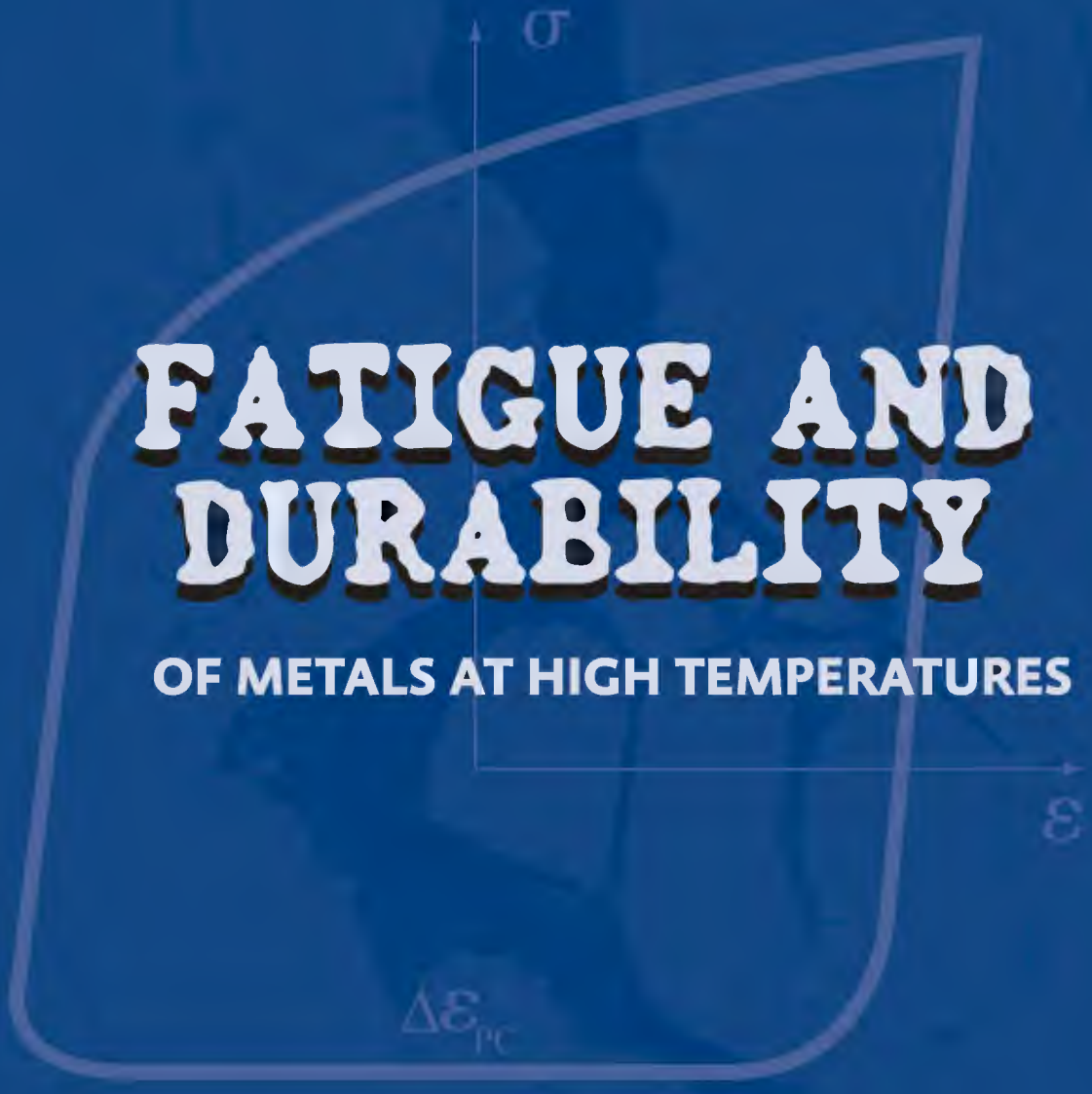

FATIGUE AND DURABILITY

OF METALS AT HIGH TEMPERATURES

S.S. MANSON
G.R. HALFORD

ASM INTERNATIONAL

Fatigue and Durability of Metals at High Temperatures

S.S. Manson
G.R. Halford

ASM International®
Materials Park, Ohio 44073-0002
www.asminternational.org

First printing, July 2009

Comments, criticisms, and suggestions are invited, and should be forwarded to ASM International.

Prepared under the direction of the ASM International Technical Book Committee (2008–2009), Lichun L. Chen, Chair.

ASM International staff who worked on this project include Scott Henry, Senior Manager of Product and Service Development; Steven R. Lampman, Technical Editor; Ann Britton, Editorial Assistant; Bonnie Sanders, Manager of Production; Madrid Tramble, Senior Production Coordinator; Diane Whitelaw, Production Coordinator; and Kathryn Muldoon, Production Assistant

Library of Congress Control Number:2009923304
ISBN-13: 978-0-87170-718-5
ISBN-10: 0-87170-718-7
SAN: 204-7586

ASM International®
Materials Park, OH 44073-0002
www.asminternational.org

Printed in the United States of America

We dedicate this book to our beloved grandchildren

Shira Lisa Entis
Jonathan Joshua Entis
Chloe Teressa Green
Alexandra Eugena Green
Jennifer Lisa Ames
Jeffrey Manson Ames
Erin Lea Stone
Jeremy Francis Stone
Paul Isaac Manson
Cecily Rose Manson
Joshua Simon Ames

Brian Patrick Packert
Brendan James Packert
Colin Richard Packert
Drew Joseph Halford
Grace Charlotte Halford
Rose Elise Halford

Contents

Preface by S. S. Manson ..... **vi**
Preface to First Volume by S. S. Manson ..... **vii**
Preface to First Volume by G. R. Halford ..... **viii**
About the Authors ..... **ix**

Chapter 1 Creep Under Monotonic and Cyclic Loading **1**

Chapter 2 Creep Rupture **21**

Chapter 3 Strain-Range Partitioning—Concepts and Analytical Methods **43**

Chapter 4 Strain-Range Conversion—An Extended View of Strain-Range Partitioning **69**

Chapter 5 Partitioning of Hysteresis Loops and Life Relations **83**

Chapter 6 Total Strain-Based Strain-Range Partitioning—Isothermal and Thermomechanical Fatigue **111**

Chapter 7 Treatment of Multiaxial Loading **155**

Chapter 8 Critique of Predictive Methods for Treatment of Time-Dependent Metal Fatigue at High Temperatures **173**

Chapter 9 Obstacles to High-Temperature Structural Durability of Continuous-Fiber Metal-Matrix Composites **223**

Chapter 10 Aerospace Applications—Example Fatigue Problems **231**

Index **259**

Preface by S.S. Manson

When Dr. Halford and I undertook to write this book, more than two decades ago, we intended that it be all-inclusive, covering our experience in the subject of fatigue and durability. We intended that it all be in one volume. As we prepared it during the ensuing years, it became clear that the subject was too broad to be contained in one volume, especially if we were to include the relevant work of our colleagues in other institutions, even in other countries. We decided that at least two volumes would be needed, with this second volume to be dedicated to high-temperature aspects, which are gaining more and more attention with the technology expanding in that direction.

This second volume gave Dr. Halford the opportunity to address in greater depth two subjects that were of intense interest to him: treatment of strain-range partitioning using the total strain-range approach (so that the application could be extended to cases involving small strains), and devoting more discussion to applications of our new technology to practical problems on which he was daily assisting industry. We intended each of these subjects to cover long chapters. He did write most of these two chapters, but his untimely passing prevented him from completing them. On Oct. 25, 2006, he passed away. I have not altered the chapters; therefore, the volume is left diminished by the sad turn of events. The entire field covered by his work is left deprived by his passing. And, I have been deprived of a colleague, and a dear friend.

The passing of Dr. Halford before the book was completed necessitated many changes in the final processing. His numerous colleagues and my former students kindly volunteered to be as helpful as they could to complete the final editing. I wish to extend my sincere gratitude to the following for their gracious contributions throughout the writing of this book and especially in the final editing:

Dr. Sissay Hailu
Dr. Michael Mitchell
Dr. Ramesh Kalluri
Dr. Robert Bill
Kejin Jung
Marvin Hirschberg (Dec)
And, of course, the ASM technical editor Steven Lampman

S.S. Manson
Feb. 5, 2009

Preface to the First Volume by S.S. Manson

The past half century has witnessed a virtual revolution in the development of two fields which are the subject of this book: the introduction of advanced materials as structural components in severely loaded machines exposed to high temperatures and temperature gradients, and the development of technology of life computation for such components, of which one of the major failure mechanisms is fatigue. This book is based on the experience of the authors during this period. Although it emphasizes our research both as individuals and as colleagues for half a century, it also includes the work of numerous others who have provided useful results that have moved progress in these fields.

My first report on fatigue appeared in 1953. An intense interest and activity in this rapidly changing field has continued since. Collaboration with Dr. Gary Halford started in 1966 when he joined NASA at its Cleveland center where I served as Chief of the Materials and Structures Division. This cooperation continued after I retired in 1974 to join the faculty of Case Western Reserve University, and even after I retired from CWRU two decades later. We started to write this book well before I left CWRU. Thus, this book has been in the making for a long time, perhaps longer than we care to admit. But to compensate for the slowness of its progress toward publication, it is fair to say that we have been continuously adding content from our own research, and from that developed elsewhere, as warranted.

Initially this book was prepared as a text on fatigue, and its content fashioned after my regular curriculum presentations at Case Western Reserve University, short course presentations at the Pennsylvania State University, and shorter presentations at MIT, The Technion in Israel, and numerous other universities. In later-year presentations it was broadened under the title *Relation of Materials to Design* to include content developed at NASA. Its current context is still largely related to fatigue but includes other subjects representative of the material presented in these courses.

I am grateful to NASA for the support it has rendered me during my employment there, and later in grants provided to continue my activities initiated there. I am also grateful to the Oak Ridge National Laboratory, the Electric Power Research Institute, and the Metals Properties Council for their grants to conduct the research described in this book. My most heartfelt gratitude is expressed toward my co-author, Gary R. Halford. It has been a genuine joy to work with him as a colleague, friend, and co-author.

As always, I express my deep appreciation to the Almighty for the gift of life and long-time participation in the developments contained in this book.

S.S. Manson
December 2005

Preface to the First Volume by G.R. Halford

This book and a planned second volume dealing with high-temperature durability represent the culmination of many years of collaborative research with my highly respected colleague, S.S. Manson. Few researchers have had the luxury of being able to work together continuously for as long as we have. And few colleagues have been able to work together as amicably as we have. We were fortunate to be involved in numerous advancements to the field through individual and joint publications spread over five decades. Our combined years of experience exceeds a century. This book provides a repository of the most significant of our contributions to the art and science of material and structural durability. Valuable contributions from other researchers are also included as appropriate.

I cannot sufficiently thank NASA for the rare opportunity provided me to have been allowed to work in this field for the duration of my employment. A prime advantage provided by a large government research organization was that we had valuable technical contacts with not only the aerospace industry, but also with many other industries, including electric power generation, off-highway and automotive manufacturing, metals producers, chemical and petroleum producers, and numerous other industries that faced serious material and structural durability issues. We were thus privileged to have exposure to countless durability issues of a diverse nature. From such a vantage point, it was possible to develop generic models having a broad range of applicability.

I would also like to thank the University of Illinois in Urbana-Champaign, its Department of Theoretical and Applied Mechanics, and in particular, Professor JoDean Morrow. I could never have been in a position to participate in this work without their providing me with the appropriate educational background. Finally, my late parents, Herbert C. and Faye S. Halford, brother Donald W. Halford, my wife, Pat M. Halford and our children, Kirk, Gwen, and Shawn must be acknowledged for instilling me with balanced senses of patience, work ethic, responsibility, dedication, and respect—all interspersed with a tinge of humor.

Gary R. Halford
December 2005

About the Authors

S.S. Manson is Professor Emeritus, Case Western Reserve University. Professor Manson joined the National Advisory Committee for Aeronautics (the precursor to NASA) at Langley, VA in 1941 and transferred to Cleveland in 1943. There, he performed cutting-edge theoretical and experimental stress analysis and durability research associated with the materials used in piston engines and the newly evolving gas turbine engines. His research interests drew him into the entirely new area of low-cycle fatigue, particularly thermal fatigue. The basic law of low-cycle fatigue that he developed remains in use 50 years later, i.e., the Manson-Coffin law. His research expanded into the study of creep, creep-rupture and time-temperature parameters, for which he created several of great practical value. He has received numerous awards for his work, including the Gold Medal from the Franklin Institute for development of the Manson-Coffin law of low-cycle fatigue, the NASA Exceptional Scientific Achievement Award, and the Nadai Award bestowed by the American Society of Mechanical Engineers. His book *Thermal Stress and Low-Cycle Fatigue* was published in 1966. He remained at NASA until 1974, serving most of the time as Chief of the Materials and Structures Division. At that time, he moved on to become Professor of Mechanical and Aerospace Engineering at Case Western Reserve University. There he continued to teach on the subject of the mechanical behavior of materials and perform research together with his students and colleagues to develop better durability lifing models. He currently lives in California.

G.R. Halford was a Distinguised Research Associate, NASA Glenn Research Center, Cleveland, Ohio. Following his education in the Department of Theoretical and Applied Mechanics at the University of Illinois under the guidance of Professor JoDean Morrow, he joined the NASA Center in 1966. Dr. Halford, in conjunction with Professor S.S. Manson, was actively involved in research and development of advanced life prediction methods for low- and high-temperature fatigue analysis of high-performance mechanical systems. Most notable is the total strain version of the method of strain-range partitioning (SRP). That methodology sees use in several industries. Dr. Halford was involved with durability issues in virtually every

propulsion system of interest to NASA. In the aeronautics arena, he dealt with subsonic, supersonic, and hypersonic propulsion systems. In space propulsion and power, he dealt with ion engines, solid propellant rockets, liquid rockets of all sizes and description, as well as solar and nuclear energy conversion and storage systems. The severe durability limitations of these systems spawned much of his research into advanced life prediction methods that are the subject of this book. Dr. Halford authored or co-authored over 200 technical papers, coordinated over 60 grant/contractor reports, edited several technical conference volumes, and delivered over 70 invited technical lectures.

CHAPTER 1

Creep Under Monotonic and Cyclic Loading

CREEP is time-dependent deformation that occurs at high temperature relative to the melting point of metallic materials. The creep regime for metals is commonly regarded to begin at a temperature of approximately half the absolute temperature (degrees Kelvin or Rankine) of the metal melting point. For example, the Rankine (R) scale of absolute temperature is related to degrees Fahrenheit as follows: R = 460 + T (in °F). Thus, the approximate onset of the creep behavior would be at temperatures greater than [(T_m/2) – 230], where T_m is the melting point in degrees Fahrenheit. For a melting temperature of 2600 °F, the creep range would begin at a temperature of approximately 1070 °F (575 °C, or 850 K).

However, this demarcation point is not exact, and the designer must often consider lower operating temperatures to preclude the occurrence of creep. Some special metals, such as titanium, creep at temperatures of only 35% of their absolute temperature melting point. Nickel- and cobalt-base superalloys, as well as eutectic alloys such as 63Sn-37Pb solder, are creep resistant up to approximately 75% of their melting point. For polymers, a melting point does not exist in the same sense as it does for metals. Instead, the dividing temperature region between being brittle (glassy) and being very pliable (rubbery) is the glass-transition temperature. Above this temperature, polymers become weak and deform readily in a time-dependent manner (creep). Unless polymers are chemically designed specifically to resist high temperature (e.g., Kevlar,™ Dupont), creep deformation typically occurs at room or slightly elevated temperatures. In any case, for each material there is a minimum temperature above which creep should be considered.

Special Aspects of Strain in the Creep Range

As emphasized in the companion volume on room-temperature fatigue (Ref 1.1), inelastic strain causes the degradation of a material that eventually results in fatigue crack initiation, propagation, and fracture. Inelastic strain also results in time-dependent creep deformation and rupture, as explained in this volume on fatigue at elevated temperatures. At room temperature, inelastic deformation is by external and internal forces that move the atoms around, causing them to assume new positions within the crystalline lattice. Dislocations assist in the atomic motion, as has been described in the Appendix and in Chapter 10 of Ref 1.1. At high temperature, atomic motion is assisted by the greater vibrational frequency of the atoms about their equilibrium positions. In fact, temperature is a measure of the amplitude of such vibrations; the higher the temperature, the greater is the amplitude of vibration. At high temperature, atoms can overcome the obstacles that prevent their movement by bringing to bear their energy of vibratory motion. The atoms can then move the obstacles in their plane, or they can even climb to a new plane to bypass an obstacle. Thus, at high temperature, inelastic deformation can occur that involves mechanisms that are not available at low temperatures.

Activation Energy. Physicists usually explain high-temperature deformation phenomena by introducing the term called activation energy. Activation energy is a measure of the resistance of a material to undergo distinct events at the atomic level. It may be regarded like the admission charge for participating in an event.

Chemical reactions, diffusion, and creep are common examples of such events. Activation energy for self-diffusion is considered first, because this energy has a major influence on the creep resistance of materials. A great amount of research has been performed by physicists using pure metals.

Activation energy, ΔH, for self-diffusion is closely related to the energy to break atomic bonds and is a material constant. High activation energy requires a high energy source, such as temperature, to provide the driving force to induce atomic events. Bonds at a free surface are easier to break than those along a grain boundary; hence, surface diffusion occurs more readily than diffusion along grain boundaries. Similarly, atomic bonds at grain boundaries are weaker than bonds within the interior of a grain, leading to higher rates of diffusion along grain boundaries.

To overcome the activation energy, it is necessary to provide excitation energy. Thermal energy is the major driving force for the processes of immediate interest. Consequently, self-diffusion and creep are referred to as thermally activated processes. Absolute temperature, T, is a direct indicator of thermal energy, expressed as kT, where k is the Boltzmann's constant. The ratio of the activation energy to the excitation energy, ($\Delta H/kT$), is a key factor in dictating the rate of occurrence of events. The actual rate is described by an exponential equation where the rate of diffusion is proportional to $exp(-\Delta H/kT)$. This classical relationship for thermally activated processes is called the Arrhenius equation, after the Swedish chemist (1859–1927) who recognized the applicability of this empirical equation to many chemical processes that are time and temperature dependent. The true behavior at the atomic level is governed by the laws of statistical quantum mechanics, a topic well beyond the scope of the current chapter.

To aid in the visualization of how thermally activated, diffusion-controlled processes involve activation energy, a highly simplified mechanical model is sketched in Fig. 1.1(a). Physicists may regard this simplistic model as unnecessary. However, it is a helpful analogy for engineers to grasp the macroscopic causes and effects due to thermally activated creep. A ball, representing an atom of mass m, sits in a trough with troughs on either side. A measure of the activation energy ΔH for the ball to jump to another trough is the potential energy required to lift the ball up and over the barrier of height h. If the troughs

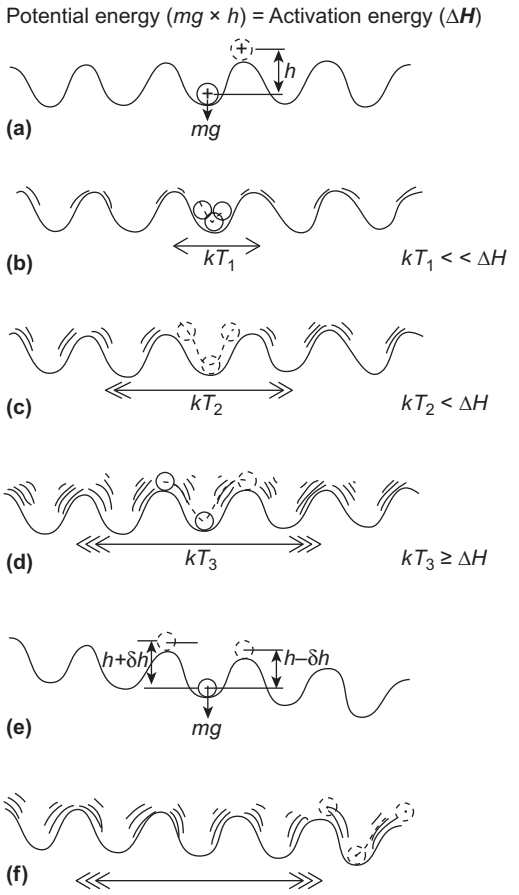

Fig. 1.1 Schematic of thermal activation energy using a mechanical energy analogy

are oscillated back and forth at temperature T_1 with an energy representative of the thermal energy kT_1, the ball will roll back and forth, forcing it to roll part way up the sides of the trough (Fig. 1.1b). With greater excitation energy, as indicated by the increased amplitude of oscillation (simulating increased atomic activity as temperature is increased to T_2), the ball will roll even further up the side of the trough (Fig. 1.1c). With further increase in temperature to T_3, the point is eventually reached where the ball can pass over the top edge of the barrier (Fig. 1.1d), enabling it to drop into the adjoining trough, and the event is completed. At this temperature, the available excitation energy kT_3 equals or exceeds the activation energy ΔH required of the event. (The simple model gives the erroneous impression that no diffusion occurs at the macroscopic level until a discrete temperature is reached. In fact, discrete behavior occurs only

for any one given atom out of the billions and billions of atoms participating in the process. While each atom requires the same action energy to permit a jump, the nature of the thermal excitation is not discrete. A broad statistical spectrum of "temperatures" exist at the individual atom level. The vibrational activity of each atom can be significantly different from its siblings. This variation gives rise to a few atoms jumping at a relatively low statistically average temperature. Similarly, it gives rise to a few atoms not jumping, even at very high temperatures. As a result, there is no discrete temperature at which diffusion commences. The Arrhenius equation describes the spectrum of temperatures over which diffusion transitions from negligible to dominant. It should also be recognized that lattice site jumping is not done by an atom but rather by a vacancy, which is the absence of an atom at a lattice site.) As either the amplitude or frequency of oscillation is further increased, the time it takes to cause the ball to change troughs will decrease. In a given period of time, more jumps will take place. After a period of oscillations, the location of the ball will be random, since, in this simplified model, there is no bias to cause the ball to move either to the right or the left. This simple aid is adequate to envision the random process of self-diffusion, a process that results in the homogenization of the state of the material at the macroscopic level. Diffusion processes are the main requisites for creep. However, creep is also significantly affected by mechanical stresses applied to the atomic lattice; the greater the stress, the higher the rate of creep. The simple model thus requires modification to accommodate the influence of mechanical stress activation.

