

V-like Formations in Flocks of Artificial Birds

Andre Nathan**
Universidade Federal
do Rio de Janeiro

Valmir C. Barbosa*,**
Universidade Federal
do Rio de Janeiro

Abstract We consider flocks of artificial birds and study the emergence of V-like formations during flight. We introduce a small set of fully distributed positioning rules to guide the birds' movements and demonstrate, by means of simulations, that they tend to lead to stabilization into several of the well-known V-like formations that have been observed in nature. We also provide quantitative indicators that we believe are closely related to achieving V-like formations, and study their behavior over a large set of independent simulations.

Keywords

Artificial birds, boids, flocking, V-like flight formations

1 Introduction

Migratory birds in flight often group into V-like formations, that is, they tend to self-organize into a flight formation having very few individuals in lead positions while the others group behind them near the edges of imaginary trailing V's whose vertices are the lead birds. While it seems to be a consensus that group flight provides greater protection against predators, the reasons for the particular grouping into laterally slanted, nearly straight lines have elicited two distinct lines of explanation, following the seminal works in [12] and [5], respectively.

The first one has evoked the aerodynamics of bird flight, whose essence is that each flying individual creates an upwash region behind it, just off the tips of its wings, such that another individual benefits greatly (in requiring less exertion during flight) if it places one of its wings in that region. For large birds, which are the ones that have been observed to group into V-like formations most often, this seems to be the main reason for formation flight [2, 6, 8, 19, 22]. The second, competing line of reasoning has been that flying in a somewhat skewed position relative to the bird in front is crucial for an individual's orientation, in addition to allowing unhindered visual communication and therefore helping avoid collisions. For small birds, the aerodynamic benefits are less relevant, and moreover, the observed lateral and longitudinal separations between nearest birds seem to be correlated in a way that strongly supports this vision-related explanation [3].

In this study, we consider simple artificial birds and address the question of whether positioning rules exist that can lead a flock to settle into some stable V-like formation during flight. Such rules should be founded on a blend of the basic explanatory trends of placing more weight on aerodynamics- or vision-related justifications, and thus consonant with the current belief that the traits that allow natural birds to take advantage of one or the other benefit evolved concomitantly [15]. They should, in addition, be inherently distributed, so that sensory information alone guides each bird's course of action, and also motivated by the birds' innate drive at flying as a flock. As we demonstrate

* Corresponding author.

** Universidade Federal do Rio de Janeiro, Programa de Engenharia de Sistemas e Computação, COPPE, Caixa Postal 68511, 21941-972 Rio de Janeiro - RJ, Brazil. E-mail: andrenth@cos.ufrj.br (A.N.); valmir@cos.ufrj.br (V.C.B.)

in the remainder of this article, rules with these characteristics that are both robust (in the sense of ensuring that stable configurations can be expected to be reached) and flexible (allowing for a rich variety of V-like formations) do exist.

Our work is preceded by important related contributions, notably by the simple rules in [16], which in essence say that birds should avoid colliding with one another while attempting to maintain the same pace as the nearest birds as well as seeking to be positioned as near to the center of the group as possible. Such rules are not meant to give rise to V-like formations, only to the flight as a group itself. Adding to them the further rule that a bird should strive to keep some portion of the visual field unobstructed is reported to have caused some of the birds to coalesce into fragmented V-like formations [4], but the whole approach seems to have remained uninformed of the sensory input that birds derive from favorable aerodynamics. We also mention the fuzzy rules introduced in [10, 11], but they fail to induce V-like formations, except in the case of very special initial conditions. Less related to this study, but very relevant nonetheless, are mathematical analyses of flocking, both in the sense of [16] (cf., for example, [21] and [13], the latter partly based on [14]) and under the assumption of a V formation (e.g., [20]).

We proceed by introducing a new set of rules and our model of an individual bird in Section 2. We then give a simulation algorithm, along with results, in Section 3. Our emphasis is on the emergence of V-like formations, so both our model of a bird and our simulation algorithm contemplate only what we deem to be essential from such a perspective. In particular, we assume for simplicity that a bird does not turn when altering its position laterally. Likewise, we also assume that flocks move on a straight path at constant speed, so it suffices for our simulation algorithm to target the birds' movements relative to one another only. We conclude with discussion and comments on further work in Section 4.

2 Positioning Rules

Our own set of rules is very succinct, being composed of only three rules, each related to one of the three guiding principles discussed earlier (birds should flock, be afforded some unobstructed view in the direction of flight, and benefit from regions of upwash). They are the following.

