GEARBOX VIBRATIONS

ANALYSIS AND REDUCTION

A.K. Rakhit

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This book is dedicated to the memory of my parents, Upendra Ch. and Charulata Rakhit, and my wife's parents, Manindra Ch. and Nirupama Neogy, for their blessings from heaven. Also, I am extremely grateful to my wife, Ratna, son, Amit, and daughter, Roma, for their love and inspiration.

Contents

Preface

Vibrations are natural phenomena. Vibration frequencies and amplitudes that are observed in rotating machinery such as gearboxes and airplanes and in structures such as bridges and buildings (due to forced excitations) can be measured. Generally, the frequencies of these vibrations do not exceed 20 kHz. Their amplitudes in rotating equipment are in the range of 10 mils or less, whereas the amplitudes of structural vibrations may exceed a few inches. Vibrations are destructive whether they arise in rotating equipment or any structures. This book discusses gearbox vibration, which has detrimental effects on the life of the gearbox. Also presented are methods to reduce vibration.

A gearbox is a simple rotating piece of equipment that consists of a stationary housing and components such as gears and shafts mounted on bearings within the housing. As the gears rotate to transfer motion from one shaft to the other, the gearbox vibrates, even at low speeds under small loads. The vibrations are attributed to minor unbalances in the rotating components. The level of such vibrations is generally low; however, as the speed and load increase, the level of vibrations increases. In addition to the microunbalance of rotating components, an error in gear tooth geometry and the dynamic complexity of the rotating components are considered the primary causes for vibration-level increases. As the speed increases to 1525 m/min (5000 ft/min) and more, even at a moderate load intensity, it becomes difficult to maintain the vibration level on a gearbox below 10g, which is an acceptable level of vibrations. In general, gearboxes subjected to vibration above this limit fail before their expected lives. Gearbox failures may cause major breakdowns of a manufacturing operation, shutdown of power generation, and disruption in propulsion of naval ships, airplanes, and many other applications where gearboxes are a vital component. To eliminate failures, the vibration level of a gearbox can be reduced by changing its design.

To design a gearbox for reduced vibrations, it is important to ensure that the rotational frequency of the components does not excite the housing structure to its natural frequency, which can create a resonance. To ensure this, a detailed analysis of the housing structure is needed to produce an optimum design that not only eliminates resonance conditions but also maintains precise spatial relationships among rotating components for kinematic stability, even at high speeds.

In addition to housing, gear tooth geometric accuracy is considered a major source of gearbox vibrations. It is important to design gears with the high geometric accuracy achievable with the current manufacturing equipment. In addition to gear geometry, there are a number of other gear design parameters that influence the level of vibration.

These include selection of a minimum number of teeth in the gears and pinions, the pressure angle of the teeth, the number of teeth in the pinion and gear for hunting tooth combinations, suitable diametral pitch, helical gears with high contact ratio, and shaft misalignment. In addition to these gear parameters, bearings that support the shafts play a significant role in raising the vibrations of gearboxes. For low speed and load, ball or roller bearings are suitable. On the other hand, precision tilt-pad sleeve bearings with high stiffness are preferred for dynamic stability and reduced vibrations at high-speed and high-load applications. Furthermore, high-speed shafts that are used for mounting gears must be designed to avoid vibrations due to rotodynamic instability.

In some turbomachinery applications, gearboxes operating at high speeds of approximately 1525 m/min (5000 ft/min) exhibit the presence of nonsynchronous and spiking-type vibrations, which are found to limit the operation of turbines at partial loads. This is due to bearing instability at high speeds and high load. To overcome this type of instability, it is possible to redesign the bearings with a taper pressure dam, which makes the bearings stable and allows the high-speed equipment to operate satisfactorily under proper hydrodynamic lubrication.

In addition, most high-speed gearboxes experience subsynchronous vibrations. Experimental studies indicate that when the level of these low-frequency vibrations rises above 1 mil, the life of Babbitted sleeve bearings is significantly reduced. An improvement in bearing design alone is not always successful in reducing these vibrations below 1 mil. For this, an in situ balancing mechanism was developed that is useful in reducing the low-frequency vibrations, particularly for the epicyclic type of parallel-axis gearbox.

This book includes eight chapters. Chapters 1 and 2 discuss the basic vibration theory applicable to gearbox vibrations and their measurement fundamentals. In Chapter 3, experimental verification and an analysis of the various factors that cause gearbox vibrations are presented. Furthermore, it is shown how vibrations that arise are related to gear tooth geometry, particularly transmission error of gear teeth. Also given in this chapter is information on a new type of gear tooth error, tooth index variation form error, observed during package testing of high-speed turbomachinery, that increases the vibration level of a gearbox.

Chapter 4 is dedicated to establishing vibration limits for different classes of gearboxes. These are helpful to design engineers for the initial design of a gearbox. Chapter 5 discusses the occurrence of nonsynchronous and spiking vibrations during a turbomachinery package test. These types of abnormal vibrations were found to have a detrimental effect on the life of tilt-pad bearings, particularly when the packages run at partial load. To overcome this limitation, some new design guidelines were developed for the bearings after systematic analysis of test results from a number of turbomachinery packages. Guidelines are discussed that minimize such vibrations, allowing the packages to operate over the full-load spectrum without the loss of any unexpected bearing life.

Chapter 6 presents several gearbox field failures. An analysis for each type of these failures is presented, identifying the cause of failure. Recommendations are made regarding design modification to avoid future failures. Chapter 7 summarizes the various design considerations for reduced vibrations of high-speed gearboxes. A graphical presentation is developed to establish a gear quality for the level of vibration desired. Chapter 8 presents the step-by-step development of an epicyclic gearbox for high efficiency and reduced vibrations.

Finally, the author firmly believes that gearbox design development is essential for the success of future high-speed gearboxes.

Acknowledgments

The author expresses his sincere thanks and gratitude to the many gear design and manufacturing organizations for their unprecedented support in providing the test facilities and manpower for the successful completion of the projects described in this book.

The author wishes to thank his colleagues in the gear industry who helped conduct tests that were run to reach conclusions on how to reduce vibrations in gearbox designs.

The author is especially thankful to Guil Cornejo, an outstanding turbomachinery test engineer, for recording and analyzing nonsynchronous vibrations observed during package testing. The author also acknowledges Joe Mihelick, Engineering Manager at Reliance Electric Company, and Sam Beaird, Manufacturing Manager of Lufkin Corporation, without whose cooperation and advice the publication of this book would not have been possible.