If a slight tilt is given to the troughs (Fig. 1.1e), the ball would have a small bias to progress more easily in the downhill direction than uphill. This is due to the slight difference in potential energy associated with uphill $h + \delta h$ and downhill $h - \delta h$ jumps. Thus, after a lengthy period of exposure to oscillations, the ball would eventually progress downhill to a lower potential energy state (Fig. 1.1f).

There is an analogy to the creep process during which diffusion is responsible for time-dependent strain occurring in the direction of the biasing creep stress. Obviously, if no stress is applied, there will be no creep; that is, the material will not change its dimensions with time. Application of stress, however, creates such a bias. This bias takes the form of a dilation of the atomic lattice, particularly at a diffusion-susceptible grain boundary having a component of normal tensile stress. The atomic dilation slightly reduces the atomic bond strength locally, thus negligibly reducing the apparent activation energy. However, the net result is that, for a constant thermal excitation, the creep rate will be greater the greater the biasing stress. When activation energies are determined experimentally, their numerical values are virtually insensitive to the minor mechanical energy contributions from stress. In fact, at low temperatures where the thermal excitation energy is very low, the application of stresses approaching the ultimate tensile strength of a material is insufficient to cause measurable diffusion-controlled creep. In other words, the mechanical excitation energy, even under extremely high-stress conditions, is usually insignificant compared to the required energy to activate significant diffusional processes associated with creep.

The tipping of the trough in the mechanical model shown in Fig. 1.1 is analogous to stress causing a biasing of the motion of the otherwise random atomic jumps during creep. At a given temperature and zero stress, atoms are in a constant stage of agitation and can jump randomly from one atomic position to another. The actual motion is really facilitated by the result of the motion of vacancies. They are present in the lattice in large numbers and are in thermodynamic equilibrium at high temperatures. The higher the temperature, the greater the number of vacancies. The end result of this random jumping process would be no net change of position of the atoms over a long period of time, unless a biasing stress is present.

The presence of a biasing tensile stress also alters the atomic spacing. Tensile stress opens the spacing of atoms in the direction of the stress (Fig. 1.1f), facilitating atomic jumps in that direction because less energy is required for diffusion in this dilated state. At the same time, the Poisson contraction perpendicular to the stress draws the atoms closer together in the transverse direction, making atomic jumps in that direction more difficult and hence less likely to occur. With more jumps of atoms into positions aligned with the tensile stress direction, the length of the material will increase in that direction. This extension is creep deformation. The greater the stress bias, a greater amount of creep strain accumulates in a given time; thus, the creep rate is greater. Similarly, at a given stress, the higher the temperature, the greater is the

thermal excitation energy and the more events that can occur in a given period of time (i.e., the creep rate increases).

The classical exponential rate equation of Arrhenius is used to relate creep rate to activation energy (ΔH) and thermal excitation energy (kT):

$$\text{Creep rate} \propto \exp(-\Delta H/kT)$$

If the temperature is decreased to a low enough level, the number of atomic jumps that are possible decreases to a point wherein the time between jumps becomes so great, from a practical engineering point of view, that creep does not occur. Thus, two major factors are involved at high temperature:

- The activation energy for diffusion
- The stress that biases the direction of the diffusion

For pure metals, there is a direct relationship between the activation energy for self-diffusion and creep, as shown in Fig. 1.2. The activation energy for creep of pure metals is approximately the same as the activation energy for self-diffusion, as demonstrated by Dorn (Ref 1.3) and his coworkers in 1956.

Types of Creep Involved in Engineering Analysis. In the engineering problems we shall discuss, consideration is given to four types of creep: primary, secondary, tertiary, and recovery, as shown in Fig. 1.3(a). The four curves, labeled curve 1 through curve 4, prominently display each of the four types of creep deformation with time at a constant tensile stress and temperature. For each curve, there is a point A that represents the instantaneous strains (both elastic and plastic) induced upon initial loading:

- *Curve 1*: For curve O_1A_1 in Fig. 1.3(a), the stress and temperature are too low to cause time-dependent deformation; only instantaneous elastic strain is present when stress is applied. Upon removal of stress, the strain would return to zero (point B_1).
- *Curve 2*: For curve $O_2A_2B_2C_2$ in Fig. 1.3(a), the instantaneous strain (OO_2) is elastic with negligible plasticity. The combination of stress and temperature, however, is sufficient to cause, with time, a form of nonlinear strain that is called primary or first-stage creep (O_2A_2). Because the rate of accumulation of primary or first-stage creep strain diminishes with time, it is also referred to as transient creep. For this curve, the stress is low enough that negligible strain is accumulated beyond A_2, and the segment of the creep curve is horizontal. At point B_2, the stress is abruptly decreased to zero, and the initial elastic strain (OO_2) is retrieved ($B_2C_2 = OO_2$). An interesting event also occurs during unloading; a portion of the original transient creep strain (B_2C_2) is recovered with time under zero applied stress. This recovered creep strain has been termed anelastic strain. Kennedy (Ref 1.4) devotes a lengthy chapter to modeling of anelasticity.
- *Curve 3*: The curve $OO_3A_3B_3C_3$ represents the most commonly encountered creep behavior of engineering interest. Segment OO_3 contains the instantaneous elastic and any plastic strain response to an applied tensile stress. Transient creep strain dominates the primary or first stage of creep deformation (O_3 to A_3). After A_3, the creep curve becomes a linear function of time until point B_3 is reached. This linear region is called secondary or steady-state creep. After point B_3, the creep rate accelerates, and the curve becomes concave upward, and rupture (failure) is imminent at C_3. This stage of creep deformation is referred to as third-stage or tertiary creep.
- *Curve 4*: This curve represents an extreme case. There is a very little steady-state (secondary) creep regime (A_4B_4), and the creep curve transitions rapidly from first-stage to third-stage creep. This case is not common, but it can occur if a very high creep stress is applied.

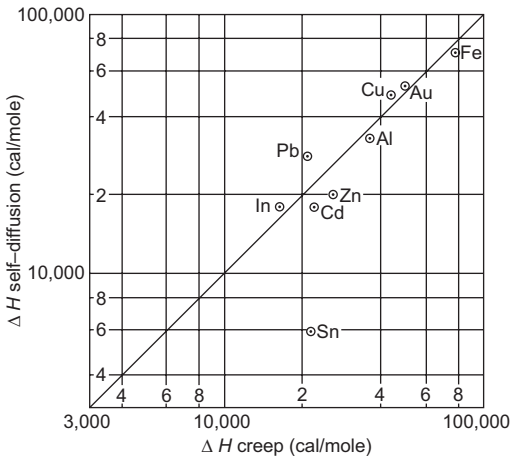

Fig. 1.2 One-to-one relationship of activation energy for self-diffusion with activation energy for creep of pure metals. Source: Ref 1.2

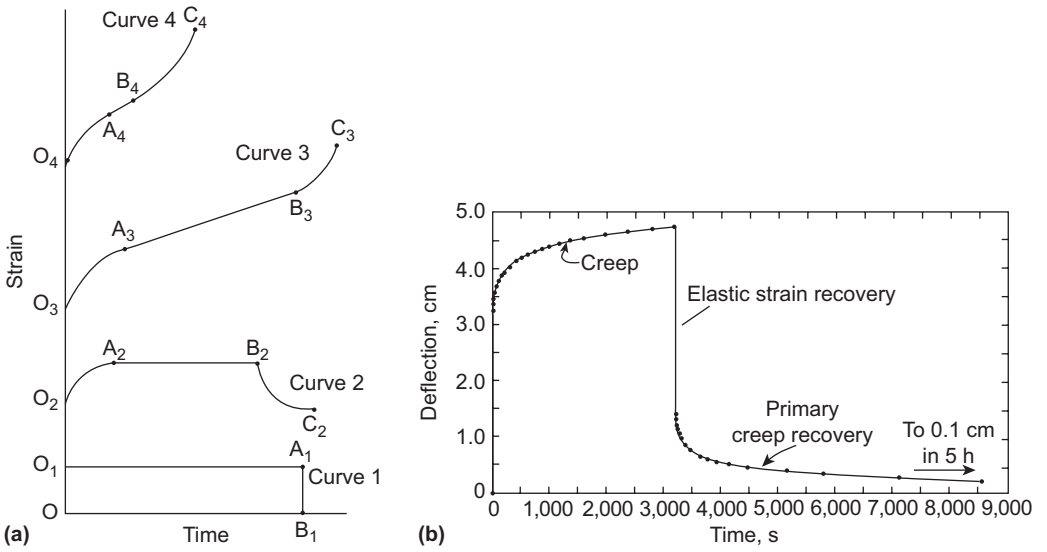

Fig. 1.3 Time-dependent deformation and recovery of primary creep. (a) Schematics of various time-dependent deformation. (b) Curve of primary creep recovery for polycrystalline aluminum in torsional creep at 175 °C (347 °F). Source: Ref 1.2

The creep behavior for most pure metals and alloys can be described by the aforementioned curves. However, alloys such as the nickel- and cobalt-base superalloys tend to exhibit limited amounts of primary creep strain, a less well-defined secondary creep regime, and a long but slowly accelerating tertiary creep behavior. These alloys have been purposely engineered to withstand very high homologous temperatures [T/T_m, where the service temperature (T) on the Kelvin scale (K) is expressed as a ratio of the melting temperature (T_m), also in absolute temperature]. Alloys with a high degree of creep resistance at higher homologous temperatures are used in applications such as aircraft gas turbine engines. To achieve their creep resistance, the microstructures are trapped in a metastable hardened state. At sufficiently high temperature and stress, creep will occur. However, under these conditions, the microstructure can also change, weakening the built-in creep resistance. The end result is a slow degradation of the original creep resistance superimposed on the more conventional creep behavior, resulting in what appears to be tertiary creep. Typical creep curves for the cast nickel-base superalloy IN-100 at 850 °C (1560 °F) are shown in Fig. 1.4 (Ref 1.5).

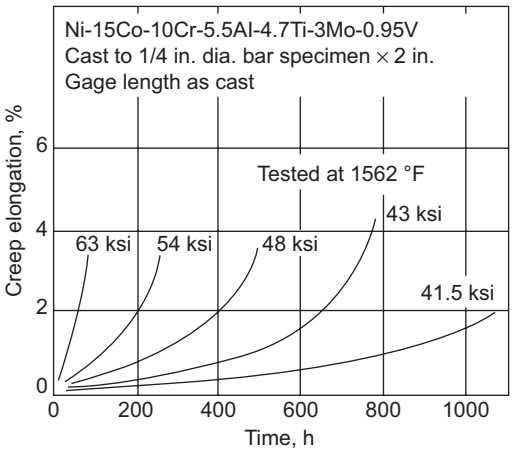

Fig. 1.4 Creep curves for a typical nickel-base superalloy, IN-100, at 850 °C (1562 °F). Source: Ref 1.5

The subject of creep is very complicated and has been studied extensively both by scientists and engineers. Numerous books have been devoted to this subject, among them Dorn (Ref 1.2), Kennedy (Ref 1.4), Garafalo (Ref 1.6), Gittus (Ref 1.7), Kocks et al. (Ref 1.8), Nabarro (Ref 1.9), Poirier (Ref 1.10), Raj et al. (Ref 1.11), Raj (Ref 1.12), Caillard and Martin (Ref 1.13), and Kassner and Perez-Prado (Ref 1.14). In this chapter, the main objective is to present just those concepts that enable an understanding of methodologies in assessing creep fatigue, creep rupture, and thermal stress. No attempt is made to provide exhaustive coverage on the subject of creep deformation, but those desiring further

exposure to this subject are referred to the aforementioned publications.

Creep Mechanisms and Representations of Special Interest

Diffusion. The intense vibration of the atoms at high temperature (by definition, temperature is a measure of the intensity of atomic vibration) means that atoms can diffuse more readily from one site to another. Strictly speaking, it is the vacancies that diffuse, but the end effect is the same. The role of the applied stress is to bias the direction of diffusion. Thus, what we observe as creep deformation is the gradual displacement of the atoms in the direction of the applied stress, changing dimension in the direction of the force as well as in the directions transverse to the stress in order to maintain approximate constancy of volume. Evidence of diffusion involvement in creep was demonstrated for pure metals in Fig. 1.2. For complex alloys, several types and sizes of atoms diffuse in accordance with their respective diffusion rates, thus complicating the interpretation of creep results in terms of a unique activation energy. Nevertheless, it can readily be accepted that diffusion is generally involved in the creep process.

Dislocation Movement. As at room temperature, the movement of dislocations at high temperature results in strain, but some dislocations can be blocked temporarily and thus are unable to contribute to strain. At high temperature, however, such immobilized dislocations can be unlocked by climbing to a plane above an obstacle. The time dependency of this process derives from the time it takes for the set of atoms along the line of the dislocation to move to other sites through the vacancy diffusion process and to allow the dislocation to climb over the obstacle and continue its motion.

The diffusion of vacancies can occur within grains or within the grain boundaries. When they occur in the regular crystal lattice within a grain, the resulting creep is referred to as Nabarro-Herring creep (as discussed by Raj and Ashby in Ref 1.15). In this case, the creep rate is proportional to stress (σ) and inversely proportional to the square of the grain size (d):

$$\dot{\varepsilon} = C\frac{\sigma}{d^2} \qquad \text{(Eq 1.1)}$$

If the diffusion is along grain boundaries, it results in Coble creep (Ref 1.16), and the creep rate is proportional to stress but inversely proportional to the cube of the grain size:

$$\dot{\varepsilon} = C'\frac{\sigma}{d^3} \qquad \text{(Eq 1.2)}$$

Thus, it is clear that materials with fine grain size are very susceptible to creep, while coarse-grained materials are more resistant to creep. It is for this reason that turbine blades were initially made from very coarse-grained castings. It is also one of the reasons why turbine blades were later made by directional solidification to minimize the number of grain-boundary numbers, especially to avoid them in a direction normal to tensile stress. It is also a reason for the subsequent introduction of single-crystal turbine blades with no grain boundaries at all. Other reasons are discussed in Chapter 6, "Total Strain-Based Strain-Range Partitioning—Isothermal and Thermomechanical Fatigue," of this book. This subject has also been briefly discussed in Chapter 11, "Avoidance, Control, and Repair of Fatigue Damage," in the companion volume (Ref 1.1).

Special Role of Grain Boundaries in Creep. To emphasize the role of grain boundaries, we include the next two illustrative figures. A small-angle tilt boundary is shown in Fig. 1.5, illustrating how edge dislocations combine to make it possible for adjacent grains to have different crystallographic orientations. Grain boundaries can, in general, be thought of as planar arrays of edge dislocations, and the greater the difference in the angle between the crystallographic orientations of adjacent crystals, the larger will be the number of dislocations required to accommodate the misfit.

Figure 1.6 shows an interesting result, which demonstrates the role of sliding in the grain boundary of a bicrystal. The white lines are straight fiducial marks scribed on the surface before the deformation is imposed. After some time under force, the two grains slide along their mutual boundary, producing steps in the fiducial lines across the grain boundary. Various methods have been used to study grain-boundary sliding in bicrystals, among them those shown in Fig. 1.7. Because of the high slip activity in the grain boundaries, it is not uncommon to observe heavy cracking in some materials after creep deformation. Figure 1.8 shows the grain-boundary cracking for aluminum and a stainless steel. Not all materials fail in an intergranular manner, and even the same

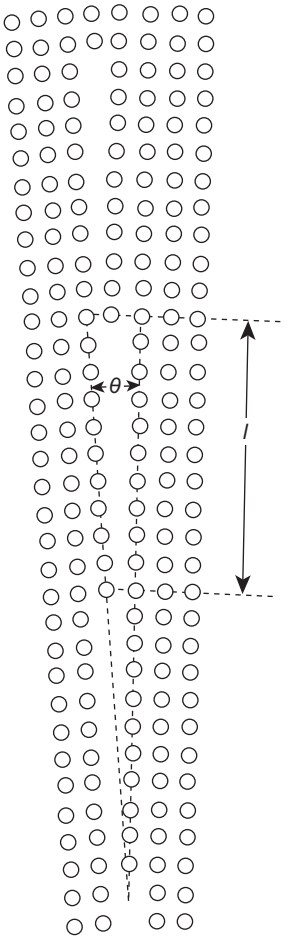

Fig. 1.5 Arrangement of atoms in a small-angle tilt boundary to demonstrate that grain boundaries contain many edge dislocations. Source: Ref 1.2

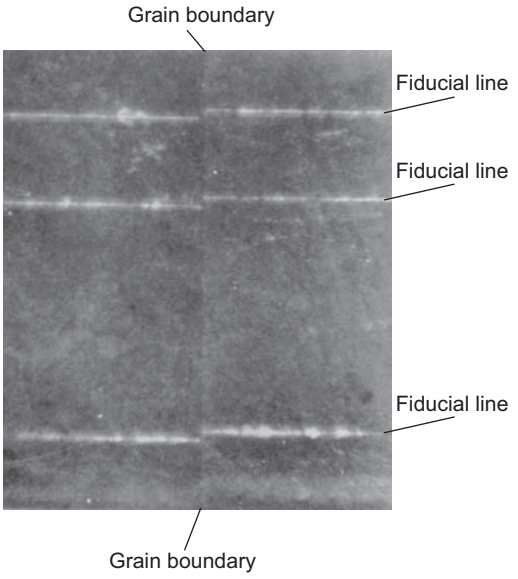

Fig. 1.6 Sliding along the boundary of tin bicrystal after 50 h under a shear stress of 590 g/cm² at 222 °C. Original magnification approximately 20×. White lines are fiducial marks. Source: Ref 1.17

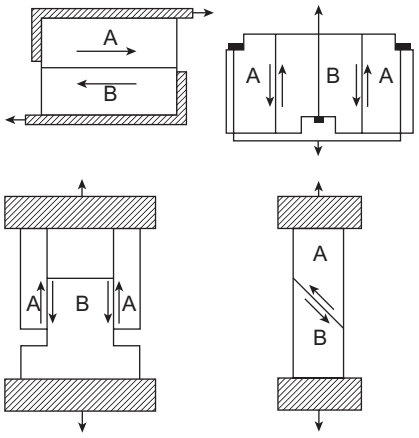

Fig. 1.7 Schematic of the types of tests used to study sliding in bicrystals. Source: Ref 1.2

material does not always fail intergranularly under all conditions, but it is a common form of failure.

Deformation Maps. Ashby and coworkers (Ref 1.20) developed a convenient map that readily displays the regions wherein the various mechanisms contribute to the complex deformation associated with the inclusive designation "creep." Figure 1.9 shows, in principle, the concept behind these maps. The horizontal coordinate is the homologous temperature (T/T_m). The vertical scale is the ratio of stress to shear modulus (at each value of homologous temperature). The map is divided into four regions:

- In region A, the deformation is basically the same as the common dislocation glide encountered at room temperature. Vacancies in the lattice are brought into configuration by the deformation itself. It is essentially plastic flow behavior dominating the entire temperature range.
- In region B, the mechanism of straining is still the same as in region A, except that the creep is enhanced by diffusion of atoms and vacancies due to the agitation of the lattice at high temperature. The main driving force is still dislocation glide, and thus, the movement requires a relatively high stress.

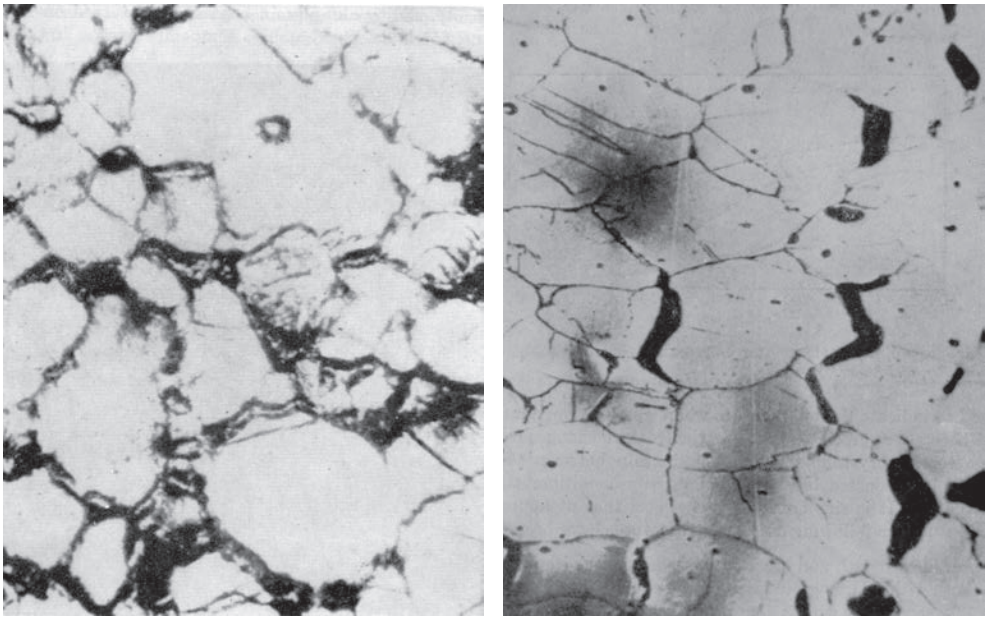

Fig. 1.8 Typical examples of grain-boundary cracking in creep tests (dark areas are voids at grain boundaries). (a) Evidence of grain-boundary activity in an aluminum specimen after 210 h creep under a stress of 19 MPa (1.4 ton/in.^{2}) at 250 °C (480 °F). Original magnification 150×. Source: Ref 1.18. (b) Intergranular fissures near fracture surface of austenitic stainless steel at 750 °C (1380 °F). Source: Ref 1.19

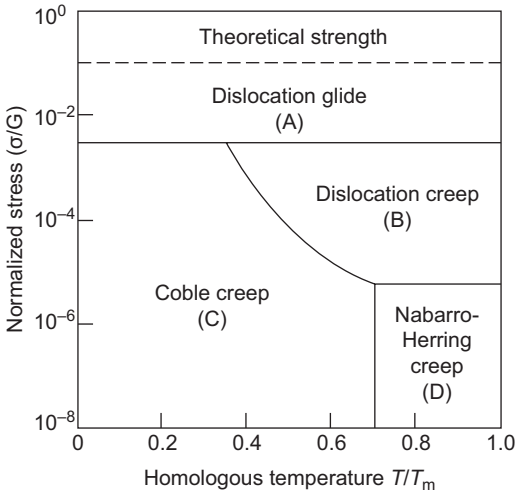

Fig. 1.9 Simplified deformation mechanism map. Source: Adapted from Ref 1.20

- In region C, the controlling mechanism is diffusion within the grain boundaries where the atoms are in considerable disarray, so that diffusion can occur even at very low stresses. This type of creep was described by Coble (Ref 1.16) and was therefore labeled in his honor.
- In region D, the deformation is controlled primarily by diffusion, because the temperature is so high that minimal stress is needed to cause the deformation, although the direction of the deformation is biased by the direction of applied stress. Nabarro (Ref 1.21) and Herring (Ref 1.22) first made reference to this type of deformation.

