), however, lithium abundance lowers with time as the star evolves. Because convective mixing is highly efficient in the convective unstable and convective overshooting zones, lithium is homogeneous there, and the profile is nearly a horizontal line at the surface layer of stars. Approaching the lower boundary of the overshooting zone where overshooting efficiency becomes lower, matter is only partially mixed, therefore there exists a lithium-abundance gradient in the region below the overshooting zone. Lithium abundance grows inwards simply due to gravitational settling, because lithium has greater atomic number than the abundant elements (H and He). Going further into stars, lithium abundance vanishes quickly due to \((p, \alpha)\) reactions involving lithium. It is made clear by Figs 4(a) and (b) that the convective zone is too shallow, therefore convective overshooting mixing is not able to bring lithium into its burning zone (see the dotted lines) for warm stars. Within the convectively unstable and convective overshooting zones, matter is fully mixed so that lithium is homogeneous over there. Such regions can be regarded as a reservoir of lithium.

Gravitational settling works as a water pump, which keeps bumping heavier lithium continuously to a deeper radiative zone to burn; this process makes the surface lithium abundance decrease as the star ages. For a warm star with \( M_0 = 1.5 M_\odot\), the lifetime is slightly shorter than for lower mass stars, therefore gravitational settling does have time to bring enough lithium effectively into the burning region, and the abundance in the whole region above the burning zone is still almost the initial value, with only a little depletion. That is to say that, for such a warm star \((M_0 = 1.5 M_\odot)\), the depletion is more like taking surface lithium to deeper radiative regions and preserving it there. For lowering stellar masses, the convective zones deepen and gravitational settling strengthens because of longer lifetimes and smaller masses of the radiative envelopes. As a result, for a slightly lighter warm star \((M_0 = 1.2 M_\odot)\), the lithium abundance in the radiative zone also decreases following the evolution process of the star (Fig. 4b).

As they are different from warm stars with \( M \geq 1.1 M_\odot\), cooler stars with \( M \leq 1.0 M_\odot\) have much stronger overshooting mixing, which supersedes gravitational settling. Figs 4(c) and (d) show lithium-abundance profiles respectively for \( M = 1.0 M_\odot\) and \( M = 0.95 M_\odot\).
Variations of lithium abundance as a function of depth \((\log \Delta M_r / M_\odot)\) for different ages (indicated on curves). \(M_0\) and \(M_r\) represent respectively stellar mass and mass within radius \(r\), \(\Delta M_r = M_0 - M_r\) and \(M_\odot\) is the mass of the Sun. The solid and dotted lines are respectively the theoretical lines with and without gravitational settling. (a) \(M_0 = 1.5 M_\odot\), (b) \(M_0 = 1.2 M_\odot\), (c) \(M_0 = 1.0 M_\odot\), (d) \(M_0 = 0.95 M_\odot\). The shaded areas mark the overshooting zone in each panel, bounded by the dashed (left) and long-dashed (right) lines, which are respectively the bottom boundaries of the convective and the overshooting zones.

stars. It is clear from these two models that the lithium hump in Figs 4(a) and (b) accumulated due to gravitational settling disappears completely and the overshooting zone extends to the burning region of lithium, i.e. overshooting in such cases can bring lithium down and get it burnt in the high-temperature zone. Overshooting mixing becomes the dominant mechanism for lithium depletion. In Fig. 4(c), the dotted and solid lines are slightly separated, i.e. gravitational settling still contributes, while in Fig. 4(d) the two lines merge to a single line; in this case overshooting mixing dominates over gravitational settling.

In the above text, we have qualitatively explained the effect of gravitational settling on lithium depletion in warm stars. Lithium depletion caused by gravitational settling can also be quantitatively measured using the surface lithium-abundance depletion time-scale \(\tau_L\). Dividing the fractional mass-conservation equation of lithium, equation (1), by \(\bar{C}\), we have

\[
\frac{D \ln \bar{C}}{D t} + \frac{1}{\bar{C}} \frac{\partial}{\partial M_t} \left[ 4 \pi^2 \bar{\rho} (U + J) \right] = -\bar{q},
\]

then multiplying the equation by \(dM_t\) and integrating \(dM_t\) from the bottom of the overshooting zone \((M_{ov})\) to the stellar surface \((M_0)\), bearing in mind that \(\bar{C}\) = constant throughout the convectively unstable and convective overshooting zones and \(4 \pi^2 \bar{\rho} (U + J) = 0\) at the surface (i.e. no injection of lithium from outside the star, see also the surface boundary-condition equations (24) and (25)), the equation becomes

