Melt Generation by Plumes: A Study of Hawaiian Volcanism

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ABSTRACT

The mantle plume underlying the Hawaiian Swell has been modelled numerically using a stationary steady axisymmetric plume under a solid conducting lid. A method of calculating the rate of melt production from the plume has been developed, and the total melt production rate, the residual depth anomaly and the geoid anomaly have been used to constrain the model. The plume has a central potential temperature of 1558 °C and the mechanical boundary layer is 72 km thick. An average of 6-6% melting occurs in a melt-producing region which has a vertical extent of ~55 km and a radial extent of ~130 km to produce 0.16 km$^3$/y of melt. A parameterization of melt composition has been developed that is consistent with laboratory experiments, with models of MORB generation, and with primitive Hawaiian tholeiites containing ~16% MgO. There is no evidence that the major and minor element concentrations in the source region of Hawaiian tholeiites differ from those in the source region of MORB. The model is consistent with the REE contents of Kilauean tholeiites if the source region has primitive REE contents. The viscosity of the low-viscosity layer is constrained to be ~10$^{16}$ m$^2$/s.

INTRODUCTION

Hotspot swells and their associated volcanism are thought to be the surface expressions of hot upwelling mantle plumes. The Hawaiian Swell is a good place to develop a model for a hotspot because of the abundance of high-quality data on both the volcanism and the physical properties of the swell.

Many models have appeared in the literature both for hotspot volcanism and for the evolution of the Hawaiian Islands. These models can be divided into chemical models, concerned with the petrogenetic evolution, and fluid dynamical models, which use physical properties of the swell to determine some property of the plume such as its temperature. This paper is an attempt to combine these two approaches to produce a fluid dynamical model that can account for the volume and composition of the melt produced by the Hawaiian plume.

The Hawaiian–Emperor chain is a 6000-km-long volcanic ridge made up from over 100 volcanoes. Most of the chain is now submarine, but the tops of most of the seamounts were subaerial when they formed. The volcanoes at the northwestern end of the chain are 75–80 Ma old, and they become progressively younger along the chain towards the currently active centres in the Hawaiian Islands at the southeastern end. Loihi seamount, to the southeast of the island of Hawaii, is thought to be in the early stages of development.

The volcanism is usually divided into four stages, based on observations from the Hawaiian Islands. Initially it appears that alkalic lavas are erupted onto the seafloor. This preshield stage is exposed only at Loihi, but is thought to be present but covered by later flows in the other volcanoes. The preshield stage is overlain by, and grades into, the main
shield stage, which produces voluminous olivine tholeiites. These make up ~98% of the volume of a Hawaiian volcano. The shield stage lasts ~400,000 y (Frey et al., 1990). It is overlain by the postshield stage, in which the lavas become transitional and then alkalic, leading to the development of a thin alkalic cap on the volcano. The eruption rate decreases by about two orders of magnitude during the transition from the shield stage to the postshield stage (Frey et al., 1990). Following a period of quiescence some of the volcanoes erupt very small volumes of alkalic and nephelinitic lavas. This posterosional stage does not appear to be present on all the volcanoes. This eruptive sequence is believed to be typical of the whole of the Hawaiian–Emperor chain, although lavas from all stages have not been recovered from any one volcano.

Associated with the volcanic ridge are prominent geoid and bathymetric highs (see Figs. 8 and 11 below). The Hawaiian Swell has a gaussian-shaped cross-section with a width of ~1200 km and a maximum height of ~1200 m (Detrick et al., 1989). The geoid anomaly has a magnitude of 6–8 m and a width similar to the bathymetric anomaly. Von Herzen et al. (1989) concluded that the anomalous heatflow associated with the Hawaiian Swell is probably no greater than 5–10 mW/m$^2$. Their difficulty in detecting a significant anomaly is probably a consequence of the high velocity of the Pacific plate (100 mm/y) in the hotspot frame of reference.

A number of hypotheses have been put forward to explain the existence of the Hawaiian–Emperor chain. These hypotheses attempt to account for the propagation of the volcanism along the chain, the fixity of the hotspot, the geophysical observations, and the details of the volcanism. The hypotheses may be grouped into four categories: (1) propagating fractures, (2) shear melting, (3) mechanical injection of heat into the lithosphere, and (4) convection.

Mechanisms associated with crustal fracturing were among the first explanations proposed for the Hawaiian–Emperor chain. Modern authors who propose propagating fracture mechanisms (e.g., Walcott, 1976) attempt to relate the fracturing to local or regional stresses within the Pacific plate. Melting occurs when mantle material upwells adiabatically through the fracture. Passive upwelling of normal-temperature mantle with a MORB source composition, a mechanism implicit in the fracture hypotheses, will not produce the volume of melt required to construct the Hawaiian Ridge (McKenzie & Bickle, 1988). There is also no mechanism for producing the residual depth and geoid anomalies in this class of models. The ideas behind these models may, however, be important in the location of individual volcanoes that appear to be arranged on a series of en-echelon fractures.

Shaw & Jackson (1973) are among the authors who put forward a model of localized shear melting. The residue from this melting formed a gravitational ‘anchor’ to fix the hotspot in the mantle. In their model the melting is produced by shear heating between the plate and the underlying mantle. This shear heating is, however, unlikely to generate enough heat to produce the observed volcanism and there is no obvious mechanism by which heat production can be localized beneath the island chain. It is also unlikely that the proposed gravitational anchor could survive in a vigorously convecting mantle.

Models of mechanical injection of heat into the lithosphere have been put forward by many authors, among them Detrick & Crough (1978). They proposed that the lithosphere is thermally ‘reset’ by the plume, an idea that requires a positive heatflow anomaly. Courtney & White (1986) showed that thermal resetting models could not explain heatflow data from the Cape Verde Rise. In addition, the rapid uplift seen in the Hawaiian and other hotspot swells requires unreasonably high heating rates in the lithosphere when it is first over the plume, and cannot be achieved using conductive heat transfer because the oceanic lithosphere has a thermal time constant of the order of 60 Ma (Sandwell & Poehls, 1980).
Finally, oceanic lithosphere will not melt to produce Hawaiian tholeiites without first undergoing significant compositional change through metasomatism, as envisaged in the model of Wright (1984).

Although there had been earlier suggestions (Menard, 1955) that the Hawaiian Swell was associated with thermally driven convection, Wilson (1963) was the first to suggest the idea of a magma source fixed under the southeastern end of the islands. The movement of the seafloor over this source led to the development of a linear island chain. Morgan (1971) proposed that the Pacific hotspots, which include Hawaii, are narrow thermal upwellings that he called plumes. Morgan considered these plumes to be of relatively low viscosity and to have a diameter of ~150 km. He argued that they originate deep in the mantle, possibly near the core, and arise because of thermal instabilities. They supplied the heat and material for the volcanism, and were considered by Morgan to be the driving force behind plate tectonics. Plumes seem to be the most reasonable explanation for the existence of hotspot swells but it is still unclear how and where they originate.

Examination of the geoid and bathymetry of the Pacific suggests the existence of mantle convection with a horizontal length scale that is considerably smaller than that of the plate (Watts et al., 1985). Plumes form the upwelling limbs of this small-scale convection, which appears to be unrelated to, and detached from, the motion of the plates. The geoid shows a series of elliptical maxima and minima with an amplitude of ~8 m separated by 1500-2000 km. These maxima and minima are thought to be associated with hot upwelling and cold downwelling plumes. The Hawaiian Swell is associated with one of the largest of the positive anomalies.

Courtney & White (1986) modelled heatflow, residual depth, and geoid anomalies over the Cape Verde Rise. Their model is of a plume rising from a heated lower boundary layer and spreading out under a solid conducting lid. They chose the Cape Verde Rise because they were principally interested in heatflow. The Cape Verde Rise is moving at <12 mm/y in the hotspot frame of reference; therefore, the heatflow there is likely to approach the equilibrium values predicted by the model. Courtney & White’s convection model is used in this study, but with a different scaling. Courtney & White specified the depth of the convecting region and used it to obtain the temperature scale. We specify the temperature scale, based on the work of McKenzie & Bickle (1988), and from it obtain the length scale. Courtney & White’s model predicted a potential temperature for normal mantle in the constant-temperature central core of the convection cell of ~1000°C, which is significantly lower than McKenzie & Bickle’s estimate of 1280°C. When we rescaled the dimensionless variables from Courtney & White’s best-fitting model (Table 1, model 4) to make the potential temperature of normal mantle 1280°C, the model no longer fitted the Cape Verde Rise data. Courtney & White did, however, obtain a similar temperature anomaly for the plume to the one calculated in this paper and their work showed that an axisymmetric plume spreading out under a solid conducting lid is likely to be a good model for a hotspot.

Estimates of the temperature anomaly associated with the plume have been made by Sleep (1987), Wyllie (1988), and Liu & Chase (1989). Sleep (1987) modelled the velocities in the plume and calculated a stagnation streamline, which he fitted to the bathymetry at the southeastern end of the Hawaiian Ridge. The distance from the centre of the plume to the stagnation point was converted into a temperature anomaly by considering the amount of heat carried by the plume, which was then used to calculate the residual depth anomaly. Sleep’s best estimate of the temperature anomaly for the Hawaiian plume is 300 K. Liu & Chase (1989) modelled the bathymetry, geoid, and heatflow along the Hawaiian Ridge. In their model an axisymmetric mantle plume introduces a gaussian-shaped thermal anomaly into the base of a conducting plate that is moving over it at a constant velocity. They did not
Convection runs (grid size is in mesh intervals for the convecting part of the model)

<table>
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<th>Run</th>
<th>Ra/Ra₀</th>
<th>Radial × Vertical</th>
<th>λ</th>
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</tr>
<tr>
<td>2</td>
<td>6000</td>
<td>96 × 64</td>
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</tr>
<tr>
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<td>96 × 64</td>
<td>0.125</td>
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<tr>
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<td>144 × 96</td>
<td>0.125</td>
</tr>
<tr>
<td>6</td>
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<td>144 × 96</td>
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<tr>
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attempt to include the fluid dynamics that controls the plume. Their plume is capable of replacing cooler material at the base of the plate with hotter mantle brought up by the plume. They call this process convective thinning. Their best-fitting model has a temperature anomaly of 250 °C in the plume under the Hawaiian Ridge. Wyllie (1988) attempted to constrain the temperature by considering melting in the plume. He used the temperature structure of Courtney & White (1986) as a starting point and calculated where this temperature structure intersects the solidus for peridotite with and without volatiles. From this constraint he concluded that the maximum plume temperature is above 1500 °C.