Deformation maps for MAR-M200, a nickel-base superalloy once commonly used for aircraft turbine blades, are shown in Fig. 1.10. In Fig. 1.10(a), the grain size is quite small (100 μm); in Fig. 1.10(b), a large grain size of 1 cm is shown. Also shown in these figures are lines of constant creep rate and the boundaries of typical turbine blade operation. It is clear that for the very small grain size, the Coble creep ranges are greatly enhanced because of the many grain boundaries, while for the coarse-grained material, there is minor Coble creep or even Nabarro-Herring creep due to the absence of many grain boundaries. For the same range of creep rates (10^{-10} to 1.0 s^{-1}), the coarse-grained material requires much higher stresses and temperatures

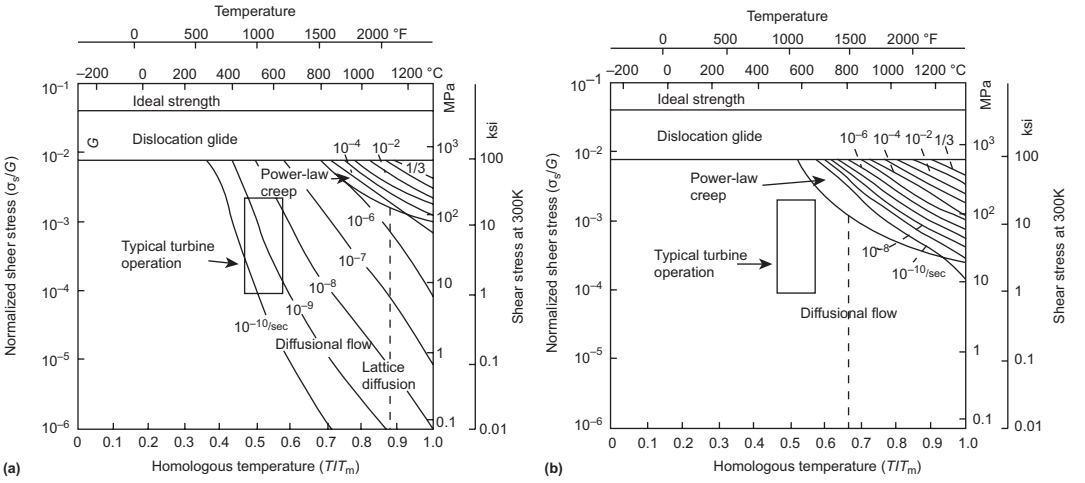

Fig. 1.10 Creep deformation maps for MAR-M200. (a) Grain size = 100 μm. (b) Grain size = 1 cm. Source: Ref 1.7

than the fine-grained material. Considerable creep can occur within the stress and temperature range encountered in typical turbine blade applications for the fine-grained material but not for the coarse-grained alloy. As discussed earlier, this reason, among others, is why a coarse grain size is more desirable for turbine blades. Even better are directionally solidified or single-crystal structures. While the fundamental understanding of the role of grain-boundaries motivated the concept of minimizing grain-boundaries as a means of reducing grain-boundary creep, representation by plots such as Fig. 1.10(b) helps to quantify the phenomenon.

Physical Damage Associated with Creep. The creep process produces various modes of internal damage in the material. One such mode is cracking in the grain boundaries, where several types of discontinuities can develop. First, there are the incompatibilities of the sliding of adjacent grains at their intersections (triple points) (Fig. 1.11). A case of actual triple-point cracking in an aluminum alloy was shown in Fig. 1.8.

Other types of intergranular cracking, designated w-type (wedge) and r-type (round), can also develop (Fig. 1.12). In w-type cracking, a crack penetrates from boundary into a grain, so that the crack is within one grain only. In r-type cracking, the cracks are simply circular holes in a grain boundary due to the agglomeration of coalesced vacancies.

In addition, the grain boundaries contain many uncombined elements that did not participate in the chemical and metallurgical reaction during formation of the base material. They are therefore available to react with other elements and compounds (sometimes impurities form brittle compounds) and consequently appear in the grain boundaries as many small cracks that result from failure of these compounds.

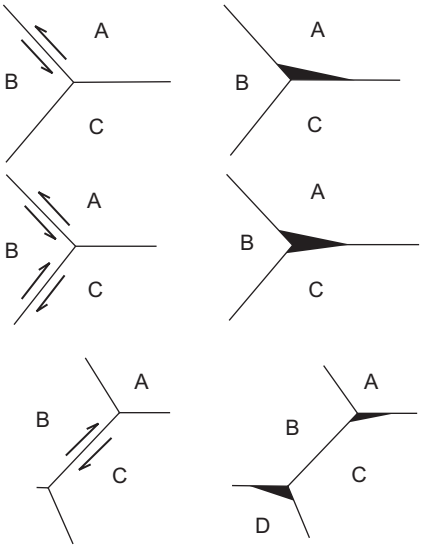

Fig. 1.11 Schematic drawings of the way intergranular cracks form due to grain-boundary sliding. Source: Ref 1.23

Ashby (Ref 1.20) has developed fracture mechanism diagrams for numerous materials. The regions delineate wherein different types of

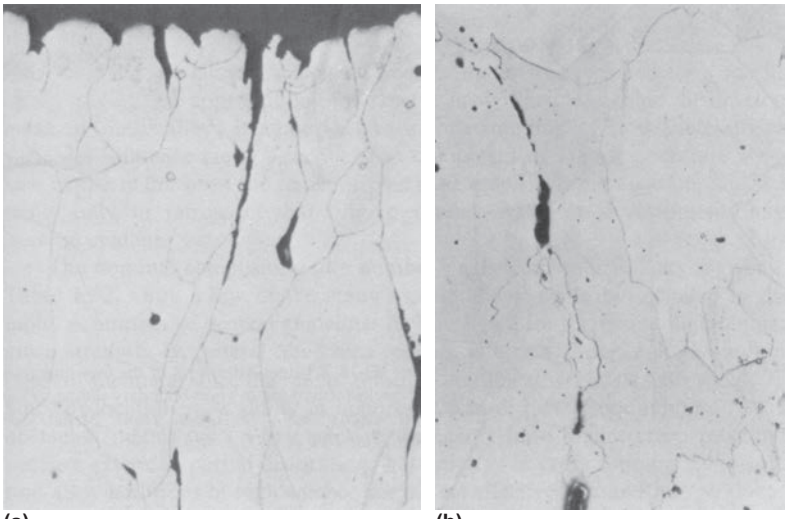

Fig. 1.12 Two types of creep cracks found in grain boundaries: (a) w-type cracks and (b) r-type cracks. Source: Ref 1.24

cracking can occur. A typical example is shown in Fig. 1.13 for nickel.

Representation of Creep Behavior

Monotonic Creep. Many attempts have been made to represent creep curves and creep-rate processes by mathematical expressions to aid analytical studies. One of the first was Andrade (Ref 1.25), who, in 1914, investigated the early stages of creep (primary creep) and who is especially noted for the $(t)^{1/3}$ term so often used in creep representation. Kennedy (Ref 1.4, pages 151–153) has summarized many proposals from the literature, as shown in Tables 1.1 to 1.3.

Conway and Sjodahl (Ref 1.55) have provided a compendium of mathematical procedures for obtaining the constants in various choices of these equations. Graham and his coworkers (Ref 1.39) have provided elaborate procedures for very accurate representation of any curve using a large number of terms. However, it should be emphasized that the mere mathematical fitting of a curve, while useful in numerical analysis, does not necessarily reflect the validity of the physical process implied by the equations.

Misleading results can sometimes be obtained from simple curve fits to the experimental data. As an example, the creep curves of

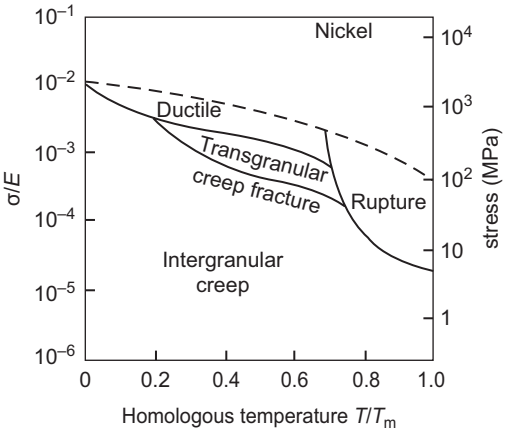

Fig. 1.13 Fracture mechanism map for nickel. Source: Ref 1.24

0.3Mo-0.23V steel in Fig. 1.14 were analyzed by Manson and Sissay Hailu (then a graduate student at Case Western Reserve University) to show how misleading results can sometimes be obtained from pure curve fitting. Initially, the purpose was simply to show how creep curves could be represented analytically. As one possibility, the following equation was chosen for curve fitting:

$$\varepsilon = A(t)^{1/3} + B(t)^{2/3} + C(t) \qquad \text{(Eq 1.3)}$$

Table 1.1 Creep strain versus time representations listed by Kennedy

Simple functions		
$\varepsilon = at/(1 + bt)$	Freudenthal	Ref 1.26
Logarithmic functions		
$\varepsilon = a + b\log t$	Phillips	Ref 1.27
	Boas and Schmid	Ref 1.28
	Smith	Ref 1.29
	Chevenard	Ref 1.30
$\varepsilon = a\{\log[1 + bt]\}$	Laurent and Eudier	Ref 1.31
$\varepsilon = \log t + bt + c$	Mott and Nabarro	Ref 1.32
	Weaver	Ref 1.33
Exponential functions		
$\varepsilon = a + bt - c\exp(-dt)$	McVetty	Ref 1.34
$\varepsilon = at + b[1 - \exp(-ct)]$	Soderberg	Ref 1.35
$\varepsilon = a[1 - \exp(-bt)] + c[1 - \exp(-dt)]$	McHenry	Ref 1.36
Power function		
$\varepsilon = a + bt^n$	Swift and Tyndall	Ref 1.37
and, in particular,	Cottrell and Aytekin	Ref 1.38
$\varepsilon = a(1 + b^{1/3})\exp kt$	Andrade	Ref 1.25
Power series		
$\varepsilon = at^m + bt^n - ct^p$	Graham	Ref 1.39
simplifying to the special case		
$\varepsilon = a + bt^{1/3} + ct + dt^3$ (thus approximating Andrade's form)		
Logarithmic plus power functions	Wyatt	Ref 1.40
$\varepsilon = a\log t + bt^n ct$	(particularly $n = 1/3$)	

ε, tensile creep strain; t, time; $d\varepsilon/dt$, minimum creep rate; e, base of natural logarithms; σ, stress; and T, absolute temperature (K). Other letters denote material constants. Note: The use of the same symbol in different equations does not mean that these constants have the same value. Source: Ref 1.4

Table 1.2 Creep strain/strain rate versus temperature-dependent relationships, from Kennedy

Exponential temperature functions		
$d\varepsilon/dt = a\exp(-Q/RT)$	Mott	Ref 1.41
$\varepsilon = a[t\exp(-Q/RT)]^n$	Kauzmann	Ref 1.42
$d\varepsilon/dt = aT\exp(-Q/RT)$	Nowick and Machlin	Ref 1.43
	Dorn	Ref 1.44
	Dushman et al.	Ref 1.45
	Stowell	Ref 1.46
Simple temperature functions		
$\varepsilon = aT^{2/3}f(t)$	Mott and Nabarro	Ref 1.47
$\varepsilon = aTf(t)$	Smith	Ref 1.48
$\varepsilon = f[T(a + \log t)]$	Larson and Miller	Ref 1.49
$\varepsilon = f[(T - a)/(\log t - b)]$	Manson and Haferd	Ref 1.50
Hyperbolic temperature functions		
$d\varepsilon/dt = a\exp(-Q/RT)\sinh(b/RT)$	Feltham	Ref 1.51
$\varepsilon = cf[t(T - T')^{-B}$	Graham	Ref 1.39

Source: Ref 1.4

Table 1.3 Creep strain/strain rate versus stress-dependent relationships, from Kennedy

Exponential stress functions		
$\varepsilon = af(t)\exp(b\sigma)$	Dorn (high stresses)	Ref 1.44
$d\varepsilon/dt = a\exp(b + c\sigma)$	Dushman et al.	Ref 1.45
$d\varepsilon/dt = a[\exp(b\sigma) - 1]$	Soderberg	Ref 1.35
Power-law stress functions		
$\varepsilon = af(t)\sigma^b$	Norton	Ref 1.52
	Bailey	Ref 1.53
	Dorn (low stresses)	Ref 1.44
	Graham	Ref 1.39
Hyperbolic stress functions		
$d\varepsilon/dt = a\sinh(b\sigma)$	Nadai	Ref 1.54
$d\varepsilon/dt = a\sinh(b\sigma/RT)$	Feltham	Ref 1.51

Source: Ref 1.4

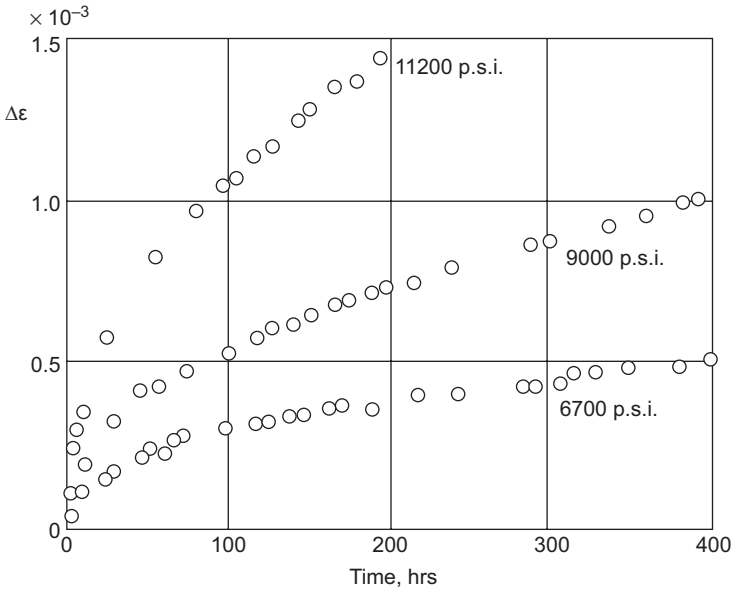

Fig. 1.14 Creep curves for 0.3Mo-0.23V steel at 645 °C. Source: Ref 1.4

where A, B, and C were to be determined as functions of stress and temperature. In one procedure, we simply chose a number of points for testing stress (e.g., at 9.0 ksi) and determined A, B, and C by a least-squares fit. This fit to the same experimental data was also very good, as shown by Fig. 1.15.

However, contemplation of the physical significance of the terms led to an inconsistency. The term $C(t)$ should, in some way, be representative of the steady-state creep behavior, and having it result in a negative analytic value, while the stress was positive, did not seem reasonable or accurate.

An alternate analysis was then made using the equation:

$$\varepsilon = A(t)^{1/3} + B(t) \qquad \text{(Eq 1.4)}$$

The constants A and B were easily determined by least-squares analysis, choosing any desired number of numerical values along a curve. An even easier procedure is to divide both sides of the equation by t, resulting in:

$$\frac{\varepsilon}{t} = A(t)^{-2/3} + B \qquad \text{(Eq 1.5)}$$

Thus, a plot of ε/t versus $(t)^{-2/3}$ should result in a straight line of slope A and intercept B. For this example, good straight lines were obtained, as shown in Fig. 1.16. The agreement between the

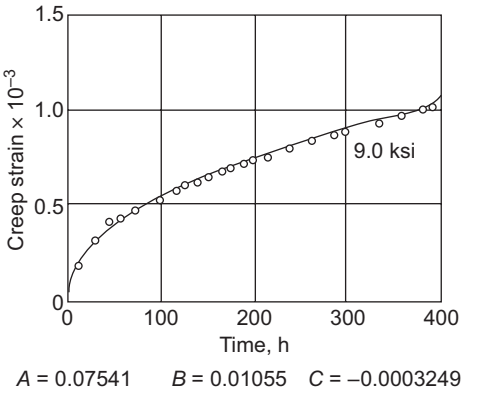

Fig. 1.15 Fitting the creep curves of 0.3Mo-0.23V steel at 9000 psi and 645 °C by an equation in the form $\varepsilon = A(t)^{1/3} + B(t)^{2/3} + C(t)$. Source: Prepared by S. Hailu

computed curves and the experimental data is again very good, as is shown in Fig. 1.17.

Additional analysis was needed to determine the variation of the constants A and B with stress, as shown in Fig. 1.18. Using a logarithmic scale for stress is seen to produce reasonably good correlation and straight lines, that is, a power-law relationship for these constants as a function of stress.

For this particular material and range of stresses, it was thus concluded that a two-term representation would be better than a three-term

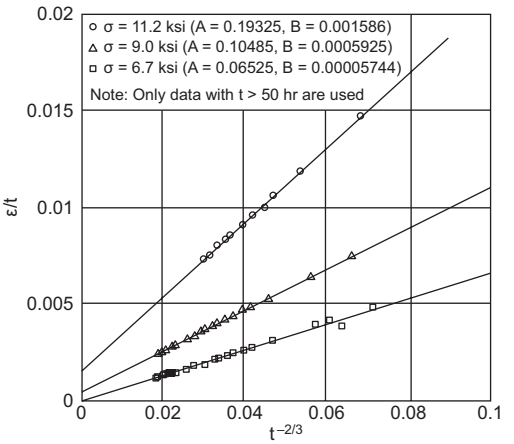

Fig. 1.16 Determination of constants A and B for creep equation $\varepsilon = A(t)^{1/3} + B(t)$ for Mo-V steel of Fig. 1.14. In this figure, only the data at times higher than 50 h were used in order to get the best results. Similar analysis using all the data also gave reasonably good results but not as well as that shown in this figure. Calculated from Fig. 1.14. Source: Prepared by S. Hailu

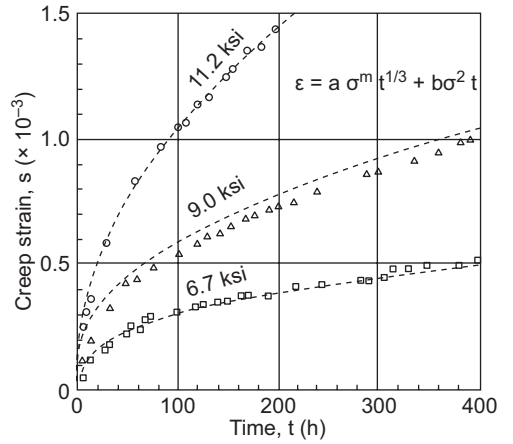

Fig. 1.17 Determination of constants A and B for creep equation $\varepsilon = A(\sigma)^{n}(t)^{1/3} + B(\sigma)^{n}(t)$ for 0.3Mo-0.23V steel of Fig. 1.14. Calculated from Fig. 1.14. Source: Prepared by S. Hailu

representation, even though the latter may be slightly better to represent all the data over the entire range. Furthermore, from a theoretical point of view, the two-term representation may be better if one regarded the $(t)^{1/3}$ term (in Eq 1.4) to be related to transient creep and the (t) term to be related to the steady-state creep.

Additional analysis of the Kennedy data, especially as it relates to temperature, is not considered. Further analysis in terms of temperature becomes too complicated for the purpose here. The complication is the recognition that the activation energy for primary creep may differ from the activation energy for steady-state creep. This would not serve the general purpose under discussion. The purpose here is to indicate that there are many choices for representing creep stress under steady loading as a function of time, stress, and temperature, but that care must be exercised in the choice of the equation selected if it is desired to relate behavior to processes.

Consideration of Creep under Cyclic Loading. While extensive literature exists for creep studies under monotonic loading, little exists for cyclic loading in which stress is held for periods of time and alternated between tension and compression. This area is, of course, quite relevant to the study of high-temperature fatigue wherein time-dependent creep deformation is well known to be deleterious to cyclic deformation (strain) resistance. The earliest reported cyclic creep studies dealt with the behavior of low-melting temperature materials, for example, alloys of lead (homologous temperature of 0.5 at room temperature) used as sheathing materials for transatlantic cables. In 1951, Eckel (Ref 1.56) and coworkers designed a unique bending test to apply very low-frequency strain cycling to arsenic-lead specimens. By testing in bending, differences between tensile and compressive stresses and their individual effect on creep rates are masked. Cyclic creep deformation as a major contributor to low-frequency failures was evidenced by the dominant intergranular cracking observed by Eckel.

