Microlensing evidence for super-Eddington disc accretion in quasars

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ABSTRACT

Microlensing by the stellar population of lensing galaxies provides an important opportunity to resolve the accretion disc structure spatially in strongly lensed quasars. Disc sizes estimated in this way are on average larger than the predictions of the standard Shakura–Sunyaev accretion disc model. An analysis of the observational data on microlensing variability suggests that some fraction of lensed quasars (primarily smaller-mass objects) are accreting in the super-Eddington regime. Super-Eddington accretion leads to the formation of an optically thick envelope scattering the radiation formed in the disc. This makes the apparent disc size larger and practically independent of wavelength. In the framework of our model, it is possible to make self-consistent estimates of mass accretion rates and black hole masses for the cases when both amplification-corrected fluxes and radii are available.

Key words: accretion, accretion discs – gravitational lensing: micro – quasars: general.

1 INTRODUCTION

Since the work of Lynden-Bell (1969), disc accretion onto super-massive black holes has been a commonly accepted interpretation for the activity of quasars, both radio-loud and radio-quiet (sometimes distinguished as quasi-stellar objects, QSOs). Among active galactic nuclei, quasars are distinguished by higher luminosities (exceeding that of host galaxies), which is probably connected with higher accretion rates.

Spectral energy distributions in the optical and UV are reasonably consistent (Elvis et al. 1994) with the predictions of the standard thin accretion disc model introduced in the seminal works of Shakura (1972), Shakura & Sunyaev (1973), Lynden-Bell & Pringle (1974) and Novikov & Thorne (1973). However, the predicted angular sizes of quasar accretion discs are too small (micro-arcseconds and less) to be resolved directly. At present, quasars remain essentially point-like (‘quasi-stellar’) objects resolved only indirectly, in particular by microlensing effects.

As shown by Agol & Krolik (1999), microlensing by the stellar population of lensing galaxies is sensitive to the size of the emitting region. Here, we adopt the argument of Mortonson, Schechter & Wambsganss (2005) that the basic quantity that microlensing amplification maps and curves are sensitive to is the half-light radius. The half-light radius \( R_{\text{1/2}} \) is defined as the radius inside which half of the observed flux is emitted at a given wavelength.

Numerous studies have aimed to probe the spatial properties of quasar accretion discs with the help of microlensing. While most early works (see Wambsganss 2006 for a review) reported reasonable agreement between the observational data and standard accretion disc theory, several important results are at odds with the theoretical predictions. By studying microlensing amplification statistics, Pooley et al. (2007) found best-fit disc sizes more than one order of magnitude larger than the theoretical predictions based on photometric data. This may in part be attributable to the mass estimates used in this study (see discussion in Abolmasov & Shakura 2012). In Morgan et al. (2010), the inconsistency is somewhat smaller (about a factor of 3) but still significant. Accretion discs seem too large for their apparent luminosities or too faint for their sizes in the UV/optical range (\( \sim 2000–4000 \) Å). Many papers (such as Jiménez-Vicente et al. 2012; Pooley et al. 2007; Morgan et al. 2010) interpret this inconsistency as evidence for the insufficiency of the standard accretion disc model, but no universal solution has been proposed so far to account for the size discrepancy. There are indications for possibly higher black hole masses in some objects (Morgan et al. 2010; Abolmasov & Shakura 2012), but no changes in masses, accretion rates and efficiencies can explain the observed sizes and fluxes simultaneously.

One of the important issues in quasar microlensing studies is whether the disc radial scale \( R_\delta \) (Morgan et al. 2010) dependence on wavelength is consistent with the power law \( R_\delta \propto \lambda^{4/3} \) predicted by the standard accretion disc theory. Several important disc models predict power-law dependences \( R(\lambda) \propto \lambda^\zeta \) with different exponents. Below we will refer to \( \zeta \) as the ‘structure parameter’. While for QSO J2237+0305 classical \( \zeta = 4/3 \) works fairly well (Eigenbrod et al. 2008; Anguita et al. 2008), other objects, such as SDSS 0924+0219, clearly require smaller \( \zeta \). Floyd, Bate & Webster (2009) propose angular momentum inflow at the inner edge of the disc in SDSS 0924+0219 (Agol & Krolik 2000) as an explanation for the apparently very steep temperature law in the disc. This model implies \( R_\delta \propto \lambda^{4/7} \), marginally consistent with the observational data.

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In the recent work by Blackburne et al. (2011), several other objects were shown to have much shallower $R(\lambda)$ dependence, some consistent with $R_\lambda = \text{const}$ for a broad range of comoving wavelengths, $0.1\mu \lesssim \lambda \lesssim 1\mu$. The only object having conventional thin-disc scaling is MG J0414+0534, which has the highest mass among the sample of 12 objects considered by Blackburne et al. (2011). All the smaller-mass ($M \lesssim 10^8 M_\odot$) black holes are characterized by $\zeta \sim 0-0.5$.

Microlensing effects in the X-ray range are more profound than in the optical (Pooley et al. 2007). Independently of the disc structure in the optical and UV ranges, X-ray properties are more or less similar for all the objects for which microlensing effects were studied in the X-ray range (Dai et al. 2010; Chen et al. 2011; Morgan et al. 2012). Evidently, X-ray emission comes from somewhere inside the inner $\sim 0.1 \times GM/c^2$ (Chen et al. 2012), and the exact mechanisms driving the formation of the X-ray continuum and lines are yet to be revealed.

Small structure parameters originate not only for very steep temperature slopes in multi-black-body models. For instance, $\zeta = 0$ is naturally reproduced if the brightness distribution does not depend on wavelength. This may be achieved if the accretion disc is surrounded by an envelope optically thick to Thomson scattering. Without affecting its spectral properties, scattering changes the spatial brightness distribution of the disc radiation. In general, the accretion disc will increase its apparent radius and lose its intrinsic radius dependence on wavelength. A possible origin for such a scattering envelope is super-Edington accretion, which leads to the formation of a Thomson-thick wind (Shakura & Sunyaev 1973).

Because there is observational evidence that some quasars accrete in this regime, especially at larger redshifts (Collin et al. 2011), we consider this scenario quite plausible.