Rule 1 (coalescing rule): Seek the proximity of the nearest bird.

Rule 2 (gap-seeking rule): If Rule 1 is no longer applicable, seek the nearest position that affords an unobstructed longitudinal view.

Rule 3 (stationing rule): Apply Rule 2 while the view that is sought is not obtained or the effort to keep up with the group decreases due to increased upwash.

These three rules are an informal expression of our understanding of how each of the guiding principles influences a bird's actions. First of all, Rules 1 and 2 imply that we make a distinction between two modes of behavior, one succeeding, or sometimes alternating with, the other: Birds are to seek the closeness of the group first, and only then seek being afforded some clear view. Secondly, Rules 2 and 3 imply that the sensory input that keeps the bird moving toward a relative position of clear view is not only visual but also originates in the ease with which it is keeping up with the group.

Rules 1–3 require further specification before we describe our computational experiments and their results. Note, first, that both Rule 1 and Rule 2 make implicit reference to the region of space where some entity is to be sought (the nearest bird in Rule 1, the nearest position with a clear view in Rule 2). We assume, in both cases, that this region is delimited, on the plane of movement and both to the right and left of the direction of movement, by an angle $\alpha/2$. We assume also that $\alpha \leq 180^\circ$ (this is in spite of the fact that many birds in nature have visual fields delimited by larger angles, since α does not delimit a bird's visual field but the region where it is to seek the input needed by Rules 1 and 2).

In a similar vein, Rules 1 and 3 allude, also implicitly, to the existence of a closed region surrounding each bird inside which proximity is attained (Rule 1) or upwash is found (Rule 3). Given the aforementioned constraints on α , and disregarding as negligible the upwash that some authors claim may be generated by a flying bird even in the space just ahead of it [1, 20], we assume that this region is as follows, with reference to Figure 1 [in this figure, as in others to come, an artificial bird is depicted as a filled circle (the bird’s body) with two protruding straight-line segments (its wings)]. If i is the bird seeking the proximity of another (say j) in Rule 1, then proximity is attained if the body of i is in $U_j^- \cup D_j \cup U_j^+$. As for finding upwash in Rule 3, all that is needed is that any portion of bird i , a body or a wing segment of any size, be in $U_j^- \cup U_j^+$ while no portion is in D_j . In this case, the optimal relative placement of i and j (i.e., the relative placement for maximum upwash) occurs when the lateral separation between them is $\lambda = (\pi/4 - 1)w/2 \approx -0.1073w$, where w is a bird’s wingspan, assumed the same for all the birds [8, 9, 17] (the birds’ longitudinal separation inside $U_j^- \cup U_j^+$ is thought to be irrelevant for maximum upwash [12, 8, 17]). So, for optimality, i and j must have laterally overlapping wings.

3 An Algorithm and Computational Results

Our simulation algorithm operates on n birds and runs from some initial positioning of the birds inside a unit square in two-dimensional space. Birds are in no way constrained to flying inside this unit square, even though by Rule 1 it seems reasonable to expect that a fixed-size square exists that, at all times, encompasses all birds. They may then be thought of as inhabiting unbounded two-dimensional space.

We assume that the lateral and longitudinal directions are parallel to the original unit square’s sides. In order to move according to Rules 1–3, a bird’s position relative to the others must undergo lateral as well as longitudinal displacements. Except for the occurrence of these displacements, we assume that birds have no lateral speed and that they all fly at the same longitudinal speed, giving rise to a common velocity vector that we henceforth refer to as the baseline velocity. Of course, each bird’s actual velocity vector at a given instant is the sum of the baseline velocity and the vector comprising the bird’s lateral and longitudinal rates of displacement at that instant.

For the sake of increased computational efficiency during simulation, it then suffices to subtract the baseline velocity off all birds’ velocity vectors and handle displacement rates only. To an observer looking down on the birds orthogonally to the plane of movement and moving synchronously with them (i.e., along with the group’s center or at the baseline velocity), a sequence of snapshots of the group will seem to result from the birds’ movements relative to one another only, even though they are indeed traveling through space and they are capable of changing their velocity vectors, both in magnitude and in direction, through time (forthcoming Figures 3 and 4 show some of these snapshots).