A decade later, two definitive papers were published at the Joint International Conference on Creep, detailing creep strain response to repeated reversals of the sign of the creep stress. The first paper in the proceeding was by Morrow and Halford (Ref 1.57), and it dealt with the behavior of chemical lead tested at room temperature in reversed torsion of tubular specimens. The nature of the stress state (simple shear) prevented assessment of the individual influences of tensile and compressive stresses. However, the effect of repeated alternating direction of torsion significantly increased the creep rate per reversal. Creep rates steadily increased by an order of magnitude from the 2nd reversal (fatigue half-cycle) to the 100th, as shown in Fig. 1.19. The cyclic torsion tests were not carried to failure during those experiments. Subsequent torsion tests conducted to failure by Kitagawa (Ref 1.58) on the same heat of chemical lead

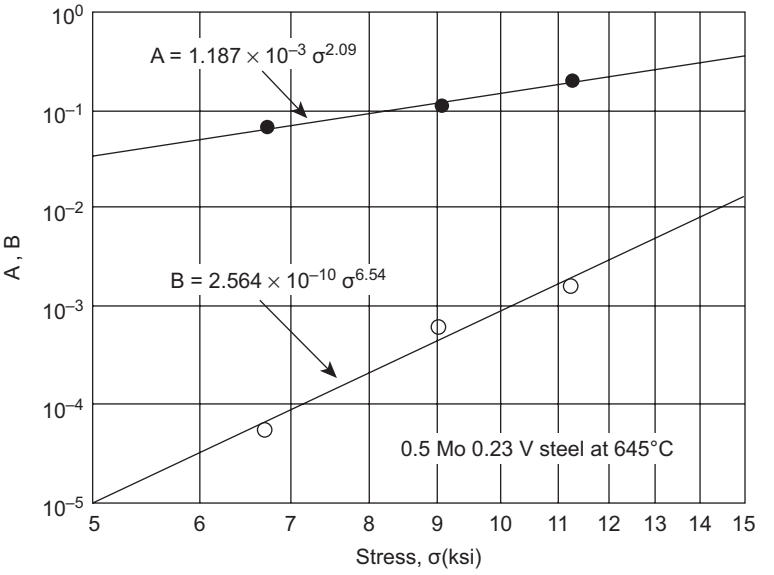

Fig. 1.18 Dependence of *A* and *B* on stress when temperature is constant (0.3Mo-0.23V steel of Fig. 1.14). Based on Fig. 1.14. Source: Prepared by S. Hailu

clearly revealed that cyclic fractures were due to intergranular creep cracking. Similar intergranular cracking was reported by Grant and his students at the Massachusetts Institute of Technology (Ref 1.59) for 99.99% Al tested at 425 °C (800 °F) in axial tension-compression strain cycling. All failures were preceded by extensive cyclic grain-boundary migration. Grain boundaries continuously migrated until they formed 45° angles with the loading axis, thus bringing them into alignment with the direction of the maximum shear stress and permitting faster creep rates. Figure 1.20 is a dramatic example of the extent of grain-boundary migration observed.

The second paper in the 1963 International Creep Conference dealing with the relationship of cyclic creep to monotonic creep was by Swindeman (Ref 1.60) of the Oak Ridge National Laboratories. Axially loaded specimens of Inconel were tested at 815 °C (1500 °F) in monotonic tensile creep-rupture and time-to-rupture recorded. The monotonic results were then compared to failure times of cyclic creep-rupture tests in which alternate tensile and compression stresses were held constant and the specimen was allowed to creep until a fixed strain limit was reached. At that point, the direction of the stress was reversed, and creep occurred in the opposite direction.

This sequence was followed until the specimen ruptured. For this Inconel alloy and test

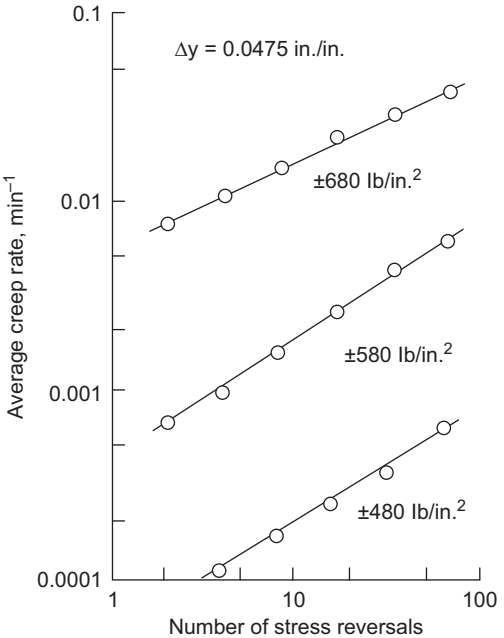

Fig. 1.19 Effect of shear stress amplitude and number of stress reversals on the average creep rate of acid lead at 29 °C (84 °F). Source: Ref 1.57

temperature, the time to rupture at a given cyclic stress was determined to be greater than the rupture time at monotonic creep. Increases of rupture time were on the order of a factor of 4 at high stresses and only 2 at the lowest stresses.

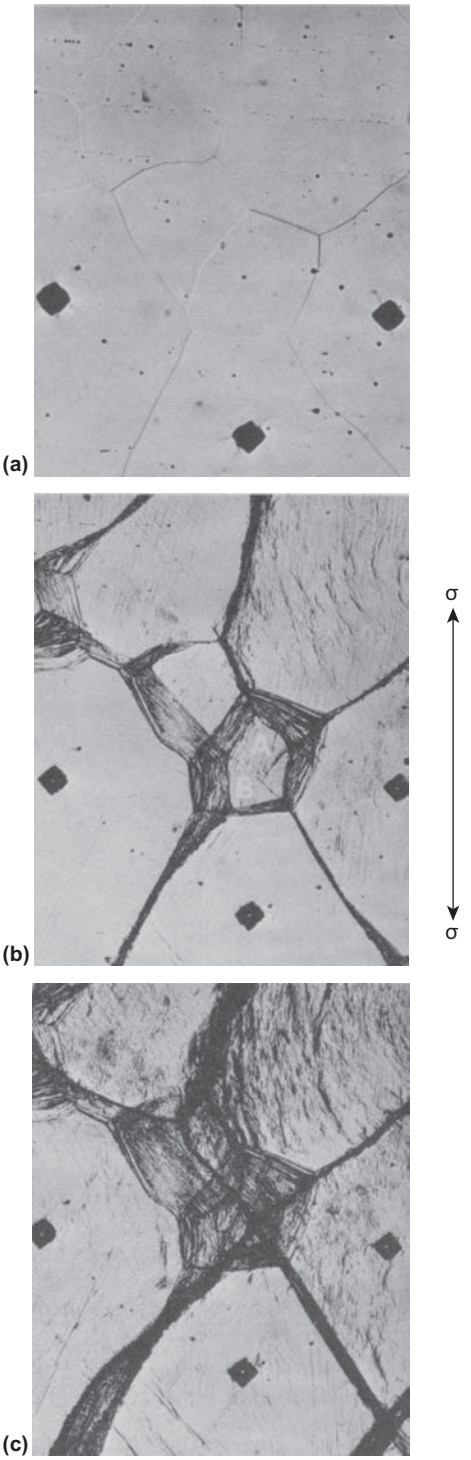

Fig. 1.20 Surface markings indicative of grain-boundary mi gration for coarse-grained pure aluminum at 427 °C (800 °F) at a cyclic strain rate of 5% per minute. Black squares are diamond-pyramid indentations used to locate the area studied. (a) Before testing. (b) After 20 cycles. (c) After 100 cycles. Original magnification approximately 40X. Source: Ref 1.59

The rationale was proposed by Swindeman that the differences in time to rupture depended on the fact that the true stress in the monotonic tests was constantly increasing as creep progressed, whereas in the cyclic tests, the true stress remained constant because net extension of the specimen was controlled to be zero.

It was observed that the cyclic creep rates increased above those for monotonic testing, and that the primary or first-stage creep was the principal contributor, although the cyclic steady-state creep rates did increase somewhat. Only tensile creep rate was measured and compared to the monotonic rate at a given stress. At that time, there was little concern for compressive creep damage, because the fledgling Time- and Cycle-Fraction Rule for creep-fatigue interaction, proposed two years earlier by Taira (Ref 1.61), considered only tensile creep. Swindeman concluded from his cyclic and monotonic creep-rupture tests that the ratio (i.e., fraction) of time under cyclic tensile stress relative to the time-to-rupture (in monotonic tension) was a reasonable measure of creep damage during creep-fatigue loading.

The cyclic creep results discussed previously raised two important issues in connection with cycling loading:

- What is the relationship between the compressive component of the cycle and the tensile component (tensile creep has been studied extensively)?
- How does compressive creep contribute to the damage of the cycle?

During research on compressive creep at the National Aeronautics and Space Administration (NASA) in Cleveland during the late 1960s, the authors contemplated how to improve creep analyses for cyclic loading. As with Swindeman, the authors recognized that as axial strain progresses in a monotonic creep test, the specimen cross-section continually contracts, thus significantly increasing the true tensile stress and the subsequent creep rate. Large cross-sectional changes rarely occur in structural components undergoing cyclic loading at high temperature.

For this reason, the authors proposed an alternating-stress, creep-rupture test (Fig. 1.21a) to determine cyclic creep-rupture resistance (Ref 1.62). Starting in compression, the stress is increased rapidly, then servo-controlled held constant upon reaching point ***A***. As compressive

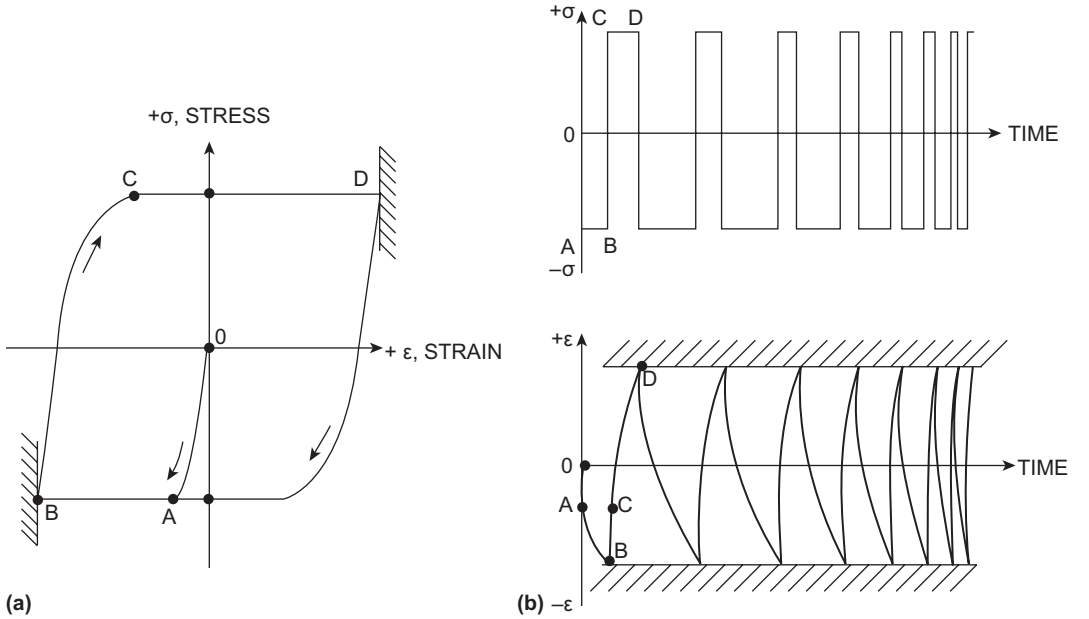

Fig. 1.21 Early concept of cyclic creep-rupture testing (a) Hysteresis loop. (b) Imposed cyclic stress history and cyclic strain response. Source: Ref 1.62

creep occurs, a compressive strain limit (***B***) is reached that triggers the servo-controller to reverse the sign of the stress to tension at ***C***. Tensile creep takes place along ***CD***. At ***D***, a tensile strain limit is reached, causing the sign of the stress to again be reversed to reach the compressive stress at ***A***. From there on, the sequence is repeated until failure occurs. Typical stress-versus-time and strain-versus-time patterns then emerge, as shown in Fig. 1.21(b). Throughout the duration of the cyclic creep-rupture test, the cross-sectional area varies only slightly about its original mean value. Thus, a nominally constant true stress is maintained for the entire test duration.

Two variables that we recognized, but at first ignored, were that the magnitude of the compressive stress could influence our results, and that we had added a component of fatigue damage to the creep damage for which we were seeking documentation. Then, two unanticipated results developed. First, within the same cycle, the compressive creep rate was always lower (by a factor of approximately 3) than the tensile creep rate; thus, the time per cycle for compressive creep was correspondingly greater than the tensile time. The relationship between the creep rate in tension and compression is shown in Fig. 1.22. Second, both the tensile and compressive creep times per cycle continuously decreased in successive cycles (i.e., there was a substantial cyclic creep softening). Despite this significant cycle-to-cycle acceleration of creep, the summed tensile creep time to failure was considerably greater than the monotonic creep-rupture time at the same value of nominal tensile stress.

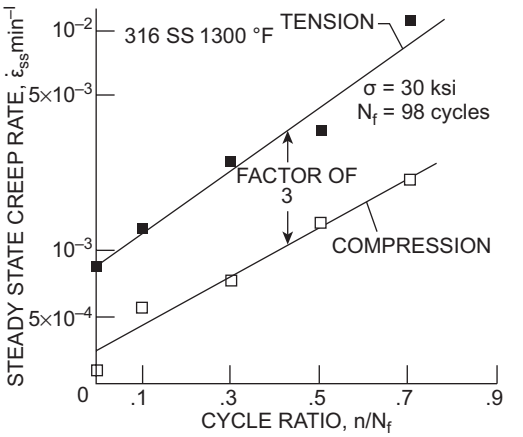

Fig. 1.22 Creep-rate response in tension and compression of a cyclic creep-rupture test of 316 stainless steel (heat 1) at 705 °C (1300 °F). Source: Ref 1.62

These results intrigued us because constitutive modeling efforts at that time assumed equal creep rates in tension and compression for a given stress and temperature. We sought to extend the program to study more variables as well as additional alloys of engineering interest. Unfortunately, we had exhausted all the specimens of heat 1 of the 316 stainless steel on which the first set of data were conducted. We subsequently procured a new heat (labeled heat 2) of the same alloy. Testing of the new heat revealed an even greater difference (by more than a factor of 6, based on the applied engineering stresses) between the tensile and compressive cyclic creep rates, as shown in Fig. 1.23 (Ref 1.62). The conclusion was incontrovertible that, at least for some classes of materials, large differences can occur between tension and compression creep rates.

Two simple mechanical models were examined that may explain why there could be differences in the tensile and compressive creep rates. First, the cross-sectional area changes at the extremes of the strain ranges involved were used to compute the true stresses. Figure 1.23 displays the creep-rate results for both true stress and engineering stress. As can be seen, making the correction for true stress does bring the tensile and compressive creep rates into closer coincidence. However, they are still different by a large factor of 5.

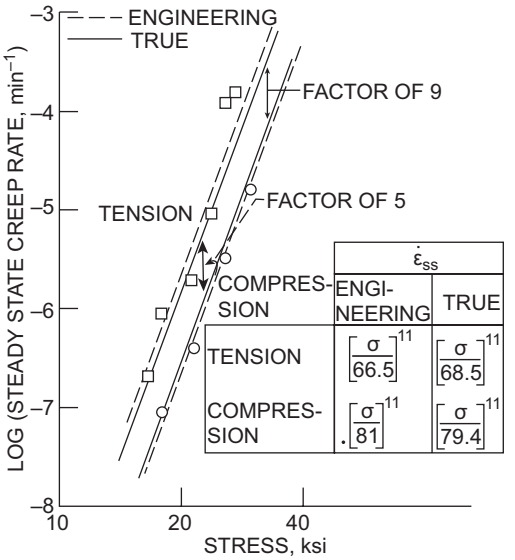

Fig. 1.23 Comparison of tensile/compressive creep rates of a cyclic creep-rupture test of 316 stainless steel (heat 2) at 705 °C (1300 °F). Source: Ref 1.62

The second simple model we considered used a friction analogy of a mass sliding on a horizontal surface. If an upward force is applied to the mass (but not enough to lift it from the surface), the frictional force to sliding is reduced. Adding downward force (compression) increases the frictional force to sliding. If grain-boundary sliding along the 45° boundaries is the creep deformation mechanism of an axially deformed specimen, then the maximum resolved shear stress in either tension or compression would be nominally identical. While an axial specimen is in tension, there is a tensile stress normal to the 45° boundary, and the shearing process is easier as the atoms along the grain boundary are being pulled apart, thus affording less "friction." Similarly, when the axial specimen is in compression, there is a compressive stress across the 45° boundary that gives rise to greater difficulty of creeping by shear, because there is more "friction." While the "friction" and "true stress" models offer qualitative explanation, quantitative answers require a more mechanistically oriented, physics-based analysis.

Although these early studies were conducted within the framework of exploring differences between creep in tension and compression, the authors later recognized that the test depicted in Fig. 1.21 was basically what was later identified as a CT-CC test, for creep in tension (CT) and creep in compression (CC) within the framework of strain-range partitioning that is introduced in Chapter 3 and detailed in Chapters 4 to 6 of this book. Subsequently, multiple CC tests were conducted on other materials. These extensive cyclic creep results have led us to believe that compressive creep rates are not approximated well enough, knowing only tensile rates. Both directions need to be evaluated separately. Ideally, the difference between the creep rates in tension and compression should be measured and incorporated into sophisticated viscoplastic models that have the capability of distinguishing behavior in tension from that in compression. Consideration of direct measurement of creep rates at various points within complex high-temperature loading cycles is discussed in subsequent chapters on strain-range partitioning.

Chapter 1: Creep Under Monotonic and Cyclic Loading

References

1.1 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, 2006

1.2 J.E. Dorn, Ed., *Mechanical Behavior of Materials at Elevated Temperatures*, McGraw-Hill Book Company, Inc., New York, 1961

1.3 J.E. Dorn, *Creep and Fracture at High Temperatures*, Her Majesty's Stationery Office, London, 1956

1.4 A.J. Kennedy, *Processes of Creep and Fatigue in Metals*, John Wiley & Sons, Inc., New York, 1963

1.5 L.J. Fritz and W.P. Koster, "Tensile and Creep Rupture Properties of (16) Uncoated and (2) Coated Engineering Alloys at Elevated Temperatures", *NASA CR-18911, Metcut Research Associates, Inc., National Aeronautics and Space Administration*, 1977

1.6 F. Garafalo, *Fundamentals of Creep and Creep-Rupture in Metals*, The Macmillan Company, New York, 1965

1.7 J. Gittus, *Creep, Viscoelasticity and Creep Fracture in Solids*, Applied Science Publishers Ltd., London, 1975, p 473

1.8 U.F. Kocks A.S. Argon and M.F. Ashby, Thermodynamics and Kinetics of Slip, *Prog. Mater. Sci.*, Vol 19, B. Chalmers J.W. Christian and T.B. Massalski, Ed., Pergamon, New York, 1975

1.9 F.R.N. Nabarro, Ed., *Theory of Crystal Dislocations*, Dover Publications, Inc., New York, 1980

1.10 J.P. Poirier, *Creep of Crystals*, Cambridge University Press, second printing, 1990

1.11 S.V. Raj I.S. Iskovitz and A.D. Freed, Modeling the Role of Dislocations Substructure during Class M and Exponential Creep, *Unified Constitutive Laws of Plastic Deformation*, A.S. Krausz and K. Krausz, Ed., Academic Press, San Diego, 1996

1.12 S.V. Raj, Power-Law and Exponential Creep in Class M Materials: Discrepancies in Experimental Observations and Implications for Creep Modeling, *Mater. Sci. Eng. A*, Vol 322, 2002, p 132–147 doi: 10.1016/S0921-5093(01)01126-1

1.13 D. Caillard and J.L. Martin, *Thermally Activated Mechanisms in Crystal Plasticity*, Pergamon, 2003

1.14 M.E. Kassner and M.T. Perez-Prado, *Fundamentals of Creep in Metals and Alloys*, Elsevier, 2004

1.15 R. Raj and M.F. Ashby, On Grain Boundary Sliding and Diffusional Creep, *Metall, Trans.*, Vol 2, April 1971, p 1113–1127 doi: 10.1007/BF02664244

1.16 R.L. Coble, A Model for Boundary Diffusion Controlled Creep in Polycrystalline Materials, *J. Appl. Phys.*, Vol 34, 1963, p 1679–1682 doi: 10.1063/1.1702656

1.17 R. King R.W. Cahn and B. Chalmers, Mechanical Behavior of Crystal Boundaries in Metals, *Nature*, Vol 161(No. 4096), 1948, p 682 doi: 10.1038/161682a0

1.18 D. Hanson and M.A. Wheeler, Deformation of Metals under Prolonged Loadings, *J. Inst., Met.*, Vol 45(No. 1), 1931, p 229–257

1.19 C. Crussard and R. Tamhankar, High Temperature Deformation of Steels: A Study of Equicohesion, Activation Energies, and Structural Modifications, *Trans. Am. Inst. Min. Metall. Eng.*, Vol 212, 1958, p 718–730

1.20 M.F. Ashby, *Acta Metall.*, Vol 20, 1972, p 887–897. See also H.J. Frost and M.F. Ashby, *Deformation-Mechanism Maps*, Pergamon Press, New York, 1982 doi: 10.1016/0001-6160(72)90082-X

1.21 F.R.N. Nabarro, *Report of a Conference on Strength of Solids*, The Physical Society of London, 1948, p 75–90

1.22 C. Herring, Diffusional Viscosity of a Polycrystalline Solid, *J. Appl. Phys.*, Vol 21, May 1950, p 437–445 doi: 10.1063/1.1699681

1.23 H.C. Chang and N.J. Grant, Mechanisms of Intercrystalline Fracture, *Trans. Am. Inst. Min. Metall. Eng.*, Vol 206(No. 5), 1956, p 545–551

1.24 G. Dieter, *Mechanical Metallurgy*, McGraw-Hill Book Company, Inc., New York, 1986

1.25 E.N. Andrade and C. Da, The Flow of Metals under Large Constant Stress, *Proc. R. Soc. (London) A*, Vol 90, 1914, p 329–342 doi: 10.1098/rspa.1914.0056