Finally, the author expresses gratitude to his maternal uncle, Loknath Bal, and to his brother, Dr. Dilip Rakhit, who was a physicist and his childhood mentor, for their blessings from the world beyond.

About the Author

Dr. Ajit K. Rakhit is an international expert on gear design and development, and machine-tool vibrations. He has more than 50 years of work experience in both fields as a mechanical engineer spending most of his time working in aerospace related industries. While designing high speed gearboxes for reduced vibration, Dr. Rakhit developed important characteristics of the gear tooth profile that allows gears to run at a high speed, over 10,000 ft/min, with an acceptable vibration level of 10 g for a minimum of 100,000 hours.

Dr. Rakhit wrote several books and technical papers published by ASM International, ASME, and AGMA. His first book published by ASM, *Heat Treatment of Gears—A Practical Guide for Engineers* was very well received by engineers. A mechanical engineering professor at the University of Waterloo, Canada considered the book best in its field.

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CHAPTER 1

Gears and Vibrations in a Gearbox

Introduction

Vibration is a concern in rotating equipment of nearly any type, especially at higher speeds. In the case of gearboxes, vibration is the primary mode of failure even at the mid-range of operating speeds. Avoiding such failures requires an understanding of gearbox design, vibration theory, and material properties, all of which are covered in the chapters of this book.

A gearbox is a mechanical system designed to transmit torque from one shaft to another, often with a reduction or increase in speed. In a typical gearbox configuration, mating gears are mounted on shafts supported by bearings in a housing made from a material with high damping capacity.

Gearboxes are loosely categorized based on the relative orientation of their input and output shafts. As shown in Fig. 1.1, the most common shaft arrangements are parallel (whether offset or concentric), intersecting (often at a right angle), and orthogonal (nonparallel and nonintersecting).

Parallel-axis gearboxes (Fig. 1.1a) are usually configured with spur or helical gears, either of which can accommodate a wide range of operating speeds.

Right-angle gearboxes (Fig, 1.1b) normally employ bevel gears with straight or spiral teeth. Straight bevels are used in low-speed applications, whereas spiral bevels are used for speeds of approximately 610 m/min (2000 ft/min). In either case, vibration levels are inherently low and rarely the cause of failure.

Gearboxes based on orthogonally oriented worm gears (Fig. 1.1c) tend to be used for very low-speed applications. As a result, their vibration characteristics are not well studied.

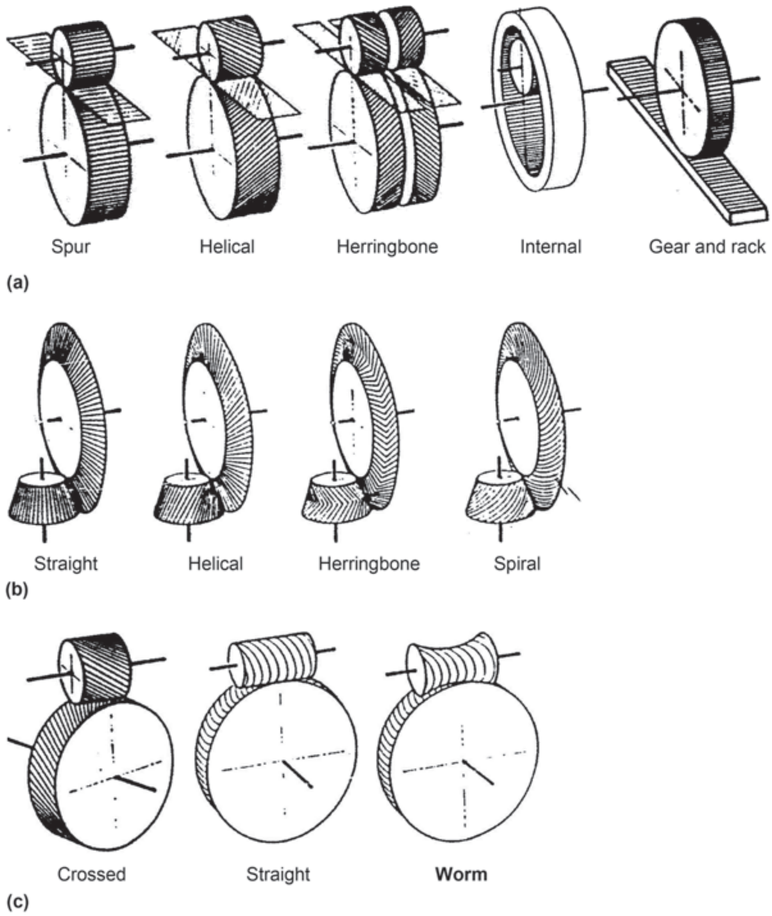


Fig. 1.1 Illustration of different axes of gear shafts, (a) parallel, (b) intersecting, (c) nonparallel, nonintersecting (skew)

Sources and Types of Gearbox Vibrations

Of the major types of gearboxes, vibration is mostly a concern in parallel-axis configurations, which are the focus of this book. Parallel-axis gearboxes are designed to reduce or increase speed. The majority are speed reducers, while speed increasers have some limited applications.

In general, two types of vibration excitations are observed in parallel-axis gearboxes:

- Torsional
- Lateral

Torsional excitations are caused by variation in the angular motion of gears due to manufacturing errors, whereas lateral excitations are produced by variations in gear tooth geometry and center distances between pinions and gears. Both torsional and lateral excitations may be present at the same time, although in parallel-axis gearboxes, excitation in one mode does not influence the other. This assumption is particularly valid for gearboxes used in turbomachinery, where internal gears have a large mass and high inertia. In applications involving low-mass, low-inertia gears, such as in aerospace, torsional excitations are much smaller in amplitude than lateral excitations.

In parallel-axis gearboxes, forced vibrations due to lateral excitations are considered the primary source of failure. The vibrations are said to be *forced* because their presence is due to an external source. In addition to forced vibrations, gearboxes may also experience self-excited vibrations due to the effect of vibrations at the natural frequencies of adjacent equipment installed near a gearbox. These are seldom observed, however, and in all further discussions, only forced excitations are considered.