Each simulation runs through T time steps of fixed duration, so the handling of displacement rates may be further simplified so that only the actual displacements per time step need in fact be handled. We implement displacements by simply adding (or subtracting) a fixed amount $\Delta x > 0$ (laterally) or $\Delta y > 0$ (longitudinally) to a bird’s position per time step.

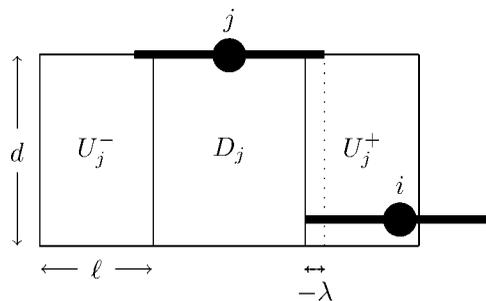


Figure 1. A flying bird j ’s upwash (U_j^- and U_j^+) and downwash (D_j) regions. Bird i is positioned for maximum upwash.

Table 1. Parameter values ($\times 1/768$).

Parameter	Value	Description
l	30	Lateral size of upwash region (cf. Figure 1)
d	50	Longitudinal size of upwash region (cf. Figure 1)
w	50	Wingspan
Δx	3	Lateral displacement per time step
Δy	3	Longitudinal displacement per time step
ϵ	9	Margin for longitudinal collision

At each time step, a bird follows Rule 1, if applicable, by altering its position via $\pm \Delta x$ or $+\Delta y$. If Rule 1 is not applicable, then applying Rule 2 (and controlling its application via Rule 3) requires a precise criterion for seeking “the nearest position that affords an unobstructed longitudinal view.” Let us define a *gap*, in the current time step, as any lateral span that contains the lateral coordinate of no portion of any bird. A gap is *maximal* if it is contained in no other gap; maximal gaps, therefore, may be regarded as inducing longitudinal stripes of empty space, each stripe delimited by some bird’s wingtip on at least one side. Our algorithm implements the criterion required by Rule 2 as follows. Let i be the bird in question, and consider all maximal gaps that are at least $w + 2\lambda$ wide (this includes the two outermost gaps, which are infinitely wide). The position that i seeks is given by the nearest wingtip delimiting such a gap, provided the width- $(w + 2\lambda)$ extension of that wing into the gap can be seen by i without obstruction. Aiming at that position, $\pm \Delta x$ is applied as needed, and eventually Rule 1 may once again become applicable. As for what remains unspecified of Rule 3, our algorithm implements the lateral separation by λ directly (i.e., Rule 2 is applied if upwash has not been found or is not optimal).

Collisions are avoided at all time steps whenever attempting a displacement. Two birds are allowed to get as close to each other as having no lateral separation between them or a longitudinal separation of ϵ . If the desired displacement infringes this, then it is not effected and the bird simply chooses randomly a longitudinal displacement of $+\Delta y$ or $-\Delta y$, which is only applied if possible. In our algorithm, this possibility of displacing a bird longitudinally by $-\Delta y$ represents the only source of longitudinal deceleration.

We have conducted extensive simulations for $n = 15, 25, 35$ and $\alpha = 170^\circ, 180^\circ$ (the production of V-like formations in large flocks is inherently difficult and is rarely observed in nature [18],¹ whence our moderate choices for the value of n). All simulations used $T = 2,000$ and the values listed in Table 1 for the distance-related parameters. For a quantitative evaluation, we concentrate on the five indicators listed below. Results for these five indicators were obtained as averages over 1,000 independent simulations, each one starting at a random placement of the n birds inside the unit square.

Time for stabilization: Number of time steps until all birds stop moving relative to one another (taken as T for the simulations in which this does not happen).

Number of lead birds: How many birds intersect no other’s upwash regions.

Number of unconnected groups: A set of birds is counted as an unconnected group if no bird outside it intersects any of its own birds’ upwash regions, and conversely [e.g., the situation in Figure 4(c) contains two groups].

¹ Except for small birds [1], in which case V-like formations, as we have noted, seem not to originate primarily from aerodynamics-related gains. In fact, large formations of small birds are characterized by lateral separations that are inherently incompatible with λ , the optimum predicted by aerodynamics-based analyses.