In the following Section 2, we describe a simple scattering envelope model that we use to account for the spatial properties of microlensed quasars. It will be shown that such an envelope may result from super-Edington accretion by a supermassive black hole. In Sections 3 and 4, we describe the observational data and interpret them in the framework of the scattering envelope model. In Section 5, we consider the possible connection between the putative class of supercritical quasars and broad absorption line (BAL) quasars. Conclusions are given in Section 6.

## 2 Spherical Envelope Model

Although super-Edington accretion is complicated, and a number of effects, such as photon trapping, should be taken into account, the simple picture of supercritical accretion introduced by Shakura & Sunyaev (1973) is sufficient for our needs. This picture is supported by comprehensive numerical simulations (Ohsuga et al. 2005; Ohsuga & Mineshige 2011).

### 2.1 Transition to the supercritical regime

Radiation pressure is the principal feedback source for disc accretion. While for a spherically symmetric source both radiation pressure force and gravity scale $\propto R^{-2}$ with distance, which leads to the universal Edington luminosity limit, accretion disc geometry makes the situation more complicated because the source is no longer isotropic, and the forces are no longer collinear. For a thin disc, the vertical gravity component grows with the vertical coordinate $z$, while the flux generated in the disc does not significantly depend on $z$ high enough in the atmosphere. A thin accretion disc supported by radiation pressure will have a thickness determined by the equilibrium of the vertical components of the two forces:

$$\frac{F\sigma}{c} = \frac{\pi G}{8} \frac{M M}{R^3} \left(1 - \sqrt{\frac{R}{R}}\right) = \frac{GM}{R^3} H.$$  

Here, $R$ is the radial coordinate; $R_m$ is the inner disc edge, which for a black hole disc can be identified with the innermost stable orbit radius; $\zeta \sim 0.4 \text{ cm}^2 \text{ g}^{-1}$ is the electron scattering opacity; $c$ is the speed of light; $G$ is the gravitational constant; and $M$ and $R$ are the black hole mass and accretion rate. $H$ is the accretion disc half-thickness, which can be expressed as

$$H = \frac{3}{8\pi} \frac{\zeta M}{c} \left(1 - \sqrt{\frac{R}{R_m}}\right) = \frac{3 G M}{2 c^2} \left(1 - \sqrt{\frac{R}{R_m}}\right).$$  

Here, we normalized the mass accretion rate as $\dot{M} = \dot{M}^* m_m$, where $\dot{L}_\text{edd} = \frac{4\pi G M c}{c \zeta}$ is the Eddington luminosity. When the thickness of the radiation-supported disc becomes comparable to its radius, the thin-disc approximation breaks down and the balance of forces is inevitably shifted: gravity scales as $(c^2 + R^2)^{-1}$, while the disc radiation decreases more slowly because at these distances the accretion disc is still a strongly extended radiation source. Hence we assume the disc to be super-Edington if its equilibrium half-thickness is $H \gtrsim R$. This condition defines the spherization radius in a non-relativistic regime but with a correction term that may play a non-negligible role near the critical accretion rate. Let us introduce the dimensionless radius $r = R/R_m$, where $R_m = x_m GM/c^2$ is the inner radius of the disc. The dimensionless inner radius $x_m$ varies between 1 (extreme Kerr case, corotating disc) and 9 (extreme Kerr, counter-rotation). The existence of a supercritical region in the disc requires the existence of a root $r > 1$ of the following equation:

$$\frac{r}{1 - \sqrt{\frac{R}{R}}} = \frac{3}{2}$$

This equation can be reduced to a cubic equation for $\sqrt{r}$ having either two or no positive real roots, depending on the right-hand side. The minimum of the left-hand side is always at $r_{cr} = 9/4$. This implies a critical accretion rate of $m_{cr} = 4.5 x_m$. The luminosity of a non-relativistic disc with this value of mass accretion rate is $L/L_{\text{edd}} = m_{cr} \eta \approx 9/4$, while the $\eta = L/M c^2$ is the accretion efficiency. This value may be thought of as the gain that the disc geometry provides for the Eddington limit. Without the correction term, the critical accretion rate should be about an order of magnitude lower, $m_{cr,0} = 2 x_m/3$. This justifies the attention we pay to the correction term. Still, the real transition to supercritical accretion may be much more complicated and influenced by relativistic effects and the exact physics of the wind acceleration.

The spherization radius for $m > m_{cr}$ can be found in a way that takes into account the correction term:

$$R_{\text{ph}} = \frac{3}{2} \frac{G M}{c^2} \psi^2 (m/x_m).$$

Here, $\psi$ is a correction multiplier that accounts for the influence of the inner radius in the disc. It can be found as the largest root of the cubic equation for $\sqrt{r}$ that follows from (2). When accretion is supercritical ($m > m_{cr,0}$, the cubic equation has three roots and its solution can be expressed using trigonometric functions (Nickalls 1993). Solving the cubic equation yields the following expression for the correction multiplier:

$$\psi(x) = \frac{2}{\sqrt{3}} \cos \left(\frac{1}{3} \arccos \left( -\frac{3}{2x} \right) \right).$$
For \( m \gg 1 \), \( \psi(n/x_m) \) rapidly approaches 1, and hence
\[
R_{\text{ph}} \simeq \frac{3}{2} \frac{GM}{c^2}
\]
which coincides with the classical definition of the spherization radius (Shakura & Sunyaev 1973).

The formation of disc winds in the supercritical accretion regime is supported by numerical simulations (Ohsuga et al. 2005; Okuda et al. 2005). If the radiation flux from the disc exceeds the local Eddington value, some part of its energy is converted to the kinetic energy of the outflow. The terminal wind velocity \( v_w \) can be estimated through the Bernoulli integral:
\[
\frac{v_w^2}{2} \approx \frac{GM}{R} \left( \frac{L}{L_{\text{Edd}}} - 1 \right).
\]

For example, setting the luminosity to the ‘disc Eddington limit’ \( L = (9/4)L_{\text{Edd}} \) implies that \( v_w = \sqrt{2.5GM/R_0} \), where \( R_0 \) is some effective radius at which the outflow is formed. Because most of the matter is ejected from \( R \sim R_{\text{ph}} \), we assume that the terminal velocity of the wind is proportional to the escape velocity at the spherization radius:
\[
v_w = \beta \sqrt{2GM/R_{\text{ph}}}, \tag{5}
\]
were \( \beta \sim 1 \) is some additional dimensionless multiplier introduced to account for the uncertain details of the wind acceleration and formation of the outflow.