Though we use a steady-state axisymmetric plume in a constant-viscosity fluid beneath an unbroken plate to model the processes occurring in the mantle beneath Hawaii, such a model is clearly a simplification. It is, however, the simplest model of a plume that takes account of the fluid dynamics. In reality, the movement of the Pacific plate breaks the circular symmetry, and may also cause the plume to become time dependent (Skilbeck & Whitehead, 1978). Though the variation of viscosity with temperature may also influence the thermal structure of the plume, at large Rayleigh numbers this effect is probably small (White, 1988). Furthermore, the velocity and variation of heat flux with Rayleigh number for our calculations agree well with those of Olsen et al. (1990), who used a fluid with a temperature dependent viscosity. In addition, the calculations described only with the behaviour of plumes where the transient effects associated with their initiation have decayed. White & McKenzie (1989) argued that some flood basalt provinces are associated with the start of a new plume, and Campbell et al. (1989) have suggested that komatiites are produced in the same way. We attempted to model plume initiation using our axisymmetric code. We found that the thermal structure of the initial hot blob was principally controlled by the initial conditions, but we could not see how to choose realistic initial conditions for the axisymmetric problem. It is probably necessary to use a three-dimensional time-dependent code to study plume initiation. We did find, however, several
examples in which the time-dependent heat flux exceeded that in steady state by a factor of
five, in agreement with White & McKenzie's suggestion.

Olsen et al. (1988) found that thermal plumes do not effectively intrude and erode the
mechanical boundary layer if upper-mantle viscosity is a function of temperature only,
unless the activation energy for the process is unusually low. Yuen & Fleitout (1985)
concluded that mantle plumes can erode the mechanical boundary layer at a rate of
kilometres per million years if secondary convection occurs. However, Rabinowicz et al.
(1990) argued that secondary convection will not occur under the conditions that we are
modelling. Our plume does not erode the mechanical boundary layer. Using a simple model
simplifies the calculations and makes it easier to see how the data we are modelling differ
from the assumptions made in the model. The major question is whether a plume that
replaces the thermal boundary layer alone can account for the geophysical and petrological
observations.

CONVECTION

The model we use has an axisymmetric plume rising from a heated lower boundary layer
and spreading out under a solid conducting lid (Fig. 1). No melting was included in the
calculations at this stage. The convection was modelled using an incompressible, Boussinesq
fluid with a constant newtonian viscosity and infinite Prandtl number. The equations
governing the convection are given in Appendix A. The convecting box was capped by a lid
in which there is no fluid flow and the heat transfer was conductive. There was no fluid flow
out of the convecting box. A constant heat flux was applied to the base and there was no flux
of heat out of the sides of the cell. The temperature at the top of the conducting lid was held
constant, and both the temperature and the heat flux were continuous between the

![Fig. 1. Model used for the convection calculations. $\lambda$ and $\gamma$ are defined in the text (dimensionless units).](https://academic.oup.com/petrology/article-abstract/32/3/501/1436328/Melt-Generation-by-Plumes-A-Study-of-Hawaiian)

by guest

on 16 September 2017
converting box and the conducting lid. There was no shear stress at the top and bottom of
the converting box and at its outer wall.

The equations were solved on a square mesh (Jones et al., 1976) using an explicit centred
second-order conservative finite difference scheme. Initially, the temperature in the convert-
ing box was almost constant, except in a hot boundary layer near the base. The temperature
in the conducting lid was set to the temperature of the box at \( r = 0, z = 1 \) (Fig. 1). The system
was given a small temperature perturbation to force the circulation to start with a hot plume
up the centre. The amplitude of this initial circulation was small compared with the
amplitude of the final fluid circulation. Because the initial velocity was small, advective heat
transport was at first small and heat accumulated in the bottom boundary layer. A rapid
increase in velocity occurred when this hot material was advected into the axial plume. This
process led to oscillations that decayed with time in all the cases we studied, and the
calculations were continued until this steady state was reached. In many of the models a cold
blob of fluid detached from the upper boundary of the converting box while the bottom
boundary layer was thickening at the end of the first cycle of oscillation. This blob perturbed
the bottom boundary layer in such a way that when the bottom boundary layer next became
unstable, hot fluid upwelled along the outer boundary of the cell instead of up the centre.
Subsequent oscillations were not violent enough to reverse the flow pattern again. In these
cases the run was stopped, the temperatures were horizontally averaged, and the model was
perturbed again as before. The model was then allowed to run to equilibrium.

A series of runs was done in which the Rayleigh number,

\[
Ra = \frac{g a F a^4}{\kappa \nu v}
\]

(1)

where \( g = 9.81 \, \text{m/s}^2, \alpha = 3.28 \times 10^{-5} \, \text{K}^{-1} \) (Parsons & Sclater, 1977), \( F \) is the basal heat flux, \( a \)
is the depth of the convecting layer, \( k \) is the thermal conductivity, \( \kappa \) is the thermal diffusivity,
and \( \nu \) is the viscosity, and \( \lambda \),

\[
\lambda = \frac{\text{conducting lid thickness}}{\text{depth of convecting box}}
\]

(2)

were specified. \( Ra \) ranged from 6000 \( Ra_{cr} \) to 35 000 \( Ra_{cr} \) (\( Ra_{cr} = 27 \pi^4/4 \)) and \( \lambda \) from 0.125 to
0.25 (see Table 1 and Fig. 1). An aspect ratio, \( \gamma \), of 1.5 was chosen for the convecting part of
the model. This choice gives calculated geoid and residual depth anomalies in the range
observed for hotspot swells. Cells with larger aspect ratios than 1.5 tend to break up into two
or more concentric cells (Courtney, 1985).

Simple boundary-layer arguments predict that the boundary-layer thickness is propor-
tional to \( Ra^{-1/4} \) when the heat flux is fixed (McKenzie et al., 1974). Richter & McKenzie
(1981) showed that, in a cartesian system, it is possible to parameterize the horizontally
averaged dimensionless temperature, \( \tilde{T}_r \), in terms of stretched variables, \( \tilde{T}_r \) and \( \tilde{z} \), where

\[
\tilde{T}_r = Ra^{1/4} \tilde{T}_r
\]

(3)

\[
\tilde{z} = (1 - \tilde{z}') Ra^{1/4}
\]

(4)

Because \( \tilde{z}' \) increases upwards (Fig. 1), \( \tilde{z} \) is zero at the top of the convecting box. \( \tilde{T}_r \) was zero at
the top of the upper boundary layer.

When Richter & McKenzie plotted \( \tilde{T}_r \) against \( \tilde{z} \) for a number of runs at different Rayleigh
numbers, the data for the upper boundary layer all lay on a curve described by an equation
of the form

\[
\tilde{T}_r = T_0 + \exp(-\tilde{z}) \sum_{n=0}^{\infty} a_n \tilde{z}^n
\]

(5)
Fig. 2. $z$ vs. $\tilde{T}_r$ for the runs in Table 1. The solid line is the best-fitting line to the data points which is described by (5) with the coefficients given in the text. Richter & McKenzie's curve for rolls (dashed line) is shown for comparison.

with

$$\left(\frac{d\tilde{T}_r}{d\tilde{z}}\right)_{\tilde{z}=0} = 1.$$

The dimensionless temperature within the convecting part of the models in this study was plotted in the same way. The best-fitting curve (Fig. 2) described by (5) has coefficients

$$T_0 = 0.721$$
$$a_0 = -0.721$$
$$a_1 = 0.279$$
$$a_2 = 0.183$$
$$a_3 = 0.242$$
$$a_4 = 0.0015.$$

The coefficients were determined by fitting (5) to the points in Fig. 2 using least squares. In all cases the value of $\tilde{T}_r$ increases slightly with decreasing $\lambda$, but no variation of $\tilde{T}_r$ with $Ra$ was apparent.

According to boundary-layer theory, the horizontal and vertical boundary layers should show the same dependence on $Ra$. The vertically averaged dimensionless temperature, $\tilde{T}_z$, stretched to produce $\tilde{T}_z$, given by

$$\tilde{T}_z = \tilde{T}_z Ra^{1/4} \tag{6}$$

was plotted against a stretched radius, $\tilde{r}$, given by

$$\tilde{r} = \left(\frac{r'}{\gamma}\right) Ra^{1/4}. \tag{7}$$
Again, all the data fall on a single curve (Fig. 3). There is now no variation with $\lambda$ and the scatter with Ra is similar to that seen for the horizontally averaged temperatures. Figure 4 shows contours of $Ra^{1/4} T'(\tilde{r}, z)$ for runs 1 (Fig. 4a) and 8 (Fig. 4b) in Table 1. As expected from the above analysis, Figs. 4a and 4b are almost identical. This dimensionless analysis shows that axisymmetric convection behaves in the same way as do the more familiar rolls. More importantly for this study, it is a valuable check on the accuracy of the convection code and suggests that all the runs were properly resolved and reached equilibrium.