Fundamentals of Periodic Vibrations

Almost all induced vibrations in parallel-axis gearboxes are periodic. Periodic vibration can be represented as an oscillatory displacement of a particle around a reference position (Fig. 1.2). Such motion may be mathematically described by a rotating vector **A** that results in two trigonometric functions: **A** sin ωt and **A** cos ωt, where ω is the angular velocity and t is time (Fig. 1.3). As the vector rotates, these two functions repeat themselves in equal intervals of time. The time between corresponding points in the vibration signal is called the *period*, T. The displacement that occurs in a single period is referred to as the *cycle*, and the number of cycles per unit of time is considered the vibration frequency. The peak value of the vibration signal is the *amplitude*, A. For a

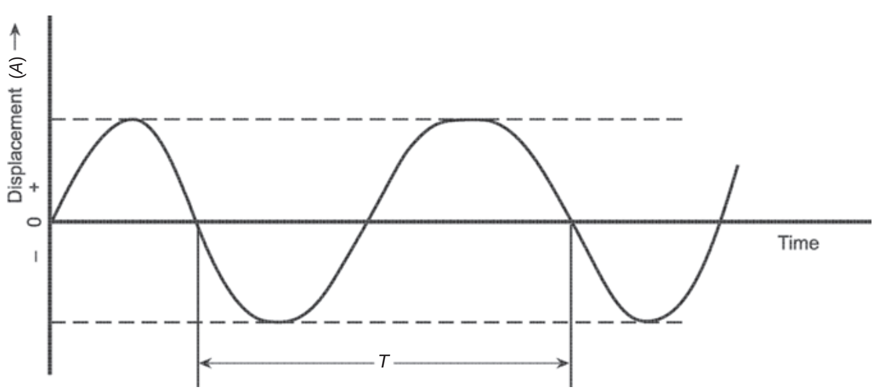


Fig. 1.2 Pure harmonic (sinusoidal) vibration signal

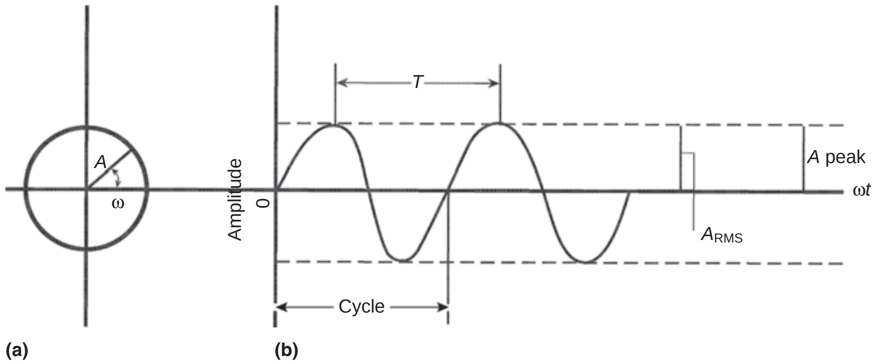


Fig. 1.3 (a) Vector representation of (b) harmonic

harmonic exciting force, the induced vibrations are at the excitation frequency and are independent of the natural frequency of any component in the system.

An important aspect of periodic vibration is that all information necessary to describe it is contained in a single period. For nonperiodic vibration, a complete description would require an infinitely long record of the vibration signal. This is not a concern, however, because parallel-axis gearbox vibrations are periodic and harmonic.

Characteristics of Gearbox Vibrations

An understanding of vibration characteristics is essential for identifying sources of gearbox vibrations and determining their frequencies. Functionally, a gearbox is a simple device that is used for low- or high-speed motion transfer. Low and high speeds are classified by the maximum surface speed of the gears used. In low-speed gearboxes, gear surface speeds are generally below 610 m/min (2000 ft/min). Above this limit, gearboxes fall into the high-speed category, where gear speeds may be 3050 m/min (10,000 ft/min) or more.

In general, gears and other rotating parts used in low-speed gearboxes are machined to tolerances that make them inherently stable over the operating range. Dynamic balancing is seldom required as a result even with gears made to AGMA quality class 8 and lower. This is not the case with high-speed gearboxes, however.

The magnitude of excitation forces in high-speed gearboxes is significantly higher and the effects are much more complex. High-speed gearboxes are also subject to vibration due to rotodynamic instability induced by fluid flow and the whirling of shafts running close to their critical speeds. Another issue to contend with is the increase in tooth-passing frequency, which can cause irregular tooth-to-tooth contact between a pinion and the mating gear. This creates an instantaneous shock to the following tooth, resulting in additional tooth load and vibration.

Vibrations due to all such excitations are ultimately transmitted to the gearbox housing, mainly through the bearings. Gearbox housings are normally made from high stiffness materials and their natural frequencies are usually much higher than vibration frequencies encountered in low-speed applications. At high speeds, however, excitation sources may excite resonant frequencies in the housing, potentially leading to failure.

Avoiding problems due to resonance requires one to not only identify all possible excitation sources, but also predict the amplitudes of vibration they are likely to produce. This requires a basic knowledge of vibration theory.

Vibration Theory

Vibration amplitudes are typically defined by one of three parameters: displacement, velocity, or acceleration. The magnitude of any one of these parameters defines the level of vibration. Because the three variables are interrelated, if one is measured, the other two can be determined by simple differentiation or integration.

For example, an instantaneous displacement of a particle can be mathematically expressed as:

$$X = x \cdot \sin\omega t \qquad \text{(Eq 1.1)}$$

where ω is the angular velocity = $2\pi f$ and $f = 1/T$, where T is the period.

To convert displacement to velocity, v, the Eq 1.2 is used:

$$v = dx/dt = \omega x \cdot \cos\omega t = v \cdot \cos\omega t = v_{\text{peak}} \cdot \cos\omega t \qquad \text{(Eq 1.2)}$$

which can be written as:

$$v_{\text{peak}} \cdot \sin(\omega t + \pi/2) \qquad \text{(Eq 1.3)}$$

Acceleration, a, which is the time-rate change of velocity, may be written as:

$$a = dv/dt = d^2x/d^2t = -\omega^2 x_{\text{peak}} \cdot \sin\omega t = -a_{\text{peak}} \cdot \sin\omega t = a_{\text{peak}} \cdot \sin(\omega t + \pi) \qquad \text{(Eq 1.4)}$$

The equations show that the form and period of vibration remain the same whether it is defined by displacement, velocity, or acceleration. The magnitude or peak value of each of these quantities is quite useful if pure harmonic vibration is considered and any of the three amplitudes may be measured and used for analysis.