### 2.2 Spherical envelope radius

Let us assume that the envelope is composed of a fully ionized spherically symmetric wind expanding at a constant velocity. The continuity equation allows us to connect the electron density with the mass-loss rate \( \dot{M}_w = f_w \dot{m} M^* \) and outflow velocity \( v_w \) calculated according to equation (5):
\[
\rho = \frac{\dot{M}_w}{4\pi R^2 v_w}.
\]

The envelope size is defined by the radius at which the radial optical depth towards the observer is unity:
\[
\tau(R) = \int_0^{\infty} \alpha \rho(R) dR = \frac{f_w \dot{m} M^*}{4\pi v_w c R},
\]
\[
R_1 = R(\tau = 1) = \frac{f_w \dot{m} M^*}{4\pi v_w c}.
\]
\[
R_1 c^2 / GM = \sqrt{\frac{3}{8}} \frac{f_w \dot{m}^{3/2} \psi(n/x_m)}{\beta}.
\]

For a moderately super-Eddington accretor with \( f_w \sim 1 \), the size of the envelope becomes comparable to and may even exceed the size of the accretion disc in the ultraviolet range. Such a scattering envelope has a radius practically independent of wavelength, while the spectral properties of the scattered radiation remain more or less unchanged. The envelope is actually a pseudo-photosphere: it expands supersonically at a large, possibly mildly relativistic velocity of \( v \sim m^{-1/3}c \).

### 2.3 Apparent intensity distribution

The half-light radius is defined by the general relation that can be used for any radially symmetric intensity distribution:
\[
\int_{R_1}^{R_2} I(R) R dR = \frac{1}{2} \sum_{R_1}^{R_2} I(R) R dR = \frac{1}{2}, \tag{7}
\]

For a standard thin non-relativistic accretion disc, monochromatic intensity scales with radius as
\[
I(R) \propto \frac{1}{\exp \left[ \left( R/R_0 \right)^{3/4} \left( 1 - \sqrt{\frac{R_0}{R}} \right) \right] - 1},
\]
where \( R_0 \) is the disc radial scale defined by the condition \( h_{\nu_{\text{em}}} = kT(R_0) \) without the correction factor. If \( R_1 \ll R_{1/2} \), to a high accuracy:
\[
R_{1/2}^{(\text{disc})} \simeq 2.44 R_0
\]
\[
\simeq 2.44 \left( \frac{45\pi^4 GM M^*}{16\epsilon_{1/3} h c^2} \right)^{1/3},
\]
\[
= 2.44 \left( \frac{45\pi^4 \epsilon_{\nu_{\text{em}}}^2 \dot{m} M^*}{4\pi^3 h c^2 GM} \right)^{1/3} GM/c^2. \tag{8}
\]

Here, \( \epsilon_{\nu_{\text{em}}} \) is the comoving (quasar reference frame) wavelength of the observed radiation (\( \nu_{\text{em}} = c\nu_{\text{obs}} \) is the corresponding frequency; observed wavelengths and frequencies are denoted as \( \lambda_{\text{obs}} \) and \( v_{\text{obs}} \)), and \( h \) is Planck’s constant.

For a spherical envelope, the brightness is nearly uniform in the centre and declines as a power law at large radii. We have taken the extended scattering atmosphere model described in Appendix A and calculated the intensity at infinity for different shooting parameter values, coming to the overall conclusion that the half-light radius in this model is proportional to the photosphere radius and \( R_{1/2} \simeq 1.06 R_1 \).

### 2.4 Disc radiation

The standard accretion disc temperature law can be written (neglecting the correction term important for the inner parts of the disc) as
\[
T(R) = \left( \frac{3}{2} \frac{G^2 M^2}{\sigma_{\nu_{\text{em}}} R^3 \dot{m}} \right)^{1/4},
\]
where \( \sigma \) is the Stefan–Boltzmann constant. The monochromatic flux is found as the integral over the picture plane:
\[
F_{\nu} = \int_{\nu_{\text{obs}}} \int_{\nu_{\text{obs}}} \frac{2\pi \cos \zeta}{D^2} (1 + z)^8 \int_{\nu_{\text{obs}}} \nu_{\text{obs}} (I(R) R dR).
\]

We use an angular size distance \( D = D(z) \), and \( i \) is the disc inclination. The assumption that the radiation generated in the disc has locally a black body spectrum leads to the following estimate for monochromatic flux valid far away from the high- and low-energy cut-offs associated with the inner and the outer disc edges:
\[
F_{\nu} = 8\pi \left( \frac{2}{3} \right)^{1/3} \Gamma(8/3) \xi(8/3) \frac{k_{\nu_{\text{obs}}}^{1/3} v_{\text{obs}}^{1/3}}{c^{1/3} h^{1/3} \nu_{\text{obs}}^{1/3}} \sigma_{\nu_{\text{em}}}^4 \frac{1}{D^2 (1 + z)^{8/3}}.
\]

Here, \( \Gamma \) and \( \xi \) are the gamma function and Riemann zeta function (not to be confused with the structure parameter \( \xi \) that is used without any argument). The above formula can be used to estimate the mass of the supermassive black hole (SMBH) using one observable quantity (flux) and one unknown parameter \( m \):
\[
M = \left( \frac{3}{2} \right)^{1/4} \left( \frac{8\pi \Gamma(8/3) \xi(8/3)}{3} \right)^{3/4} c^{3/4} k_{\nu_{\text{obs}}}^{1/2} v_{\text{obs}}^{1/2} \frac{G^2 \nu_{\text{obs}}}{D^2 (1 + z)^{3/2}} \nu_{\text{obs}}^{1/2} \frac{1}{D^2 (1 + z)^{3/2}} \right) \left( \frac{1}{D^2 (1 + z)^{3/2}} \right). \tag{10}
\]

The spherical envelope scatters the radiation generated in the disc and makes it roughly isotropic. The effective inclination cosine
in this case is $\cos i_{\text{eff}} = 1/2$, as the initially anisotropic flux $F_v \propto \cos i$ is redistributed isotropically with the total luminosity conserved.