The dimensionless variables obtained from the convection runs were scaled according to the scheme given in Appendix A. McKenzie & Bickle (1988) showed that the potential temperature of normal mantle is 1280 °C. The depth of the convecting box, $a$, and the viscosity, $\nu$, were varied to constrain the potential temperature at the central point in the convecting box to be 1280 °C. From the nondimensionalization scheme given in Appendix A
the temperature, $\Delta T$, is related to the dimensionless temperature, $\Delta T^*$, by

$$\Delta T = \frac{Fa\Delta T^*}{k}$$  \hspace{1cm} (8)

and

$$\kappa = \frac{k}{\rho_0 C_p}$$  \hspace{1cm} (9)

where $\rho_0 = 3300 \text{ kg/m}^3$ and $C_p = 1200 \text{ J/(kg K)}$. The basal heat flux, $F$, was taken to be $51 \text{ mW/m}^2$ [based on a plate age of $\sim 85$ Ma (Parsons & Sclater, 1977)] and $\kappa$ to be $8 \times 10^{-7} \text{ m}^2/\text{s}$. Equation (8) is used to obtain $a$:

$$a = \frac{k\Delta T}{F\Delta T^*}.$$  \hspace{1cm} (10)

Figure 2 shows that $\delta' \propto \text{Ra}^{-1/4}$, therefore

$$\delta = a\delta' = aC\text{Ra}^{-1/4}$$

$$= C \left( \frac{g\alpha F}{kkv} \right)^{-1/4}.$$  \hspace{1cm} (11)

where $C$ is a constant. Because $\delta$ and $F$ are independent of $a$, the true temperature structure is similarly independent. The thermal structure and velocities within the plume are, therefore, independent of the thickness of the convecting layer if the heat flux is kept constant, and hence the arguments below are not affected by whether the plumes extend through the whole of the mantle or only the upper mantle. Conversely, observations dependent on the thermal structure of the plume head, such as the geoid and residual depth anomalies, cannot provide constraints on the depth to which the plume circulation extends. The viscosity, $\nu$, was then obtained from (1). Therefore, once the interior average temperature is specified, the length scale and the viscosity are fixed. The geoid and residual depth anomalies over the models were calculated using Green's functions (Parsons & Daly, 1983). The method is described in appendix C of Courtney & White (1986).

**MELT PRODUCTION**

Melting within the plume and the extraction of that melt will affect the dynamics of the plume. Latent heat of melting will decrease the plume temperature and the separation of melt will affect the buoyancy forces that drive the convection. No attempt was made to include the effects of melting in the convection calculations described in the previous section because too little is known about the physics of melt separation, and how this process affects the convection, for it to be modelled satisfactorily. In addition, inclusion of melting effects in the convection calculation would make the convection modelling specific to the Hawaiian plume rather than able to be scaled to any application.

Throughout our analysis of melting, we assume that the melt and matrix move together until the melt fraction stops increasing (batch melting). The melt is then assumed to separate totally from the matrix, so that no melt freezes as the matrix moves out of the melting region. McKenzie (1985) showed that it is unlikely that more than $\sim 1\%$ melt can remain in contact with the solid for any length of time, so the assumption that melt and matrix move together is unlikely to be valid. We should therefore use the relationship between $T$, $P$, and melt fraction for Rayleigh (or fractional) melting, and not that for batch melting. Unfortunately, calculations must be based on parameterization of experiments, and all the experiments
involve batch melting. It is not clear how to obtain the relevant Rayleigh melting parameters, which we require to do the calculation properly, from the batch melting data.

If melting occurs in a plume, and the melt and matrix move together, an element of mantle that melts will pass through a region in which the melt fraction is increasing, then through one in which the melt fraction is decreasing. The rate of melt production depends on the rate at which material passes through the melting region and on the temperature gradients that it encounters as it does so. To calculate the rate at which melt is being produced we could integrate the change in melt fraction along the $P-T$ path of each streamline. Such a calculation is slow. It is better to use the instantaneous melt production rate, which is the melt production rate associated with a change in melt fraction from $X$ to $X + dX$, obtained from McKenzie & Bickle's (1988) parameterizations. The instantaneous melt production rate is easy to calculate using the $X-T$ relationships given by McKenzie & Bickle (1988). The total melt production rate, $R$, from the plume is then given by

$$ R = \int_0^V \Gamma dV \quad (12) $$

where $\Gamma$ is the instantaneous melt production rate and $V$ is the volume of the melting region. $\Gamma$ was calculated using the expressions in Appendix B, and depends on the gradient of real temperature at each point. The volume integral was taken only over the region where $\Gamma > 0$. To obtain real temperatures we must calculate the adiabatic decompression path for each particle within the melting region [appendix D of McKenzie (1984)], which is time consuming. The results of the adiabatic decompression calculation were, therefore, parameterized. We first obtained $\Delta T$, given by

$$ \Delta T = T - T_s(P) \quad (13) $$

where $T$ is the temperature calculated from decompression melting, and $T_s(P)$ is the solidus temperature at the point. $\Delta T$ was parameterized in terms of $P$, the pressure at the point, and $\Delta \Theta$, defined as

$$ \Delta \Theta = \Theta - \Theta_s(P) \quad (14) $$

where $\Theta$ is the potential temperature of the solid before the onset of melting and $\Theta_s(P)$ is the potential temperature of the solidus.

$\Delta T$, $\Delta \Theta$, and $P$ are related by an equation of the form

$$ \Delta T = \Delta \Theta \left[ \sum_{m=0}^{3} \sum_{n=0}^{3} a_{mn} \Delta \Theta^m P^n \right] \quad (15) $$

with non-zero coefficients, obtained by least-squares fitting, of

$$ \begin{align*}
    a_{00} &= 0.5192 \\
    a_{10} &= 0.0002 \\
    a_{01} &= -0.0387 \\
    a_{02} &= -0.0072 \\
    a_{03} &= 0.0009 \\
    a_{11} &= 0.0001
\end{align*} $$

for $\Delta S = 250 \text{ J/(kg}^{\circ}\text{C)}$. The r.m.s. error in this parameterization is 0.45%. The incompressible convection calculations give a temperature, $T$, which corresponds to $\Theta$ for the
compressible case (Jarvis & McKenzie, 1980). Hence, we can find $\Delta \Theta$ from the convection results and use (15) to obtain $T$. The melt fraction, $X$, is then calculated using $X = X(P, T)$ (McKenzie & Bickle, 1988), and $\Gamma$ follows from the expressions in Appendix B.

At any point in the melting region, $\Gamma$ should satisfy

$$\Gamma \propto u \cdot [V(T - T_s)]$$

$$\propto v \left[ \frac{(\Delta T - T_s)}{\delta} \right]$$

(16)

where $u$ is the velocity vector, $\delta$ is the boundary layer thickness, $v$ is the velocity, and $T_s$ is the solidus temperature. The total melt production rate, $H$, also depends on the volume of the melting region, so

$$H \propto \Gamma \delta^3.$$  

(17)

Using (17) we obtain

$$H \propto v(\Delta T - T_s)\delta^2.$$  

(18)

From McKenzie et al. (1974),

$$v = \frac{\kappa Ra^{1/2}}{a}$$  

(19)

$$\delta = a Ra^{-1/4}$$  

(20)

$$\Delta T = \left( \frac{\kappa v}{g \alpha a^2} \right) Ra^{3/4}.$$  

(21)

Substituting (20)–(22) into (19) gives

$$H \propto a \left( \frac{\kappa v}{g \alpha} \right)^{1/4} \left( \frac{F}{\rho_0 C_P} \right)^{3/4} - C_0 a$$  

(22)

where $C_0$ is a constant. Using the scheme in the previous section,

$$a = \frac{kT_1}{F(\lambda + C Ra^{-1/4})}$$  

(23)

where $C$ is a constant and $T_1$ is the temperature in the centre of the cell. Substituting (24) into (1) gives

$$v = \left[ \frac{kT_1}{F(\lambda + C Ra^{-1/4})} \right]^{1/4} \frac{g \alpha F}{k \kappa Ra}.$$  

(24)

Substituting (24) and (25) into (23) and rearranging gives

$$H = A \left[ \left( \frac{kT_1}{F(\lambda + C Ra^{-1/4})} \right)^2 \left( \frac{g \alpha F}{k \kappa Ra} \right)^{1/4} \right] - B \left( \frac{kT_1}{F(\lambda + C Ra^{-1/4})} \right)$$  

(25)

where $A$ and $B$ are constants. $G$, the term in square brackets, varies by a factor of $\sim 3$ and the other term varies by a factor of $\sim 1.4$ in the range of data we are considering. Figure 5 shows a plot of $H$ vs. $G$, which suggests that the relationship derived above is correct. The total rate of melt production from melting in the thermal boundary layer can be therefore adequately described by (26).
MODELLING THE PLUME BENEATH HAWAII

The Hawaiian Swell was chosen to constrain the model because the geoid and residual depth anomalies associated with it are well known, it has both the best studied and most voluminous volcanism of the intraplate hotspots, and it is far removed from any continental crust.

The residual depth anomaly, $h$, has a conductive and a convective component. The conductive component depends on the thickness of and the temperature change across the conducting layer, which are both constants in our models. Parsons & Daly (1983) showed that the convective component, $h_c$, should vary as

$$h_c \propto C_1 a \delta \Delta T$$  \hspace{1cm} (27)

where $C_1$ is a constant. Using (21) and (22) gives

$$h_c \propto \frac{y}{a} \text{Ra}^{1/2}.$$  \hspace{1cm} (28)

Substituting in (24) and (25) gives

$$h_c \propto \frac{\text{Ra}^{1/2}}{\left(\lambda + C \text{Ra}^{-1/4}\right)^3}.$$  \hspace{1cm} (29)

Figure 6 shows a plot of total uplift against $\text{Ra}^{-1/2}/(\lambda + C \text{Ra}^{-1/4})^3$. This plot confirms the relationship derived above, and the intercept suggests that the conductive component of the uplift is 250–300 m. As Parsons & Daly also showed that the admittance only varies slowly with $\text{Ra}$ and $\lambda$, the melt production rate was used as an additional constraint.

Frey et al.'s (1990) data from the currently active shield volcanoes, Kilauea and Mauna Loa, show that the eruption rate is $\sim 0.03 \text{ km}^3/\text{y}$ with a magma supply rate for Kilauea of $\sim 0.1 \text{ km}^3/\text{y}$ during shield formation. The postshield stage at Mauna Kea involves a decrease in eruption rate from 0.01 to 0.002 $\text{km}^3/\text{y}$. The posterosional stage contributes...
<1% of the volcano's volume and the preshield stage is thought to account for only a few percent of the volcano in volumetric terms (Clague & Dalrymple, 1989). Clearly, the tholeiitic shield stage is the dominant stage in Hawaiian volcanism, and any modelling of the melting in the plume should concentrate on this stage. The figures quoted above are mainly for the erupted products, but the seismic reflection data show that construction of the Hawaiian Ridge requires the melt production rate to be larger than the measured eruption rate. If the Pacific plate is assumed to be moving at 100 mm/y in the hotspot frame of reference, the cross-sectional area of $1.6 \times 10^9$ m$^2$ (Watts & ten Brink, 1989) requires the current melt production rate from the Hawaiian plume to be 0.16 km$^3$/y, and therefore only ~20% of the melt production is extruded as lava.