To convert from one parameter to another for any particular frequency, there are several relationships to be used for convenience:

$$d = (19,100/f) \cdot v$$

$$d = (8383/f)^2 \cdot a$$

$$v = (f/19,100) \cdot d$$

$$v = (3687/f) \cdot a$$

$$a = (f/8383)^2 \cdot d$$

$$a = (f/3687) \cdot v$$

where d is peak-to-peak displacement in mils, v is peak velocity in in./s, a is peak acceleration in g, f is frequency in cycles/min, 1 mil = 0.001 in., and $1g = 386.087$ in./s^2.

Figure 1.4 illustrates the relationship among these three parameters. As shown, velocity and acceleration are offset from displacement by 90° and 180°, respectively. This is a physical phenomenon that is true for all types of vibrations.

Many gearbox vibrations, although not purely harmonic, are characterized as periodic. This allows useful information to be obtained such as peak and average values of the vibration signal. It also facilitates the use of frequency analysis, one of the most powerful tools for the study of vibration.

Frequency analysis is based on a mathematical theorem developed by French mathematician, Joseph Fourier. The theorem states that any

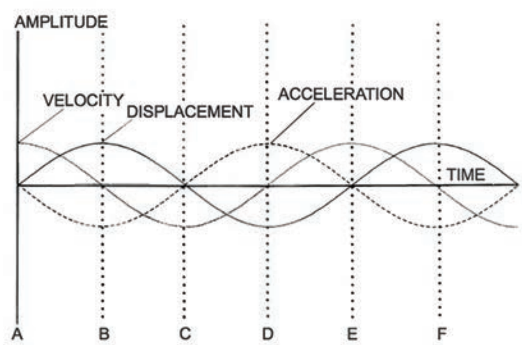


Fig. 1.4 Velocity and acceleration of the vibration are offset 90° and 180° in phase from displacement

periodic curve, no matter how complex, may be broken into pure sinusoidal curves with harmonically related frequencies. The more sinusoidal curves used, the better the approximation.

Fourier approximation helps analyze both the causes and effects of gearbox vibrations. For example, vibration amplitudes with the same frequency can be algebraically added to find their total effect. Fourier analysis is also helpful for identifying gearbox vibration sources based on relative frequency for which there are two general classes: *synchronous* and *nonsynchronous*.

Synchronous gearbox vibrations are related to the rotational speed of an internal or connected component. Common sources include:

- Rotor unbalance, such as prime-mover-driven equipment
- Gear tooth mesh
- Bearing vibrations
- Shaft whirl within a bearing
- Rotors running close to their critical speeds
- Shaft misalignment
- Subsynchronous vibrations

Nonsynchronous gearbox vibrations are not directly related to the rotational speed of any component. Common sources include:

- Viscosity of lubricating oil used in sleeve bearings
- Nonlinear bearing stiffness
- Rotor rub within the bearings
- Spiking
- Bent shaft
- Mechanical looseness of components

All such vibration sources must be considered in the design of a high-speed gearbox. In addition to anticipated operating speeds and loads, other design factors of importance include gear tooth meshing forces, housing stiffness, tooth contact ratio, and hunting tooth combinations for pinions and gears.

Parallel-Axis Gearboxes

Parallel-axis gearboxes used in a majority of industrial applications are of three basic configurations:

- *Offset parallel:* Input and output shafts are offset from each other.
- *Concentric:* Input and output shafts are arranged on a common centerline.
- *Epicyclic:* Input and output shafts are concentric, but the internal arrangement of gears (sun, planet, and ring) traces an epicyclic curve in space.

Figures 1.5(a and b) show single- and double-stage offset parallel-axis gearboxes, while Figure 1.5(c) shows the gear arrangement in an epicyclic gearbox. Concentric gearboxes are similar to offset parallels except that their input and output shafts are not only parallel but also concentric, necessitating a layout with at least two gearing stages.

Of the different gearbox configurations, the offset parallel design offers the most advantages. It is simpler than concentric and epicyclic designs, takes fewer components, and achieves excellent power transmission efficiency. Its main drawback is that it is not as compact as an epicyclic for the same gear ratio. It also uses relatively larger gears, which tend to produce higher levels of vibration.

Although epicyclic gearboxes incorporate many more gears than the other types, they are typically smaller and put less of a load on each gear

