An analytic model for the strong-/weak-shock transition in a spherical blast wave

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ABSTRACT
The solution of a non-radiative, strong, spherical blast wave has been derived from an approximate model of a low-density hot bubble which drives a thin shell (which contains most of the displaced environmental material). We show that this model can be extended to the case in which the blast wave is no longer strong, leading to a full analytic solution. We then compare this analytic model with a numerical solution of the full spherically symmetric Euler equations. Finally, we discuss possible applications of the analytic model to supernovae that explode within pre-existing hot bubbles, or inside dense molecular clouds.

Key words: hydrodynamics – ISM: evolution – ISM: kinematics and dynamics – ISM: supernova remnants.

1 INTRODUCTION
Supernova (SN) explosions typically have three distinct phases: an initial ‘ejectum-dominated’ phase, which is followed by the Taylor–Sedov phase, and by a later, radiative phase (in which the outer shock becomes radiative). For example, Truelove & McKee (1999) studied (both analytically and numerically) the transition between the ‘ejectum-dominated’ and the Taylor–Sedov phases. The radiative phase has been studied analytically by Blinnikov, Imshennik & Utrobin (1982) and numerically by Falle (1981). This paper focuses on the Taylor–Sedov phase.

The problem of a non-radiative blast wave generated by a point explosion was first studied theoretically by Taylor (1946, 1950) and by Sedov (1959, who found the full, self-similar analytic solution). The self-similar solution explored by these authors applies to a blast wave in the ‘strong-shock’ regime (in which the shock wave moves outwards at a highly supersonic velocity).

A simplified model, in which it is assumed that the swept-up environmental material is piled up into a ‘thin shell’ (pushed out by an inner low-mass hot bubble), was proposed by Chernyi (1957). This method is described in detail in Zel’dovich & Raizer (1967) and leads to an expansion law which is very similar to the one of Sedov (1959). In this ‘thin-shell model’, it is also assumed that the expansion drives a strong shock.

In this paper, we show that this ‘thin-shell model’ can be generalized to the case in which the outer shock has a transition from a strong-shock (in the early evolution of the expansion) to a weak-shock regime (in the later evolution). This generalization is done with the ‘thick-shell’ prescription which we have previously used to model the expansion of an H II region (Raga, Cantó & Rodríguez 2012a) and of a wind-driven bubble (Raga, Cantó & Rodríguez 2012b).

We find that the resulting model has a full analytic solution, which at short evolutionary times coincides with the solution of Chernyi (1957), and at longer evolutionary times has a weak shock which moves out sonically from the centre of the explosion.

It is somewhat surprising that this solution appears to have been missed in the substantial literature on the subject of blast waves driven by explosions (see, e.g., the review of Dewey 2010). In the astrophysical context, Tang & Wang (2005) presented numerical simulations and an analytic fit for SN explosions within pre-existing hot bubbles, which do have the strong-/weak-shock transition (and at the same time remain non-radiative). These authors discuss the literature on analytic models of explosions and conclude that there apparently has been no work on weak-shock blast waves.

Additionally to the literature discussed by Tang & Wang (2005), we have found the interesting paper of Deb Ray (1957), who presents an analytic, self-similar solution valid for the case of an explosion in an environment with a spherically symmetric, $R^{-3}$ density stratification (where $R$ is the spherical radius measured from the initial point explosion). This solution (with a constant expansion velocity) is valid both for the strong- and the weak-shock cases, and is relevant for astrophysical situations with an appropriately stratified density structure.

This work, however, as far as we can tell gives the first analytic model of an explosion driving an initially strong-shock wave, which becomes weak at later evolutionary times. The paper is organized as follows. In Section 2, we derive the model equations (Section 2.1) and derive the strong-shock and general analytic solutions for the
hot bubble (Section 2.2) and for the outer shock (Section 2.3). In Section 3, we compare the analytic solution with a 1D, spherically symmetric numerical solution of the full Euler equations. Finally, in Section 4, we discuss the possible astrophysical applications of our solution.

2 THE ANALYTIC MODEL

2.1 The model equation

In order to derive an analytic model for an explosion in an environment with a non-negligible pressure, we use the ‘expanding shell’ model described in Zel’dovich & Raizer (1967) and, in the astrophysical context, in Dyson & Williams (1980).