\[
\frac{D \ln \bar{C}}{D t} (M_0 - M_{ov}) = \left[ \frac{1}{\bar{C}} 4 \pi^2 \bar{\rho} (U + J) \right]_{ov} - \int_{M_{ov}}^{M_0} q dM_t.
\]
The subscript ‘0’ marks surface quantities, while subscript ‘ov’ marks quantities at the bottom of the overshooting zone. Notice that 
\(4\pi r^2 \rho U_{\text{ov}}\) is the total amount of lithium flowing into (when \(> 0\)) or out of (when \(< 0\)) the convective and overshooting zones, hence
\[
\tau_{\text{ov}} = -(M_0 - M_{\text{ov}}) \left[4\pi r^2 \rho U / C\right]_{\text{ov}}^{-1},
\]
(38)
is the e-folding time of lithium depletion caused by the overshooting mixing process.

For the same reason, and using the same algorithm, we have
\[
\tau_g = -(M_0 - M_{\text{ov}}) \left[4\pi r^2 \rho J / C\right]_{\text{ov}}^{-1},
\]
(39)
which is the e-folding time of lithium depletion owing to gravitational settling.

\[
\tau_{\text{nuc}} = (M_0 - M_{\text{ov}}) / \int_{M_{\text{ov}}}^{M_0} q \, dM,
\]
(40)
is the e-folding time of lithium depletion due to nuclear reactions in the convective and overshooting zones. Following equation (37), the general lithium-depletion time-scale at the surface can be expressed as
\[
\tau^{-1} = \frac{D \ln C}{D r} = \tau_{\text{nuc}}^{-1} + \tau_{\text{nuc}}^{-1} + \tau_{\text{nuc}}^{-1}.
\]
(41)

Bearing in mind that \(d \ln C / dr \approx 0\) at the bottom of the overshooting zone, putting equation (2) into equation (39) we then have
\[
\tau_\Delta \approx (M_0 - M_{\text{ov}}) \left[4\pi r^2 \rho \left(7 - \frac{4\mu_0}{1 + \Phi} \frac{GM}{R T}\right)\right]_{\text{ov}}^{-1}.
\]
(42)

The dotted and solid lines in Figs 2(a) and (b) represent respectively the e-folding times of lithium depletion as functions of time calculated using the averaged slopes of the dotted lines (considering only overshooting mixing) and solid lines (both gravitational settling and overshooting mixing are considered) shown in Figs 1(a) and (b), while the dashed lines are the difference between the lithium-depletion time-scale \(\tau\) calculated using equations (38)–(42) and the solid lines in Figs 2(a) and (b).

\[\Delta = \log (\tau_\Delta / \tau).\]

As shown by the dashed lines in Figs 2(a) and (b), the lithium-depletion time-scale estimated from equations (38)–(42) agrees perfectly with the actual lithium-depletion time-scale calculated normally. Figs 2(a) and (b) show that, for cool stars with \(M \leq 1.0 \, M_\odot\), overshooting mixing is the primary depletion mechanism, while for warm stars with \(M > 1.1 \, M_\odot\) gravitational settling becomes the main cause of lithium depletion at the stellar surface. 1.0 \(M_\odot \leq M \leq 1.1 \, M_\odot\) is a transition range in stellar mass. Overshooting mixing and gravitational settling both contribute to lithium depletion at the stellar surface; this is exactly the reason why the lithium-abundance plateau level near \(T_e \sim 6000\, \text{K}\) decreases with higher cluster ages, as shown in Fig. 3(b). Without gravitational settling (the dotted lines in the plot), the warm stars with \(T_e \geq 6100\, \text{K}\) would not have their surface lithium abundance depleted.

### 4 COMPARISON BETWEEN THEORY AND OBSERVATIONS

A number of well-observed Galactic open clusters with different ages are selected to be compared with the results of our theoretical calculations. Table 1 gives the parameter of the selected clusters and data sources.

#### Table 1. Open clusters used in this work.

<table>
<thead>
<tr>
<th>Name</th>
<th>Age ((10^8, \text{yr}))</th>
<th>[Fe/H]</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a Per</td>
<td>0.5</td>
<td>0.05</td>
<td>1–6</td>
</tr>
<tr>
<td>Pleiades</td>
<td>0.7–1.0</td>
<td>0.034–0.06</td>
<td>6–12</td>
</tr>
<tr>
<td>Coma Berenices</td>
<td>4–5</td>
<td>0.052</td>
<td>27</td>
</tr>
<tr>
<td>Hyades</td>
<td>6–7</td>
<td>0.13 ± 0.06</td>
<td>16,13–18</td>
</tr>
<tr>
<td>Praesepe</td>
<td>6–7</td>
<td>0.13 ± 0.07</td>
<td>16,15,17,19</td>
</tr>
<tr>
<td>NGC 752</td>
<td>17</td>
<td>0.01 ± 0.04</td>
<td>20,22</td>
</tr>
<tr>
<td>M67</td>
<td>50</td>
<td>0.03 ± 0.01</td>
<td>23,24</td>
</tr>
</tbody>
</table>