### 2.5 Mass and accretion rate estimates

Microlensing studies are unique for distant quasars in that they allow the size of the emitting region in the continuum to be estimated independently of the observed flux. In the case of a standard accretion disc, both observables, $F_v$ and $R_{1/2}$, can be used to estimate the black hole mass and mass accretion rate only in combination $M^2 \dot{m}$ (see also the discussion in Abolmasov & Shakura 2012). The existence of a scattering envelope allows us to break this degeneracy and make self-consistent estimates of both principal parameters ($M$ and $\dot{m}$).

Solving the system of two equations (10) and (6) for $M$ and $\dot{m}$ allows us to estimate both the black hole mass and the dimensionless accretion rate. We also set $\cos i = \cos i_{\text{eff}} = 1/2$:

$$\dot{m} = \frac{\sqrt{7}}{3\sqrt{3}} \left(8\pi \Gamma(8/3)\xi(8/3)\right)^{3/4} \frac{k_B^2 h^{1/2}}{\lambda^2 c^2 \kappa_{\text{en}}^{1/2} \sigma_{\text{en}}^{1/2}}$$

$$\times F_v^{-3/4} D^{-3/2} (1+z)^{-2} \frac{\beta}{f_{\nu}} \frac{R_1}{\psi (\tilde{m}/\tilde{x}_\text{m})}^{-3/4}$$

$$\simeq 169 \left(\frac{\lambda_{\text{abs}}}{0.79 \mu}\right)^{-1/4} \times 10^{0.3(1-19)} \left(\frac{D}{1 \text{ Gpc}}\right)^{-3/4}$$

$$\times (1+z)^{-2} \frac{\beta}{f_{\nu}} \frac{R_1}{10^{15} \text{ cm}} \frac{1}{\psi (\tilde{m}/\tilde{x}_\text{m})}. \ (11)$$

Here, we expressed the observed flux through the magnitude $I$ in the $HST$ F814W band ($\lambda_{\text{abs}} \approx 0.79 \mu$), adopting the zero-point flux equal to $F_v = 2.475 \times 10^{-20} \text{ erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1}$ (Holtzman et al. 1995) as we use the amplification-corrected magnitudes from Morgan et al. (2010) obtained in this photometric band by the $HST$. Because for the left-hand side of (11) the dependence on mass accretion rate is much stronger than for the right-hand side. Once $\tilde{m}$ is found, the black hole mass can be estimated following (10) as

$$M \simeq 4.6 \times 10^7 \times 10^{0.3(1-19)} \left(\frac{D(z)}{1 \text{ Gpc}}\right)^{3/2} (1+z)^2 \tilde{m}^{-1/2} M_\odot.$$  

### 2.6 Disc and envelope sizes

Depending on the wavelength range, a supercritical disc can be either observed directly (if its size is larger than the photosphere of the wind) or covered by the photosphere of the supercritical wind. The equality of the half-light radii set by the two radial scales leads to the following condition for the observational importance of the envelope:

$$R_{1/2} (\text{disc}) = R_{1/2} (\text{envelope})$$

$$2.44 R_{1} = 1.06 R_{1}.$$  

After substituting equations (8) and (6), one obtains the following mass limit:

$$M_{\text{lim}} = \left(\frac{2.44}{1.06}\right)^3 \left(\frac{4}{3}\right)^{\frac{3}{2}} \frac{45}{4\pi^5} \left(\frac{\beta}{f_{\nu}}\right)^3$$

$$\times \frac{c^3}{\chi hG} \frac{\lambda_{\text{em}}^2 \dot{m}^{-7/2} \psi^{-3}(\tilde{m}/\tilde{x}_\text{m})}{\frac{\lambda_{\text{em}}}{0.25 \mu}} \simeq 3.8 \times 10^{10} \left(\frac{\lambda_{\text{em}}}{0.25 \mu}\right)^4 \frac{\tilde{m}}{10} \psi^{-3}(\tilde{m}/\tilde{x}_\text{m}) M_\odot. \ (12)$$

For higher masses (at a given wavelength, for fixed $\tilde{m}$), the size of the envelope is smaller than the half-light radius of the disc, and the appearance of the quasar will be close to the thin disc case. The limit is shown in Figs 1 and 2 with solid lines. This limit evidently depends on wavelength. For the sample of Morgan et al. (2010), the comoving-frame wavelength changes in the range 0.2–0.5 μ, which corresponds to an upward shift of about a factor of 2 of the limit in Fig. 1.

### 3 MULTIWAVELENGTH DATA

#### 3.1 Disc radii and amplification-corrected fluxes

We make use of the amplification-corrected fluxes and microlensing-based radii collected and published by Morgan et al. (2010). The sample of Morgan et al. (2010) overlaps with that of Blackburne et al. (2011). In the original work, disc radii are given

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Quasar parameters (mass and dimensionless mass accretion rate) recovered in the framework of a supercritical envelope. The left and the right panels correspond to $a = 0$ ($x_{\text{m}} = 6$) and $a = 0.9$ ($x_{\text{m}} = 2.32$), respectively. The horizontal dotted line shows the actual Eddington limit: accretion is subcritical below the line and supercritical above it. The solid inclined line marks the limit where the envelope becomes larger than the disc monochromatic size (for $\lambda_{\text{em}} = 0.25 \mu$). Hereafter, solid circles show objects that are also present in the sample of Blackburne et al. (2011), and the recent result for Q J0158–4325 (Morgan et al. 2012) is shown by a star. Errors correspond to $1\sigma$ uncertainties in flux and radius.
Several methods are used to estimate the masses of supermassive black holes. Most of them are model-dependent and suffer from biases of various natures. For bright distant quasars, masses are usually estimated either through photometric data (bolometric luminosity is restored from multiwavelength observations) or by measuring the widths of broad emission lines and the size of the emitting region by reverberation mapping (Blandford & McKee 1982). While the first method relies heavily on ad hoc assumptions about the mass accretion rate and accretion efficiency, the second has a fundamental uncertainty connected to the geometry of the emitting region. The virial mass is estimated as