In this model, one assumes that a hot, low-mass bubble pushes out the surrounding uniform environment into an outwardly moving shell. The initial energy $E$ of the explosion is divided between thermal energy of the hot bubble and kinetic energy of the shell, so that at any time in the expansion we have

$$E = \frac{P V}{\gamma - 1} + \frac{1}{2} M_s v_s^2,$$

where $\gamma$ is the specific heat ratio, $P$ and $V$ are the pressure and volume (respectively) of the bubble (at an arbitrary time) and $M_s$ and $v_s$ are the mass and velocity (respectively) of the swept-up shell.

Following the classical derivation (see Dyson & Williams 1980), we use the estimates

$$P \approx \frac{2}{\gamma + 1} \rho_0 v_s^2,$$

$$v_s \approx \frac{\gamma + 1}{2} v_s,$$  

$$M_s \approx V \rho_0,$$

where $R$ is the outer radius of the bubble (so that the volume of the bubble is $V = 4\pi R^3/3$), $M_s$ the shell mass, $v_s = R$ the shell velocity, $\gamma$ the specific heat ratio and $\rho_0$ the density of the uniform, undisturbed environment. In principle, the specific heat ratio $\gamma$ for the swept-up environment (equations 2–3) could be different from the specific heat ratio of the gas within the hot bubble (equation 1). Equation (2) is the strong-shock jump relation and equation (3) gives the velocity $v_s$ of a strong shock pushed out by a plane piston of velocity $v_s$. Equation (4) gives the shell mass assuming that the shell is thin, and that it incorporates all of the environmental material swept up by the expansion of the hot bubble.

Combining equations (1)–(4), we obtain an energy conservation equation of the form

$$E = \Gamma R^3 P,$$  

with

$$\Gamma = \frac{8\pi}{3} \frac{\gamma}{\gamma - 1}$$

for a bubble of uniform pressure $P$ and radius $R$. Equation (5) indicates that a constant fraction $(\gamma + 1)/2\gamma$ of the initial energy $E$ remains as thermal energy of the hot bubble at all evolutionary times [also, a constant fraction $(\gamma - 1)/2\gamma$ of this energy is in the form of shell kinetic energy; see equation 1).

We will now assume that this energy distribution also holds for the regime in which the outer shock is no longer strong. This is of course not correct, but as $\Gamma$ (see equation 6) appears only as $\Gamma^{1/3}$ in the final solution, the assumption of a constant value for $\Gamma$ is not likely to introduce large errors.

We now consider the general (i.e. no longer ‘strong’) shock jump relations:

$$P_1 = \frac{2}{\gamma + 1} \rho_0 v_s^2 - \frac{\gamma - 1}{\gamma + 1} P_0,$$  

$$v_1 = \frac{\gamma - 1}{\gamma + 1} v_s + \frac{2}{\gamma + 1} v_s,$$

where $c_0$ is the adiabatic sound speed, $P_0 = \rho_0 c_0^2/\gamma$ the pressure of the undisturbed environment, $P_1$ the post-shock pressure and $v_1$ the post-shock velocity (in the shock reference system).

If we assume that the hot bubble acts like a plane piston (i.e. the shell is not very thick), the relation between the shell velocity $v_s$ and $R$ is equal to the radius of the hot bubble, see above) and the shock velocity $v_s = v_s + v_1$. Combining this relation with equation (8), we obtain the ‘piston relation’

$$\frac{R}{c_0} = \frac{2}{\gamma + 1} \left( \frac{v_s}{c_0} - \frac{c_0}{v_s} \right).$$

If we now assume that the post-shock pressure $P_1$ is equal to the pressure $P$ of the hot bubble, combining equations (5) and (7) we obtain

$$\left( \frac{v_s}{c_0} \right)^2 = \frac{1}{c_0} R^2 = \frac{1}{2\gamma} \left[ (\gamma + 1) \left( \frac{R}{c_0} \right)^3 + \gamma - 1 \right],$$

with

$$R_i = \left( \frac{\gamma E}{\Gamma \rho_0 c_0^4} \right)^{1/3}. $$

In equation (10), $R_i$ is the (time-dependent) radius of the outer shock. It is clear that if we set $R = R_i$ (where $R$ is the radius of the hot bubble, see above), from equation (10) we obtain $v_s = c_0$, and inserting this result in (9) we obtain $R = 0$. This is the regime of large evolutionary times, in which the hot bubble attains a final radius $R_t$, and the (weak) outer shock travels sonically into the surrounding environment.