Figure 5. The distribution of lithium abundance with effective temperature for member stars in the Pleiades (solid dots) and α Per (open circles). The solid and dotted lines are respectively the theoretical isochrones with and without gravitational settling for a solar abundance cluster at age = 0.5 Gyr.

4.1 Young clusters (age ≲ 0.1 Gyr)

α Per and the Pleiades are the two close neighbours to us that have been intensively observed. At a given effective temperature (or colour), there is a great dispersion in lithium abundance among the member stars. This fact used to be regarded as convincing evidence for the mechanism of rotationally induced mixing. Later investigations show that this is not completely true, and at least a large part of the dispersion is due to atmospheric phenomena involved in stellar surface activity. The lithium–stellar rotation correlation is likely a reflection of the lithium–stellar-surface activity correlation (King et al. 2000; Xiong & Deng 2005a,b, 2006). Fig. 5 shows the lithium abundance versus effective temperature for stars in these two young open clusters. The lithium abundance of stars in α Per is taken from Balachandran, Lambert & Stauffer (1996); a few suspicious photometric binaries, abnormally reddened stars and stars showing abnormal properties in the colour–colour diagram have been removed from the original catalogue. Member stars in the Pleiades from table 1 of King et al. (2000) are adopted, the original observed linewidths of which for lithium absorption lines were from Soderblom et al. (1993). Similarly to what we did for α Per, all spectroscopic and photometric binaries are removed. Very high colour anomalies indicate that stars in these two clusters are highly active. In the catalogue of King et al. (2000), as many as 17 out of 26 stars (over 60 per cent) with $T_e < 5000$ K (or $B - V > 0.92$) in the Pleiades are actually flare stars! Considering the difficulties in detecting flare stars, the real ratio may be even higher. The flare stars are marked by asterisks (*) in the plot, while the solid and dotted lines are respectively the theoretical isochrones with and without gravitational mixing at age 0.05 Gyr.

4.2 Intermediate open clusters (0.1 Gyr < age < 1 Gyr)

Three well-studied clusters are selected in this work to represent intermediate-age Galactic open clusters. Coma Berenices has an age between 0.4 and 0.5 Gyr, the Hyades and Praesepe between 0.6 and 0.7 Gyr. The lithium abundance versus temperature relation of all their member stars is plotted in Fig. 6. The V and Δ symbols express the estimated upper limits of lithium abundance for stars without sure determinations. The two pairs of solid and dotted lines in the plot are respectively the theoretical isochrones with and without gravitational settling at age = 0.4 and 0.6 Gyr. It follows from Fig. 6 that it is not possible to explain the lithium-abundance turn-over at $T_e > 6000$ K without considering gravitational settling (shown by the dotted lines). The lithium-depletion pattern in these three clusters can be approximately reproduced when both overshooting mixing and gravitational settling are taken into account. Quantitatively, there are sizable deviations between the observations and the theoretical predictions: the depletion in warm stars with $T_e > 6000$ K is predicted to be a little too slow, while near $T_e \sim 5800$ the theoretical curves are all higher than observations. The former difference hints that maybe the gravitational settling is underestimated, while the latter is somewhat difficult to explain, and this deserves further investigation.

4.3 Old open clusters (age > 1 Gyr)

M67 is one of the most studied old open clusters in the Galaxy, the age and metallicity of which are both close to the Sun. NGC 752 seems to be the only well-studied old cluster with an age between that of the Hyades and M67, and a fairly old cluster. Fig. 7 demonstrates the lithium abundance versus effective temperature for member stars in NGC 752. The abundance data for the solid dots and solid inverse triangles are taken from Balachandran (1995)’s re-analysed values, and the equivalent widths of lithium lines were from Hobbs & Pilachowsky (1986a,b) and Pilachowsky & Hobbs (1988), while the open circles are the lithium abundance from the latest observations of Sestito et al. (2004).
Lithium depletion in late-type dwarfs

Figure 6. Lithium abundance versus effective temperature plot for member stars in the Hyades (solid dots and inverse triangles), Praesepe (open circles and inverse triangles) and Coma Berenices (open squares and triangles). The triangles and inverse triangles are the estimated upper limits. The solid and dotted lines are respectively the theoretical isochrones with and without gravitational settling for a solar abundance cluster at age = 0.4 and 0.7 Gyr.