To convert from dimensionless to real variables requires values of the physical quantities in the governing equations. The values used for the heat flux, thermal expansion coefficient, thermal conductivity, and diffusivity were:

$$F = 51 \text{ mW/m}^2$$
$$\alpha = 3.28 \times 10^{-5} \text{ K}^{-1}$$
$$k = 3.168 \text{ W/(m K)}$$
$$\kappa = 8 \times 10^{-7} \text{ m}^2/\text{s}.$$  

Then for any convective model with given values of $Ra$ and $\lambda$ the viscosity is given by (1) and the layer depth by (10). The best-fitting model was found by varying $Ra$ and $\lambda$ until the calculated and observed melt production rates and geoid and residual depth anomalies agreed. Decreasing $Ra$ or $\lambda$ increases the value of $v$. Melt production is limited by the velocity in the melting region, which decreases with increasing viscosity, in cases where the viscosity is greater than $\sim 1.5 \times 10^{16}$ m$^2$/s. Model 6 (Table 1) has a viscosity of $1.7 \times 10^{16}$ m$^2$/s, geoid and residual depth anomalies of 7.9 and 1549 m, and a melt production rate of 0.16 km$^3$/y. Although the melt production rate fits the Hawaiian data, the geoid and residual depth

![Figure 6](https://academic.oup.com/petrology/article-abstract/32/3/501/1436328/Melt-Generation-by-Plumes-A-Study-of-Hawaiian)}
anomalies are too large. Model 15 (Table 1), with a viscosity of $8.8 \times 10^{15}$ m$^2$/s, has geoid and residual depth anomalies of 61 and 1233 m, which are closer to the Hawaiian values, but the melt production rate of 0.14 km$^3$/y is slightly too small. The maximum melt production rate is 0.18 km$^3$/y from model 8, which does not have the highest potential temperature. The model that best fits the Hawaiian data is number 17 in Table 1 (Fig. 7). This model has a central plume potential temperature of 1558°C, which gives a temperature anomaly of 278°C before melting is taken into account. The melt production rate is 0.16 km$^3$/y, in good agreement with the observed value, and the residual depth anomaly is $\sim$1320 m. The geoid anomaly is 6.7 m and the best fit to the Hawaiian geoid data positions the maximum in the anomaly $\sim$100 km northeast of the islands (Fig. 8). Though analysis of the geoid is complicated by the removal of the long-wavelength component, the correlation between the calculated uplift and geoid (Fig. 9) falls within the range of Hawaiian data.

![Fig. 7. Potential temperature structure for the best-fitting model (contours are 300, 700, 1100, 1300, and 1500°C).](https://academic.oup.com/petrology/article-abstract/32/3/501/1436328/Melt-Generation-by-Plumes-A-Study-of-Hawaiian)

![Fig. 8. Part of Geosat track D129 relative to GEM 9 (dotted line) and calculated geoid anomaly (solid line).](https://academic.oup.com/petrology/article-abstract/32/3/501/1436328/Melt-Generation-by-Plumes-A-Study-of-Hawaiian)
The mechanical boundary layer thickness obtained from the calculations is 72 km, and convection extends to a depth of 473 km, but, as explained above, this depth is not constrained by the available data. The best-fitting model has a viscosity of $9.5 \times 10^{15} \text{ m}^2/\text{s}$. The plume transports $5.8 \times 10^{10} \text{ W}$. Melt production consumes $\sim 5.6 \times 10^9 \text{ W}$, and hence should not significantly modify the temperature structure.

Sleep's (1987) calculations of the stagnation streamline for the Hawaiian plume assumed that $v \propto 1/r$, where $v$ is the radial velocity and $r$ is the distance from the centre of the plume. Figure 10 shows that the expression $v \propto 1/r^{0.779}$ better describes the horizontal velocity variation in the numerical model except near the axis and the outer edge of the cell. This relationship was obtained by minimizing the variation in the quantity $vr^n$, in the region well away from the axis and the outer edge of the cell, by varying $n$, using least squares. We calculated a stagnation streamline for our model from the radial component of the velocity at the top of the convecting box using the definition of the stagnation point given by Sleep (1987). Figure 11 shows that the shape of the stagnation streamline calculated from our model is very similar to his. Both Sleep's and our stagnation streamlines fit the bathymetry better if the centre of the plume is moved northeast by $\sim 100 \text{ km}$, in agreement with the best fit to the geoid data.

McKenzie & Bickle (1988) obtained empirical expressions for the dry peridotite solidus and liquidus temperatures and for the melt fraction, $X$. We used these expressions to calculate the melt fraction present at each point in the plume. The melt fractions were then averaged over the region in which $\Gamma > 0$. Figure 12a shows that the region in which mantle material is above the solidus has a maximum vertical extent of $\sim 65 \text{ km}$ and a maximum radial extent of $\sim 230 \text{ km}$, but that the melt-producing region is smaller. The average melt fraction involved in melt production is 6.6%. Melt is produced between depths of $\sim 136$ and 82 km on the axis, and melt production extends up to $\sim 132 \text{ km}$ radially (Fig. 12a). The maximum degree of melting is $\sim 22\%$. In our model, which has reached equilibrium and has therefore had time for heat to be conducted into the mechanical boundary layer, melting...
FIG. 10. $n, r^{0.779}$ vs. $r$.

FIG. 11. Simplified map of the residual depth anomaly in metres associated with the Hawaiian Swell. The dashed line shows the stagnation streamline calculated for our model. The dotted line shows Sleep's (1987) stagnation streamline, positioned as close to the position in Sleep (1987) as his figure allows. The straight lines directed northwestwards are the axis of symmetry of the stagnation streamlines and represent the direction of plate motion.
extends ~4 km into the mechanical boundary layer. The melt fraction associated with this melting is only ~0.5%, so the contribution to the overall melt production rate is negligible, both because of the small melt fraction involved and because the material within the mechanical boundary layer is not moving. No melting from the mechanical boundary layer is included in our estimates of the melt production rate. Such melting is probably important in the later stages of Hawaiian volcanism. Contours of melt production rate (Fig. 12b) show

Fig. 12. (a) Contours of melt fraction for the melting region. The base of the mechanical boundary layer is at a depth of 72 km and the figure is symmetrical about the plume axis. No vertical exaggeration. (b) Contours of instantaneous melt production rate in units of $\text{yr}^{-1}$ for the melting region. The base of the mechanical boundary layer is at a depth of 72 km and the figure is symmetrical about the plume axis. The melting region is that where $\Gamma > 0$ and the freezing region where $\Gamma < 0$. No vertical exaggeration.
that melt production is concentrated on the axis in regions where the melt fraction is low. The melt-producing region is capped by a region in which the melt fraction is decreasing. It is assumed that no melt is lost through freezing in this region, which is shown in Fig. 12b. Little melt is generated in those regions where the melt fraction exceeds 10%. Equation (16) shows that the instantaneous melt production rate at a point depends on the temperature gradient and the velocity at the point. Melt production is concentrated in the low melt fraction region on the axis because this is the region in which the material is moving fastest and the potential temperature is decreasing slowest. The average melt fraction involved in shield tholeiite production is therefore $<10\%$ (the main calculations include the whole plume rather than just the region on the axis that produces the tholeiites). About $2.4 \times 10^6$ km$^3$ of mantle is processed through the melting region per million years. The large volumes of melt required to construct the Hawaiian Ridge can only be produced by small degrees of melting because such large volumes of mantle are involved.

We stated in the Introduction that the Hawaiian Swell is not an ideal place for constraining a model of a mantle plume because of the problems that arise from the northwestward movement of the lithosphere beneath Hawaii relative to the plume. We are now in a position to consider how this movement will affect our modelling and the results presented above. For much of its passage through the upper mantle the plume will be unaffected by the movement of the plate above. However, when the plume meets the moving plate above it the horizontal mushrooming shown in our model will be modified. The plume's nose will be abruptly decelerated and the top will be sheared over and entrained in the horizontal motion of the overlying plate. Therefore the true temperature structure and melting structure under the Hawaiian Swell will not be axisymmetric. However, the geometry of the plume in the plane perpendicular to the plate movement should be similar to that shown in our model.

MELT COMPOSITIONS

McKenzie & Bickle (1988) showed that the major element compositions of melts can be calculated from a simple parameterization of experimental results. This parameterization successfully predicts the composition of mid-ocean ridge basalt (MORB), but it has a number of shortcomings at high pressures and small melt fractions. If the results of the previous section are to be believed, this is the region of $P-X$ space involved in the melt production by the plume. At pressures of $>2$ GPa their parameterization produces highly nepheline normative melts at melt fractions of $<25\%$, which are not observed in the shield stage of Hawaiian volcanism. At pressures $>3$ GPa, O'Hara (1968) suggested that the 'cpx-out' point returns to the tholeiite field from the nepheline normative field, but in McKenzie & Bickle's parameterization it does not do so. This problem stems from the absence of reliable experimental data at high pressures and low melt fractions. When the melt fraction is small, it is difficult to quench the charge to produce a glass that is representative of the equilibrium liquid. As a result, the estimated volume and composition of the melt is unlikely to be well determined. If an improved parameterization is to be developed, the only option available is to include data from the Hawaiian tholeiites. The resulting parameterization scheme will then predict the observed composition and the argument is circular. The inclusion of the Hawaiian data point has little effect, however, on the fit to the experimental data points, and if the resulting parameterization can reproduce the results of McKenzie & Bickle (1988), we have a valid scheme. Until the experimental database improves significantly, this is the only route we can follow. Its success does at least show that the laboratory experiments are consistent with our melting model for the plume.
The pressure and melt fraction of the Hawaiian primitive magma was obtained from the model in the previous section. Though there are compositional differences among the shield-building tholeiites from different volcanoes, these are smaller than the variation between different laboratory experiments. Therefore a composition representative of all the tholeiite data was used. The final problem is the question of the MgO content of the primary tholeiite. A value of 16% was used (F. Frey, pers. comm.) and the resulting composition is given in Table C1 below. The composition of the source material is determined by the parameterization.