2.2 Solutions for the bubble radius

Equations (9) and (10) can be straightforwardly combined to obtain a differential equation for the radius $R$ of the hot bubble:

$$\frac{1}{c_0} \frac{dR}{dr} = \frac{2}{\gamma + 1} \left[ \left( \frac{\gamma + 1}{2\gamma} \right)^{1/3} + \frac{\gamma - 1}{2\gamma} \right] \left[ \left( \frac{\gamma + 1}{2\gamma} \right)^{1/3} + \frac{\gamma - 1}{2\gamma} \right].$$

Using the boundary condition $R(t = 0) = 0$, the solution to this equation is

$$\frac{c_0 \gamma^{1/3}}{R_t} \left( \frac{2}{\gamma + 1} \right)^{4/3} t = \frac{2}{5} \frac{\gamma^{1/2}}{\gamma - 1} \left[ \frac{\gamma - 1}{2\gamma} \right]^{1/3} F_i \left[ \frac{\gamma - 1}{2\gamma}, 1, 1 - a^2, (1 - a)^2 \right],$$

where $F_i$ is the Appell hypergeometric function of two variables,

$$R = \left( \frac{2\gamma}{\gamma + 1} \right)^{1/3} \frac{R_t}{R}.$$


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with $R_f$ given by equation (11) and
\[ a = \frac{\gamma - 1}{2\gamma}. \] (15)

For the $\gamma = 1$ case, equation (12) has the simpler integral
\[ \frac{c_0}{R_f} t = \frac{1}{6} 2\sqrt{3} \tan^{-1} \left( \frac{\sqrt{3}r}{r - 1} \right) \]
\[ + \ln \left[ \frac{(\sqrt{7} + 1)^2 (1 + \sqrt{7} + r)}{(\sqrt{7} - 1)^2 (1 - \sqrt{7} + r)} \right], \] (16)
with $r$ given by equation (14).

The radius $R$ of the hot bubble as a function of time $t$ (actually computed as $t$ versus $R$ from equations 13 and 16) is shown for $\gamma = 5/3, 7/5$ and 1 in Fig. 1. It is clear from this figure that (if plotted in the correct dimensionless form) the solutions for the different values of $\gamma$ differ by at most a few per cent. Therefore, for an arbitrary $\gamma$ (within the $5/3 \rightarrow 1$ range shown in Fig. 1), it is a good approximation to compute the time evolution of the hot bubble through
\[ t \approx \frac{\gamma + 1}{2} t_1(R), \] (17)
with $r$ given as a function of the dimensional radius $R$ of the bubble by equation (14) and $t_1(R)$ obtained through equation (16). With this approximation, the more complex evaluation of the Appell hypergeometric function of two variables (equation 13) is avoided.

We end by pointing out that if one takes the $R \ll R_f$ limit in equation (12), the resulting differential equation can be straightforwardly integrated to obtain
\[ \frac{R_{ts}}{R_f} = \left[ \frac{25}{2\gamma(\gamma + 1)} \right]^{1/5} \left( \frac{c_0 t}{R_f} \right)^{2/5}, \] (18)
which corresponds to the ‘strong-shock’ Taylor–Sedov solution in which the hot bubble expands as time to the $2/5$ power. It can be shown that equations (13) and (16) coincide with equation (18) for $R \ll R_f$.

### 2.3 Solution for the shock radius

In order to find the radius $R_c$ of the outer shock as a function of time, we combine equations (9) and (10) to obtain
\[ \frac{dR_c}{dR} = \frac{1 + \alpha (R/R_c)^{\gamma}}{(1 - \alpha)(1 - (R/R_c)^{\gamma})}. \] (19)
where $R$ is the radius of the hot bubble (given as a function of time by equations 13–17), $R_f$ is given by equation (11) and
\[ \alpha = \frac{\gamma - 1}{\gamma + 1}. \] (20)

This equation can be directly integrated to obtain
\[ \frac{R_c}{R_f} = \frac{(\gamma - 1)}{2} \left( \frac{R}{R_f} \right) + \frac{\gamma}{\sqrt{3}} \tan^{-1} \left( \frac{\sqrt{3}R/R_c}{2 + R/R_c} \right) \]
\[ + \frac{\gamma}{6} \ln \left[ \frac{1 + R/R_c + (R/R_c)^{2\gamma}}{(1 - (R/R_c)^{2\gamma})} \right]. \] (21)

We have taken $\gamma = 7/5, 5/3$ and 1 $R(t)$ solutions of equation (13) shown in Fig. 1, and used equation (21) to compute the corresponding $R_c(t)$ shock radii. The obtained results are shown in Fig. 2.