1.26 A.M. Freudenthal, *International Association of Bridge and Structural Engineers*, Vol 4, 1939, p 249

1.27 F. Phillips, *Philos. Mag.*, Vol 9, 1905, p 513

1.28 W. Boas and E. Schmid, *Z. Phys.*, Vol 100, 1939, p 463 doi: 10.1007/BF01344260

1.29 C.L. Smith, *Proceedings of the Physical Society of London*, Vol 64, 1948, p 201 doi: 10.1088/0959-5309/61/3/301

1.30 P. Chevenard, *Rev. Metall.*, Vol 31, 1934, p 473

1.31 P. Laurent and M. Eudier, *Rev. Metall.*, Vol 47, 1950, p 39

1.32 N.F. Mott and F.R.N. Nabarro, *Report of a Conference on the Strength of Solids*, The Physical Society of London, 1948, p 46

1.33 S.H. Weaver, *Trans. Am. Soc. Mech. Eng.*, Vol 58, 1936, p 745

1.34 P.G. McVetty, *Trans. Am. Soc. Mech. Eng.*, Vol 55, 1933, p 99 doi: 10.1115/1.4012192

1.35 C.R. Soderberg, *Trans. Am. Soc. Mech. Eng.*, Vol 58, 1936, p 735

1.36 D. McHenry, *Proceedings of the American Society for Testing and Materials*, Vol 43, 1943, p 1069

1.37 I.H. Swift and E.P.T. Tyndall, *Phys. Rev.*, Vol 61, 1942, p 359 doi: 10.1103/PhysRev.61.359

1.38 A.H. Cottrell and V. Aytekin, *Nature*, Vol 160, 1947, p 328 doi: 10.1038/160328a0

1.39 A. Graham, The Phenomenological Method in Rheology, *Research*, Vol 6, 1953, p 92–96

1.40 O.H. Wyatt, *Proc. Phys. Soc. (London) B*, Vol 66, 1953, p 495

1.41 N.F. Mott, A Theory of Work-Hardening of Metals, Part 2: Flow without Slip-Lines, Recovery and Creep, *Philos. Mag.*, Vol 44(No. 354), 1953, p 742–765 doi: 10.1080/14786440708521052

1.42 W. Kauzmann, Flow of Solid Metals from Standpoint of Chemical-Rate Theory, *Trans. Am. Inst. Min. Metall. Eng.*, Vol 143, 1941, p 57–81

1.43 A.S. Nowick and E.S. Machlin, Dislocation Theory as Applied by N.A.C.A. to the Creep of Metals, *J. Appl. Phys.*, Vol 18, Jan 1947, p 79–87 doi: 10.1063/1.1697556

1.44 J.E. Dorn, Some Fundamental Experiments on High Temperature Creep, *J. Mech. Phys. Solids*, Vol 3(No. 2), Jan 1955, p 85–116 doi: 10.1016/0022-5096(55)90054-5

1.45 S. Dushman L.W. Dunbar and H. Huthsteiner, Creep of Metals, *J. Appl. Phys.*, Vol 15, Feb 1944, p 108–124 doi: 10.1063/1.1707401

1.46 E.Z. Stowell, "A Phenomenological Relation between Stress, Strain Rate, and Temperature for Metals at Elevated Temperatures," *NACA Technical Note 4000*, 1957

1.47 N.F. Mott and F.R.N. Nabarro, Dislocation Theory and Transient Creep, *Report of a Conference on the Strength of Solids*, The Physical Society of London, 1948, p 1–19

1.48 C.L. Smith, A Theory of Transient Creep in Metals, *Proc. Phys. Soc. London*, Vol 61, 1948, p 201–205 doi: 10.1088/0959-5309/61/3/301

1.49 F.R. Larson and J. Miller, Time-Temperature Relationship for Rupture and Creep Stresses, *Trans. Am. Soc. Mech. Eng.*, Vol 74(No. 2), 1952, p 765–771

1.50 S.S. Manson and A.M. Haferd, "A Linear Time-Temperature Relation for Extrapolation of Creep and Stress-Rupture Data," *NACA Technical Note 2890*, March 1953

1.51 P. Feltham, The Plastic Flow of Iron and Plain Carbon Steels Above the A_3-Point, *Proc. Phys. Soc. (London) B*, Vol 66, Oct 1953, p 865–883 doi: 10.1088/0370-1301/66/10/307

1.52 F.H. Norton, *The Creep of Steel at High Temperature*, McGraw-Hill, New York, 1929

1.53 R.W. Bailey, *Engineering*, Vol 129, 1930, p 265

1.54 A. Nadai, The Influence of Time upon Creep, The Hyperbolic Sine Creep Law, *Stephen Timoshenko 60th Anniversary Volume*, Macmillan, New York, 1938, p 155–170

1.55 J.B. Conway and L.H. Sjodahl, *Analysis and Representation of Fatigue Data*, ASM International, 1991

1.56 J.F. Eckel, The Influence of Frequency on the Repeated Bending Life of Acid Lead, *Proceedings of the American Society for Testing and Materials*, Vol 51, 1951, p 745–756

1.57 J. Morrow and G.R. Halford, Creep under Repeated Stress Reversals, Paper 21, *Proceedings of the Joint International Conference on Creep* (London), 1963, p 3–43 to 3–47 doi: 10.1243/PIME_CONF_1963_178_039_02

1.58 M. Kitagawa, Intergranular Fatigue Fracture of Chemical Lead at Room Temperature, *Cyclic Stress-Strain Behavior— Analysis, Experimentation, and Failure Prediction, STP 519*, American Society for Testing and Materials, 1973, p 58–69 doi: 10.1520/STP38024S

1.59 N.J. Grant J.T. Blucher and D.L. Ritter, "Research on the Role of Strain Rate and Temperature in Fatigue," *Air Force Materials Laboratory Technical Report AFMLTR-66-39, Wright Patterson Air Force Base, Jan 1967, Grant AF 33(615)-1143, Project 7351, Task*

735106, Massachusetts Institute of Technology. See also J.T. Blucher and N.J. Grant, Low Strain Rate, High Strain Fatigue of Aluminum as a Function of Temperature, *Trans. Am. Inst. Min. Metall. Eng.*, Vol 239(No. 6), June 1967, p 805–813, and D.L. Ritter and N.J. Grant, Modes of Deformation and Fracture in Low-Cycle Fatigue of Aluminum as a Function of Temperature, *International Conference on Thermal and High Strain Fatigue*, Institution of Metals, London, June 6–7, 1967, preprint p 80–99

1.60 R.E. Swindeman, The Interrelation of Cyclic and Monotonic Creep Rupture, *Proceedings of the Joint International Conferenceon Creep* (London), 1963, p 3–71 to 3–76 doi: 10.1243/PIME_CONF_1963_178_043_02

1.61 S. Taira, Lifetime of Structures Subjected to Varying Load and Temperature, *Colloquium on Creep in Structures*, N.J. Hoff, Ed., Stanford University, International Union of Theoretical and Applied Mechanics (IUTAM), Springer-Verlag, 1962, p 96–119 doi: 10.1007/978-3-642-86014-0_6

1.62 S.S. Manson U. Muralidharan and G.R. Halford, Tensile and Compressive Constitutive Response of 316 Stainless Steel at Elevated Temperatures, *Nonlinear Constitutive Relations for High Temperature Applications*, NASA CP-2271, May 1982, p 13–42. See also NASA TM-83506, 1983

Chapter 2: Creep Rupture

References

2.1 M.F. Ashby C. Gandhi and D.M.R. Taplin, Fracture-Mechanism Maps and Their Construction for F.C.C. Metals and Alloys, *Acta Metall.*, Vol 27, 1979, p 699–729 doi: 10.1016/0001-6160(79)90105-6

2.2 M.F. Ashby and L.M. Brown, *Perspectives in Creep Fracture*, Pergamon Press, Oxford, 1983, p 13

2.3 F.R. Larson and J. Miller, A Time-Temperature Relationship for Rupture and Creep Stresses, *Trans. Am. Soc. Mech. Eng.*, Vol 74, 1952, p 765–771

2.4 J.H. Holloman and L.D. Jaffee, Time-Temperature Relations in Tempering Steel, *Trans. Am. Inst. Metall. Eng.*, Vol 162, 1945, p 22

2.5 S.S. Manson and A.M. Haferd, "A Linear Time-Temperature Relation for Extrapolation of Creep and Stress-Rupture Data," *NACA Technical Note 2890*, Lewis Flight Propulsion Laboratory, Cleveland, National Advisory Committee for Aeronautics, Washington, D.C., 1952

2.6 R.L. Orr O.D. Sherby and J.E. Dorn, Correlations of Rupture Data for Metals at Elevated Temperature, *Trans. Am. Soc. Met.*, Vol 46, 1954, p 113

2.7 H.P. van Leeuwen, "Predicting Material Behavior under Load, Time and Temperature Conditions," *AGARD Report N67-19241*, June 1965

2.8 S.S. Manson and W.F. Brown, Time-Temperature Stress Relations for Correlation and Extrapolation of Stress-Rupture Data, *Proceedings, American Society for Testing and Materials*, Vol 53, 1953, p 693–719

2.9 S.S. Manson and G. Succop, Stress Rupture Properties of Inconel 700 and Correlation on the Basis of Several Time-Temperature Parameters, *STP 174, American Society for Testing and Materials*, 1956, p 60

2.10 Manson and Murry, 1963

2.11 Barrett, Ardell, and Sherby, 1968

2.12 M. Grounes, A Reaction-Rate Treatment of the Extrapolation Methods in Creep Testing, *ASME J. Basic Eng.*, 1969 doi: 10.1115/1.3571028

2.13 S.S. Manson and C.R. Ensign, "Interpolation and Extrapolation of Creep Rupture Data by the Minimum Commitment Method, Part II," *Metal Properties Council, MPC-7, American Society of Mechanical Engineers*, New York, 1978

2.14 H.R. Voorhees and M. Prager, Assessment and Use of Creep-Rupture Properties, *Mechanical Testing and Evaluation*, Vol 8, *ASM Handbook*, ASM International, 2000 doi: 10.31399/asm.hb.v08.a0003289

2.15 S.S. Manson and U. Muralidharan, "Analysis of Creep Rupture Data for Five Multi-Heat Alloys by the Minimum Commitment Method Using Double Heat Term Centering Technique," *EPRI CS 317*, Electric Power Institute, Palo Alto, CA, 1983

2.16 Y. Monma, Creep and Stress Rupture— Long Term, *Superalloys, Supercomposites and Superceramics*, Academic Press, Inc., 1989, p 339 doi: 10.1016/B978-0-12-690845-9.50017-8

2.17 J. Park, "Creep Rupture Data Analysis by Association with a Large Database on Numerous Materials Tested to Long Times," *Doctoral thesis, Department of Mechanical and Aerospace Engineering*, Case Western Reserve University, May 1993

2.18 S.S. Manson and J. Park, "Concurrent Field Service and Laboratory Testing as a Means of Improving Reliability in Creep-Rupture Applications," Proceedings, Sixth International Conference on Mechanical Behavior of Materials(Kyoto, Japan), July 28– Aug 2, 1991

2.19 S.S. Manson, A Quarter-Century of Progress in the Development of Correlation and Extrapolation Methods for Creep Rupture Data, *J. Eng. Technol.*, Vol 101, October 1979, p 317 doi: 10.1115/1.3443696

Chapter 3: Strain-Range Partitioning—Concepts and Analytical Methods

References

3.1 S.S. Manson G.R. Halford and M.H. Hirschberg, Creep-Fatigue Analysis by Strain-Range Partitioning, *Symposium on Design for Elevated Temperature Environment*, ASME, 1971, p 12–28. See also NASA TM X-67838, 1971

3.2 M.H. Hirschberg and G.R. Halford, "Use of Strain-Range Partitioning to Predict High-Temperature Low-Cycle Fatigue Life," NASA TN D-8072, 1976

3.3 S.S. Manson G.R. Halford and R.E. Oldrieve, "Relation of Cyclic Loading Pattern to Microstructural Fracture in Creep-Fatigue," NASA TM-83473, 1983

3.4 S.S. Manson and G.R. Halford, Relation of Cyclic Loading Pattern to Microstructural Fracture in Creep-Fatigue, *Proceedings of the Second International Conference on Fatigue and Fatigue Thresholds (Fatigue 84)*, Vol 3, C.J. Beevers, Ed., Engineering Materials Advisory Services Ltd. (Warley, U.K.), 1984, p 1237–1255

3.5 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, 2006

3.6 K.D. Sheffler and G.S. Doble, "Influence of Creep Damage on the Low Cycle Thermal-Mechanical Fatigue Behavior of Two Tantalum Base Alloys," NASA-CR-121001 (TRW ER-7592), *Contract NAS 3-13228*, TRW Materials Technology Laboratories, Cleveland, OH, Final Contractor Report NAS-3-13228, May 1972

3.7 *Proceedings of the NATO AGARD Conference on Characterization of Low Cycle High Temperature Fatigue by the Strain-Range Partitioning Method*, Advisory Group for Aerospace Research & Development, AGARD CP-243 (Aalborg, Denmark), 1978

3.8 K. Hirakawa and K. Tokimasa, "Environmental Effects on Creep Fatigue Properties of Austenitic Stainless Steel," *Paper C47/80, Institution of Mechanical Engineering*, 1980, p 315–320

3.9 K. Hirakawa and K. Tokimasa, Fatigue-Creep Interaction and Fracture Morphology, *J. Soc. Mater. Sci., Jpn.*, Vol 30(No. 328), Jan 1981, p 65–71

3.10 J. He Y. Ning and D. Zhao, Strain Energy Partitioning and Its Application to GH33A Nickel-Based Superalloy and 1Cr-18Ni-9Ti Stainless Steel, *Proceedings, ASME International Conference on Advances in Life Prediction Methods* (New York), April 1983, p 27–32

3.11 K. Tokimasa and I. Nitta, "High Temperature Fatigue Properties of Large-Diameter Thick-Walled Pipe Materials for Boiler Power Plants," The Sumitomo Search No. 28 (Amagasaki, Japan), Nov 1983

3.12 K. Setoguchi M. Yamauchi T. Igari and Y. Wakamatsu, Creep-Fatigue Life Prediction of Normalized and Tempered 2-1/4Cr-1Mo Steel by Life Fraction Rule and Strain-Range Partitioning Method, *Trans. Iron Steel Inst. Jpn.*, Vol 24, 1984, p 1063–1071 doi: 10.2355/isijinternational1966.24.1063

3.13 J. He Z. Duan Y. Ning and D. Zhao, Strain Energy Partitioning and Its Application to GH33A Nickel-Base Superalloy and 1Cr18Ni9Ti Stainless Steel, *Acta Metall. Sin.*, Vol 21(No. 1), Feb 1985, p A54–A63

3.14 T. Inoue T. Igari M. Okazaki M. Sakane and K. Tokimasa, Fatigue Creep Life Prediction of 2-1/4Cr-1Mo Steel by Inelastic Analysis—Results of Joint Work (B), *Transactions of the Ninth International Conference on Structural Mechanics in Reactor Technology* (Lausanne), F.H. Wittman, Ed., Aug 17–21, 1987, p 261–266

3.15 M. Yamauchi T. Igari K. Setoguchi and H. Yamanouchi, Comparison of Creep-Fatigue Life Prediction by Life Fraction Rule and Strain-Range Partitioning Methods, *Low Cycle Fatigue*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 399–413 doi: 10.1520/STP24494S

3.16 Z. Duan J. He Y. Ning and Z. Dong, Strain Energy Partitioning Approach and Its Application to Low Cycle Fatigue Life Prediction for Some Heat-Resistant Alloys, *Low Cycle Fatigue*, STP 942, H.D. Soloman G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 133–1143

3.17 Z. Dong Z. Duan and J. He, Application of Damage Rules to Time-Dependent Fatigue Life Prediction, *J. Aerosp. Power*, Vol 3, July 1988, p 274–276, 287–288

3.18 B.N. Leis, An Energy-Based Fatigue and Creep-Fatigue Damage Parameter, *J. Pressure Vessel Technol. (Trans. ASME)*, Vol 99(No. 4), 1977, p 524–553 doi: 10.1115/1.3454571

3.19 K.D. Sheffler, "Vacuum Thermal-Mechanical Fatigue Testing of Two Iron-Base High-Temperature Alloys," NASA CR-134524, TRW ER-7697, Contract NAS3-6010, 1974

3.20 K.D. Sheffler, Vacuum Thermal-Mechanical Fatigue Behavior of Two Iron-Base Alloys, *Thermal Fatigue of Materials and Components*, STP 612, D.A. Spera and D.F. Mowbray, Ed., American Society for Testing and Materials, 1976, p 214–226 doi: 10.1520/STP27893S

3.21 A.J. Opinsky, Bend Fatigue of Two Iron-Nickel-Base Superalloys at Elevated Temperature, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 451–461 doi: 10.1520/STP38861S

3.22 R.C. Bill M.J. Verrilli M.A. McGaw and G.R. Halford, "A Preliminary Study of the Thermomechanical Fatigue of Polycrystalline MAR M-200," NASA TP-2280 (AVSCOM TR 83-C-6), Feb 1984

3.23 C.G. Annis M.C. VanWanderham and R.M. Wallace, "Strain-Range Partitioning Behavior of an Automotive Turbine Alloy," NASA CR-134974, Contract NAS3-18930, Pratt & Whitney, 1976

3.24 S.S. Manson, The Strain-Range Conversion Principle for Treating Cumulative Fatigue Damage in the Creep Range, *Proceedings, Fourth National Conference on Pressure Vessel and Piping Technology*, PVP Vol 12, American Society of Mechanical Engineers (New York), 1983, p 1–29

3.25 C.G. Annis M.C. VanWanderham and R.M. Wallace, "Strain-Range Partitioning Behavior of an Automotive Turbine Alloy", NASA CR-134974, Contract NAS3-18930, 1976

3.26 S.S. Manson, The Challenge to Unify Treatment of High Temperature Fatigue – A Partisan Proposal Based on Strain-Range Partitioning, *Fatigue at Elevated Temperatures*, ASTM STP 520, A.E. Carden A.J. McEvily C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 744–775 doi: 10.1520/STP38885S

3.27 S.S. Manson G.R. Halford and M.H. Hirschberg, Creep-Fatigue Analysis by Strain-Range Partitioning, *Symposium on Design for Elevated Temperature Environment*, ASME, 1971, p 12–28. See also NASA TM X-67838, 1971

3.28 M.H. Hirschberg and G.R. Halford, "Use of Strain-Range Partitioning to Predict High-Temperature Low-Cycle Fatigue Life", NASA TN D-8072, 1976

3.29 J.G. Conway R.H. Stentz and J.T. Berling, "Fatigue, Tensile, and Relaxation Behavior of Stainless Steels", TID-26135, United States Atomic Energy Commission under contract to Mar-Test, Inc., 1975 doi: 10.2172/4239699

Chapter 4: Strain-Range Conversion—An Extended View of Strain-Range Partitioning

References

4.1 S.S. Manson, The Strain-Range Conversion Principle for Treating Cumulative Fatigue Damage in the Creep Range, *Proceedings, Fourth National Conference on Pressure Vessel and Piping Technology*, PVP Vol 12, American Society of Mechanical Engineers (New York), 1983, p 1–29

4.2 R. Wesling, tests done as part of Case Western Reserve University Masters Thesis Program

4.3 S.S. Manson, "Some Useful Concepts for the Designer in Treating Cumulative Fatigue Damage at Elevated Temperatures," *presented at the Third International Conference on Mechanical Behavior of Materials* (University of Cambridge, U.K.), 1979 doi: 10.1016/B978-1-4832-8414-9.50012-2

4.4 S. Majumdar and P.S. Maiya, A Mechanistic Model for Time-Dependent Fatigue, *J. Eng. Mater. Technol. (ASME)*, Vol 102, Jan 1980, p 159–167 doi: 10.1115/1.3224774

4.5 D. Roulette, Masters Thesis at Case Western Reserve University, Cleveland, OH

4.6 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, Materials Park, OH, 2006

Chapter 5: Partitioning of Hysteresis Loops and Life Relations

References

5.1 A.K. Miller, A Realistic Model for the Deformation Behavior of High-Temperature Materials, *Fatigue at Elevated Temperatures, STP 520*, American Society for Testing and Materials, 1973, p 613–624

5.2 S. Bodner and Y. Partom, Constitutive Equations for Elastic Viscoplastic Strain Hardening Materials, *J. Appl. Mech. (Trans. ASME)*, Vol 42, 1975, p 385–389 doi: 10.1115/1.3423586

5.3 D.A. Robinson, "A Unified Creep-Plasticity Model for Structural Metals at High Temperature," ORNL/TM-5969, Oak Ridge National Laboratory, Oct 1978 doi: 10.2172/6546103

5.4 S. Bodner I. Partom and Y. Partom, Uniaxial Cyclic Loading of Elastic Viscoplastic Material, *J. Appl. Mech. (Trans. ASME)*, Vol 46, 1979, p 651–664 doi: 10.1115/1.3424658

5.5 D.C. Drucker and L. Palgen, On the Stress-Strain Relations Suitable for Cyclic and Other Loading, *J. Appl. Mech. (Trans. ASME)*, Vol 48, 1981, p 479–485 doi: 10.1115/1.3157660

5.6 K.P. Walker, "Research and Development Program for Non-Linear Structural Modeling with Advanced Time-Temperature Dependent Constitutive Relationships," *NASA CR-165533, Contract NAS3-22055 with United Technologies Research Center*, National Aeronautics and Space Administration, Nov 1981

5.7 D.N. Robinson and R.W. Swindeman, "Unified Creep-Plasticity Constitutive Equations for 2-1/4 Cr-1 Mo Steel at Elevated Temperature," ORNL/TM-8444, Oak Ridge National Laboratory, Oct 1982 doi: 10.2172/708802