\[ M = \frac{\sigma^2 R_{\text{BLR}}}{G}, \]

where \( \sigma^2 \) is the velocity dispersion corresponding to the observed line width, \( R_{\text{BLR}} \) is the size of the emitting region (determined with the help of reverberation mapping), and the coefficient \( f \) is calibrated using better-studied nearby active galaxies where \( f \approx 5.5 \) (Onken et al. 2004). Sometimes only a limited number of spectra are available, and reverberation analysis is impossible. In this case, empirical virial relations (Vestergaard & Peterson 2006) are used. These two types of mass estimates will be hereafter referred to as virial. Because we use the microlensing-based disc radii from Table 1, but instead of the dimensionless mass accretion rate \( \dot{m} \), the dimensional mass accretion rate \( \dot{M} \) is given.

Table 1. Properties of the microlensed quasars from the sample of Morgan et al. (2010). Black hole mass \( M_\bullet \) and accretion rate \( \dot{M} \) are calculated for the case of supercritical accretion for \( a = 0.9 \).

<table>
<thead>
<tr>
<th>Object</th>
<th>( M_\bullet ) [10^9 M_\odot]</th>
<th>( \dot{M} ) [10^9 M_\odot/yr]</th>
<th>( \dot{M}/\dot{M}_\text{Edd} )</th>
<th>( \dot{M}/\dot{M}_\text{crit} )</th>
<th>( \dot{M}/\dot{M}_\text{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE J0435–1223</td>
<td>0.50</td>
<td>20.76 ± 0.25</td>
<td>12.23 ± 0.79</td>
<td>9 ± 12</td>
<td>0.028 ± 0.03</td>
</tr>
<tr>
<td>SDSS 0924+0219</td>
<td>0.11</td>
<td>21.24 ± 0.25</td>
<td>2.44 ± 0.24</td>
<td>4.6 ± 0.24</td>
<td>0.026 ± 0.017</td>
</tr>
<tr>
<td>FB J0951+2635</td>
<td>0.89</td>
<td>17.16 ± 0.11</td>
<td>30.72 ± 0.46</td>
<td>–</td>
<td>0.33 ± 0.11</td>
</tr>
<tr>
<td>SDSS J1004+4112</td>
<td>0.39</td>
<td>20.97 ± 0.44</td>
<td>1.94 ± 0.97</td>
<td>–</td>
<td>0.344 ± 0.02</td>
</tr>
<tr>
<td>HE J1104–1805</td>
<td>2.37</td>
<td>18.17 ± 0.31</td>
<td>19.38 ± 0.34</td>
<td>–</td>
<td>0.4 ± 0.25</td>
</tr>
<tr>
<td>PG 1115+080</td>
<td>1.23</td>
<td>19.52 ± 0.27</td>
<td>97.14 ± 0.47</td>
<td>57 ± 60</td>
<td>0.047 ± 0.02</td>
</tr>
<tr>
<td>RXJ 1131–1231</td>
<td>0.06</td>
<td>20.73 ± 0.11</td>
<td>4.87 ± 0.85</td>
<td>2.4 ± 0.14</td>
<td>0.009 ± 0.003</td>
</tr>
<tr>
<td>SDSS J1138+0314</td>
<td>0.04</td>
<td>21.97 ± 0.19</td>
<td>1.94 ± 0.45</td>
<td>8.9 ± 0.17</td>
<td>0.03 ± 0.014</td>
</tr>
<tr>
<td>SBS J1520+530</td>
<td>0.88</td>
<td>18.92 ± 0.13</td>
<td>12.23 ± 0.51</td>
<td>9.3 ± 0.3</td>
<td>0.17 ± 0.05</td>
</tr>
<tr>
<td>QSO J2327+0305</td>
<td>0.9</td>
<td>17.90 ± 0.44</td>
<td>9.71 ± 0.85</td>
<td>–</td>
<td>0.4 ± 0.22</td>
</tr>
<tr>
<td>Q 0158–4325</td>
<td>0.16</td>
<td>19.09 ± 0.12</td>
<td>1.94 ± 0.97</td>
<td>–</td>
<td>0.168 ± 0.05</td>
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<tr>
<td>Q 0158–4325</td>
<td>0.16</td>
<td>19.09 ± 0.12</td>
<td>1.94 ± 0.97</td>
<td>–</td>
<td>0.07 ± 0.04</td>
</tr>
</tbody>
</table>

For their sample of objects, Morgan et al. (2010) also provide magnitudes corrected for strong lensing amplification (see also Table 1) based on HST observations in the F814W filter. Flux calibration is based on Holtzman et al. (1995); see Section 2.5.

3.2 Masses and emissivity slopes

Several methods are used to estimate the masses of supermassive black holes. Most of them are model-dependent and suffer from biases of various natures. For bright distant quasars, masses are usually estimated either through photometric data (bolometric luminosity is restored from multiwavelength observations) or by measuring the widths of broad emission lines and the size of the emitting region by reverberation mapping (Blandford & McKee 1982). While the first method relies heavily on ad hoc assumptions about the mass accretion rate and accretion efficiency, the second has a fundamental uncertainty connected to the geometry of the emitting region. The virial mass is estimated as

\[ M = \frac{\sigma^2 R_{\text{BLR}}}{G}, \]

where \( \sigma^2 \) is the velocity dispersion corresponding to the observed line width, \( R_{\text{BLR}} \) is the size of the emitting region (determined with the help of reverberation mapping), and the coefficient \( f \) is calibrated using better-studied nearby active galaxies where \( f \approx 5.5 \) (Onken et al. 2004). Sometimes only a limited number of spectra are available, and reverberation analysis is impossible. In this case, empirical virial relations (Vestergaard & Peterson 2006) are used. These two types of mass estimates will be hereafter referred to as virial. Because we use the microlensing-based disc radii from

Figure 2. As Fig. 1, but instead of the dimensionless mass accretion rate \( \dot{m} \), the dimensional mass accretion rate \( \dot{M} \) is given.
Morgan et al. (2010) we also make use of the virial masses given in this work (see references in this paper, especially Peng et al. 2006).