Finally, we note that in the strong shock, $R \ll R_f$ limit, both equations (19) and (21) give
\[ \frac{R_c}{R_f} = \frac{(\gamma + 1)}{2} \frac{R}{R_f}. \] (22)

Therefore, using equation (18), we recover the Taylor–Sedov solution in which the shock wave radius grows as $t^{2/5}$.

### 3 Gasdynamic Simulation

In order to evaluate the accuracy of the analytic solution derived above (see Section 2), we have computed a numerical integration...
of the spherically symmetric, Lagrangian gasdynamic equations in the form
\begin{equation}
\frac{\partial}{\partial t} \left( \frac{1}{\rho} \right) - \frac{\partial}{\partial m} \left( r^2 u \right) = 0,
\end{equation}
\begin{equation}
\frac{\partial u}{\partial t} + r \frac{\partial P}{\partial m} = 0,
\end{equation}
\begin{equation}
\frac{\partial}{\partial t} \left[ \frac{P}{(\gamma - 1)\rho} + \frac{u^2}{2} \right] + \frac{\partial}{\partial m} \left( r^2 P u \right) = 0,
\end{equation}
where \( \rho \) is the density, \( u \) the flow velocity, \( P \) the pressure and \( r \) the spherical radius, and the mass coordinate \( m \) is defined as \( 1/4\pi \) times the mass within a sphere of radius \( r \).

The integration is carried out with a first-order Godunov method, using the ‘exact’ Riemann solver described by Toro (1999). At the resolution of the simulation that we have carried out (see below), a second-order algorithm gives almost indistinguishable numerical results.

In this simulation, we set \( \gamma = 5/3 \), \( R_t = 1 \) (where \( R_t \) is the final, pressure equilibrium radius of the hot bubble, see equation 11), \( \rho_0 = c_0 = 1 \) and \( P_0 = 1/\gamma \) (where \( \rho_0, c_0 \) and \( P_0 \) are the density, sound speed and pressure of the undisturbed environment, respectively). For the hot bubble, we choose an initial radius \( R_t = 10^{-2} R_f \), and fill this sphere with gas of density \( \rho_1 = 1 \) and pressure \( P_1 = (\gamma - 1) E / V_i \); 
\begin{equation}
E = \frac{8\pi}{3(\gamma^2 - 1)},
\end{equation}
where \( V_i = 4\pi R_f^3 / 3 \) is the initial volume of the hot bubble, and the second equality is obtained by setting \( R_t = \rho_0 = c_0 = 1 \) in equation (11).

The simulation is done in a grid with \( 2 \times 10^4 \) grid points between \( r = 0 \) and an outer radius of \( 3R_f \). The grid points are initially distributed with a constant spacing \( \Delta r = 3R_f / 2 \times 10^4 \), so that the mass within the successive computational cells increases upwards. A reflection condition is applied in the centre of the spherical coordinate system, and a transmission condition at the outer boundary. The pressure (top frame) and temperature \( T = P/\rho \) structures (bottom frame) resulting from an integration up to a time \( t = 2R_t/c_0 \) are shown in Fig. 3.

In this figure, we also show the new analytic solutions obtained for the shock wave (black solid line in the top frame) and for the outer radius of the hot bubble (black dashed line in the bottom frame). We also show the strong-shock solutions (white solid and dashed lines) of equations (18) and (22). From the top frame of Fig. 3, it is clear that while the strong-shock solution (white solid line) shows strong deviations from the numerical simulation for \( t > R_t/c_0 \), the new ‘strong-/weak-shock’ solution (equations 13 and 21) agrees to within \( \sim 5 \) per cent with the simulated shock position for all of the displayed times.

It is less clear what is the feature in the numerical simulation that corresponds to the outer radius of the ‘hot bubble’. In the bottom frame, we see that the radius \( R \) of the hot bubble (obtained from equation 13) encloses a hot region with a strongly stratified temperature. For larger radii, the temperature has values similar to the temperature of the undisturbed environment (except for a narrow hotter region just behind the outer shock).