5.8 D.N. Robinson, "Constitutive Relationships for Anisotropic High-Temperature Alloys," NASA TM-83437, National Aeronautics and Space Administration, 1983

5.9 D.A. Wilson and K.P. Walker, "Constitutive Modeling of Engine Materials," *AFWAL-TR-84-4073, Contract F33615-81-C-5040 with United Technologies, Pratt & Whitney*, Air Force Wright Aeronautical Laboratories, July 1984

5.10 V.G. Ramaswamy, "A Constitutive Model for the Inelastic Multiaxial Cyclic Response of a Nickel-Base Superalloy René 80," *NASA CR-3998, Contract NAS3-23927 with the University of Cincinnati*, National Aeronautics and Space Administration, July 1986

5.11 M.Y. Sheh and D.C. Stouffer, "Anisotropic Constitutive Model for Nickel-Base Single Crystal Superalloys," *NASA CR-182158, Grant NAG3-511 with University of Cincinnati*, National Aeronautics and Space Administration, July 1988

5.12 J.L. Chaboche, Constitutive Equations for Cyclic Plasticity and Cyclic Viscoplasticity, *Int. J. Plast.*, Vol 5(No. 3), 1989, p 247–302 doi: 10.1016/0749-6419(89)90015-6

5.13 J.L. Chaboche and D. Nouaihas, A Unified Constitutive Model for Cyclic Viscoplasticity and Its Applications to Various Stainless Steels, *J. Eng. Mater. Technol.*, Vol III, Oct 1989, p 424–430 doi: 10.1115/1.3226490

5.14 D. Slavik and T.S. Cook, A Unified Constitutive Model for Superalloys, *Int. J. Plast.*, Vol 6, 1990, p 651–664 doi: 10.1016/0749-6419(90)90037-F

5.15 S.S. Manson G.R. Halford and A.J. Nachtigall, Separation of the Strain Components for Use in Strain-Range Partitioning, *Advances in Design for Elevated Temperature Environment*, S.Y. Zamrik and R.I. Jetter, Ed., American Society of Mechanical Engineers, New York, 1975, p 17–28. See also NASA TM X-71737, 1975

5.16 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, 2006 doi: 10.31399/asm.tb.fdsm.9781627083447

5.17 H. Conrad, The Role of Grain Boundaries in Creep and Stress Rupture, *Mechanical Behavior of Materials at Elevated Temperatures*, J.E. Dorn, Ed., McGraw-Hill Book Company, New York, 1961

5.18 S.S. Manson, The Challenge to Unify Treatment of High Temperature Fatigue—A Partisan Proposal Based on Strain-Range Partitioning, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 744–775 doi: 10.1520/STP38885S

5.19 G.R. Halford J.F. Saltsman and M.H. Hirschberg, Ductility Normalized Strain-Range Partitioning Life Relations for Creep-Fatigue Life Prediction, *Proceedings of the Conference on Environmental Degradation of Engineering Materials*, R. P. McNitt and M.R. Louthan Jr., Ed., V. P.I. and State University, 1977, p 599–612. See also NASA TM X-73737, 1977

5.20 S.S. Manson, Fatigue—A Complex Subject, *Exp. Mech.*, Vol 5(No. 7), 1965, p 193–226 doi: 10.1007/BF02321056

5.21 R.M. Curran and B.M. Wundt, "Interpretive Report on Notched and Unnotched Creep Fatigue InterspersionTests of Cr-Mo-V, 2-1/4 Cr-1Mo and Type 304 Stainless Steel," *presented*

at the symposium on Ductility and Toughness Consideration in Elevated Temperature Service, ASME-MPC Meeting (San Francisco), Dec 1978

5.22 J.F. Saltsman and G.R. Halford, Strain-range Partitioning Life Predictions of the Long Time Metal Properties Council Creep-Fatigue Tests, *Methods for Predicting Material Life in Fatigue*, American Society for Mechanical Engineers, New York, 1979, p 101–132. See also NASA TM-79260, 1979

5.23 S. Kalluri and S.S. Manson, "Time Dependency of SRP Life Relationships," NASA CR-174946, NASA Grant NAG3-337, Case Western Reserve University, Aug 1984

5.24 S. Kalluri, "The Effect of Creep Rate on Strainrange Partitioning Life Relationships," *M.S. thesis, Department of Mechanical and Aerospace Engineering*, Case Western Reserve University, Jan 1984

5.25 S. Kalluri, "Generalization of the Strain-range Partitioning Method for Predicting High Temperature Low Cycle Fatigue at Different Exposure Times," Ph.D. dissertation, Case Western Reserve University, Jan 1987

5.26 S. Kalluri S.S. Manson and G.R. Halford, Environmental Degradation of AISI Type 316 Stainless Steel in High Temperature Low Cycle Fatigue, *Proceedings of the Conference on Environmental Degradaton of Engineering Materials III*, (University Park, PA), M.R. Louthan R.P. McNitt and R.D. Sisson Jr., Ed., 1987, p 503–519

5.27 E.C. Ellison, Strainrange Partitioning in Cyclic Creep of a 1Cr Mo V Steel, *Proceedings of the NATO AGARD Conference on Characterization of Low Cycle High Temperature Fatigue by the Strainrange Partitioning Method*, Advisory Group for Aerospace Research & Development, AGARD CP-243 (Aalborg, Denmark), 1978

5.28 R.H. Priest and E.C. Ellison, Estimation of Cyclic Creep Damage by Strain and Strainrate Considerations, *Proceedings, International Conference on Engineering Aspects of Creep*, Vol 1 (London), Institution of Mechanical Engineers, 1980, p 185–192

Chapter 6: Total Strain-Based Strain-Range Partitioning—Isothermal and Thermomechanical Fatigue

References

6.1 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, 2006 doi: 10.31399/asm.tb.fdsm.9781627083447

6.2 S.S. Manson and R. Zab, "Treatment of Low Strains and Long Hold Times in High Temperature Metal Fatigue by Strainrange Partitioning," *ORNL/Sub-3988/1 Report, prepared by Case Western Reserve University for Oak Ridge National Laboratory*, Aug 1977 doi: 10.2172/5244848

6.3 S.S. Manson and G.R. Halford and M. H. Hirschberg, Creep-Fatigue Analysis by Strain-Range Partitioning. *Proceedings of First Symposium on Design for Elevated Temperature Environment*, S.Y. Zamrik, Ed., American Society of Mechanical Engineers, 1971, p 12–24

6.4 J.B. Conway R.H. Stentz and J.T. Berling, "Fatigue, Tensile, and Relaxation Behavior of Stainless Steels," United States Atomic Energy Commission, 1975 doi: 10.2172/4239699

6.5 S.S. Manson and G.R. Halford and A.J. Nachtigall, Separation of the Strain Component for Use in Strain-Range Partitioning, *Advances in Design for Elevated Temperature Environment*, S.Y. Zamrikand R.I. Jetter, Ed., American Society of Mechanical Engineers, 1975, p 17–28

6.6 G.R. Halford and J.F. Saltsman, Strain-Range Partitioning—A Total Strain-Range Version. *Proceedings, International Conference on Advances in Life Prediction Methods*, D.A. Woodford and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1983, p 17–26

6.7 J.F. Saltsman and G.R. Halford, An Update on the Total Strain Version of SRP, *Low Cycle Fatigue—Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 329–341. See also NASA TP-2499, Oct 1985

6.8 J.F. Saltsman and G.R. Halford, "Procedures for Characterizing an Alloy and Predicting Cyclic Life Using the Updated Total Strain Version of Strain-Range Partitioning," NASA TM-4102, June 1989

6.9 J.F. Saltsman and G.R. Halford, "Life Prediction of Thermomechanical Fatigue Using Total Strain Version of Strain-Range Partitioning (SRP)—A Proposal," NASA TP-2779, Feb 1988

6.10 G.R. Halford and A.J. Nachtigall, The Strain-Range Partitioning Behavior of an Advanced Gas Turbine Disk Alloy, AF2-1DA, *J. Aircr.*, Vol 17, (No. 8), 1980, p 598–604 doi: 10.2514/3.57945

6.11 J.F Saltsman, personal communication, 1985

6.12 K.P. Walker, "Research and Development Program for Nonlinear Structural Modeling with Advanced Time-Temperature Dependent Constitutive Relationships," NASA CR-165533, 1981

6.13 D.N. Robinson and R.W. Swindeman, "Unified Creep-Plasticity Constitutive Equations for 2-¼Cr-1Mo Steel at Elevated Temperature," ORNL/TM 8444, Oak Ridge National Laboratory, Oct 1982 doi: 10.2172/708802

6.14 S.S. Manson, The Challenge to Unify Treatment of High Temperature Fatigue— A Partisan Proposal Based on Strain-Range Partitioning, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 744–775 doi: 10.1520/STP38885S

6.15 S. Kalluri S.S. Manson and G.R. Halford, Environmental Degradation of 316 Stainless Steel in High Temperature Fatigue, *Proceedings, Third International Conference on Environmental Degradation of Engineering Materials*, M.R. Louthan Jr., R.P. McNitt and R.D. Sisson Jr., Eds., 1987, p. 503–519. See also NASA TM-89931

6.16 J. Morrow, Fatigue Properties of Metals, Section 3.2, *Fatigue Design Handbook*, Society of Automotive Engineers, 1968 (Section 3.2 is a Summary of a Paper Presented at a Meeting of Division 4 of the SAE Iron and steel Technical Commmittee, Nov 4, 1964.)

6.17 G.R. Halford, Low-Cycle Thermal Fatigue. *Thermal Stresses II*, R. B. Hetnarski, Ed., Elsevier, 1987, p 329–428

6.18 G.R. Halford J.F. Saltsman and M.H. Hirschberg, Ductility-Normalized Strain-range Partitioning Life Relations for Creep-Fatigue Life Prediction. *Environmental Degradation of Engineering Materials*, M.R. Louthan and R.P. McNitt, Ed., Virginia Polytechnic Institute and State University, 1977, p 599–612

6.19 G.R. Halford M.A. McGaw R.C. Bill and P. D. Fanti, Bithermal Fatigue, A Link between Isothermal and Thermomechanical Fatigue. *Low Cycle Fatigue—Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 625–637 doi: 10.1520/STP24510S

6.20 T.Y. Chang and R.L. Thompson, "A Computer Program for Predicting Nonlinear Uniaxial Material Responses Using Viscoplastic Models," NASA TM-83675, 1984

6.21 S.S. Manson and U. Muralidharan, "A Single Expression Formula for Inverting Strain-Life and Stress-Strain Relationships," NASA CR 165347, 198l

6.22 J.M. Hyzak and H.L. Bernstein, An Analysis of the Low Cycle Fatigue Behavior of the Superalloy René 95 by Strain range Partitioning. *Characterization of Low Cycle High Temperature Fatigue by the Strain-range Partitioning Method*, AGARD-CP-243, 1978, p 11-1 to 11-25

6.23 H.L. Bernstein, "An Evaluation of Four Current Models to Predict the Creep-Fatigue Interaction in René 95," AFML-TR-79-4075, 1979

6.24 S.S. Manson, "Behavior of Materials under Conditions of Thermal Stress," *Lecture presented at Symposium on Heat Transfer, University of Michigan*, June 27–28, 1952. See also, NACA TN 2933, July 1953, and NACA Report 1170, 1954

6.25 K.D. Sheffler and G. S. Doble, "Influence of Creep Damage on the Low-Cycle Thermal-Mechanical Fatigue Behavior of Two Tantalum Base Alloys," NASA-CR-121001, May 1972

6.26 G.R. Halford M.J. Verrilli S. Kalluri F.J. Ritzert R.E. Duckert and F.A. Holland, Thermomechanical and Bithermal Fatigue Behavior of Cast B-1900+Hf and Wrought Haynes 188. *Advances in Fatigue Lifetime Predictive Techniques*, STP 1122, M.R. Mitchell and R.W. Landgraf, Ed., American Society for Testing and Materials, 1992, p 120–142 doi: 10.1520/STP24156S

6.27 G.R. Halford J.F. Saltsman M.J. Verrilli and V. Arya, Application of Thermal Fatigue Life Prediction Model to Two High-Temperature Aerospace Alloys. *Advances in Fatigue Lifetime Predictive Techniques*, STP 1122, M.R. Mitchell and R.W. Landgraf, Ed., American Society for Testing and Materials, 1992, p 107–119

6.28 G.R. Halford B.A. Lerch and V.K. Arya, Thermal Strain Fatigue Modeling of a Matrix Alloy for a Metal Matrix Composite. *Third Symposium on Thermo-Mechanical Fatigue Behavior of Materials*, STP 1371, H. Sehitoglu and H. Maier, Ed., American Society for Testing and Materials, 2000, p 186–203 doi: 10.1520/STP15261S

6.29 G.-Y. Lui M.B. Behling and G.R. Halford, Bithermal Low-Cycle Fatigue Evaluation of Automotive Exhaust System Alloy SS409. *Fatigue Fract of Eng. Mater. Struct*., Vol. 23, 2000, p 787–794 doi: 10.1046/j.1460-2695.2000.00310.x

6.30 J.F. Saltsman and G.R. Halford, "Ability of the Total Strain Version of Strainrange Partitioning to Characterize Thermomechanical Fatigue Behavior," NASA TP-4556, July 1994

Chapter 7: Treatment of Multiaxial Loading

References

7.1 S.S. Manson and G.R. Halford, Treatment of Multiaxial Creep-Fatigue by Strain-range Partitioning. *1976 ASME-MPC Symposium on Creep-Fatigue Interaction*, MPC-3, R.M. Curran, Ed., American Society of Mechanical Engineers, New York, 1976, p 299–322

7.2 M.W. Brown and K.J. Miller, A Theory for Fatigue Failure Under Multiaxial Stress-Strain Conditions, *Proc. Inst. Mech. Eng.*, Vol 187(65/73), 1973, p 745–755, discussion, p D229–D243 doi: 10.1243/PIME_PROC_1973_187_069_02

7.3 A.R. Kallmeyer A. Krgo and P. Kurath, Evaluation of Multiaxial Fatigue Life Prediction Methodologies for Ti-6Al-4V. *J. Eng. Mater. Technol.*, Vol 124, April 2002, p 220–237 doi: 10.1115/1.1446075

7.4 S.S. Manson and M.H. Hirschberg, Crack Initiation and Propagation in Notched Fatigue Specimens. *Proceedings, First International Conference on Fracture*, Vol 1 (Sendai, Japan), 1965 p 479–498

7.5 S.S. Manson, *Thermal Stress and Low-Cycle Fatigue*. McGraw-Hill, Inc., New York, 1966 doi: 10.1115/1.3625225

7.6 S.S. Manson G.R. Halford and A.J. Nachtigall, Separation of the Strain Components for Use in Strain-range Partitioning. *Advances in Design for Elevated Temperature Environment*, S.Y. Zamrik R.I. Jetter, Eds., American Society of Mechanical Engineers, New York, 1975, p 17–28. See also NASA TM X-71737, 1975

7.7 G.R. Halford and S.S. Manson, Life Prediction of Thermal-Mechanical Fatigue Using Strain-range Partitioning, Thermal Fatigue of Materials and Components, ASTM STP 612, D.A. Spera D.F. Mowbray, Ed., American Society for Testing and Materials, 1976, p 239–254. See also NASA TM X-71829, 1975

7.8 S.Y. Zamrik and T. Goto, The Use of Octahedral Shear Strain Theory in Biaxial Low Cycle Fatigue. *Proceedings of the Inter-American Conference on Materials* (*ASME*), New York, 1968, p 551–562

7.9 S.Y. Zamrik, "The Application of 'Strain-Range Partitioning Method' to Torsional Creep-Fatigue Interaction," NASA CR-134817, National Aeronautics and Space Administration, 1975

7.10 J.F. Saltsman and G.R. Halford, "Application of Strain-range Partitioning to the Prediction of Creep-Fatigue Lives of AISI Types 304 and AISI Type 316 Stainless Steel," NASA TM X-71898, National Aeronautics and Space Administration, 1976 doi: 10.1115/1.3454531

7.11 S.S. Manson, The Challenge to Unify Treatment of High Temperature Fatigue—A Partisan Proposal Based on Strain-range Partitioning. *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 744–775 doi: 10.1520/STP38885S

7.12 P.W. Bridgeman, *Studies in Large Plastic Flow and Fracture with Special Emphasis on the Effects of Hydrostatic Pressure*, 1st ed., *Metallurgy and Metallurgical Engineering Series*, McGraw-Hill Book Company, New York, 1952, p 118–124

7.13 C.D. Richards, *Engineering Materials Science*, Wadsworth Publishing Company, Inc., San Francisco, 1961, p 222–227

7.14 E.A. Davis and F.M. Connelly, Stress Distribution and Plastic Deformation in Rotating Cylinders of Strain-Hardening Materials, *J. Appl. Mech. (Trans. ASME)*, Vol 81, 1959, p 25–30 doi: 10.1115/1.4011918

7.15 M.J. Manjoine, Ductility Indices at Elevated Temperatures, *J. Eng. Mater. Technol. (Trans. ASME)*, Vol 97 (Series H, No.2), April 1975, p 156–161 doi: 10.1115/1.3443276

7.16 S.S. Manson and G.R. Halford, discussion to paper by J.J. Blass and S.Y. Zamrik, Multiaxial Low-Cycle Fatigue of Type 304 Stainless Steel, *1976 ASMEMPC Symposium on Creep-Fatigue Interaction*, R.M. Curran, Ed., American Society of Mechanical Engineers, (New York), 1976, p 129–159. Also published in *J. Eng. Mater. Technol.*, Vol 99, July 1977, p 283–286 doi: 10.1115/1.3443540

7.17 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, 2006

7.18 R.W. Weeks D.R. Diercks and C.F. Cheng, ANL Low-Cycle Fatigue Studies—Programs, Results, and Analysis," *Argonne National Laboratories Report ANL-8009*, Nov 1973

Chapter 8: Critique of Predictive Methods for Treatment of Time-Dependent Metal Fatigue at High Temperatures

References

8.1 S.S. Manson, A Critical Review of the Predictive Methods for Treatment of Time-Dependent Metal Fatigue at High Temperatures, *Pressure Vessel and Piping Design Technology – 1982 – A Decade of Progress*, S.Y. Zamrik and D. Dietrich, Ed., American Society of Mechanical Engineers, 1982, p 203–225

8.2 G.R. Halford, Low-Cycle Thermal Fatigue, *Thermal Stresses II*, R.B. Hetnarski, Ed., Elsevier Science Publishers B. V., 1987, p 329–428

8.3 G.R. Halford, Evolution of Creep-Fatigue Life Prediction Models, *Creep-Fatigue Interaction at High Temperature*, AD Vol 21, G.K. Haritos and O.O. Ochoa, Ed., American Society of Mechanical Engineers, 1991, p 43–57

8.4 G.R. Halford, Creep-Fatigue Interaction, *Heat Resistant Materials*, *ASM Specialty Handbook*, ASM International, Oct 1997, p 499–517

8.5 Anonymous, "Time-Dependent Fatigue of Structural Alloys: A General Assessment," ORNL-5073, Oak Ridge National Laboratory, Jan 1977

8.6 S.S. Manson, Observations and Correlations Emphasizing Wave Shape and Other Salient Features, *Time-Dependent Fatigue of Structural Alloys*, ORNL-5073, Jan 1977, p 155–332

8.7 S.S. Manson, Behavior of Materials under Conditions of Thermal Stress, *Heat Transfer Symposium*, June 27– 28, 1952, University of Michigan. See also NACA-TN-2933, July 1953

8.8 L.F. Coffin Jr., A Study of the Effects of Cyclic Thermal Stresses on a Ductile Metal, *Trans., American Society of Mechanical Engineers*, Vol 76, 1954, p 931–950

8.9 S.S. Manson, Thermal Stress in Design, Part 19, Cyclic Life of Ductile Materials, *Mach. Des.*, Vol 32(No. 14), July 7, 1960, p 139–144

8.10 S.S. Manson, Fatigue: A Complex Subject—Some Simple Approximations, *Exp. Mech.*, Vol 5(No. 7), July 1965, p 193–226 doi: 10.1007/BF02321056

8.11 S.S. Manson, Interfaces between Fatigue Creep and Fracture, *Int. J. Fract. Mech.*, Vol 2(No. 1), 1966, p 327–363 doi: 10.1007/BF00698478

8.12 S.S. Manson and G.R. Halford, A Method of Estimating High-Temperature Low-Cycle Fatigue Behavior of Materials, *Proc., International Conference on High-Strain Fatigue*, The Metals and Metallurgy Trust, London, 1967, p 154–170

8.13 G.R. Halford and S.S. Manson, Application of a Method of Estimating High-Temperature Low-Cycle Fatigue Behavior of Materials, *Trans., American Society of Metals*, Vol 61(No. 1), 1968, p 94–102

8.14 S.S. Manson G.R. Halford and M.H. Hirschberg, Creep-Fatigue Analysis by Strain-Range Partitioning, *Proc. of First Symposium on Design for Elevated Temperature Environment*, May 10–12, 1971, S.Y. Zamrik, Ed., p 12–24 (discussion, p 25–28)

8.15 R.E. Oldrieve, "Fractographic Evaluation of Creep Effects on Strain-Controlled Fatigue-Cracking of AISI 304LC and 316 Stainless Steel," NASA TM-78913, National Aeronautics and Space Administration, June 1978

8.16 S.S. Manson, The Challenge to Unify Treatment of High Temperature Fatigue—A Partisan Proposal Based on Strain-Range Partitioning, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 744–782 doi: 10.1520/STP38885S

8.17 Anonymous, *Proc., AGARD Conference on Characterization of Low-Cycle High-Temperature Fatigue by the Strain-Range Partitioning Method*, AGARD CP-243, 1978

8.18 M.H. Hirschberg and G.R. Halford, "Use of Strain-Range Partitioning to Predict High-Temperature Low-Cycle Fatigue Life," NASA TN D-8072, 1976