Like McKenzie & Bickle (1988), we found that it was not easy to parameterize the experimental results. The approach taken here (Appendix C) is similar to theirs, and the final minimization required 19 parameters. We made two determinations of the parameters, without (a) and with (b) the data of Falloon & Green (1988) and Falloon et al. (1988). For parameterization (a) the average misfit is 0.96% (0.92% before the correction to CaO) for the major element oxides and 0.28% for the minor element oxides. Without the Hawaiian point the misfit for the major element oxides is 0.90%. This difference is not significant. For parameterization (b) the average misfit is 0.98% for the major element oxides and 0.32% for the minor element oxides.

The composition of the melt being produced by the plume was calculated using a similar approach to that used to calculate the melt production rate (see the section on 'Melt Production'). The instantaneous melt composition, \( c \), is related to the point average melt composition, \( C \), by

\[
C(X) = \frac{\int_V \Gamma c(X') dV}{\int_V \Gamma dV}.
\]

This equation is used to calculate the composition of the melt being produced by the plume (Appendix B).

Figure 13 shows that the agreement between the predicted and observed concentrations is good for the major elements and Na\(_2\)O (see Table C1 below). This agreement is to be expected, because the parameterization was required to fit the composition of a primitive Hawaiian tholeiite. There is, however, only a negligible penalty for fitting the Hawaiian point. A simple model of dry melting of a peridotite source can therefore account for the observed concentrations of these oxides, and no melt from the mechanical boundary layer is required. It is less successful at predicting the concentrations of K\(_2\)O and TiO\(_2\), probably because the experimental concentrations are so variable. The predicted Ni content of the primitive Hawaiian tholeiite is 360 ppm. Figure 13 and Tables C1 and C2 show that a single parameterization is able to predict the compositions of both MORB and the primitive Hawaiian tholeiite. There is, therefore, no evidence that the source regions of MORB and the Hawaiian tholeiites have different major and minor element concentrations.

Figure 14 shows the REE predictions of our plume model. The values were calculated assuming Rayleigh melting and were summed in the same way as the major and minor element concentrations. The partition coefficients and proportions of minerals and REE content in the source region were taken from McKenzie & O'Nions (submitted). The source region was taken to have primitive REE concentrations. It was assumed that the source region is composed of spinel peridotite and garnet peridotite in equal proportions, because McKenzie & O'Nions showed that the stability field of spinel peridotite extends into the melt-producing region of our model. The partition coefficients used were, therefore, the average of the spinel and garnet peridotite values. The Kilauean tholeiite shown in Fig. 14
Fig. 13 (a–h)
has 14.36% MgO whereas the predicted melt from the plume contains 15.82% MgO. Some fractionation is, therefore, required to produce the Kilauean tholeiite in Fig. 14 from our primitive melt. This fractionation increases the REE concentrations by ~ 5%. Figure 14 also suggests that, as McKenzie & O’Nions suspected, their partition coefficients for Yb and Lu may be too high.

If the above calculation is repeated for Ti an enrichment of a factor of ~ 11 is predicted. Taking the concentration of TiO$_2$ in the source region that is given in Table C3, this enrichment factor predicts that the primitive Hawaiian tholeiite should contain 1.88% TiO$_2$.
This figure is in agreement with the Hawaiian data. Therefore, it appears that the parameterizations given in Appendix C cannot accurately predict the TiO$_2$ concentration.

We can now understand the relationship between the stages of volcanism and the structure of the plume. Loihi is about 60 km from Kilauea. It is currently erupting small volumes of lava of the preshield stage. The melt production rate and the degree of melting will both increase, building the main shield, as the volcano moves towards the axis of the plume. The shield stage typically lasts about 400 000 y, during which time the plate moves a distance of 40 km at 100 mm/y. The diameter of the region on the axis of the plume where the melt production is concentrated is about the same size. Mauna Kea is 54 km from Kilauea and is an example of a volcano that apparently finished erupting lavas of the postshield stage 4000 y ago (Frey et al., 1990). As the volcano passes from the shield stage to the postshield stage the degree of melting and melt production rate both decrease as the volcano moves away from the zone of high melt production. This behaviour is consistent with the arguments of Chen & Frey (1985) and Feigenson (1986). These models require a second source component to model the observed change in isotopic signature with time seen in some volcanoes. By this time plume material may have heated and mixed with the surrounding mantle material to produce the change to a MORB isotopic signature. The posterosional stage erupts ~160 km downstream of the tholeiite activity (Clague & Dalrymple, 1989), and, unlike the shield tholeiites, has a MORB isotopic signature. It is separated from the postshield stage by a time gap of a few million years and probably represents melts from the base of the mechanical boundary layer. By this time the base of the mechanical boundary layer will probably have heated up enough for the melting seen there in our model to have developed. The time gap is probably caused by the time required to extract such a small melt fraction from the base of the mechanical boundary layer (Gurriet, 1987). Figure 15 shows the relationship between the potential temperature structure, the melt production rate, the vertical velocity, and the observed crustal thickening in the Hawaiian Islands. It also shows the narrowness of the plume and how closely melt production and vertical velocity are related. One problem with these proposals is that the geoid anomaly and the stagnation streamline suggest that the Hawaiian Islands are situated.
Fig. 15. Summary figure showing the geometry of the plume relative to the Hawaiian Ridge. The profile across the islands is taken from Watts & ten Brink (1989). The vertical velocity profile (dotted line) is the vertical component of the velocity at the base of the figure.

~100 km southwest of the plume centre. Melt production is presumably concentrated beneath the islands, so the evidence for the location of the centre of the plume from melt production does not agree with that from the geoid and residual depth anomalies. Such inconsistencies are perhaps not surprising, because a stationary axisymmetric plume is unlikely to provide an accurate model of the circulation beneath a plate moving at 100 mm/y.

Plumes are found in many settings, and hence must exist beneath mechanical boundary layers of different thicknesses. Changes in the mechanical boundary layer thickness can be approximately modelled by changing the pressure in the melting region. This modelling is not correct in detail, because the temperature gradient must change when the mechanical boundary layer thickness is changed, but it should provide an indication of what will happen. If the mechanical boundary layer thickness is increased to 125 km (Fig. 16); all melt production from the plume stops. When the mechanical boundary layer is absent the melt production rate increases to 106 km³/y, or more than sufficient to generate the largest aseismic ridges. This rate of melt production is not sufficient to generate continental flood
basalt provinces: the Deccan Volcanic province requires a melt production rate of up to 10 km$^3$/y (White & McKenzie, 1989). Studies of the initiation of plumes in the laboratory and numerical studies of convection indicate that plumes are generally hotter when they first initiate than in steady state. Such flood basalt provinces are believed to be generated by rifting over a plume as it starts. The extra melt production associated with plume initiation is required to produce a high enough melt production rate. The area affected by continental flood basalts often has a diameter of \( \sim 2000 \) km but the diameter of the plume in our model is only 1200 km, so some phenomenon such as the spreading out under the mechanical boundary layer of the initial hot blob, which has a larger diameter than the main steady-state plume, is required. Our modelling therefore supports the conclusions of White & McKenzie.

The calculations illustrated in Fig. 16 also suggest an alternative explanation for the hotspot distribution found by Weinstein & Olsen (1989). They showed that the hotspot locations listed by four different groups tended to be near ridges, and argued that this behaviour occurred because plumes in the mantle are swept towards ridges by the large-scale flow. But Fig. 16 shows that the melt produced by the plume is strongly dependent on the thickness of the mechanical boundary layer. Therefore plumes beneath young plates near ridges will tend to be associated with larger aseismic ridges and volcanoes than those beneath older plates, and the correlation observed by Weinstein & Olsen may be an artefact. Such an explanation is also consistent with Houseman's (1983) numerical experiments, which showed that most ridges are moving too rapidly to trap plumes.

Table 2 shows that the composition is insensitive to the mechanical boundary layer thickness, whereas the melt production rate is very sensitive to it (Fig. 16). An increase in the mechanical boundary layer thickness, from 72 to 109 km, is enough to decrease the melt production rate to 0.015 km$^3$/y, a rate appropriate for hotspots such as the Cape Verde or Kerguelen. These hotspots are unlikely to produce less melt than Hawaii solely as a result of lower plume temperatures. Plumes with a central plume temperature of 1520°C, or only 38°C less than that of the Hawaiian plume, are unable to generate the geoid and residual depth anomalies associated with these hotspots, yet still have a melt production rate of
### Table 2

*Melt compositions, calculated with parameterization (a), and production rates (in km$^3$/y) for mechanical boundary layer thicknesses given in km*

<table>
<thead>
<tr>
<th>Thickness</th>
<th>SiO$_2$</th>
<th>TiO$_2$</th>
<th>Al$_2$O$_3$</th>
<th>FeO</th>
<th>MgO</th>
<th>CaO</th>
<th>Na$_2$O</th>
<th>K$_2$O</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>49.06</td>
<td>0.75</td>
<td>11.61</td>
<td>10.62</td>
<td>16.34</td>
<td>9.32</td>
<td>1.58</td>
<td>0.21</td>
<td>1.06</td>
</tr>
<tr>
<td>35</td>
<td>48.43</td>
<td>0.87</td>
<td>11.80</td>
<td>11.01</td>
<td>16.17</td>
<td>9.41</td>
<td>1.69</td>
<td>0.22</td>
<td>0.49</td>
</tr>
<tr>
<td>72</td>
<td>48.27</td>
<td>1.13</td>
<td>11.56</td>
<td>11.79</td>
<td>15.83</td>
<td>8.96</td>
<td>1.86</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td>109</td>
<td>49.88</td>
<td>1.64</td>
<td>10.82</td>
<td>12.20</td>
<td>14.69</td>
<td>8.15</td>
<td>2.11</td>
<td>0.20</td>
<td>0.015</td>
</tr>
</tbody>
</table>

~0.13 km$^3$/y. Plumes under older plates will produce less melt because $F$ will be smaller, which causes the mechanical boundary layer thickness to increase [equation (10)].