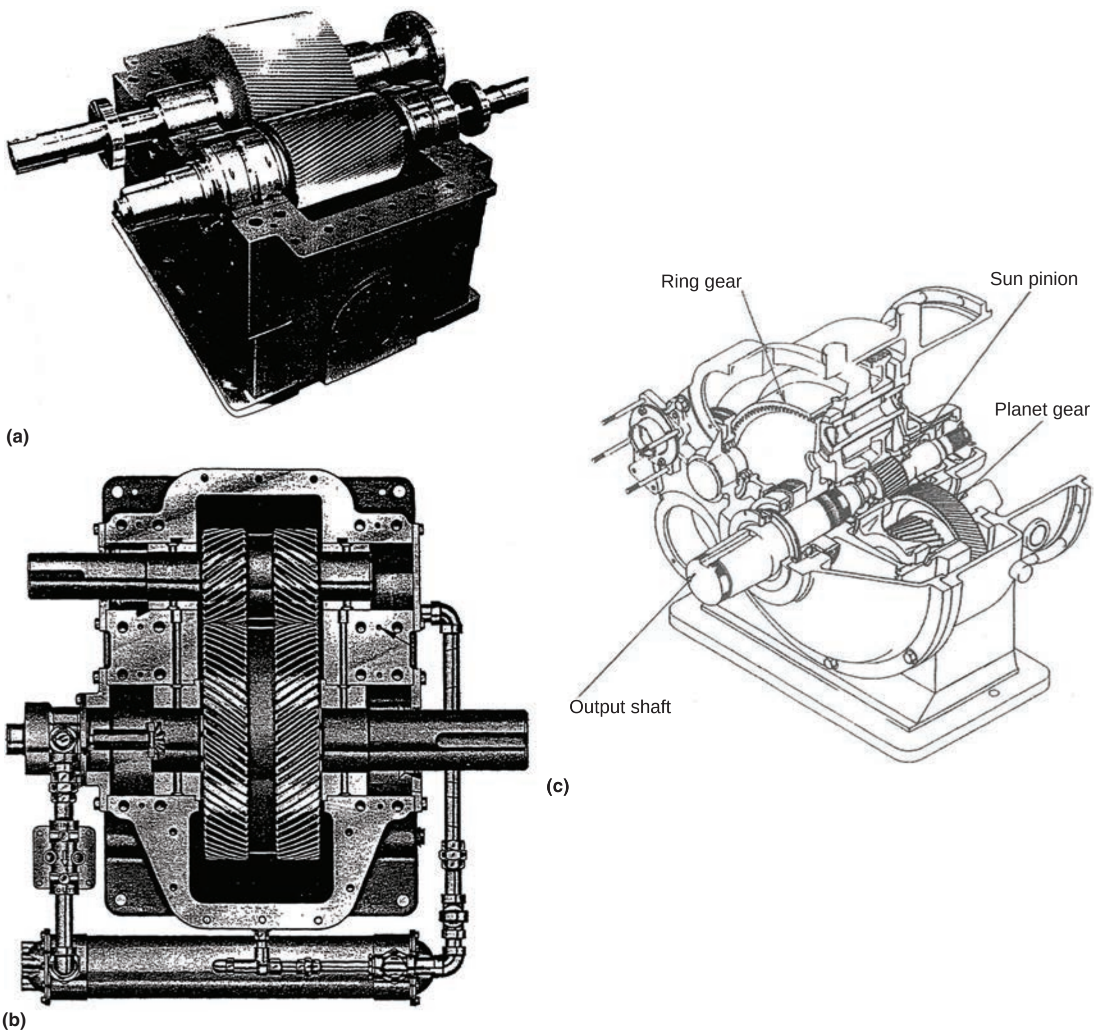


Fig. 1.5 Offset parallel gearbox with (a) single and (b) double helical gears. (c) Epicyclic gearbox

tooth because of load sharing. In addition, synchronous and subsynchronous vibrations that occur as a result of dynamic unbalance are easier to control. The main drawback of epicyclic gearboxes is the complex dynamics of the floating ring gear connected to the output shaft through a spline coupling and its effect on vibration.

Housing Design for Single-Stage Offset Parallel Gearboxes

It is assumed that the housing to be designed is for a single-stage offset parallel reducer. For given input and output speeds, power ratings for all gears, shafts, and bearings may be specified using AGMA design guidelines. Center distances between shafts can be established based on gear size and a preliminary housing design may be completed that holds all components in their proper locations.

The housing must be horizontally split to facilitate the assembly of gears and should have sufficient bending and torsional rigidity for the power to be transmitted. The material for the housing is selected either as a casting or weldment based on the quantity to be manufactured. Castings are favored for large production runs, while steel fabrication is preferred when only a few units are produced at a time. Appropriate casting materials include ductile and gray iron. Gearbox housings, whether cast or welded, are heat treated for stress relief to ensure dimensional stability after machining. The final machining of the housing is usually done with the two halves bolted together for improved accuracy.

Housing Design for High-Speed Gearboxes

Housing design for high-speed gearboxes follows the same general process as that used for low-speed gearboxes, but with the added need to maintain precise spatial relationships among components subjected to higher dynamic loads. High-speed gearbox housings, therefore, require much higher structural and torsional rigidities which, in turn, calls for materials with high damping capacity.

As a first step, preliminary rigidities are obtained for an initial housing design. Often, a prototype is made, preferably a steel weldment, to assess frequency response. The results are then used to optimize the rigidities of the housing to prevent resonance in the frequency range of interest.

Optimum Housing Design

An offset parallel arrangement is the simplest configuration for a parallel-axis gearbox, but the development process for optimizing the housing can be quite complex.

Housings are made via casting or steel fabrication. Cast iron housings have higher vibration-damping properties than housings fabricated from steel. In some applications, ductile iron is preferred to cast iron for additional damping. The relative damping capacities of cast iron, ductile iron, and steel are plotted in Fig. 1.6.

To ensure that a housing design meets both dimensional and functional specifications, a prototype steel weldment is often made. This allows quick modification of housing dimensions in order to shift the natural frequency of the housing to avoid resonance with excitation frequencies induced by rotating components in the gearbox. A minimum of 20% separation is recommended between excitation frequencies and the natural frequency of the housing.

To meet this requirement, a trial-and-error method may be used to select prototype dimensions. In most cases, both halves of the prototype structure are reinforced in small increments for increased wall thickness. With each increment of wall thickness, a hammer test is conducted until the design satisfies a 20% separation between the lowest housing frequency and the frequency of gear tooth meshing.

Figure 1.7 shows a typical hammer test setup. The hammer (either handheld or mounted independently on a stand) is used to strike the housing, and with sufficient impact energy, all natural frequencies are usually excited. To minimize uncertainties, hammer tests may be conducted in all three (x, y, z) directions using vibration-measuring instruments to record responses. Even greater measurement accuracy can be achieved by suspending the prototype housing using flexible wire or rope. This method offsets the effects of mass and is thus more sensitive to stiffness.