8.19 G.R. Halford B.A. Lerch and V.K. Arya, Thermal Strain Fatigue Modeling of a Matrix Alloy for a Metal Matrix Composite, *Third Symposium on Thermo-Mechanical Fatigue Behavior of Materials*, STP 1371, H. Sehitoglu and H. Maier, Ed., American Society for Testing and Materials, 2000, p 186–203 doi: 10.1520/STP15261S

8.20 G.-Y. Lui M.B. Behling and G.R. Halford, Bithermal Low-Cycle Fatigue Evaluation of Automotive Exhaust System Alloy SS409, *Fatigue Fract. Eng. Mater. Struct.*, Vol 23, 2000, p 787–794 doi: 10.1046/j.1460-2695.2000.00310.x

8.21 G.R. Halford M.H. Hirschberg and S.S. Manson, Temperature Effects on the Strain-Range Partitioning Approach for Creep-Fatigue Analysis, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 658–669 doi: 10.1520/STP38877S

8.22 G.R. Halford J.F. Saltsman and M.H. Hirschberg, Ductility Normalized Strain-Range Partitioning Life Relations for Creep-Fatigue Life Predictions, *Proc. of the Conference on*

Environmental Degradation of Engineering Materials, R.P. McNitt and M.R. Louthan Jr., Ed., Virginia Polytechnic Institute and State University, Oct 1977, p 599–612

8.23 S.S. Manson G.R. Halford and A.J. Nachtigall, Separation of the Strain Components for Use in Strain-Range Partitioning, Advances in Design for Elevated Temperature Environment, *Proc. of ASME Second National Congress on Pressure Vessels and Piping* (San Francisco), S.Y. Zamrik and R.I. Jetter, Ed., American Society of Mechanical Engineers, 1975, p 17–28

8.24 G.R. Halford and S.S. Manson, Life Predictions of Thermal-Mechanical Fatigue Using Strain-Range Partitioning, *Thermal Fatigue of Materials and Components*, STP 612, D.A. Spera and D.F. Mowbray, Ed., American Society for Testing and Materials, 1976, p 239–254 doi: 10.1520/STP27895S

8.25 S.S. Manson, The Development and Application of Strain-Range Partitioning as a Tool in the Treatment of High Temperature Metal Fatigue, *AGARD Specialist Meeting on Evaluation of Strain-Range Partitioning*, AGARD Conf. Proc. No. 243, April 1978, p K-1 to K-11

8.26 S.S. Manson G.R. Halford and D.A. Spera, The Role of Creep in High-Temperature Low-Cycle Fatigue, *Advances in Creep Design, The A.E. Johnson Memorial Volume*, A.I. Smith and A.M. Nicholson, Ed., Applied Science Publisher Ltd., London, 1971, p 229–249

8.27 G.R. Halford and A.J. Nachtigall, The Strain-Range Partitioning Behavior of an Advanced Gas Turbine Disk Alloy, AF2-1DA, *J. Aircr.*, Vol 17(No. 8), 1980, p 598–604 doi: 10.2514/3.57945

8.28 G.-Y. Lui M.B. Behling and G.R. Halford, Bithermal Low-Cycle Fatigue Evaluation of Automotive Exhaust System Alloy SS409, *Fatigue Fract. Eng. Mater. Struct.*, Vol 23, 2000, p 787–794 doi: 10.1046/j.1460-2695.2000.00310.x

8.29 G.R. Halford B.A. Lerch and V.K. Arya, Thermal Strain Fatigue Modeling of a Matrix Alloy for a Metal Matrix Composite, *Third Symposium on Thermo-Mechanical Fatigue Behavior of Materials*, STP 1371, H. Sehitoglu and H. Maier, Ed., American Society for Testing and Materials, 2000, p 186–203 doi: 10.1520/STP15261S

8.30 L.F. Coffin Jr., A Generalized Equation for Predicting High-Temperature, Low-Cycle Fatigue, Including Hold Times, *Proc. of the Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials*, AF FDL TR70-144 (Miami Beach), Dec 1969, p 301–309

8.31 L.F. Coffin Jr., The Concept of Frequency Separation in Life Prediction for Time-Dependent Fatigue, *Symposium on Creep-Fatigue Interaction*, ASME MPC-3, American Society of Mechanical Engineers (New York), 1976, p 349–363

8.32 T. Udoguchi Y. Asada and I. Ichino, A Frequency Interpretation of Hold-Time Experiments on High-Temperature Low-Cycle Fatigue of Steels for LMFBR, *Proc. of the 1973 International Conf. on Creep and Fatigue* (Philadelphia), April 1974, p 211.1–211.9

8.33 W.J. Ostergren, A Damage Function and Associated Failure Equations for Predicting Hold Time and Frequency Effects in Elevated Temperature Low Cycle Fatigue, *J. Test. Eval.*, Vol 4(No. 5), Sept 1976, p 327–339 doi: 10.1520/JTE10520J

8.34 E.L. Robinson, Effect of Temperature Variation on the Long-Time Strength of Steels, *Trans. ASME*, Vol 74, 1952, p 777–780, discussion, p 780–781

8.35 S. Taira, Lifetime of Structures Subjected to Varying Load and Temperature, *Creep in Structures*, N.J. Hoff, Ed., Springer Verlag, Berlin, 1962, p 96–119, discussion, p 119–124 doi: 10.1007/978-3-642-86014-0_6

8.36 M. Kitagawa and R.W. Weeks, Analysis of Hold Time Fatigue Test Results of AISI 304 Stainless Steel by Five Existing Methods, *Proc., Symposium on Mechanical Behavior of Materials* (Kyoto, Japan), 1973, p 233–338 doi: 10.2172/4246350

8.37 C.E. Jaske H. Mindlin and J.S. Perrin, Combined Low-Cycle Fatigue and Stress Relaxation of Alloy 800 and Type 304 Stainless Steel at Elevated Temperature, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 365–376, discussion, p 375–376 doi: 10.1520/STP38854S

8.38 Anonymous, Boiler and Pressure Vessel Code, Case N-47-25, American Society of Mechanical Engineers, Dec 1986

8.39 R.W. Swindeman, The Interrelation of Cyclic and Monotonic Creep Rupture, *Proc. of the Joint International Conference on Creep*, ASME, ASTM, and the Institution of Mechanical Engineers, Aug and Oct 1963, p 3-71 to 3-76 doi: 10.1243/PIME_CONF_1963_178_043_02

8.40 J.D. Morrow and G.R. Halford, Creep under Repeated Stress Reversals, *Proc. of the Joint International Conference on Creep*, ASME, ASTM, and the Institution of Mechanical Engineers, Aug and Oct 1963, p 3-42 to 3-48 doi: 10.1243/PIME_CONF_1963_178_039_02

8.41 G.R. Halford, Cyclic Creep-Rupture Behavior of Three High-Temperature Alloys, *Metall. Trans.*, Vol 3(No. 8), 1972, p 2247–2256 doi: 10.1007/BF02643239

8.42 S. Majumdar and P.S. Maiya, "A Unified and Mechanistic Approach to Creep-Fatigue Damage," presented at the Second International Conf. of Mechanical Behavior of Materials, Aug 16–20, 1976 (Boston). See also Argonne National Laboratory Report ANL-79-58, 1976

8.43 S. Majumdar and P.S. Maiya, A Damage Equation fo167r Creep-Fatigue Interaction, *Symposium on Creep-Fatigue Interaction*, ASME MPC-3, American Society of Mechanical Engineers, 1976, p 323–335

8.44 S. Majumdar and P.S. Maiya, Creep-Fatigue Interactions in an Austenitic Stainless Steel, *Can. Metall. Q.*, Vol 18, 1979, p 57–64 doi: 10.1179/cmq.1979.18.1.57

8.45 S. Majumdar and P.S. Maiya, A Mechanistic Model for Time-Dependent Fatigue, *Trans. ASME*, Vol 102, Jan 1980, p 159–167 doi: 10.1115/1.3224774

8.46 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, 2006 doi: 10.31399/asm.tb.fdsm.9781627083447

8.47 L.M. Kachanov, "On the Time to Failure under Creep Conditions," *Izv. AN SSR, Otd Tekn. Nauk.*, No. 8, 1958, p 26–31 (in Russian)

8.48 Y.N. Rabotnov, On the Equations of State of Creep, *Progress in Applied Mechanics—The Prager Anniversary Volume*, Macmillan Co., New York, 1963, p 307–315. See also Creep Rupture, *12th International Congress of Applied Mechanics*, 1968

8.49 J. Memaitre, *A Course on Damage Mechanics*, 2nd ed., Springer, 1996 doi: 10.1007/978-3-642-18255-6

8.50 J. Memaitre and J.-L. Chaboche, *Mechanics of Solid Materials*, Cambridge University Press, reprinted 2000

8.51 G.Z. Voyiadjis and P.I. Kattan, *Advances in Damage Mechanics: Metals and Metal Matrix Composites*, Elsevier, 1999 doi: 10.1016/B978-0-08-043601-2.50005-2

8.52 P.I. Kattan and G.Z. Voyiadjis, Damage Mechanics with Finite Elements, *Practical Application with Computer Tools*, Springer-Verlag, Berlin, 2002 doi: 10.1007/978-3-642-56384-3

8.53 D. Krajcinovic, *Damage Mechanics*, North-Holland, 1996

8.54 J.-L. Chaboche, Une Loi Différentielle D'Endommagement de Fatigue avec Cumulation non Linéaire, *Extrait de la Revue Francaise de Mécanique*, No. 50– 51, 1974, p 71–82 (in French)

8.55 J.-L. Chaboche, Le Concept de Contrainte Effective Applique a L'Élasticité et a la Viscopasticité en Présence D'Un Endommagement Anisotrope, *Colloque International du CNRS No. 295 (Euromach 115) sur le Comportement Mécanique des Solides Anisotropes*, Villard de Lans, June 1979

8.56 J.-L. Chaboche H. Kaczmarek and P. Raine, Hardening and Fatigue Damage Interaction in 316L Steel, *Rech. Aérosp.*, No. 1980-3, May– June 1980, p 35–55

8.57 J.-L. Chaboche, Continuous Damage Mechanics: A Tool to Describe Phenomena before Crack Initiation, *Nucl. Eng. Des.*, Vol 64, 1981 p 233–247 doi: 10.1016/0029-5493(81)90007-8

8.58 G. Cailletaud and J.-L. Chaboche, Lifetime Predictions in 304 Stainless Steel by Damage Approach, *ASME Pressure Vessel and Piping Conf.* (Orlando, FL), June–July 1982

8.59 J.-L. Chaboche, Anisotropic Damage in the Framework of Continuum Damage Mechanics, Seventh International Conf. on Structural Mechanics in Reactor Technology (SMIRT 7) (Chicago), Aug 1983

8.60 J.-L. Chaboche and P.M. Lesne, A Non-Linear Continuous Fatigue Damage Model, *Fatigue Fract. Eng. Mater. Struct.*, Vol 11(No. 1), 1988, p 1–17 doi: 10.1111/j.1460-2695.1988.tb01216.x

8.61 S. Kruch P. Prigent and J.-L. Chaboche, A Fracture Mechanics Based Fatigue-Creep-Environment Crack Growth Model for High Temperature, *J. Int. Pressure Vessels and Piping*, Vol 59, 1994, p 141–148 doi: 10.1016/0308-0161(94)90149-X

8.62 S. Kruch and S.M. Arnold, Creep Damage and Creep-Fatigue Damage Interaction Model for Unidirectional Metal-Matrix Composites, *Applications of Continuum Damage Mechanics to Fatigue and Fracture*, STP 1315, D.L. McDowell, Ed., American Society for Testing and Materials, 1997, p 7–28. See also NASA TM 1-5213, Nov 1991

8.63 F. Gallerneau D. Nouailhas and J.-L. Chaboche, "A Fatigue Damage Model Including Interaction Effects with Oxidation and Creep Damages," ONERA reprint of paper presented at Fatigue '96 (Berlin), May 1996

8.64 S.S. Manson and G.R. Halford, Complexities of High Temperature Metal Fatigue—Some Steps Toward Understanding, *Isr. J. Technol.*, Vol 21, 1983, p 29–53. See also NASA TM-83507, 1983

8.65 R. Neu and H. Sehitoglu, Thermo-Mechanical Fatigue, Oxidation, Creep, Part I: Experiments, *Metall. Trans. A*, Vol 20, 1989, p 1755–1767 doi: 10.1007/BF02663207

8.66 R. Neu and H. Sehitoglu, Thermo-Mechanical Fatigue, Oxidation, Creep, Part II: Life Prediction, *Metall. Trans. A*, Vol 20, 1989, p 1769–1783 doi: 10.1007/BF02663208

8.67 H. Sehitoglu and D.A. Boismier, Thermo-Mechanical Fatigue of Mar-M247, Part I: Experiments, *Trans. ASME, J. Eng. Mater.*, Vol 112, 1990, p 68–80

8.68 H. Sehitoglu and D.A. Boismier, Thermo-Mechanical Fatigue of Mar-M247, Part II: Life Prediction, *Trans. ASME, J. Eng. Mater.*, Vol 112, 1990, p 80–90 doi: 10.1115/1.2903191

8.69 M. Karayaka and H. Sehitoglu, Thermo-Mechanical Fatigue of Particulate Reinforced Aluminum 2 *xxx*-T4, *Metall. Trans. A*, Vol 22, 1991, p 697–707 doi: 10.1007/BF02670292

8.70 Y. Kadioglu and H. Sehitoglu, Thermo-mechanical and Isothermal Fatigue Behavior of Bare and Coated Superalloys, *Trans. ASME, J. Eng. Mater.*, Vol 118, 1996, p 94–102 doi: 10.1115/1.2805939

8.71 M.P. Miller D.L. McDowell R.L.T. Oehmke and S.D. Antolovich, A Life Prediction Model for Thermomechanical Fatigue Based on Microcrack Propagation, *Thermomechanical Fatigue*

Behavior of Materials, STP 1185, H. Sehitoglu, Ed., American Society for Testing and Materials, 1993, p 35–49 doi: 10.1520/STP24248S

8.72 D. Slavik and H. Sehitoglu, A Constitutive Model for High Temperature Loading, Part I—Experimentally Based Forms of the Equations, Thermal Stress, Material Deformation, and Thermomechanical Fatigue, *Proc. of the Pressure Vessel and Piping Conference, ASME PVP*, Vol 123, 1987, p 65–73

8.73 G.R. Halford, Evolution of Creep-Fatigue Life Prediction Models, *Creep-Fatigue Interaction at High Temperature*, AD-Vol 21, G.K. Haritos and O.O. Ochoa, Ed., American Society of Mechanical Engineers, 1991, p 43–57

8.74 H.J. Tapsell P.G. Forrest and G.R. Tremain, Creep Due to Fluctuating Stresses at Elevated Temperatures, *Engineering*, Vol 170, 1950, p 189

8.75 D.S. Wood, The Effect of Creep on the High-Strain Fatigue Behavior of a Pressure Vessel Steel, *Weld. J. Res. Suppl.*, Vol 45, 1966, p 92s–96s

8.76 J.F. Eckel, The Influence of Frequency on the Repeated Bending Life of Acid Lead, *Proc. ASTM*, Vol 51, 1951, p 745–756

8.77 C.G. Annis M.C. VanWanderham and R.M. Wallace, "Strainrange Partitioning Behavior of an Automotive Turbine Alloy," NASA CR-134974, National Aeronautics and Space Administration, 1976

8.78 R. Lagneborg and R. Attermo, The Effect of Combined Low-Cycle Fatigue and Creep on the Life of Austenitic Stainless Steels, *Metall. Trans.*, Vol 2, 1971, p 1821–1827

8.79 L.F. Coffin Jr., The Effect of Frequency on High Temperature, Low-Cycle Fatigue, *Proc. Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials*, AFDL-TR70-144, 1970, p 301–312

8.80 G.R. Halford and J.F. Saltsman, Strain-range Partitioning—A Total Strainrange Version, *Advances in Life Prediction Methods*, D.A. Woodford and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1983, p 17–26

8.81 J. Lemaitre J.-L. Chaboche and Y. Munakata, Method of Metal Characterization for Creep and Low Cycle Fatigue Prediction in Structures—Example of Udimet 700, *Proc., Symposium on Mechanical Behavior of Materials* (Kyoto), Society of Materials Science, 1974, p 239–249

8.82 A. Plumtree and J. Lemaitre, "Application of Damage Concepts to Predict Creep-Fatigue Failure," presented at the joint ASME/CSME PVP Conf.(Montreal), 1978

8.83 A. Berkovits, "Investigation of Three Analytical Hypotheses for Determining Material Creep Behavior under Varied Loads," NASA Technical Note D799, National Aeronautics and Space Administration, 1961

8.84 D.A. Spera, "A Linear Creep Damage Theory of Thermal Fatigue of Materials," Ph.D. dissertation, University of Wisconsin, 1968

8.85 Anonymous, Code Case 1331-5 (predecessor of current Code Case N-47), "Boiler and Pressure Vessel Code," Section III, American Society of Mechanical Engineers, 1971

8.86 D.P. Timo, Designing Turbine Components for Low-Cycle Fatigue, *Proc., International Conf. on Thermal Stresses and Thermal Fatigue*, D.J. Littler, Ed., 1971, p 453–469

8.87 P. Marshall and T.R. Cook, Prediction of Failure of Materials under Cyclic Loading, *Proc., International Conf. on Thermal Stresses and Thermal Fatigue*, D.J. Littler, Ed., 1971, p 81–88

8.88 T. Bui-Quoc, An Engineering Approach for Cumulative Damage in Metals under Creep Loading, *J. Pressure Vessel Technol., Trans., ASME*, Vol 101, 1979, p 337–343 doi: 10.1115/1.3443699

8.89 T. Bui-Quoc and A. Biron, A Phenomenological Approach for the Analysis of Combined Fatigue and Creep, *Nucl. Eng. Des.*, Vol 71, 1982, p 89–102 doi: 10.1016/0029-5493(82)90172-8

8.90 A. Plumtree and J. Lemaitre, *Advances in Fracture Research*, D. Francois, Ed., Pergamon Press, 1982, p 2379

8.91 C. Heng J.M. Grandemange and A. Morel, Rules for Design and Construction of Nuclear Components, Nuclear Engineering Design, selected papers from SMiRT-8 Seminar on Construction Codes and Engineering Mechanics (Paris), Aug 26–27, 1984, Vol 98(No. 3) Jan 1985, p 265–277 doi: 10.1016/0029-5493(87)90003-3

8.92 B.J. Lazan, Dynamic Creep and Rupture Properties under Tensile Fatigue Stress, *Proc., ASTM*, Vol 49, 1949, p 757

8.93 A. Coles and D. Skinner, Assessment of Thermal-Fatigue Resistance of High Temperature Alloys, *J. R. Aeron. Soc. Lon.*, Vol 69, 1965, p 53–55 doi: 10.1017/S0001924000060693

8.94 J.T. Berling and J.B. Conway, A Proposed Method for Predicting Low-Cycle Fatigue Behavior of 304 and 316 Stainless Steel, *Trans., Metall. Soc. Min. Metall. Eng.*, Vol 245, 1969, p 1137–1140

8.95 K. Wellinger and S. Sautter, Der Einfluss von empertur, Dehnungsgeschwindigkeit und Haltezeit auf das Zeitfestigkeitsverhalten von Stiihlen, *Archives Eisenhiittenwes.*, Vol 44, 1973, p 47–55 (in German) doi: 10.1002/srin.197304496

8.96 W.J. Ostergren, Correlation of Hold Time Effects in Elevated Temperature Low Cycle Fatigue Using a Frequency Modified Damage Function, *ASMEMPC Symposium on Creep-Fatigue Interaction*, Publication G00112 (MPC-3), R.M. Curran, Ed., American Society of Mechanical Engineers, 1976, p 179–202

8.97 W.J. Ostergren and E. Krempl, A Uniaxial Damage Accumulation Law for Time-Varying Loading Including Creep-Fatigue Interaction, *J. Pressure Vessel Technol., Trans. ASME*, Vol 101, 1979, p 118–124 doi: 10.1115/1.3454610

8.98 K.C. Valanis, On the Effect of Frequency on Fatigue Life, *Mechanics of Fatigue*, T. Mura, Ed., American Society of Mechanical Engineers, 1981, p 21–32

8.99 S.S. Manson and M.H. Hirschberg, Fatigue Behavior in Strain Cycling in the Low- and Intermediate-Cycle Range, *Fatigue—An Interdisciplinary Approach*, J.J. Burke N.L. Reed and V. Weiss, Ed., Syracuse University Press, 1964, p 133–178

8.100 J. Morrow, "An Investigation of Plastic Strain Energy as a Criterion for Finite Fatigue Life," report to the Garrett Corporation, 1960

8.101 J.T. Berling and J.B. Conway, A New Approach to the Prediction of Low-Cycle Fatigue Data, *Metall. Trans.*, Vol 1, 1970, p 805–809

8.102 T. Udoguchi and T. Wada, Thermal Effect on Low-Cycle Fatigue Strength of Steels, *Proc., International Conf. on Thermal Stresses and Thermal Fatigue*, D.J. Littler, Ed., 1971, p 109–123

8.103 D. Sunamoto T. Endo and M. Fujihara, Hold Time Effects in High Temperature, Low Cycle Fatigue of Low Alloy Steels, *Creep and Fatigue in Elevated Temperature Applications*, Institution of Mechanical Engineers, London, 1974, p C252

8.104 M. Morishita K. Taguchi T. Asayama A. Ishikawa and Y. Asada, Application of the Overstress Concept for Creep-Fatigue Evaluation, *Low Cycle Fatigue— Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 487–499 doi: 10.1520/STP24501S

8.105 B.N. Leis, An Energy-Based Fatigue and Creep-Fatigue Damage Parameter, *J. Pressure Vessel Technol., Trans., ASME*, Vol 99, 1977, p 524–533 doi: 10.1115/1.3454571

8.106 V.M. Radhakrishnan, Damage Accumulation and Fracture Life in High-Temperature Low-Cycle Fatigue, *Low-Cycle Fatigue and Life Prediction*, STP 770, C. Amzallag B.N. Leis and P. Rabbe, Ed., American Society for Testing and Materials, 1982, p 135–151 doi: 10.1520/STP32426S