### Discussion

A stationary axisymmetric plume in the mantle with a viscosity of $9.5 \times 10^{15}$ m$^2$/s under a mechanical boundary layer of 72 km accounts for the principal features of the Hawaiian Islands and Swell. The modelling constrains the structure of the upper boundary layer of the convecting system relatively well, but places no constraints on the thickness of the convective layer. Though the viscosity is constant in the convecting part of the model, there is good evidence that a thin low-viscosity layer exists in the mantle at the base of the lithosphere. The viscosity determined for our model is the viscosity in this low-viscosity layer. Our estimate of $9.5 \times 10^{15}$ m$^2$/s for this viscosity compares well with Craig & McKenzie's (1986) estimate of $4 \times 10^{15}$ m$^2$/s from modelling of the geoid at fracture zones. Parsons & McKenzie (1978) and Houseman & McKenzie (1982) modelled the change in the age-depth relationship in the oceans when the lithosphere reaches an age of 70 Ma, and estimated that the average viscosity in the thermal boundary layer is $2 \times 10^{16}$ and $5 \times 10^{16}$ m$^2$/s respectively.

The models we use have reached steady state whereas the Hawaiian Swell has not. Therefore, we would not expect to see the conductive component of the uplift fully developed in the Hawaiian Swell. The mechanical boundary layer, however, contributes only ~300 m to the residual depth anomaly, or less than a quarter of the total observed anomaly.

The Introduction described the method used by Liu & Chase (1989) to model the geoid, bathymetry, and heatflow on the Hawaiian Swell. Their best-fitting model has a plume that is 250 ± 30 °C hotter than the background mantle, but, like us, they did not take account of the effect of melting on the geoid and residual depth. Our estimate of 278 °C is in agreement with theirs. Liu & Chase (in press) added melting to their earlier model and obtained a potential temperature of 1530 °C for the plume. They also estimated that melting starts at a depth of 130 km.

Because of the effects of the latent heat of fusion, melting causes the temperature to fall from the solid adiabat. Therefore the estimate of 1558 °C from the geoid and bathymetry for the potential temperature of the plume is a maximum estimate for the true potential temperature of the plume. The temperature drop caused by latent heat of melting is up to 85 °C, and is probably similar to the uncertainty resulting from errors in $\Delta S$. This temperature drop is included in the melting calculations.

Liu & Chase estimated that up to 30% melting occurs in the plume centre and that up to 20% melting occurs in the mechanical boundary layer (their lithosphere). In our model no melt is produced from the mechanical boundary layer. Liu & Chase used the same $X$-$T$ relationship as us and they assumed that the temperature in the plume increased with depth...
as 1/depth. The melting region extends to a depth of ~ 50 km in their model even though their initial lithosphere (our mechanical boundary layer) thickness is 90 km, whereas in ours the shallowest melt production is at 82 km. Melting extends to shallower depths in Liu & Chase’s model because the mechanical boundary layer is rapidly eroded and replaced by hot plume material whereas in our model there is no thermal erosion of the mechanical boundary layer. There is not enough time for the mechanical boundary layer to be heated by conduction as it passes over the plume axis.

The plume model in the section ‘Modelling the plume beneath Hawaii’ requires $2.4 \times 10^6$ km$^3$/Ma of mantle material to pass through the melting zone. The upper mantle can therefore support 100 plumes, with the melt production rate of the Hawaiian plume for > 1 Ga without reprocessing material that has passed through the melting region. The debate about whether mantle plumes have an upper or a lower mantle origin has been long and controversial. Isotopic data, such as the $^3$He/$^4$He data of Kurz et al. (1983), provide evidence for a primitive, undegassed component in the tholeiite source. Figure 14 shows that a primitive mantle source is required to produce the REE enrichments seen in Kilauean tholeiites. Though we believe that the lower mantle is the most likely source of this primitive component, our modelling adds nothing new to the debate about how this primitive material enters the plume.

The section on modelling the plume shows that no melt is produced from the mechanical boundary layer during the shield stage of volcanism. Using McKenzie & Bickle’s (1988) definition, the base of the lithosphere is at a depth of 92 km, or well within the thermal boundary layer. Therefore, a considerable volume of melt is produced from within the lithosphere, but the material that is melting has been transported by the plume and has not obtained its isotopic ratios by decay within the lithosphere. Even though the lithosphere is melting, no thermal erosion of the lithosphere has occurred! These contradictions confuse much of the previous discussion of the Hawaiian plume and its near-surface thermal structure: a confusion that results from the absence of any generally accepted definition of what is meant by the term ‘lithosphere’.

Figure 13 shows tholeiite data from Kilauea and Mauna Loa. The tholeiites generally range in MgO value from ~5-5% to ~20%. Lavas with more than ~7% MgO show olivine control, whereas those with <7% MgO show the effects of augite and plagioclase removal as well. Mauna Loa is distinguished from Kilauea by the presence of hypersthene (Wright & Clague, 1989). Glasses occur with up to ~10% MgO and the most magnesian olivine phenocrysts are in equilibrium with liquids of 13-14% MgO (Wright, 1984).

Hawaiian tholeiites differ from MORB tholeiite by having higher FeO, TiO$_2$, P$_2$O$_5$, and K$_2$O, and lower CaO and Al$_2$O$_3$, at a given MgO content. They also have higher amounts of incompatible elements and more LREE enrichment and higher initial $^{87}$Sr/$^{86}$Sr ratios (Wright, 1984). The generation of MORB is now fairly well understood (Klein & Langmuir, 1987; McKenzie & Bickle, 1988), so the differences between Hawaiian and MORB tholeiites can be used to discover how the processes generating them differ. Melts with the amount of LREE enrichment shown by the Hawaiian tholeiites can be produced by high degrees (>20%) of melting of an enriched source, or by small degrees (<10%) of melting of a source with a chondritic REE pattern (Frey & Roden, 1987). As a result, two contrasting views have emerged on the amount of melting involved in tholeiite generation. Budahn & Schmitt (1985) proposed that <5% melting is involved in tholeiite generation. If they are correct, the source is not required to be enriched relative to chondrites, in agreement with the Nd isotopic data. Models that propose that the tholeiites are generated by small degrees of melting, however, suffer from a significant volume problem: the large volumes of tholeiite generated appear to require an unreasonably large source region (Frey & Roden, 1987).
The opposite view was taken by Wright (1984). He proposed that the melting is relatively shallow (60–90 km), just below the deepest Hawaiian earthquakes. The source of the tholeiites must, therefore, be partially within the mechanical boundary layer, which has already partially melted to yield MORB tholeiites in his model. This source region undergoes metasomatism and then melts by 35–42% to produce primitive Hawaiian tholeiites with ~25% MgO. The metasomatism required involves the addition of 15–20% of a highly alkalic, incompatible element rich melt (30%) combined with amphibole (65%) and other minor components. Wright argued that these fluids ascend from the low-velocity zone. Clearly, the high degree of melting in this model solves the problem of the size of the source region, but, in so doing produces another problem: the source of the metasomatic component. This source must supply the metasomatic fluid at a rate sufficient to complete the metasomatism in the 85 Ma between formation of the oceanic lithosphere and the eruption of the Hawaiian volcanoes.

From studies of trace element patterns, Feigenson (1986) concluded that garnet and clinopyroxene are residual phases in the tholeiite source region. If the source region is a peridotite that has not been enriched, the degree of melting must be less than ~15%, provided that garnet is present in the residua. Similar, more detailed, calculation by McKenzie & O’Nions (submitted) suggested that a melt fraction of no more than 6-7% is required to account for the rare earth abundances.

Only a small fraction of a Hawaiian volcano is available for study. The shield stage is buried beneath later lavas in the majority of the volcanoes. Even Mauna Loa is beginning to show signs of the start of the transition to postshield volcanism in its lavas (McKenzie & O’Nions, submitted). Kilauea is likely to be the best guide to the shield stage, as data from the other candidate, Loihi, are limited. The lavas that are erupted are the end-products of a complex and well-studied differentiation process. The primitive tholeiite lava is rarely, if ever, sampled, and models such as ours can only predict compositions for the parental tholeiite that are compatible with the available data.

The internal structure and mechanics of eruption of Hawaiian volcanoes such as Kilauea have been extensively studied (Ryan, 1988). By the time it has ascended to 34 km the magma appears to have collected into a central conduit. It is then delivered to the magma chambers and surface by a complex network of dykes. Differentiation occurs at many stages in the above process and eruption often appears to be controlled by density considerations. The processes by which melt separating from the plume is collected into conduits remains unclear. During transport from the melting region, processes such as those envisaged by Navon & Stolper (1987) are likely to be important.

Our model falls in the low degree of melting category, with an average of 6-6% melting. It is consistent with the work of Budahn & Schmitt (1985) and of Feigenson (1986), rather than with that of Wright (1984), but it does not suffer from the problem of requiring an unreasonably large source region (Frey & Roden, 1987). The melt-producing region has a maximum vertical extent of ~55 km and a maximum radial extent of ~132 km with the melt production concentrated in a region of ~30 km in height and ~40 km in diameter on the plume axis, but is able to produce the required melt volume because the calculated vertical velocity on the plume axis is up to 340 mm/y.

CONCLUSIONS

The model in Fig. 15 is consistent with both the petrological and the geophysical data from the Hawaiian Swell. It produces the observed volumes and major element compositions of the melts that built the Hawaiian Ridge. The average degree of melting in the plume is 6-6%, and melting is restricted to the thermal boundary layer and convecting mantle. A
small degree of melting is able to generate the volumes of melt required to produce the Hawaiian Ridge because the velocities on the axis of the plume are large: up to 340 mm/y on the axis of the plume. Melt production is concentrated on the axis of the plume in the deeper parts of the melting region. This region has a vertical extent of ~30 km and a diameter of ~40 km. We have produced a parameterization of melt compositions from experiments that is consistent with the laboratory experiments, with models for the generation of MORB and with primitive Hawaiian tholeiites containing ~16% MgO. There is no evidence to suggest that the major and minor element concentrations in the source regions of Hawaiian tholeiites differ from those in the source region of MORB. The model predicts REE contents for the primitive Hawaiian tholeiite that are consistent with Kilauean tholeiites if the source region has primitive REE concentrations. The rate of melt production from the plume depends on the plume temperature and, in particular, the thickness of the mechanical boundary layer. When the mechanical boundary layer is absent the model can produce enough melt to generate the largest observed aseismic ridges, yet a relatively modest increase in mechanical boundary layer thickness, from 72 to 125 km, is sufficient to stop all melt production.