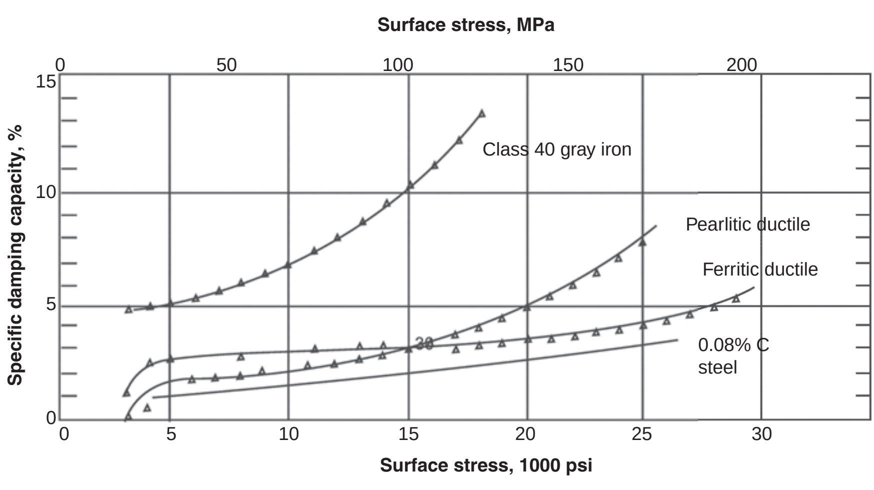


Fig. 1.6 Damping capacity of different gearbox materials

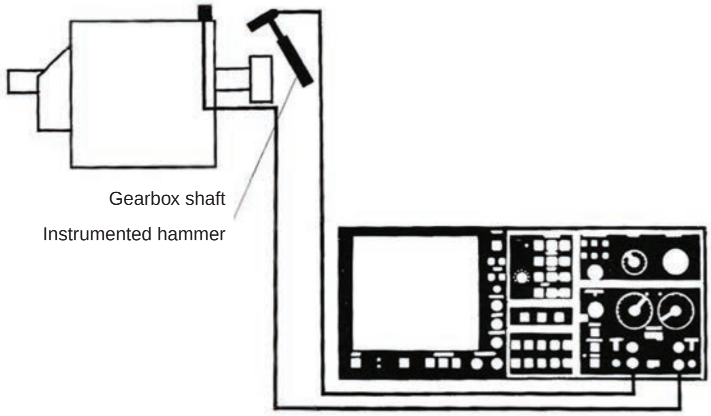


Fig. 1.7 The transfer function of a gearbox can be measured with an instrumented hammer and a two-channel dynamic signal analyzer.

Prototype Housing and the Finite-Element Model

Despite advances in measurement technology, finalizing the design of a prototype housing by trial-and-error alone can take a considerable amount of time. A more efficient approach is one that makes use of numerical models based on finite-element analysis (FEA). Finite-element analysis models may be used to determine vibration characteristics such as natural frequencies and associated mode shapes of the housing structure. To ensure that the finite-element model correlates with the actual housing structure, a prototype is constructed via steel fabrication and its dynamic response is evaluated by hammer testing.

The results obtained are then compared with those calculated using finite-element analysis. If the results are within 5% of each other, the FEA model is accepted as valid. If the data indicate a deviation of more than 5%, the model is reworked by adding more elements to critical areas of the structure. Increasing or reducing wall thickness in the model usually helps to achieve better correlation.

Once validated, the model can be used to make prototypes with the required structural and torsional rigidities. At this point, the gearbox housing drawing can be finalized for steel. In cases where a casting is selected, however, a new finite-element model is required. A similar process is then followed to generate new housing drawings.

Final Machining of the Housing

Precision machining of the housing is usually necessary to meet design tolerances for centerline dimensions between shafts and concentricity requirements for bearing bores. Before finish machining, each housing half is machined individually then bolted together guided by locating pins.

Material Treatment for Dimensional Stability of Housing Structure

To maintain dimensional stability over the life of a gearbox, it is important to heat treat the housing before finish machining. This applies to housings made of ductile and cast iron as well as steel.

Heat Treating Cast Iron

A process proven effective for stabilizing castings made from ductile and nickel-containing gray cast iron includes:

- Heating both halves of castings to 845-870 °C (1550-1600 °F) for 2 h with an additional 1 h/in. of casting thickness.
- Furnace cooling no faster than 55 °C/h (100 °F/h) down to 525-550 °C (975-1025 °F), holding at this temperature for 1 h/in. of casting section thickness, then cooling uniformly to room temperature.
- After rough machining, reheating casting halves to 455-480 °C (850-900 °F) and holding for 1 h/in. of casting section thickness.
- Cooling uniformly to room temperature, preferably in the same furnace with the heating switched off.

This heat treatment ensures maximum metal stability and complete removal of induced stresses and strains. It also helps to maintain precise spatial relationships among gearbox components after assembly.

Heat Treatment for Steel Housings

For housings made of steel, each half is fully annealed after welding, then rough machined. The two halves are then assembled and stress relieved before final machining.

Housing Design for Epicyclic Gearboxes

The housing design process for offset parallel gearboxes can be used for any parallel-axis configuration, including epicyclic gearboxes with planetary, star, or solar gear arrangements. Epicyclic gearbox housings are vertically split to facilitate the assembly of planet and ring gears. Due to inherently high structural rigidity, the natural frequencies of epicyclic gearbox housings are far above any gear mesh or component natural frequency, providing an effective safety margin.

Analysis and Selection of Design Factors for Vibration Reduction

Other design factors beyond the housing can have a significant effect on gearbox vibration and thus play a role in its reduction. One such factor is the type of gearing used. For low-level vibration, for example, helical gears are preferred in parallel-axis gearboxes. The design of helical gears makes it possible to maintain tight tolerance during the meshing of teeth and to operate with minimum friction between the involute helicoid profiles of the pinion and gear teeth. Helical gears are also able to maintain more than one tooth in contact during mesh; this is known as helical overlap.

Figure 1.8 shows typical spur gear and helical gear sets. Figure 1.9 summarizes the advantages and disadvantages of helical gears with a single helix angle. Figure 1.10 illustrates how a spur gear can be replaced by helical gears with a different helix angle. It can be shown that the larger the helix angle, the higher the gear tooth contact ratio. A higher contact ratio reduces vibration, but there is a limit to which the helix angle and contact ratio can be increased.