Figure 7. The same as Fig. 6 but for NGC 752. The isochrones are at age = 1.5 and 2.0 Gyr. The solid dots and open circles are respectively taken from Balachandran (1995) and Sestito et al. (2004). The inverse triangles are upper limits.

two solid and dotted lines in the plot are the theoretical isochrones with and without gravitational settling at age = 1.5 and 2 Gyr. It is clear from the plot that our theoretical results agree fairly well with observations. Near $T_e \approx 6000$ K, the theory seems to be systematically higher than observations.

The lithium abundance of stars in M67 shows a very large dispersion, and this was believed to be observational evidence supporting rotationally induced mixing. In fact, the sample of some early observations of lithium abundance for this cluster (Hobbs & Pilachowsky 1986a,b, 1988; Spite et al. 1987; García López et al. 1988; Deliyannis et al. 1994) contains some evolved stars above the turn-off point. At a given temperature (or colour), MS stars co-exist with evolved ones with slightly higher mass. The spread shown by those observations does not represent the intrinsic dispersion in lithium abundance of the sample stars; at least a part of that spread can be attributed to stellar-mass dependence of the abundance. To avoid such a problem, a more recent observational sample by Jones, Fischer & Soderblom (1999), the...
programme stars of which are all below the turn-off point with $5480 \text{ K} \leq T_e \leq 6160 \text{ K}$ (or $0.6 < (B - V)_0 < 0.74$) and $13.70 < V < 14.87$, is selected to be compared with theoretical results. Fig. 8 shows lithium abundance versus effective temperature for Jones et al.’s sample. The solid and dotted lines are respectively the theoretical isochrones with and without gravitational settling, and the position of the Sun is marked by $\odot$, while the observational uncertainties in temperature and lithium abundance are expressed by the error bar in the upper right corner of the plot. A few stars located to the right side of the curves (solid dots) have lithium abundance slightly higher than the theoretical predictions. Near $T_e \approx 5800 \text{ K}$, however, stellar lithium abundance depends sensitively on effective temperature. Lithium abundance can change by more than 0.5 dex for 50 K variation in temperature. An error of $\pm 0.01 \text{ mag}$ in $(B - V)$ due to photometry and reddening correction can cause an error in effective temperature of $\pm 43 \text{ K}$. For M67, the typical uncertainty in effective temperature estimation is about $\pm 130 \text{ K}$ (Hobbs & Pilachowsky 1986a). The stars at $T_e \sim 5800 \text{ K}$ deviating from the isochrones are all within the observational uncertainty range. There are five other stars with $T_e < 5600 \text{ K}$ in the plot (three marked as solid dots, two with only upper limit estimates as solid inverse triangles), the lithium abundances of which are rather higher than our predictions. Two of those stars, marked by I-2a and I-2b, are the two components of a spectroscopic binary, and their lithium abundance and effective determination are both quite uncertain. The only wired star without any good explanation is the one marked by S963. In Jones et al.’s table 1, the effective temperature of S963 is lower than the Sun by 80 K, while its lithium abundance higher by 0.8 dex! If this were true, then the Sun would be a very special star, the lithium of which would have been seriously overdepleted. This is, of course, not acceptable. In our theoretical calculations, the bottom of the solar convective zone has a temperature of about $2.3 \times 10^6 \text{ K}$ and a radius of 0.715 $R_\odot$; both are compatible with the helioseismic measurement. In fact, there are too few non-evolved cool stars ($T_e < 5700 \text{ K}$) with reliable observed lithium-abundance values in M67. It becomes a very important issue as to whether there are any cool MS stars with lithium abundance as high as S963. Such a problem deserves further investigation; no firm conclusion can be made at this time.

5 CONCLUSION AND DISCUSSION

Using a self-consistent dynamical theory of non-local convection, we have calculated two sets of lithium depletion for stellar evolutionary models. One set of lithium-depletion models considers only overshooting mixing, while the other considers both gravitational diffusion and overshooting mixing. The main results can be summarized as follows.

(i) When only overshooting mixing is considered, all warm stars with $M > 1.1 M_\odot$ (or $T_e > 6100 \text{ K}$) will not have lithium depletion in their atmospheres.