ACKNOWLEDGEMENTS

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REFERENCES


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**APPENDIX A: CONVECTION**

The convection code used in this study is that used by Jones *et al.* (1976). Courtney (1985) added a conducting lid and modified the code to allow the Prandtl number to be infinite. In a Boussinesq fluid, the velocity, u, and density, ρ, satisfy

\[ \nabla \cdot u = 0 \]  \hspace{1cm} (A1)

\[ \rho = \rho_0 [1 - \alpha(T - T_0)] \]  \hspace{1cm} (A2)
where $\alpha$ is the coefficient of thermal expansion, $T$ is the temperature, and $\rho_0$ is the density at $T = T_0$. The vorticity, $\omega = \nabla \times u$, satisfies

$$\frac{\partial \omega}{\partial t} = \nabla \times (u \times \omega) - \alpha \nabla T \times g + \nu \nabla^2 \omega$$

(A3)

where $g$ is the acceleration due to gravity and $\nu$ is the kinematic viscosity.

Conservation of energy requires that

$$\frac{\partial T}{\partial t} = -\nabla \cdot (Tu) + \kappa \nabla^2 T$$

(A4)

where $\kappa$ is the thermal diffusivity.

The equations are solved in cylindrical polar coordinates using a Stokes stream function $\psi$ such that

$$u = \frac{1}{r} \left( \frac{\partial \psi}{\partial z}, \frac{\partial \psi}{\partial r} \right)$$

(A5)

It is convenient to rewrite the equations in terms of the potential vorticity, $\Omega$, where

$$\Omega = \omega \left( \frac{a}{r} \right)$$

(A6)

where $a$ is the layer depth, and to use dimensionless variables

$$r, z = a(r', z')$$

$$t = \frac{\alpha^2 t'}{\kappa}$$

$$u = \frac{\kappa u'}{a}$$

$$T = \frac{FaT'}{k}$$

(A7) (A8) (A9) (A10)

where $k$ is the thermal conductivity and $F$ is the basal heat flux.

Dropping the primes, the equations governing the convection are

$$\nabla \cdot \left( \frac{1}{r^2} \nabla \psi \right) = -\Omega$$

(A11)

$$\nabla \left[ \frac{1}{r^2} \nabla (r^2 \Omega) \right] = Ra \frac{1}{r} \frac{\partial T}{\partial r}$$

(A12)

$$\frac{\partial T}{\partial t} + \nabla \cdot (uT) = \nabla^2 T$$

(A13)

where

$$Ra = \frac{g \alpha Fa^4}{\kappa \nu}$$

(A14)

is the Rayleigh number.

The axisymmetric convecting cell is capped by a solid conducting lid in which $\psi = 0$ and the temperature is governed by

$$\frac{\partial T}{\partial t} = \nabla^2 T.$$  

(A15)

For simplicity, the boundaries of the convecting region are assumed to be stress free, so the boundary conditions are (see also Fig. 1)
\[
\psi = 0 \quad z = 0, 1
\]
\[
\psi = 0 \quad r = 0, \gamma
\]
\[
\Omega = 0 \quad z = 0, 1
\]
\[
\Omega = 0 \quad r = 0, \gamma
\]
\[
\frac{\partial T}{\partial r} = 0 \quad r = 0, \gamma
\]
\[
T = -1 \quad z = 0
\]
\[
T = -\lambda \quad z = 1 + \lambda
\]

where the outer edge of the cell is at \( r = \gamma \).

These equations were solved using a second-order finite difference scheme on a staggered mesh, properly centred in time and space (Jones et al., 1976), with equations (A13) and (A15) expressed in conservative form. The timestep was required to satisfy either

\[
\Delta t < \frac{h}{|u|_{\infty}}
\]  

(A16)

where \(|u|_{\infty}\) is the maximum velocity of the fluid at the previous time level and \( h \) is the spacing between gridpoints, or

\[
\Delta t < 2h^2
\]  

(A17)

whichever was smaller.

**APPENDIX B: MELT PRODUCTION RATE**

The melt production rate, \( \Gamma \), can be expressed as

\[
\Gamma \equiv \frac{dX}{dt} = \left( \frac{\partial X}{\partial z} \right)_t + \left( \frac{\partial X}{\partial \rho} \right)_t = w \left( \frac{\partial X}{\partial z} \right)_t + u \left( \frac{\partial X}{\partial r} \right)_t
\]  

(B1)

where \( w \) and \( u \) are the vertical and radial velocities that may be calculated from the stream function (Appendix A). The partial differentials in (B1) can be expressed as

\[
\left( \frac{\partial X}{\partial z} \right)_r = \left( \frac{\partial X}{\partial \rho} \right)_T \left( \frac{\partial P}{\partial \rho} \right)_T \left( \frac{\partial T}{\partial \rho} \right)_T + \left( \frac{\partial X}{\partial \rho} \right)_T \left( \frac{\partial T}{\partial z} \right)_T
\]  

(B2)

where

\[
\left( \frac{\partial P}{\partial z} \right)_r = -\rho_0 g
\]  

(B3)

and \( \rho_0 \) is the density and \( g \) is the acceleration due to gravity, and, as

\[
\left( \frac{\partial P}{\partial r} \right)_z = 0
\]  

(B4)

\[
\left( \frac{\partial X}{\partial r} \right)_z = \left( \frac{\partial X}{\partial \rho} \right)_T \left( \frac{\partial T}{\partial \rho} \right)_T
\]  

(B5)

McKenzie & Bickle (1988) showed that \( X = X(T^*) \) to a good approximation, where

\[
T^* = \frac{T - (T_s + T_l)/2}{T_l - T_s}
\]  

(B6)

where \( T_s(P) \) is the solidus temperature and \( T_l(P) \) the liquidus temperature. The partial differentials in (B2) and (B5) can then be written as

\[
\left( \frac{\partial X}{\partial \rho} \right)_T = \frac{dX}{dT^*} \left( \frac{\partial T^*}{\partial \rho} \right)_T
\]  

(B7)
\[
\frac{\partial X}{\partial T} = \frac{dX}{dT^*} \left( \frac{\partial T^*}{\partial T} \right)_p.
\]  
(B8)

Combining (B1), (B2), (B5), (B7), and (B8) gives the instantaneous melt production rate at a point,

\[
\Gamma = \frac{dX}{dT^*} \left[ -w_{pD} \left( \frac{\partial T^*}{\partial P} \right)_T + u \cdot \nabla T \left( \frac{\partial T^*}{\partial T} \right)_p \right].
\]  
(B9)

In (B9), \(\partial T^*/\partial P\) and \(\partial T^*/\partial T\)_p are calculated from McKenzie & Bickle's expressions.

**APPENDIX C: PARAMETERIZATION OF MELT COMPOSITIONS**

That a different parameterization from the one proposed by McKenzie & Bickle is required is clearly illustrated in Table C1 and Fig. 14, which show the calculated composition from their parameterization C and our estimate of the Hawaiian primitive melt. The predicted concentrations of \(\text{Al}_2\text{O}_3\) and \(\text{Na}_2\text{O}\) are too large and the predicted \(\text{SiO}_2\) concentration is too small. Such a failure is scarcely surprising. As McKenzie & Bickle pointed out, there are few experimental constraints on the composition of small melt fractions generated at high pressures. We therefore decided to use the estimated composition of the primitive magma that produced the Hawaiian tholeiites as a constraint on the parameterization, and to discover whether this constraint was compatible with the laboratory experiments by calculating the resultant increase in misfit.

We parameterized the instantaneous melt composition, \(c\), using

\[
c(X) = a + b(1 - X)^{1 - D/D}
\]  
(C1)

where \(X\) is the melt fraction and \(D\) is the partition coefficient. The melting region, \(0 < X < 1\), was divided into three regions, \(0 < X < X_1\), \(X_1 < X < X_2\), and \(X_2 < X < 1\), to provide enough adjustable parameters to fit the data. The point average composition, \(C\), is related to \(c\) by

\[
c = \frac{d}{dX}(XC)
\]  
(C2)

so

\[
C = a + \frac{bD}{X} [1 - (1 - X)^{1/P}] + \frac{d}{X}
\]  
(C3)

\(C(X)\), but not \(c(X)\), was required to be continuous at \(X_1\) and \(X_2\). As \(C(0)\) is finite, \(d\) must be zero in the first interval for all oxides. In all parameterizations all oxides other than \(\text{SiO}_2\), \(\text{TiO}_2\), \(\text{Al}_2\text{O}_3\), \(\text{FeO}\), \(\text{MgO}\), \(\text{CaO}\), \(\text{Na}_2\text{O}\), and \(\text{K}_2\text{O}\) were lumped together as 'the rest'. \(\text{SiO}_2\) was calculated by requiring that the sum of all the concentrations should equal 100%.

We first attempted to calculate \(\text{FeO}\), \(\text{MgO}\), and 'the rest' in the same way as did McKenzie & Bickle, and allowed \(X_1\) for all other elements to vary linearly with pressure. We found that \(a\) for all elements and \(D\) for \(\text{K}_2\text{O}\) were poorly constrained, and therefore set \(a\) to be zero for all elements and \(D\) for \(\text{K}_2\text{O}\) to be 0.5. For \(\text{TiO}_2\), \(\text{Al}_2\text{O}_3\), and \(\text{K}_2\text{O}\), \(X_1\) and its pressure dependence were also poorly constrained, and therefore a single value of \(X_1\) and \(dX_1/dP\) was required to fit \(\text{Al}_2\text{O}_3\) and \(\text{Na}_2\text{O}\), and another to fit \(\text{TiO}_2\), \(\text{CaO}\), and \(\text{K}_2\text{O}\).

The experimental data are all taken from batch melting experiments, whereas the processes we are trying to model are more likely to be Rayleigh melting processes. Equation (C1) is a Rayleigh melting equation. As the parameterization was unable to fit the \(\text{TiO}_2\) concentration of the Hawaiian data point, the \(\text{TiO}_2\) and \(\text{Na}_2\text{O}\) concentrations were calculated using the expressions for batch melting in the experimental data but as Rayleigh melting for Hawaii. This change did not improve the fit to the Hawaiian \(\text{TiO}_2\) concentration.