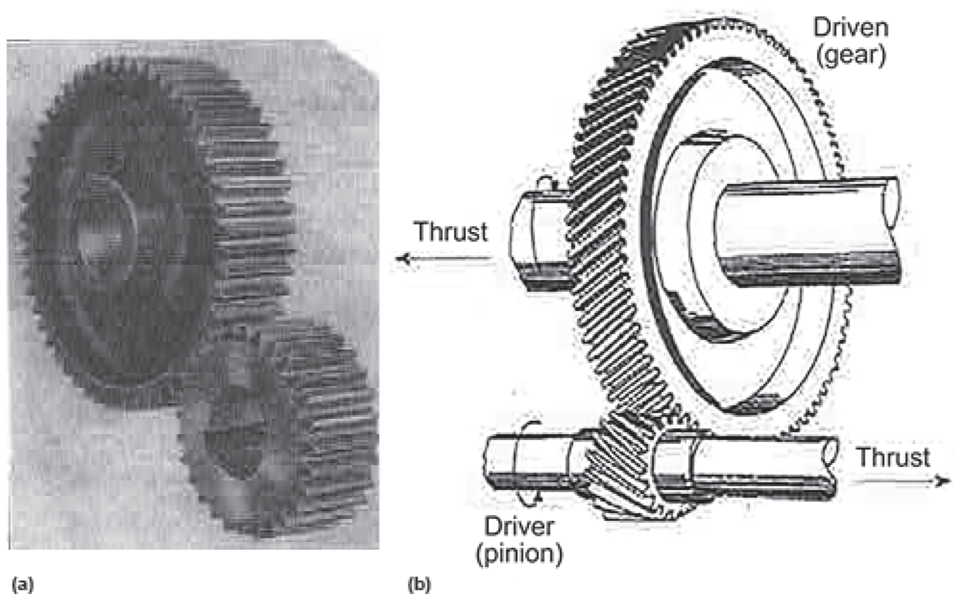


Fig. 1.8 Gear and pinion; (a) spur, (b) helical

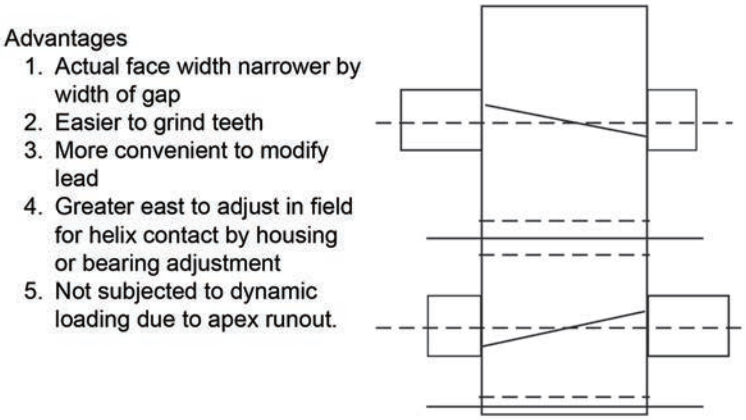


Fig. 1.9 Advantages and disadvantages of a single helical gear

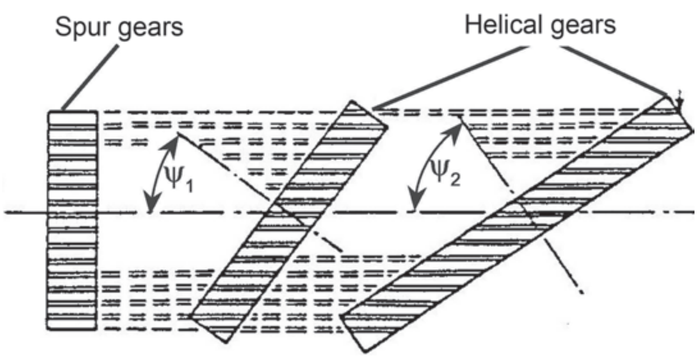


Fig. 1.10 By varying the helix angle, Ψ, a single gear spur can be replaced by a number of helical gears.

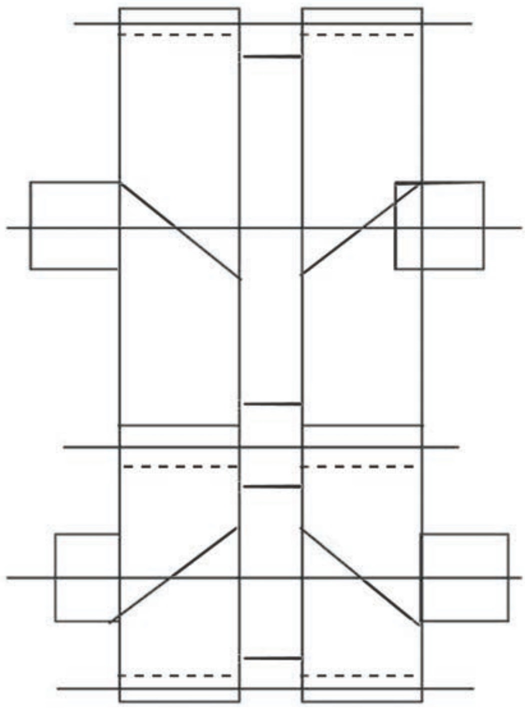
To avoid high helix angles, double helical gears (Fig. 1.11) are preferred and are more effective in reducing vibration. Double helical gears also eliminate axial thrust, which allows the use of antifriction ball or roller bearings.

Double helical gears are not without drawbacks, however. Apex runout during gear mesh, as shown in Fig. 1.12, can be a major problem. A small variation in apex runout coupled with the axial inertia of the gear prevents equal load sharing, causing gears in mesh to shuttle back and forth in the axial direction. Increased vibration due to this effect can defeat the advantages of the double helical design.

Once a selection is made on the type of gear, the next consideration is choosing a gear tooth profile that maintains constant velocity transfer

Advantages

1. Higher helix angle results in greater contact ratio
2. Divided load allows the use of greater face width, better contact
3. For the same hardness, shorter center distance and lower pitch line velocity
4. Shorter oil exit paths
5. Less tendency to distort housing walls
6. Low lateral and torsional vibrations
7. Lower windage loss
8. Less tooth deformation under load



Disadvantages

1. Longer total element length due to necessary gap
2. Subject to dynamic load fluctuation due to apex runout
3. Two setups for each gear tooth operation
4. More difficult to grind teeth
5. Sensitive to gear misalignment

Fig. 1.11 Advantages and disadvantages of double helical gears

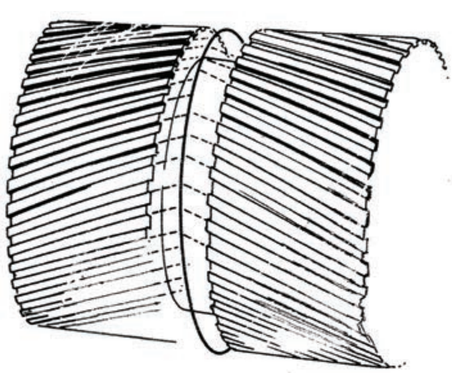


Fig. 1.12 Schematic of apex runout in double helical gears

between mating gears. Two profiles satisfy this requirement, involute and cycloid, with the former being preferred for most gears. Cycloidal tooth profiles are used only in certain applications because they require precise center distances between gears to maintain conjugacy and constant velocity transfer. In that respect, the involute gear tooth profile is quite forgiving.

Gear Ratio and Number of Stages for Parallel-Shaft Gearboxes

Investigations carried out by several researchers show that the gear ratio in a single stage of helical gears should not exceed 8:1 to avoid dynamic problems at high speeds. In gearboxes with ratios in that range, gear size, pitch line velocity, and mesh stiffness were within acceptable limits for reduced vibration levels.