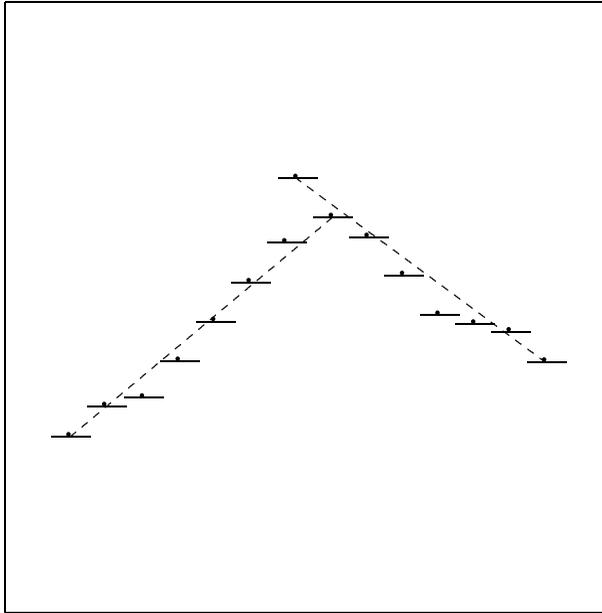


Figure 2. Flight formation with two straight-line segments (shown in dashes).

Number of straight-line segments: A straight-line segment joins a trailing bird—one whose upwash regions are not intersected by another bird—to either a lead bird or a *bifurcation* bird—one whose upwash regions are intersected by two other birds (e.g., the situation in Figure 2 contains two straight-line segments).

Mean distance to nearest straight-line segments: Each of the n birds contributes one distance to the mean, unless it is a lead or bifurcation bird, in which case, it contributes the average distance to the nearest two segments.

Except for the time for stabilization, all indicators refer to the end of the simulation.

Two snapshot sequences, in the sense discussed earlier, are shown in Figures 3 and 4. The former sequence depicts, for six time steps during the same simulation, the evolution of the birds' positions from their initial, randomly determined, situation through the eventual V-like formation. The latter sequence is a sampler of the formations achieved after T time steps. The averages for the five indicators are shown in Figure 5.

4 Discussion and Further Work

The plots in Figure 5 reveal that, for $\alpha = 180^\circ$, the average formation into which the n birds settle seems to become stable relatively quickly, and also to be evocative of the canonical V formation: one lead bird (hence one single group), and two straight-line segments. Decreasing α to 170° delays stabilization significantly while allowing for an increased number of lead birds and of unconnected groups, and consequently of straight-line segments as well.² Behind all these effects are the facts that, with the smaller angle, a bird tends to have a more limited set of choices for the nearest bird needed in Rule 1, and also fewer maximal gaps for use in the implementation of Rule 2. Upon reaching stability, however, the number of birds clustered around each straight-line segment tends to be smaller, and thus, so does the mean distance between the birds and the nearest segments.

² We remark that our results for $\alpha = 170^\circ$, in contrast with those for $\alpha = 180^\circ$, are qualitatively representative of what we have observed down through roughly $\alpha = 120^\circ$. Thus $\alpha = 180^\circ$ is a special case.