4 RESULTS

4.1 Masses and accretion rates

The mass and dimensionless mass accretion rates estimated from the observables by the method introduced in Section 2.5 are given in Table 1 (for $a = 0.9$ and $x_{\text{in}} = 2.32$), and shown in Figs 1 and 2 for two values of $a$ and $x_{\text{in}}$. All the masses and mass accretion rates were determined assuming the existence of an optically thick scattering envelope. They apply only to objects for which the disc is surrounded by an envelope larger than the disc itself (i.e. above both the dotted and the solid lines in Fig. 1).

Errors given in Table 1 and Figs 1 and 2 were calculated using direct non-linear error propagation. We used the $\sigma$ uncertainties given by Morgan et al. (2010) for the radii and fluxes. Because we do not know the exact probability distributions of these quantities, it seems to be the most reasonable approach. We substituted both the dotted and the solid lines in Fig. 1).

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The properties of most of the objects shown in Fig. 1 are consistent with accretion in a moderately super-Eddington regime. If the accretion efficiency is high ($\eta \gtrsim 0.9$), all the objects may be interpreted as super-Eddington. Most of the objects are, however, difficult to identify as super- or sub-Eddington sources owing to large uncertainties in $\dot{m}$. More probable super-Eddington objects such as RXJ 1231–1231 and PG 1115+080 tend also to have lower $\zeta$ (see Section 4.2). Einstein’s cross, one of the most probable subcritical discs from the sample, is evidently among the lowest-$\dot{m}$ objects. According to our model, Q J0158–4325 should best conform to the thin-disc model. Indeed, the disc size for this object reported in Morgan et al. (2010) is in good agreement with the theoretical predictions. However, the recent work of Morgan et al. (2012) reports evidence for a larger disc size, several times larger than (but still consistent with at about the $1.5\sigma$ confidence level) the standard model predictions based on the measured amplification-corrected flux. Because the dimensionless mass accretion rate is proportional to the envelope radius, the larger radius makes the properties of Q J0158–4325 consistent with our supercritical disc model.

Morphisms determined in the spherical envelope model are generally smaller than virial masses (see Fig. 3). Indeed, the disc size for this object reported in Morgan et al. (2010) is in good agreement with the theoretical predictions. However, the recent work of Morgan et al. (2012) reports evidence for a larger disc size, several times larger than (but still consistent with at about the $1.5\sigma$ confidence level) the standard model predictions based on the measured amplification-corrected flux. Because the dimensionless mass accretion rate is proportional to the envelope radius, the larger radius makes the properties of Q J0158–4325 consistent with our supercritical disc model.

4.2 Structure parameter correlation with mass

Several quasar microlensing studies have used multiwavelength data to trace the disc size dependence on wavelength. Fitting this dependence with a power law allows the validity of several accretion disc models to be checked, for example thin discs ($\zeta = 4/3$), slim or irradiated discs ($\zeta = 2$) and a thin disc with a strong torque at the inner radius ($\zeta = 8/7$). Available data are collected in Table 2 and in Fig. 4.

Interestingly, all the objects for which the standard disc slope holds have masses $M \gtrsim 10^8 M_\odot$. All the smaller-mass black holes show intensity distributions with much lower $\zeta$. In spite of the large fitting errors, objects in Fig. 4 can be separated into two groups: high-mass black holes surrounded by accretion discs similar to standard, and lower-mass objects for which $\zeta \sim 0-0.5$. An evident qualitative solution is to propose that at least some quasars accrete in the super-Eddington regime. If all the bright lensed quasars accrete at $M \sim 30 M_\odot$ yr$^{-1}$, Eddington luminosity will be reached for $M \sim 10^9 M_\odot$ for accretion efficiency $\eta \sim 0.1$. Lower-mass objects are expected to enter the super-Eddington accretion phase more easily and lose excess accreting matter (Shakura & Sunyaev 1973). We propose that the supercritical wind does not have a significant effect on the spectral energy distribution of a QSO, but changes its spatial properties, acting as a lampshade that changes only the visible size and shape of the lamp.

5 DISCUSSION

In the previous sections we have shown that some lensed quasars are surrounded by scattering envelopes of moderate optical depths.
Microlensing of super-Eddington quasars

Figure 4. Structure parameters for quasars of various masses. Virial masses are shown by crosses. For several objects (HE J0435−1223, SDSS 0924+0219, HE J1104−1805, PG 1115+080, RXJ 1131−1231, SDSS J1138+0314, QSO J2237+0305), we use our envelope model to estimate masses (shown by stars).

\[ \tau \sim R_i/R_{\text{out}} \sim m^{1/2} \sim 1-10. \] Expected outflow velocities are \( v \sim m^{1/3}c. \) At the same time, the Doppler widths of broad emission lines are significantly lower: \( \Delta v/\Delta \lambda \approx 0.01. \) In addition, the expected emission-line luminosities are several orders of magnitude lower than that of observed broad emission lines in quasars. If we propose a constant filling factor \( f, \) a recombination line in a supercritical outflow should have a luminosity estimated as the following volume integral:

\[ L_{\text{line}} = \frac{1}{f} \alpha h \nu \int n_e n_i \, dV. \]

Here, \( \alpha \) is the recombination coefficient, and \( n_e \) and \( n_i \) are the electron concentration and concentration of the particular ion emitting the line. It is convenient to express the concentrations as \( n_i = n_e x_i = p x_i m. \) Below, we fix the values of \( x_i \) and the effective particle mass \( m. \) For a completely ionized hydrogen-rich gas, \( m \) is about equal to the proton mass.

\[ \frac{L_{\text{line}}}{L_{\text{Edd}}} \approx 10^{-7} \left( \frac{f_w^2 \lambda_{\text{line}}}{f \beta^2} \right) \frac{\alpha}{10^{-13} \text{cm}^3 \text{s}^{-1}} \frac{1000 \text{ Å}}{R_{\text{out}}} \frac{R_{\text{out}}}{R_{\text{in}}} \]