With the exception of the \(\text{TiO}_2\) concentrations, this parameterization successfully reproduced the experimental batch melt and the Hawaiian melt compositions, but did not produce nepheline normative melts at low melt fractions and low pressures. Nepheline normative melts are common in fracture zones, and 10% of MORB glasses with \(\text{MgO}>9\%\) are nepheline normative. Klein & Langmuir (1987) also found that the \(\text{Na}_2\text{O}\) concentration in the melts increased as the melt fraction decreased. McKenzie & Bickle's parameterization produced such melts at low melt fractions, and we wished to retain this behaviour in our new parameterization. The value of \(X_1\) was therefore required to
FIG. C1. Projection from diopside onto the Ol-Pl-Qz plane of Walker et al.'s (1979) tetrahedron. The crosses represent glass compositions collected by Elthon (1987) with MgO > 9%. The curves are point and depth averages calculated from parameterization (a) shown in Table C3, the small open circles mark melt fractions of 0.05, 0.15, 0.245, and 0.35 and the dots inside large circles mark point and depth averages. The ellipse shows the projection of the 2σ eight-dimensional ellipse of the error of the electron probe analysis.

FIG. C2. (a) Calculated curves for parameterization (a) marked with pressures in GPa projected from Pl (Walker et al., 1979). The dashed line marks 'cpx-out' from Stolper (1980). (b) Plot taken from Stolper (1980) for comparison.

be constant at pressures of <1.5 GPa, and $dX_1/dP$ to be different for $\text{Al}_2\text{O}_3$ and $\text{Na}_2\text{O}$. The values of $D$ for $\text{Al}_2\text{O}_3$, $\text{CaO}$, and $\text{Na}_2\text{O}$ were then poorly constrained, and were fixed to be 0.5, 0.8, and 0.2 respectively. It was also necessary to fix $\text{CaO}/\text{Al}_2\text{O}_3$: if $\text{CaO}/\text{Al}_2\text{O}_3$ was not constrained, the fitting process produced an unrealistically large value. The minimization then gave a value of 0.806 for $\text{CaO}/\text{Al}_2\text{O}_3$, but the CaO concentrations predicted by the parameterization were still too small to fit the MORB and Hawaiian observations. This behaviour seemed to result from the data of Takahashi &
Kushiro (1983) which have unusually low CaO concentrations. The CaO/Al₂O₃ ratio was therefore increased to 0.8436, a value consistent with that of 0.847 ± 0.125 from chondritic meteorites. The parameterization then reproduced the observed CaO concentrations in MORB glasses and the fit to the other experiments was little changed.

The final constraint imposed was that

$$K_B = \frac{(\text{MgO/FeO})_s}{(\text{MgO/FeO})_l}$$

where the subscripts s and l refer to solid and liquid, should be constant at all melt fractions less than $X = X_2$ and at all pressures. This condition was not imposed by McKenzie & Bickle, and $K_B$ calculated from their expressions varies between 0.13 at $X = 0$, $P = 0$ and 0.41 at $X = 0.245$, $P = 4$ GPa. When (C4) was not applied the parameterization produced values of $K_B$ which varied from 0.16 at $X = 0$, $P = 0$ to 0.45 at $X = 0.28$, $P = 4$ GPa. The value of $K_B$ was calculated by fitting the experiments and the Hawaiian composition. Takahashi & Kushiro (1983) suggested that $K_B$ depended on pressure; but when we allowed for a linear dependence of $K_B$ on pressure, the value of $dK_B/dP$ so obtained was negligible. We therefore required $X_5$ to be constant and independent of pressure for $X < X_2$. Though some of the experimental charges probably underwent iron loss to the platinum capsules, this process will only affect the value of $K_B$ if the charges were in chemical disequilibrium at the end of the run.

The function that was minimized was

$$F = (1 - \alpha) \sum_{i=1}^{N} \frac{W_i}{N} \sum_{j=1}^{M} h_{ij} + \alpha \sum_{i=1}^{N} m_{i} \sum_{j=1}^{M} e_{ij}$$

where $\alpha$ is a constant between 0 and 1, $N$ and $M$ refer to the number of concentrations used from experiments, $e_{ij}$ and Hawaii, $h_{ij}$, respectively, the subscript $i$ is 1 for major elements and ‘the rest’, 2 for TiO₂, 3 for Na₂O, and 4 for K₂O, $W_i$ are weighting factors, and $e$ and $h$ are the absolute values of the misfit between the observed and calculated compositions. We used the same weights, $W_1 = 20, W_2 = 10, W_3 = 30$, as did McKenzie & Bickle. When the Hawaiian values were excluded from the fitting, by using $\alpha = 0$, the mean value of the misfit was 0.90% for the major elements and 0.28% for the minor elements.

### Table C1

Estimated composition of the primitive Hawaiian tholeiite, used to constrain the parameterizations, the composition predicted using McKenzie & Bickle’s (1988) parameterization C, and that predicted by our parameterizations

<table>
<thead>
<tr>
<th></th>
<th>SiO₂</th>
<th>TiO₂</th>
<th>Al₂O₃</th>
<th>FeO</th>
<th>MgO</th>
<th>CaO</th>
<th>Na₂O</th>
<th>K₂O</th>
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<tr>
<td>Hawaiian Point</td>
<td>48.5</td>
<td>1.8</td>
<td>11.2</td>
<td>11.5</td>
<td>160</td>
<td>8.6</td>
<td>1.8</td>
<td>0.35</td>
</tr>
<tr>
<td>McKenzie &amp; Bickle</td>
<td>42.62</td>
<td>1.09</td>
<td>13.44</td>
<td>11.21</td>
<td>19.79</td>
<td>8.85</td>
<td>2.39</td>
<td>0.26</td>
</tr>
<tr>
<td>Predicted</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(a)</td>
<td>48.27</td>
<td>1.13</td>
<td>11.56</td>
<td>11.79</td>
<td>15.83</td>
<td>8.96</td>
<td>1.86</td>
<td>0.21</td>
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<tr>
<td>(b)</td>
<td>48.38</td>
<td>1.19</td>
<td>11.20</td>
<td>11.75</td>
<td>15.77</td>
<td>9.04</td>
<td>1.91</td>
<td>0.34</td>
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### Table C2

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<th>MgO</th>
<th>CaO</th>
<th>Na₂O</th>
<th>K₂O</th>
<th>Total</th>
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<tbody>
<tr>
<td>McKenzie &amp; Bickle, 1280°C</td>
<td>51.89</td>
<td>0.92</td>
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<td>8.53</td>
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<td>11.01</td>
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<tr>
<td>(a)</td>
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<td>11.41</td>
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<tr>
<td>(b)</td>
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<td>0.97</td>
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<td>11.20</td>
<td>2.27</td>
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</tr>
<tr>
<td>SAVE (Pallister, 1984)</td>
<td>51.1</td>
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<td>9.2</td>
<td>12.8</td>
<td>2.3</td>
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<td>17.27</td>
<td>9.04</td>
<td>1.63</td>
<td>0.21</td>
<td>99.59</td>
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<tr>
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<td>9.19</td>
<td>1.60</td>
<td>0.35</td>
<td>99.60</td>
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<td>Clarke (1970), no. 3</td>
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<td>0.76</td>
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Table C3

Parameters used in the parameterization

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'Misfit' is the misfit to all the experiments when α = 0.05 and 'misfit 1' is the misfit before the concentration of CaO was increased by 5%. In both parameterizations C(1) for CaO determined by the minimization is 2.66. Our preferred values are labelled (a). Values labelled (a): the experimental database of McKenzie & Bickle (1988) was used. Values labelled (b): these include the data of Falloon & Green (1988) and Falloon et al. (1988). Experimental data taken from Arndt (1977), Bickle et al. (1977), Green et al. (1979), Jaques & Green (1980), Stolper (1980), Takahashi & Kushiro (1983). Fujii & Scarfe (1985), Takahashi (1986), Falloon & Green (1988), and Falloon et al. (1988).
If $\alpha = 0.05$ the values were 0.92 and 0.28%, before the CaO/Al$_2$O$_3$ ratio was increased to 0.847. After this increase the major element misfit was 0.96% for $\alpha = 0.05$.

We made two determinations of the parameters. The first, (a), used the experimental database of McKenzie & Bickle (1988) and the second, (b), also used the results of Falloon & Green (1988) and Falloon et al. (1988). The results of parameterization (a) are used in all the figures and the errors associated with it are given above.

Walter & Presnall (1990) found that, at constant melt fraction, as pressure increases the concentrations of Na$_2$O and Al$_2$O$_3$ increase in the melt and the residual cpx becomes more aluminous. This behaviour results from the increased stability of the jadeitic component of the cpx at the high pressures, and is consistent with the decrease in the value of $X_1$ for Na$_2$O and Al$_2$O$_3$ at high pressures in the parameterization.

Figure C1 shows curves for $T_p = 1280, 1380, 1480$, and 1580°C, calculated using the new parameterization (a) and projected from diopside, together with MORB glass compositions with MgO > 9%. Figure C2 shows $X = X_1$ (CaO), corresponding to 'cpx-out', calculated with the new parameterization (a), together with Stolper's (1980) estimates. Table C2 shows the point and depth average for $T_p = 1280°C$ and 1480°C for the new parameterizations and for McKenzie & Bickle's together with Pallister's (1984) estimate of the mean composition of the Oman ophiolite (SAVE) and Clarke's (1970) measurements of the major element concentrations in the picritic basalts from Baffin Bay. These figures and tables show that the calculated compositions from the new parameterizations are in agreement with the observations and differ little from McKenzie & Bickle's except when $X \leq 0.1$ and $P > 2$ GPa. The position of the 'cpx-out' curve in Fig. C2 agrees less well with Stolper's figure (though not with his observed compositions) than does that of McKenzie & Bickle, probably because of solid solution of cpx in opx in Stolper's experiments. This effect will cause 'cpx-out' to occur at melt fractions smaller than $X_1$(CaO). The constants used in the new parameterizations are listed in Table C3.