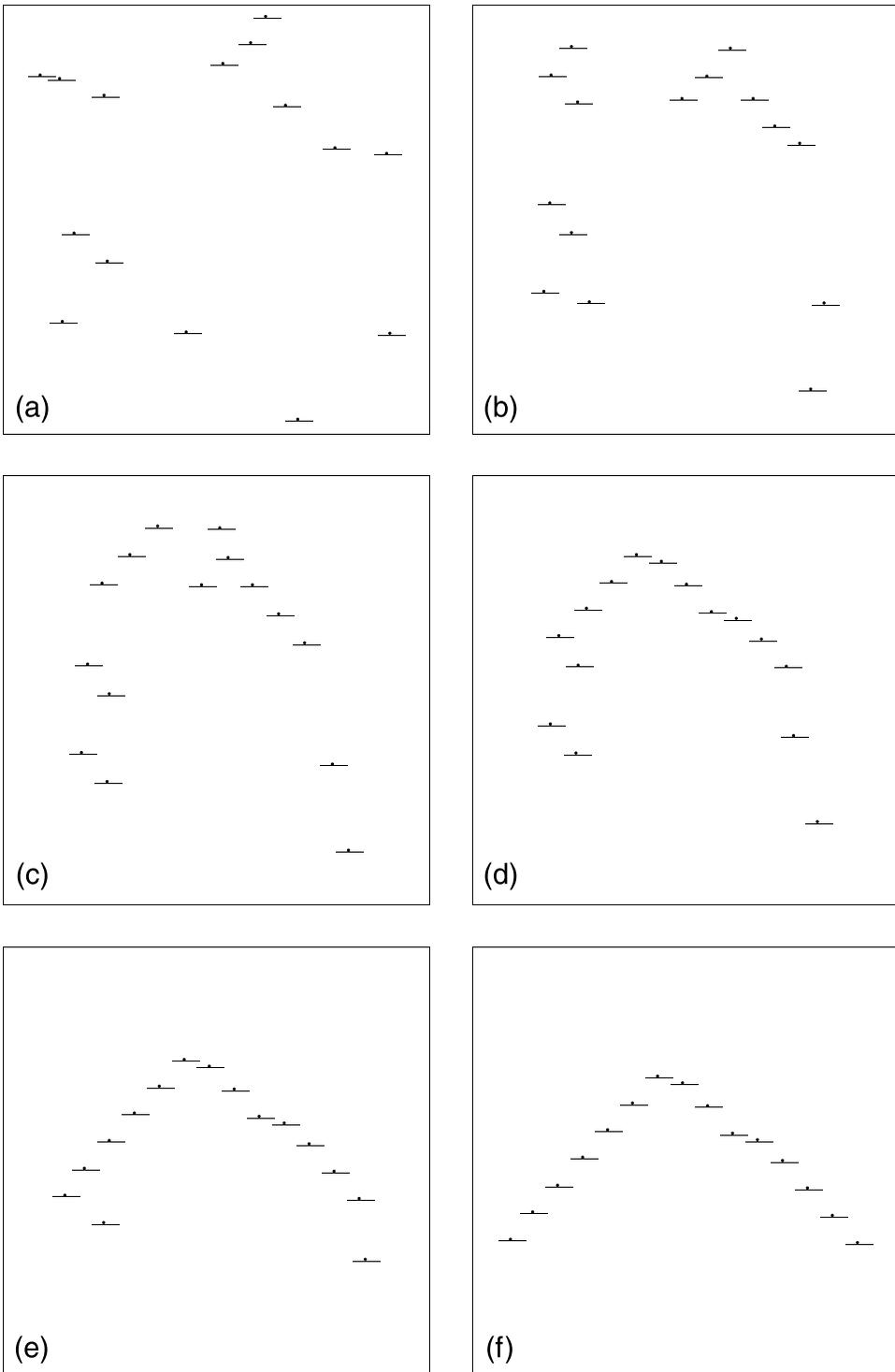


Figure 3. Sample evolution of relative positions for $n = 15$ and $\alpha = 180^\circ$. Time increases from (a) to (b) by 40 time steps, from (b) to (c) by another 40, and so on.