Integration is performed from some inner radius \( R_{\text{in}} \) to infinity. Because the inner parts of the flow are considerably ionized, the above luminosity is an upper estimate. Predicted equivalent widths are of the order \( \sim 10^{-4} \text{ Å}, \) about 5–6 orders lower than the observed equivalent widths of broad emission lines. Conditions are much more favourable for the formation of absorption lines, as the wind is thick to the electron scattering and \( N_H \sim \tau \sigma_T \approx 10^{22} m^1 \text{cm}^{-2}. \)

Outflows are expected to manifest themselves in blue-shifted absorptions and P Cyg lines. BAL quasars (Turnshek 1984) show strongly blue-shifted absorption components of UV and sometimes X-ray spectral lines. Two of the objects of our sample, PG 1115+080 and SBS J1520+530, are known as BAL quasars. PG 1115+080 also demonstrates signatures of moderately relativistic outflows. In particular, Chartas, Brandt & Gallagher (2003) found two relativistic absorption components at \( \sim 0.1 \) and \( \sim 0.3c \) of highly ionized iron species and an \( \text{OVI} \) absorption component at \( \sim 0.02c. \) Similar X-ray absorption systems were found for some other high-redshift luminous quasars such as APM 08279+5255 (Chartas et al. 2002), HE J1414+117 (Chartas et al. 2007) and HS 1700+6416 (Lanzuisi et al. 2012). Absorption components with relativistic velocities were also found in the UV spectra of some BAL, ‘mini-BAL’ and narrow-absorption-line quasars (see Narayanan et al. 2004 and references therein). Relativistic outflows often coexist with slower absorption systems, and UV and X-ray absorption lines usually show different profiles and velocities.

Discrepancies in wind velocities for different absorption components suggest that the wind is highly inhomogeneous, with different components having different velocities. Furthermore, its structure may be far from spherical symmetry, and even in the spherically symmetric case the shape of the visible photosphere is distorted if the wind is relativistic (Abramowicz, Novikov & Paczynski 1991). For relativistic winds, there are two effects important for their spatial properties: (i) relativistic beaming makes the visible size of the emitting region \( \sim \gamma^2 \) times smaller, where \( \gamma \) is the Lorentz factor, and (ii) the optical depth along the wind flow direction is smaller. Both effects are expected to produce a wavelength-dependent limb-darkening effect that may be responsible for deviations of \( \xi \) from zero for some objects.

For several objects, the size of the X-ray-emitting region was studied using microlensing effects (see Pooley et al. 2007; Chen et al. 2011, 2012; Morgan et al. 2012, and references therein). Independently of the UV/optical structure parameter \( \xi, \) the X-ray sizes of all the studied quasars are estimated as several gravitational radii, which is sometimes 1–2 orders of magnitude smaller than the proposed envelope size. This is difficult to account for in the spherical envelope model because the size of the envelope (as well as the size of the accretion disc in the optical/UV range) is generally much larger.

However, the outflows formed by super-Eddington accretion discs are expected to possess high net angular momentum that leads to the formation of an avoidance cone also known as the supercritical funnel (Shakura & Sunyaev 1973). This picture is supported by numerical simulations (Ohsuga et al. 2005; Okuda et al. 2005): for a large range of inclinations, the angular size of the source in X-rays is considerably smaller than the size of the outer photosphere of the wind. The optical depth of an envelope with a funnel is expected to be much lower (if the disc is viewed at low inclinations, see Poutanen et al. 2007) for the innermost parts of the disc where the X-ray component is supposed to be formed. The non-monotonic optical depth dependence on radius predicted for supercritical accretion disc winds can qualitatively explain the decrease in the disc size at smaller wavelengths observed for some objects with \( \xi \sim 0, \) such as PG 1115+080 and WF1 J2033−4723 (Blackburne et al. 2011).

As long as we use virial masses, the properties of the X-ray radiation indicate that the radiation is formed near the last stable orbit (Morgan et al. 2012). However, smaller masses recovered in the framework of our supercritical envelope model shift the last stable orbit towards smaller sizes. For instance, the X-ray-emitting region of RXJ 1131−1231 has a size of \( \sim 2 \times 10^{14} \text{ cm} \) (Dai et al. 2010). For a virial mass estimate of \( 6 \times 10^7 \text{ M}_\odot \), this corresponds
to \(\sim 7 \text{Gm/c}^2\), while the envelope-based mass is about an order of magnitude larger, and the estimated X-ray size becomes several tens of \(\text{Gm/c}^2\). This difference can hardly be used to distinguish between the individual mass estimates and individual models of X-ray emission separately. However, it should be borne in mind that virial masses are consistent with the models in which X-ray emission is formed near the last stable orbit, while the envelope-based mass estimates allow the X-ray emission to be extended for tens of gravitational radii. This is consistent, for example, with models such as that by Liu et al. (2012), in which the X-rays are produced by the hot gas of the corona present in the inner parts of the accretion flow. In any case, the size of the X-ray-emitting region is expected to be considerably smaller than both the disc size at \(\sim 2500 \text{Å}\) and the scattering wind photosphere size.

The existence of true absorption processes should also affect the apparent size of the spherical envelope. Assuming total thermalization in a spherical relativistic wind, Fukue & Iino (2010) find that the visible photosphere surface follows the law \(T(R) \propto R^{-1}\), which implies \(\zeta \sim 1\). More elaborate studies taking into account the temperature and ionization structure of the wind are needed to explain the observed \(\sim 0.5\) values of most of the putative supercritical quasars.

True absorption processes are also important for wind acceleration. In the super-Eddington regime, wind is efficiently launched by resonance lines even in the presence of strong X-ray radiation (Proga et al. 2000; Proga & Kallman 2004). Resonance lines do not alter the measured photosphere size significantly, as the wind is opaque to absorption only in a narrow wavelength range. However, their contribution to wind acceleration through \(\beta\) and \(f_w\) may be important.