The radius of the ‘hot bubble’ of the analytic model does not correspond to the position of the outer edge of the initial bubble imposed in the numerical simulation. At time \( t = 1.5 c_0 / R_t \), the initial bubble has expanded to a radius of \( \approx 0.17R_t \) (seen as the saturated high-temperature region in the bottom frame of Fig. 3).

The ‘hot bubble’ of the analytic model has approximately five times this radius, and its mass is dominated by environmental material heated by the blast wave at early evolutionary times.

In Fig. 4, we show the shock velocity of the blast wave as a function of logarithmic time obtained from the numerical simulation, from the Taylor–Sedov ‘strong-shock’ solution and from our ‘strong-/weak-shock’ analytic solution. The ‘numerical’ shock velocity versus time dependence converges to the Taylor–Sedov solution at \( t \approx 2 \times 10^{-3} R_t/c_0 \) evolutionary time and follows it out to \( t \approx 0.1R_t/c_0 \). For larger evolutionary times, the numerically derived shock velocity deviates from the Taylor–Sedov solution, approaching its asymptotic \( v_t = c_0 \) value. The ‘strong-/weak-shock’ analytic solution shows qualitatively similar deviations from the Taylor–Sedov solution.

In order to give a more quantitative view of the deviations from the Taylor–Sedov solution at large evolutionary times, we show in Fig. 5 the density and pressure jumps at the shock as a function of time (now with a linear time axis, so that the deviations in the pressure jump in the early evolution of the expansion are not seen). In order to compute the compression and the pressure jump from the Taylor–Sedov solution, we have taken the strong-shock solution (equations 18 and 22), obtained the time derivative to compute the shock velocity and then inserted this velocity in the general
‘strong/weak’ shock jump equations to obtain the compression and the pressure jump. In the same way, we compute the compression and pressure jump from our new analytic blast wave solution.

From Fig. 5, we see that for $t < 0.1 R_t/c_0$ the numerically computed compression and pressure jumps agree well with the two analytic solutions. For larger times, the numerical solution gives higher compressions and pressure jumps than the analytic solutions, with the new ‘strong/weak’ solution following more closely the numerical results.

For example, at $t = 0.25 R_t/c_0$ the numerical simulation gives a pressure jump $P_f/P_0 \approx 3.9$, the strong-/weak-shock solution gives $P_f/P_0 \approx 3.6$ (a good prediction) and the Taylor–Sedov solution gives $P_f/P_0 \approx 2.4$. At $t = 0.5 R_t/c_0$, the numerical pressure jump is $\approx 2.4$, the strong/weak solution gives $\approx 1.8$ (still a reasonable prediction) and the Taylor–Sedov solution gives an incorrect value of $P_f/P_0 = 1.0$.

Therefore, for evolutionary times $t \sim 0.5 R_t/c_0$, the strong-/weak-shock analytic model predicts the overpressure $P_f/P_0 - 1$ produced by the shock to be within $\sim 40$ per cent. Therefore, for shorter evolutionary times, the new analytic solution gives a realistic prediction of the damage inflicted on the environment by the blast wave.

4 CONCLUSIONS

We have extended the analytic ‘hot bubble/swept-up shell’ model for non-radiative, spherical blast waves of Chernyi (1957, described in Zel’dovich & Raizer 1967) to the case in which the outer shock has a strong-/weak-shock transition. The resulting model has a full analytic solution, which gives both the radius of the hot bubble and of the outer shock as a function of time (see Section 2).

The analytic model has an initial behaviour which is similar to the ‘strong-shock’ Taylor–Sedov solution. For radii of the hot bubble $R \sim R_t$ and times $t \sim R_t/c_0$ (where $R_t$ is given by equation 11), the analytic solution has a transition to a ‘weak-shock’ regime, in which the hot bubble stops growing (the bubble remaining in approximate pressure equilibrium with the undisturbed environment) and the outer shock velocity approaches $c_0$ (the environmental sound speed).

We have compared this analytic solution with a spherically symmetric numerical simulation (for the $\gamma = 5/3$ case) and found good agreement between the analytic and numerical results for times up to $\sim 0.5 R_t/c_0$. For longer evolutionary times, the compression and pressure jump (at the shock wave) predicted by the analytic model approach unity faster than the ‘correct’ solution (obtained from a numerical integration of the full Lagrangian gasdynamic equations). These deviations from the correct solution could clearly be important for some applications.