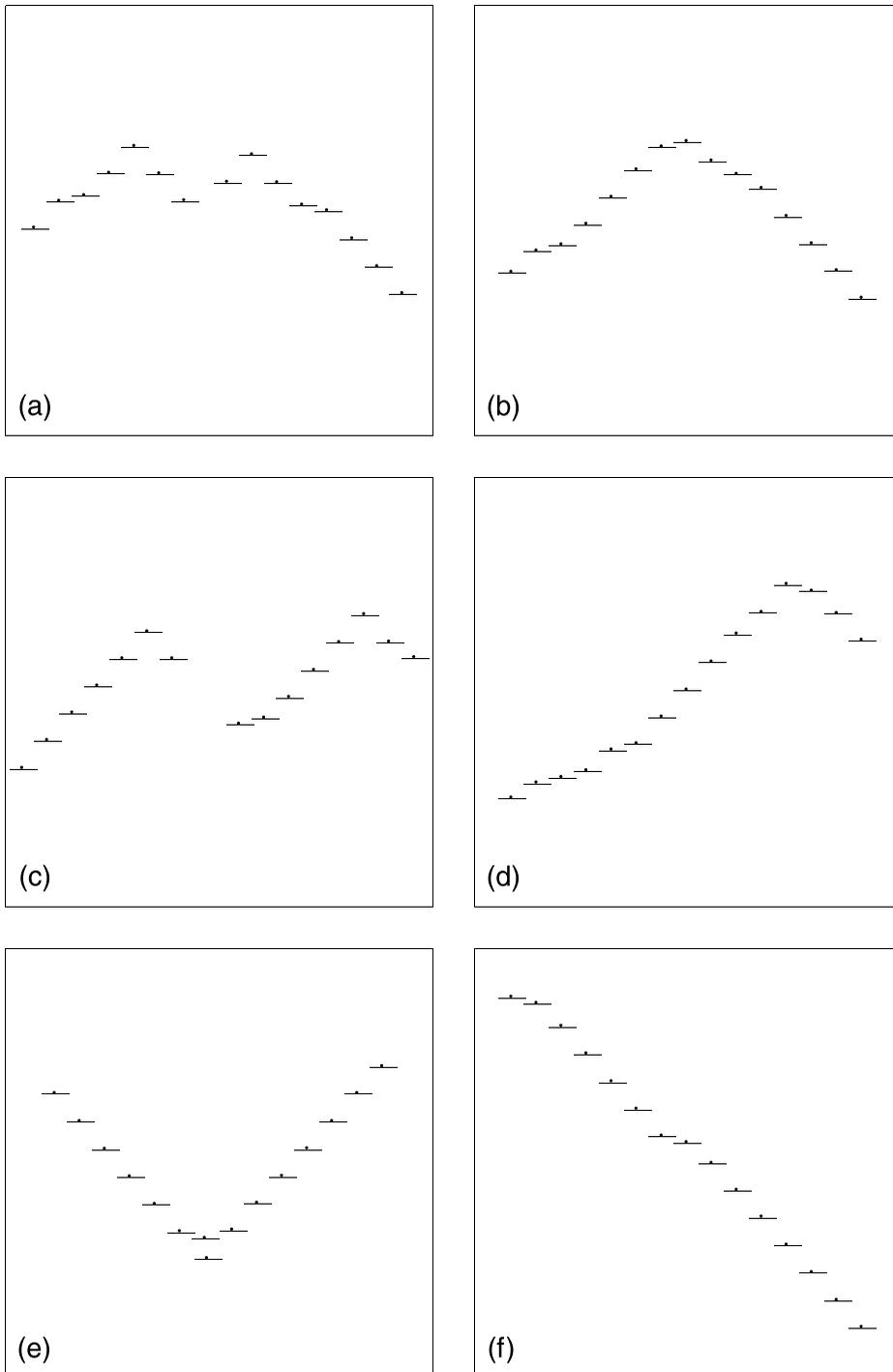


Figure 4. Sample formations after $T = 2,000$ time steps from independent, random initial placements for $n = 15$, with $\alpha = 170^\circ$ (a, c, e) and $\alpha = 180^\circ$ (b, d, f). Samples include the celebrated W (a) and V (b) formations, in addition to a formation into multiple unconnected groups (c) and those formations that in [7] have been called the J (d), the inverted V (e), and the echelon (f).