It is tempting to compare the population of super-Eddington quasars with the few known and well-studied supercritical black hole X-ray binaries, primarily with SS433 (Fabrika 2004; Cherepashchuk et al. 2005). For SS433, the dimensionless mass accretion rate is of the order of several thousands, which implies a much slower outflow velocity of \(\sim 1000 \text{km s}^{-1}\) and thermalized emission from the wind pseudo-photosphere. On the other hand, mass accretion rates estimated in the present work, as well as the values given by Collin et al. (2002), are considerably smaller. Maximum values are of the order of \(\dot{m} \sim 100–200\). Note that these values are only moderately supercritical, Eddington luminosity is exceeded a factor of \(\sim 10–20\) (depending on the unknown accretion efficiency \(\eta \sim 0.06–0.4\)). It is more instructive to compare the population of super-Eddington quasars with the high-luminosity states of X-ray binaries such as GRS J1915+105 (Vierdayanti, Mineshige & Ueda 2010) rather than with persistent strongly supercritical accretors such as SS433 or with sources such as V4641 (Revnivtsev et al. 2002) suffering strongly super-Eddington outbursts.

6 CONCLUSIONS

The scattering envelope formed by a super-Eddington accretion disc is a plausible model for the spatial properties of the emitting regions in some lensed quasars. Large spatial sizes (\(R \sim 10^{16}–10^{17} \text{cm}\)) practically independent of wavelength are an expected outcome of a moderately super-Eddington \((\dot{m} \sim 10–100)\) mass accretion rate. Black hole masses and mass accretion rates may be determined self-consistently if both disc size and flux estimates are available.

The small sizes of X-ray-emitting regions of microlensed quasars may be explained by the existence of an avoidance cone, or supercritical funnel, in the disc wind.

Some of our super-Eddington objects are BAL quasars, and at least one (PG 1115+080) shows the signature of a mildly relativistic outflow.

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REFERENCES

Lynden Bell D., 1969, Nat, 223, 690

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Moments, defined as:

\[ \frac{\mu}{\cos \theta} I(\mu) = -\kappa(R) (S - I) \]

Here, \( I = I(R, \mu) \) is the monochromatic intensity, \( \mu = \cos \theta \) is the cosine of the angle between the radius vector and the radiation propagation direction, and \( S = S(R) \) is the source function. We consider only isotropic coherent scattering that allows us to equate the source function to intensity averaged over solid angle.

We use the moment approach, and use the first three radiation intensity moments, defined as:

\[ J = \frac{1}{2} \int_{-1}^{+1} I d\mu, \quad (A1) \]

The first radiation moment \( J \) has the physical meaning of mean intensity, and hence \( S = J \) in our approximation. The system of moment equations is closed by Eddington’s assumption \( K = J \), where \( f = 1/3 \) is valid in the diffusion approximation. The questionability of the Eddington approximation for extended atmospheres is well known (Chapman 1966), but is sufficient for our purposes. Here, we consider pure electron scattering by a spherical atmosphere with electron density \( n \propto \rho \propto R^{-2} \). In this case, the system of moment equations takes the form

\[
\begin{align*}
\frac{1}{R} \frac{d}{dR} \left( \frac{R^2 H}{\pi} \right) &= 0 \\
\frac{d}{dR} \left( f J \right) + \frac{3f - 1}{R} J &= -\kappa \rho H. \quad (A4)
\end{align*}
\]

The system is simplified if \( f = 1/3 \) (inner parts, \( \tau \gg 1 \)) and if \( f = 1 \) (opposite limit, \( \tau \rightarrow 0 \)). The mean intensity (and hence source function) scales in these two approximations as \( \propto C_1 R^{-3} \) and \( \propto C_2 R^{-2} (1 + \tau) \), respectively. It is convenient to use the second formula and to set \( C_1 = C_2 = H_0 = H(\tau = 1) \). Both asymptotics are then naturally reproduced. \( H_0 \) may be connected to the physical flux at the photosphere as \( F(\tau = 1) = 4\pi H_0 \), and the luminosity as \( L = (4\pi)^2 R_1^2 H_0 \). However, this approximate formula does not conserve the total flux (intensity integrated over the solid angle deviates from the total radiation flux calculated as \( F(\tau = 1) = 4\pi H_0 \)), which may result in systematic errors, and hence we adopt the following form for the source function:

\[ S(r) = H_0 r^{-2} (1 + d r^{-1/2} + r^{-1}), \]

where \( r = R/R_1 = 1/\tau \), and \( d \) is a free parameter. Integrating the source function for some shooting parameter \( P \) yields the observed intensity:

\[ I = \int \int \int_{-\infty}^{+\infty} S(\sqrt{P^2 + l^2}) e^{-\tau(P,l)} \rho(\sqrt{P^2 + l^2}) dl \]

where \( \tau(P, l) \) is the optical depth along the current line of sight:

\[ \tau = \int_{-\infty}^{l} \rho \left( \frac{\sqrt{P^2 + l^2}}{P} \right) dl \]

\[ = \frac{R_1}{P} \left( \frac{l}{P} + \pi/2 \right). \]

**Figure A1.** Principal scheme illustrating the integration along the line of sight performed in the Appendix.
The coordinates and designations are shown in Fig. A1.

Finally, the intensity distribution can be expressed as the following definite integral:

\[ I(p) = H_0 \left( u_2(p) + u_3(p) + d u_{5/2}(p) \right), \quad (A5) \]

where \( x = l/R_1 \) and \( p = P/R_1 \), and

\[ u_k(p) = p^{-(k+1)} \int_0^\pi e^{-\theta/p} \sin^k \theta \, d\theta. \]

The constant \( d \) is tuned in a way to fit the integral flux value,

\[ 2\pi \int_0^\infty I(p) \, dp = 4\pi H_0. \]

Numerical integration allows us to estimate the value of \( d \approx -0.097 \).

The half-light radius for this model can be estimated as \( R_{1/2} = 1.063 R_1 \) to an accuracy of about \( 10^{-3} \). Setting \( d = 0 \) is also a reasonable approximation: it overestimates the flux by only about 5 per cent, while \( R_{1/2} \approx 1.05 R_1 \) in this assumption.