While the application of the present model to ‘ground-based’ explosions is evident, it also has astrophysical applications. Tang & Wang (2005) showed that SN explosions within pre-existing hot bubbles can reach the ‘weak-shock’ regime. In the context of our analytic model, this can be seen by setting $n_0 \sim 1 \text{ cm}^{-3}$, $c_0 \sim 1000 \text{ km s}^{-1}$ (the environmental number density and sound speed, respectively) and $E \sim 10^{50}$ erg in equation (11), to obtain $R_t \sim 2.3 \text{ pc}$. As can be seen from Fig. 1, the bubble reaches a radius $R_t$ at a time $t_t \sim R_t/c_0 \sim 2200 \text{ yr}$. At a time $t_t$, the outer shock also becomes weak (see Fig. 1).

Therefore, if we have a pre-existing hot bubble of radius $R_0 > R_t$, the explosion will generate an inner higher temperature core (of radius $\sim R_t$), and the outer shock will perturb only weakly the remaining outer region of the initial bubble. This hot core will eventually be dissipated as a result of thermal conduction. If the initial

- **Figure 4.** Shock velocity versus time obtained from the gasdynamic simulation (solid line), from the Taylor–Sedov solution (long dash) and from our new ‘strong/weak’ shock solution (short dash). One can see that by $t \approx 2 \times 10^{-5} R_t/c_0$, the numerical blast wave has reached the Taylor–Sedov phase, and that it starts deviating again from the strong-shock solution for $t > 0.1 R_t/c_0$. The strong/weak-shock solution shows qualitatively similar deviations from the Taylor–Sedov solution for $t > 0.1 R_t/c_0$.

- **Figure 5.** Compression (top) and pressure jump (bottom) across the blast wave versus time obtained from the gasdynamic simulation (solid line), from the Taylor–Sedov solution (long dash) and from our new ‘strong/weak’ shock solution (short dash). One can see that for $t < 0.5 R_t/c_0$, the strong-/weak-shock solution predicts the overdensity and overpressure produced by the shock to be within $\sim 30$ per cent. The strong shock Taylor–Sedov solution results in substantial deviations from the numerical simulation for $t > 0.2 R_t/c_0$. 


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hot bubble is stratified, the hot core generated by the SN explosion would migrate, ‘floating’ towards the direction of maximum (negative) pressure gradient. This discussion of SN explosions in pre-existing hot bubbles does not differ substantially from the one of Tang & Wang (2005).

A second possible astrophysical application of our model is for SN explosions within molecular clouds. This situation has been explored theoretically by Chevalier (1999). In order to evaluate whether an SN explosion will reach the weak-shock regime within a molecular cloud, we set $n_0 \simeq 10^6 \text{ cm}^{-3}$, $c_0 \simeq 1 \text{ km s}^{-1}$ and $E \simeq 10^{50} \text{ erg}$ in equation (11), obtaining $R_f \simeq 2.3 \text{ pc}$. The evolution of the remnant to the weak-shock regime will take place in $t_f \simeq R_f/c_0 \simeq 2.2 \times 10^6 \text{ yr}$. Therefore, if we have an SN explosion within a molecular cloud of radius larger than $\sim 2.3 \text{ pc}$, it will contain the hot bubble generated by the explosion. This contained bubble will generate convective instabilities through which it will eventually mix with the outer regions of the molecular cloud.

We should note that though the present model is strictly non-radiative, it is still possible to apply it to the case in which the outer shock is highly radiative. This can be done by considering $\gamma = 5/3$ for the non-radiative hot bubble (i.e. setting $\gamma = 5/3$ in equation 6) and $\gamma = 1$ in the equations resulting from the shock relations (i.e. in equations 13–16 and 19–20). In this way, the model can be applied to the case of an SN within a molecular cloud, in which the transition from a non-radiative to a radiative outer shock takes place in the early stages of the evolution of the remnant.

However (at least in its present form), the model cannot be applied to a ‘normal’ SN remnant, in which substantial parts of the evolution are both in the non-radiative and radiative outer shock regimes. An analytic model for this transition was described by Toledo-Roy et al. (2009), and it might be worthwhile to try to extend this model to the weak outer shock regime.

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