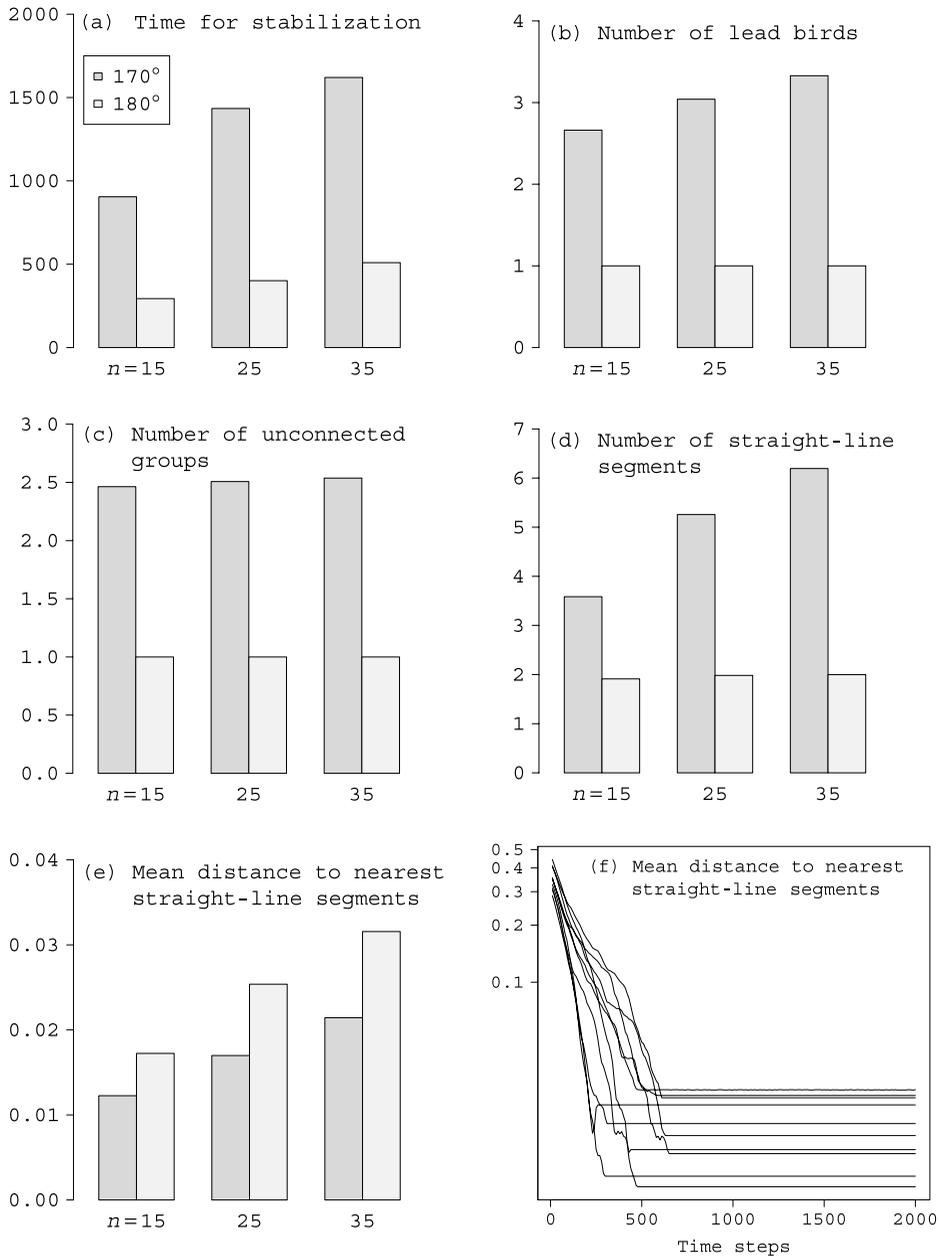


Figure 5. The five indicators, averaged over 1,000 independent simulations (a–e), and the time evolution of the fifth indicator for ten randomly chosen simulations with $n = 25$ and $\alpha = 170^\circ$ (f).

Figure 5(f) offers a closer glimpse into 10 (out of the 1,000) simulations for $n = 25$ and $\alpha = 170^\circ$. The figure indicates that, even though, at this reduced angle, stability is on average harder to achieve, there are simulations in which the mean distance to the nearest straight-line segments drops nearly vertiginously. This suggests, as in fact we have confirmed by examining all the data more closely, that the average time for stabilization is being influenced by some simulations that did not fully stabilize each contributing $T = 2,000$ time steps to the average.

While we believe our results support the view that Rules 1–3 are successful in helping explain the emergence of V-like formations, we also recognize that the possibilities for further investigation

are numerous. These include a more detailed study of how different parameter values in our simulation algorithm affect the results,³ and also the possibility of studying other algorithmic approaches to realizing the same set of rules. Some of these other approaches might, for example, introduce a new parameter to quantify the “unobstructed longitudinal view” mentioned in Rule 2; such a parameter would, essentially, substitute bounded regions of space for the maximal gaps of our current algorithm, which are longitudinally unbounded. They might also adopt a perturbation model to continually effect small fluctuations in the birds’ positions so that stability would never really be achieved; interesting questions in this case would be whether V-like formations would still occur, how persistent they would be, and how they would change with time.

Rules 1–3 themselves may be put through ever more stringent tests. One clear possibility is to consider them in three-dimensional space and to investigate whether roughly planar V-like formations still emerge. We find it unclear at this point whether the three rules will hold unchanged since a three-dimensional proximity model will be needed for use with Rule 1, and likewise, three-dimensional models of vision obstruction and of flight aerodynamics will be needed for use with the other two rules.

To conclude, we remark on the interesting possibility of lifting the constraints, imposed at the beginning, that birds do not turn and that flocks are restricted to moving at constant speed on a straight path. These were adopted because the flexibility that they preclude seems to us to be only mildly related to the emergence of V-like formations, and also because they afford greater simplicity during simulation. But clearly it is desirable to probe further by abolishing the two assumptions and investigating how close Rules 1–3 come to affecting the flock as we have observed.

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³ For example, allowing λ to deviate from its purported optimal value near $-0.1073w$, or varying the value of d , is expected to influence the various angles in the resulting V-like formations.

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