(ii) When both overshooting mixing and gravitational settling are taken into account, the overall observational properties of lithium depletion in stars can be approximately reproduced: (i) for all stars with $T_e < 6600 \text{ K}$, lithium abundance decreases as they age; (ii) for cool stars with $T_e < 6000 \text{ K}$, lithium decreases with lower effective temperature. On the other hand, for warm stars with $T_e \geq 6000 \text{ K}$, lithium abundance decreases when effective temperature goes up. Therefore a lithium plateau around $T_e \sim 6000 \text{ K}$ can be seen when considering all stars. Such a lithium plateau becomes narrower and lower when the stars (in clusters) become older.

(iii) Overshooting mixing extends greatly the completely mixed zone in the outer layers of stars, and therefore slows down the lithium depletion in warm stars due to gravitational settling. This naturally removes the difficulty created by the microdiffusion process, by which
alone the settling was too fast in these stars. This also removes the arbitrary addition of turbulent diffusion below the bottom of a convective zone (Proffitt & Michaud 1991; Richard et al. 2005). In other words, overshooting mixing is the physics underlying turbulent diffusion.

(iv) For warm stars with $T_e \geq 6100\text{K}$, the convective zones are not deep enough and the radiative acceleration neglected in the present work is no longer negligible. For this reason, our work may overestimate lithium depletion in warm stars. The depletion rates shown in Figs 1(b), 2(a) and (b) and 3(b) would be suitably lowered by considering the radiative acceleration. Nevertheless, our results will not be altered qualitatively. Quantitatively, this area needs further work, but this should not be an impossible barrier. Actually, adding radiation force into the microdiffusion equation (2) should in principle do the work.

Our conclusions are that overshooting mixing together with microdiffusion induced by gravitational settling and radiative acceleration are the leading mechanisms for lithium depletion in the lower part of the main sequence. For cool stars with $M \leq 1.1\ M_\odot$ (or $T_e \leq 5800\text{K}$), overshooting mixing dominates; for warm stars with $M > 1.1\ M_\odot$ (or $T_e \geq 6100\text{K}$), the microdiffusion process takes its turn.

Qualitatively, our theoretical predictions for lithium depletion in stellar atmospheres agree with observations. Quantitatively, there are some minor deviations.

(i) For stars with $T_e \sim 5900\text{K}$ in intermediate to old open clusters (age $> 0.1\text{Gyr}$), the theoretical predictions run at higher values than observations.

(ii) For warm stars with $T_e > 6300\text{K}$ in intermediate-age clusters Coma Berenices, Hyades and Praesepe, the observed lithium abundance is lower than theoretical predictions.

(iii) Observationally, the lithium abundance of stars in the low-temperature region ($T_e < 5700\text{K}$) is obviously higher than theoretical results. Our theoretical model predicts that there should be almost no observable lithium in such cool stars in the old open cluster M67.

The first two points infer that our models have somehow underestimated gravitational settling, or that there is some other unknown depletion mechanism. As for the problem revealed by the third point, we consider that no firm conclusion may be drawn because there are too few stars in M67 with $T_e < 5700\text{K}$ that have reliable lithium-abundance and effective-temperature determinations. This important and interesting problem deserves further investigation, both in theory and in observation.

Throughout this work, radiative acceleration and diffusion of helium and metals are neglected, all of which are important for warm stars with $T_e > 6200\text{K}$. Therefore our theoretical results cannot explain observations for such stars. Gravitational settling of helium in the envelopes of evolving main-sequence A and F stars has been investigated by Richer, Michaud & Proffitt (1992), who show that the largest effect of the settling of helium is to reduce the size of the convection zone. Inclusion of the settling of helium will accelerate the Li diffusion process and therefore will improve our theoretical predictions.

Taking diffusion and acceleration into account, Turcotte, Richer & Michaud (1998) modelled the consistent evolution of stars and studied abundance anomalies, and they showed that the diffusion of metals has a large contribution to the opacity in F stars. Changing the opacity will directly affect the depth of the convection zone in F stars, and therefore the depletion of lithium. Even with enormous computational effort, the predicted abundance anomalies are still much larger than observed in most F stars. They concluded that in order to match observations some additional hydrodynamical processes must be considered. In this work, we have shown that if convective overshooting is considered then the completely mixed region in the stellar outer envelope is largely extended, and the abundance anomalies in F stars are lowered. All these arguments tend to show that the study of warm stars with $T_e > 6200\text{K}$ in the current work is incomplete. Nevertheless, the current work demonstrates that the general pattern of the observed lithium-abundance depletion in late-type stars can possibly be reproduced by the interplay of overshooting mixing and diffusion induced by gravitational settling and radiative acceleration, while at the same time a solution can be provided for the abundance anomalies in F stars, which are overestimated by diffusion theory. The additional hydrodynamical processes hypothesized by Turcotte et al. (1998) may well simply be convective overshooting, if properly calculated following our model.

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