8.107 V.M. Radhakrishnan, Life Prediction in Time Dependent Fatigue, *Advances in Life Prediction Methods*, D.A. Woodford and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1983, p 143–150

8.108 P.W. Whaley, A Thermodynamic Approach to Material Fatigue, *Advances in Life Prediction Methods*, D.A. Woodford and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1983, p 41–50

8.109 J. He Z. Duan Y. Ning and D. Zhao, Strain Energy Partitioning and Its Application to GH33A Nickel-Base Superalloy and 1Cr18Ni9Ti Stainless Steel, *Advances in Life Prediction Methods*, D.A. Woodford and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1983, p 27–32

8.110 D.M. Nissley T.G. Meyer and K.P. Walker, "Life Prediction and Constitutive Models for Engine Hot Section Anisotropic Materials Program," Final report, NASA CR-189223, National Aeronautics and Space Administration, 1992

8.111 H.G. Edmunds and D.J. White, Observations of the Effect of Creep Relaxation on High-Strain Fatigue, *J. Mech. Eng. Sci.*, Vol 8(No. 3), 1966, p 310–321 doi: 10.1243/JMES_JOUR_1966_008_040_02

8.112 J.F. Polhemous C.E. Spaeth and W.H. Vogel, Ductility Exhaustion for Prediction of Thermal Fatigue and Creep Interaction, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 625–636 doi: 10.1520/STP38874S

8.113 R.H. Priest and E.G. Ellison, A Combined Deformation Map-Ductility Exhaustion Approach to Creep-Fatigue Analysis, *Mater. Sci. Eng.*, Vol 49, 1981, p 7–17 doi: 10.1016/0025-5416(81)90128-2

8.114 D.A. Miller C.D. Hamm and J.L. Phillips, A Mechanistic Approach to the Prediction of Creep-Dominated Failure during Simulated Creep-Fatigue, *Mater. Sci. Eng.*, Vol 53, 1982, p 233–244 doi: 10.1016/0025-5416(82)90056-8

8.115 R.H. Priest D.J. Beauchamp and E.G. Ellison, Damage during Creep-Fatigue, *Advances in Life Prediction Methods*, D.A. Woodford and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1983, p 115–122

8.116 R. Hales, Fatigue, *Eng. Mater. Struct.*, Vol 6, 1983, p 121 doi: 10.1111/j.1460-2695.1983.tb00329.x

8.117 B. Tomkins, Fatigue Crack Propagation—An Analysis, *Philos. Mag.*, Vol 18, 1968, p 1041–1066 doi: 10.1080/14786436808227524

8.118 J. Wareing B. Tomkins and G. Sumner, Extent to Which Material Properties Control Fatigue Failure at Elevated Temperatures, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 123–138 doi: 10.1520/STP38834S

8.119 R.D. Solomon, Frequency Modified Low Cycle Fatigue Crack Propagation, *Metall. Trans.*, Vol 4, 1973, p 341–347 doi: 10.1007/BF02649635

8.120 A.E. Carden, Parametric Analysis of Fatigue Crack Growth, *Proc., Conf. on Creep and Fatigue in Elevated Temperature Environment*, ASTM, 1974

8.121 B. Tomkins, The Development of Fatigue Crack Propagation Models for Engineering Applications at Elevated Temperatures, *J. Eng. Mater. Des., Trans., ASME*, Vol 97, 1975, p 289–297 doi: 10.1115/1.3443301

8.122 J. Wareing, *Metall. Trans. A*, Vol 8, 1977, p 163

8.123 C.J. Franklin, in *High Temperature Alloys for Gas Turbines*, D. Coutsouradis P. Felix H. Fischmeister L. Habraken Y. Lindblom and M.D. Speidel, Ed., Applied Science, London, 1978, p 513–547

8.124 J. Janson, Damage Model of Creep-Fatigue Interaction, *J. Eng. Fract. Mech.* Vol 11, 1979, p 397–403 doi: 10.1016/0013-7944(79)90014-6

8.125 S. Taira R. Ohtani and T. Komatsu, Application of J-Integral to High-Temperature Crack Propagation, Part 1: Creep Crack Propagation, *J. Eng. Mater. Technol., Trans. ASME*, Vol 101, 1979, p 154–161 doi: 10.1115/1.3443668

8.126 A. Saxena, A Model for Predicting the Effect of Frequency on Fatigue Crack Growth Behavior at Elevated Temperature, *Fatigue Eng. Mater. Struct.*, Vol 3(No. 3), 1981, p 247–255 doi: 10.1111/j.1460-2695.1980.tb01364.x

8.127 K.S. Chan and A.K. Miller, A Unified Phenomenological Model for Predicting Fatigue Crack Initiation and Propagation, *Int. Conf. on Advances in Life Prediction Methods*, American Society of Mechanical Engineers, 1983, p 1–16

8.128 H. Riedel, Crack-Tip Stress Fields and Crack Growth under Creep-Fatigue Conditions, Elastic-Plastic Fracture, Second Symposium, Vol 1, *Inelastic Crack Analysis*, STP 803, C.F. Shih and J.P. Guda, Ed., American Society for Testing and Materials, 1983, p 505–520 doi: 10.1520/STP37312S

8.129 E. Renner H. Vehoff and P. Neumann, Prediction for Creep-Fatigue Based on the Growth of Short Cracks, *Fatigue Fract. Eng. Mater. Struct.*, Vol 12(No. 6), 1989, p 569–584 doi: 10.1111/j.1460-2695.1989.tb00564.x

8.130 D.L. McDowell and M.P. Miller, "Physically Based Microcrack Propagation Laws for Creep-Fatigue Environment Interaction," presented at ASME Winter Annual Meeting(Atlanta), American Society of Mechanical Engineers, 1991

8.131 S.S. Manson and G.R. Halford, Treatment of Multiaxial Creep-Fatigue by Strainrange Partitioning, *ASMEMPC Symposium on Creep-Fatigue Interaction*, Publication 000112 (MPC-3), R.M. Curran, Ed., American Society of Mechanical Engineers, 1976, p 299–322

8.132 W.H. Vogel R.W. Soderquist and B.C. Schlein, Application of Creep-LCF Cracking Model to Combustor Durability Prediction, *Fatigue Life Technology*, T.A. Cruse and J.P. Gallagher, Ed., American Society of Mechanical Engineers, 1977, p 22–31

8.133 S.S. Manson and G.R. Halford, discussion of paper by J.J. Blass and S.Y. Zamrik, Multiaxial Low-Cycle Fatigue of Type 304 Stainless Steel, *J. Eng. Technol., Trans. ASME*, Vol 99, 1977, p 283–286 doi: 10.1115/1.3443540

8.134 S. S. Manson, Some Useful Concepts for the Designer in Treating Cumulative Fatigue Damage at Elevated Temperatures, *Proc. Third International Conf. on Mechanical Behavior of Materials*, Vol 1, K.J. Miller and R.F. Smith, Ed., Pergamon Press, 1979, p 13–45 doi: 10.1016/B978-1-4832-8414-9.50012-2

8.135 S.S. Manson, The Strainrange Conversion Principle for Treating Cumulative Fatigue Damage in the Creep Range, Random Fatigue Life Prediction, *J. Pressure Vessels and Piping*, Vol 72, Y.S. Shin and M. K. Au-Yang, Ed., American Society of Mechanical Engineers, 1983, p 1–30

8.136 C.W. Lawton, Use of Low-Cycle Fatigue Data for Pressure Vessel Design, *Low-Cycle Fatigue and Life Prediction*, STP 770, C. Amzallag B.N. Leis and P. Rabbe, Ed., American Society for Testing and Materials, 1982, p 585–599 doi: 10.1520/STP32447S

8.137 O.T. Saugerud, Advances in Life Prediction of Thermally Loaded Diesel Engine Components, *Advances in Life Prediction Methods*, D.A. Woodford and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1983, p 229–240

8.138 W. Hoffelner K.N. Melton and C. Wuthrich, On Life Time Predictions with the Strain Range Partitioning Method, *Fatigue Eng. Mater. Struct.*, Vol 6, 1983, p 77–87 doi: 10.1111/j.1460-2695.1983.tb01140.x

8.139 P.H. Wirsching and Y.T. Wu, "Reliability Considerations for the Total Strainrange Version of Strainrange Partitioning," NASA CR-174757, National Aeronautics and Space Administration, 1984

8.140 V. Bicego, A Nonstandard Technique for the Evaluation of the Basic Laws of the Strain Range Partitioning Method in Creep-Fatigue Life Prediction, *Proc., Second International Conf. on Fatigue and Fatigue Thresholds*, Fatigue '84, Vol 3, C.J. Beevers, Ed., 1984, p 1257–1268

8.141 S. Kalluri and S.S. Manson, "Time Dependency of SRP Life Relationships," NASA CR-174946, NASA Grant NAG3-337, Case Western Reserve University, 1985

8.142 S. Kalluri S.S. Manson and G.R. Halford, Environmental Degradation of 316 Stainless Steel in High Temperature Low Cycle Fatigue, *Third International Conf. on Environmental Degradation of Engineering Materials*, R.P. McNitt and M. R. Louthan Jr., Ed., 1987, p 503–519

8.143 S. Kalluri S.S. Manson and G.R. Halford, Exposure Time Considerations in High Temperature Low Cycle Fatigue, *Proc., Fifth International Conf. on Mechanical Behavior of Materials*, Vol 2, M.G. Yan S.H. Zhang and Z.M. Zheng, Ed., Pergamon Press, 1987, p 1029–1036 doi: 10.1016/B978-0-08-034912-1.50139-9

8.144 H.D. Solomon, Low-Frequency, High-Temperature Low Cycle Fatigue of 60Sn-40Pb Solder, *Low Cycle Fatigue— Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 342–370 doi: 10.1520/STP24491S

8.145 J.F. Saltsman and G.R. Halford, "A Model for Life Prediction of Thermomechanical Fatigue Using the Total Strain Version of Strainrange Partitioning (SRP)—A Proposal," NASA TP-2779, National Aeronautics and Space Administration, 1988

8.146 T. Kitamura and G.R. Halford, "High Temperature Fracture Mechanics Basis for Strainrange Partitioning," NASA TM-4133, National Aeronautics and Space Administration, 1989

8.147 E.P. Esztergar and J.R. Ellis, Cumulative Fatigue Damage Concepts in Creep-Fatigue Life Predictions, *Proc., International Conf. on Thermal Stresses and Thermal Fatigue*, D.J. Littler, Ed., 1971, p 128–156

8.148 E. Krempl, The Temperature Dependence of High-Strain Fatigue Life at Elevated Temperature in Parameter Representation, *Proc., International Conf. on Thermal Stresses and Thermal Fatigue*, D.J. Littler, Ed., 1971, p 36–46

8.149 H. Bernstein, A Stress-Strain-Time Model (SST) for High Temperature Low Cycle Fatigue, *Methods for Predicting Material Life in Fatigue*, W.J. Ostergren and J.R. Whitehead, Ed., American Society of Mechanical Engineers, 1979, p 89–100

8.150 R.V. Miner J. Gayda and M.G. Hebsur, Creep-Fatigue Behavior of Ni-Co-Cr-Al-Y Coated PW A 1480 Superalloy Single Crystals, *Low Cycle Fatigue— Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 371–384 doi: 10.1520/STP24492S

8.151 M. Satoh and E. Krempl, An Incremental Life Prediction Law for Creep-Fatigue Interaction, *J. Pressure Vessels and Piping*, Vol 60, American Society of Mechanical Engineers, 1982, p 71–79

8.152 A. Zhang T. Bui-Quoc and R. Gomuc, Procedure for Low Cycle Fatigue Life Prediction for Various Temperatures and Strain Rates, *J. Eng. Mater. Technol., Trans., ASME*, Vol 112(No. 4), 1990, p 422–428 doi: 10.1115/1.2903352

8.153 V. Moreno D.M. Nissley G.R. Halford and J.F. Saltsman, "Application of Two Creep-Fatigue Life Prediction Models for the Prediction of Elevated Temperature Crack Initiation of a Nickel-Base Alloy," AIAA preprint 85-1420, American Institute of Aeronautics and Astronautics, 1985 doi: 10.2514/6.1985-1420

8.154 R.S. Nelson, "A High Temperature Fatigue Life Prediction Computer Code Based on the Cyclic Damage Accumulation (CDA) Model," presented at ASME Winter Annual Meeting(Atlanta), American Society of Mechanical Engineers, 1991

8.155 D. McLean and A. Pineau, Grain-Boundary Sliding as a Correlating Concept for Fatigue Hold-Times, *Metall. Sci.*, Vol 12, 1978, p 313–316 doi: 10.1179/msc.1978.12.7.313

8.156 B.K. Min and R. Raj, Hold-Time Effects in High Temperature Fatigue, *Acta Metall.*, Vol 26, 1978, p 1007–1022 doi: 10.1016/0001-6160(78)90051-2

8.157 K.D. Challenger A.K. Miller and C.R. Brinkman, An Explanation for the Effects of Hold Periods on the Elevated Temperature Fatigue Behavior of 2Cr-1Mo Steel, *J. Eng. Mater. Technol., Trans., ASME*, Vol 103, 1981, p 7–14 doi: 10.1115/1.3224979

8.158 K.D. Challenger A.K. Miller and R.L. Langdon, *J. Mater. Energy Syst.*, Vol 3, 1981, p 51–61 doi: 10.1007/BF02833529

8.159 S.D. Antolovich S. Liu and R. Baur, Low Cycle Fatigue Behavior of Rene 80 at Elevated Temperature, *Metall. Trans. A*, Vol 12, 1981, p 473–481 doi: 10.1007/BF02648545

8.160 J.R. Weertman, "A Study of the Role of Grain Boundary Cavitation in the Creep-Fatigue Interaction in High Temperature Fatigue," final technical report, Northwestern University, 1982

8.161 S. Baik and R. Raj, Wedge Type Creep Damage in Low Cycle Fatigue, *Metall. Trans. A*, Vol 13, 1982, p 1207–1214 doi: 10.1007/BF02645503

8.162 S. Baik and R. Raj, Mechanisms of Creep-Fatigue Interaction, *Metall. Trans. A*, Vol 13, 1982, p 1215–1221 doi: 10.1007/BF02645504

8.163 J. Reuchet and L. Remy, Fatigue Oxidation Interaction in a Superalloy— Application to Life Prediction in High Temperature Low Cycle Fatigue, *Metall. Trans. A*, Vol 14, 1983, p 141–149 doi: 10.1007/BF02643747

8.164 V.C. Nardone, Doctoral dissertation, Columbia University, 1983

8.165 K.-T. Rie R.-M. Schmidt B. Ilschner and S.W. Nam, A Model for Predicting Low Cycle Fatigue Life under Creep-Fatigue Interaction, *Low Cycle Fatigue— Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 313–328

8.166 G.R. Romanoski S.D. Antolovich and R.M. Pelloux, A Model for Life Predictions of Nickel-Base Superalloys in High-Temperature Low Cycle Fatigue, *Low Cycle Fatigue— Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 456–467 doi: 10.1520/STP24497S

8.167 Y. Oshida and H.W. Liu, Grain Boundary Oxidation and an Analysis of the Effects of Oxidation on Fatigue Crack Nucleation Life, *Low Cycle Fatigue— Directions for the Future*, STP 942, H.D. Solomon G.R. Halford L.R. Kaisand and B.N. Leis, Ed., American Society for Testing and Materials, 1988, p 1199–1217

8.168 A.D. Freed and K. Diethelm, Fractional Calculus in Biomechanics: A 3-D Viscoelastic Model Using Regularized Fractional Derivative Kernels with Application to the Human Calcaneal Fat Pad, *Biomechan. Model. Mechanobiol.*, Vol 5(No. 4), Nov 2006, p 203–215 (DOI: 10.1007/s10237-005-0011-0) doi: 10.1007/s10237-005-0011-0

Chapter 9: Obstacles to High-Temperature Structural Durability of Continuous-Fiber Metal-Matrix Composites

References

9.1 C.W. Richards, *Engineering Materials Science*, Wadsworth Publishing, San Francisco, 1961, p 27, 468

9.2 G.R. Halford B.A. Lerch J.F. Saltsman and V.K, Arya, "Proposed Framework for Thermomechanical Fatigue (TMF) Life Prediction of Metal Matrix Composites (MMCs)," STP 1186, American Society for Testing and Materials, 1993, p 176–194 doi: 10.1520/STP24256S

9.3 B.A. Lerch and G.R. Halford, Effects of Control Mode and R-Ratio on the Fatigue Behavior of a Metal Matrix Composite, *Materials Science and Engineering Symposium on Mechanisms and Mechanics of MMC Fatigue* (Rosemont, IL), 1994

9.4 G.R. Halford, *Thermal Stress II*, North-Holland, Amsterdam, p 329–428

9.5 D.W. Petrasek, NASA-Lewis Research center, Cleveland, OH, personal communication

9.6 M.G. Castelli and J. Gayda, *DE-Vol 55*, American Society of Mechanical Engineers, 1993, p 213–221

Chapter 10: Aerospace Applications—Example Fatigue Problems

References

10.1 J.F. Newall, A Note of Appreciation for the MUS, *Material Durability/Life Prediction Modeling—Materials for the 21st Century*, PVP-Vol 290, S.Y. Zamrik and G.R. Halford, Ed., American Society of Mechanical Engineers, 1994, p 57–58

10.2 J. Newman *et al.*, "LaRC's NASCRAC Computer Code for Cyclic Crack Propagation"

10.3 S.S. Manson, Fatigue: A Complex Subject—Some Simple Approximations, *Exp. Mech.*, Vol 5(No. 7), 1965, p 193–226 doi: 10.1007/BF02321056

10.4 S.S. Manson and G.R. Halford, *Fatigue and Durability of Structural Materials*, ASM International, 2006 doi: 10.31399/asm.tb.fdsm.9781627083447

10.5 S.M. Tipton and J.R. Sorem Jr., A Reverse Plasticity Criterion for Specifying Optimal Proof Load Levels, *Advances in Fatigue Lifetime Predictive Techniques: Second Volume*, STP 1211, M.R. Mitchell and R.W. Landgraf, Ed., American Society for Testing and Materials, 1993, p 186–202 doi: 10.1520/STP15086S

10.6 G.R. Halford and A.J. Nachtigall, The Strainrange Partitioning Behavior of an Advanced Gas Turbine Disk Alloy, AF2-1DA, *J. Aircr.*, Vol 17(No. 8), 1980, p 598–604 doi: 10.2514/3.57945

10.7 E.A. Davis and F.M. Connelly, Stress Distributions and Plastic Deformation in Rotating Cylinders of Strain Hardening Materials, *J. Appl. Mech*., *Trans., ASME*, Vol 81, 1959, p 25–30 doi: 10.1115/1.4011918

10.8 S.S. Manson and G.R. Halford, discussion of paper by J.J. Blass and S.Y. Zamrik, Multiaxial Low-Cycle Fatigue of Type 304 Stainless Steel, *J. Eng. Technol., Trans. ASME*, Vol 99, 1977, p 283–286 doi: 10.1115/1.3443540

10.9 M.J. O'Neill, "A Review of Some Cumulative Damage Theories," ARL/SM-Report-326, Aeronautical Research Laboratories, Melbourne, Australia, 1970

10.10 S.S. Manson and G.R. Halford, Re-Examination of Cumulative Fatigue Damage Analysis—An Engineering Perspective, *Eng. Fract. Mech.*, Vol 25(No. 5/6), 1986, p 539–571 doi: 10.1016/0013-7944(86)90022-6

10.11 S.S. Manson and G.R. Halford, Practical Implementation of the Double Linear Damage Rule and Damage Curve Approach for Treating Cumulative Fatigue Damage, *Int. J. Fract.*, Vol 17, 1981, p 169–192 doi: 10.1007/BF00053519

10.12 V. Moreno, "Combustor Liner Durability Analysis," NASA CR-165250, NASA Contract NAS3-21836, 1981.

10.13 V. Moreno G.J. Meyers A. Kaufman and G.R. Halford, Nonlinear Structural and Life Analyses of a Combustor Liner, *Comput. Struct.*, Vol 16(No. 1–4), 1983, p 509–515 doi: 10.1016/0045-7949(83)90191-8

10.14 S.S. Manson G.R. Halford and M.H. Hirschberg, Creep-Fatigue Analysis by Strain-Range Partitioning, *Proc. of First Symposium on Design for Elevated Temperature Environment*, S.Y. Zamrik, Ed., May 10–12, 1971, American Society of Mechanical Engineers, New York, p 12–28

10.15 S.S. Manson, The Challenge to Unify Treatment of High Temperature Fatigue—A Partisan Proposal Based on Strain-Range Partitioning, *Fatigue at Elevated Temperatures*, STP 520, A.E. Carden A.J. McEvily and C.H. Wells, Ed., American Society for Testing and Materials, 1973, p 744–782 doi: 10.1520/STP38885S

10.16 G.R. Halford, Low-Cycle Thermal Fatigue, *Thermal Stresses II*, R.B. Hetnarski, Ed., Elsevier Science Publishers B. V., 1987, p 329–428

10.17 R.L. McKnight J.H. Laflen G.R. Halford and A. Kaufman, Turbine Blade Nonlinear Structural and Life Analysis, *J. Aircr.*, Vol 20(No. 5), May 1983, p 475–480 doi: 10.2514/3.44896

10.18 G.R. Halford and A.J. Nachtigall, Strain-range Partitioning Behavior of the Nickel-Base Superalloys, René 80 and IN-100, *Proc., AGARD Conf. on Characterization of Low Cycle High Temperature Fatigue by the Strainrange Partitioning Method*, AGARD CP-243, 1978, p 2-1 to 2-14

10.19 G.R. Halford J.F. Saltsman and M.H. Hirschberg, Ductility Normalized-Strain-range Partitioning Life Relations for Creep-Fatigue Life Prediction, *Proc., Conf. on Environmental Degradation of Engineering Materials* (Blacksburg, VA), R.P. McNitt and M.R. Louthan Jr., Ed., 1977, p 599–612