Number of Teeth, Pressure Angle, and Diametral Pitch

For any gear ratio, the number of teeth in the pinion and gear should be selected so that the only common factor between them is 1. This allows every tooth on the pinion to mesh with every tooth on the gear, which corrects for minor imperfections in tooth surfaces and equalizes wear, resulting in ideal tooth contact patterns. Sometimes, for high-quality gears, a partial hunting tooth combination may be used as long as the pinion and gear teeth add up to a prime number. For a hunting or partial hunting tooth combination, it is important to consider the minimum number of teeth that can be cut on a gear blank without any undercut at the root of the teeth. Gear-cutting machines limit the minimum number of teeth for different pressure angles (PAs):

- 32 teeth for 14½° PA
- 17 teeth for 20° PA
- 12 teeth for 25° PA

Gears without undercut ensure meshing of teeth without any interference, which helps to further reduce vibration.

Tooth-to-Tooth Spacing

Tooth-to-tooth spacing is the distance between two teeth measured at the pitch circle. Any variation in tooth-to-tooth spacing causes gears to accelerate and decelerate as adjacent teeth go in and out of mesh, producing torsional vibration at a frequency corresponding to the periodicity error.

Diametral Pitch

Diametral pitch (DP) is the number of teeth per unit pitch diameter. The higher the DP, the thinner the gear teeth, and vice versa. A higher DP also corresponds to a greater number of teeth in contact and a high overlap ratio, a parameter that aids vibration reduction. There is a practical limit, however, to the selection of a high DP. With current measuring equipment, it may not be possible to inspect tooth geometry higher than 15 DP.

Pressure Angle

Gear tooth profile designs with a pressure angle of 14½° produce the least amount of noise, but are rarely used in industrial or highly loaded gears because the number of teeth required to prevent undercutting is too high for many applications. A 25° PA, on the other hand, requires the fewest teeth, but can significantly increase gearbox vibration at high speeds due to loss of gear conjugacy. As a result, 20° is the preferred PA for most industrial and aerospace gears.

Contact Ratio

Contact ratio determines the number of teeth in contact at any time. A contact ratio of 2.0 is considered high. Helical gears can be designed to have a large contact ratio if needed. This reduces the overall vibration level of a gearbox, thus making helical gears highly desirable in high-speed applications.

Recess Action Gears

Gears can be designed for high recess action, which reduces impact during mesh and thus vibration.

Elastic Tooth Deformation

Elastic deformation of gear teeth is another factor that must be considered, particularly for gears subjected to high loads. As each tooth goes through the mesh, it deflects elastically, which changes its profile due to combined bending, shear, and Hertzian stresses. Variations in elastic deformation may be minimized by selecting a smaller DP tooth (8 or below) and a PA higher than 20° with a contact ratio above 2.0.

Backlash between Pinion and Gear Teeth

The distance between pinion and gear teeth at their pitch points, as shown in Fig. 1.13, is a type of backlash due to gear manufacturing limitations. Backlash may also result when the centerline dimension between a gear and pinion shaft exceeds the calculated value. In a gear set with backlash, the pinion starts to rotate ahead of the gear and does not make contact with the gear tooth surface until it takes up the slack. This causes various problems ranging from high impacts between pinion

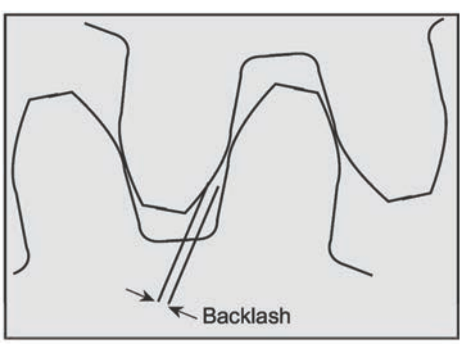


Fig. 1.13 Backlash between pinion and gear teeth

and gear teeth to the modulation of gear tooth mesh frequency. Forces generated during gear mesh also contribute to vibration and may cause premature bearing failure.

Figure 1.14 shows a sectional view of a right-angle double-reduction gear unit with an excessive amount of backlash (0.13 mm, or 0.005 in.) in the spiral bevel gear set. At the intermediate shaft running at 16.09 rev/s, the velocity of vibration was recorded at more than 4.3 mm/s (0.17 in./s). By adjusting the bevel gear relative to the pinion, backlash was reduced to 0.076 mm (0.003 in.), corresponding to an acceptable vibration velocity of 2.5 mm/s (0.1 in./s).

Tooth Misalignment

Tooth misalignment due to various assembly and dimensional errors contributes to increased tooth loads as well as noise and vibration. In Fig. 1.15, gear vibration is plotted as a function of tooth misalignment for different values of transmitted torque. The data show a significant increase in vibration, approximately 10 dB, for a gearbox with 0.05 mm/mm (0.002 in./in.) of tooth misalignment.

Misalignment of Shafts

Shaft misalignment, as shown in Fig. 1.16, may occur when bearing bores in a gearbox housing are not concentric. It may also be due to preload from a bent shaft improperly seated in a bearing. When a shaft turns, the characteristic frequencies of the bearing may appear in the vibration signal, making it possible to differentiate between shaft misalignment and unbalance.

Mechanical Looseness

Vibration due to mechanical looseness usually involves loose bearings that result in a large number of harmonics in the vibration spectrum.

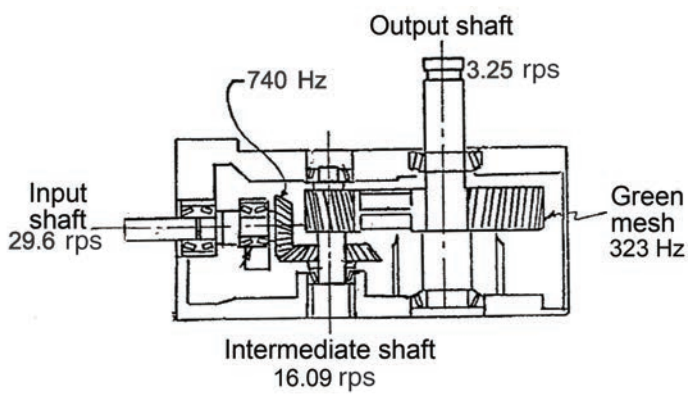


Fig. 1.14 Right-angle double-reduction gear unit; rps, revolutions per second

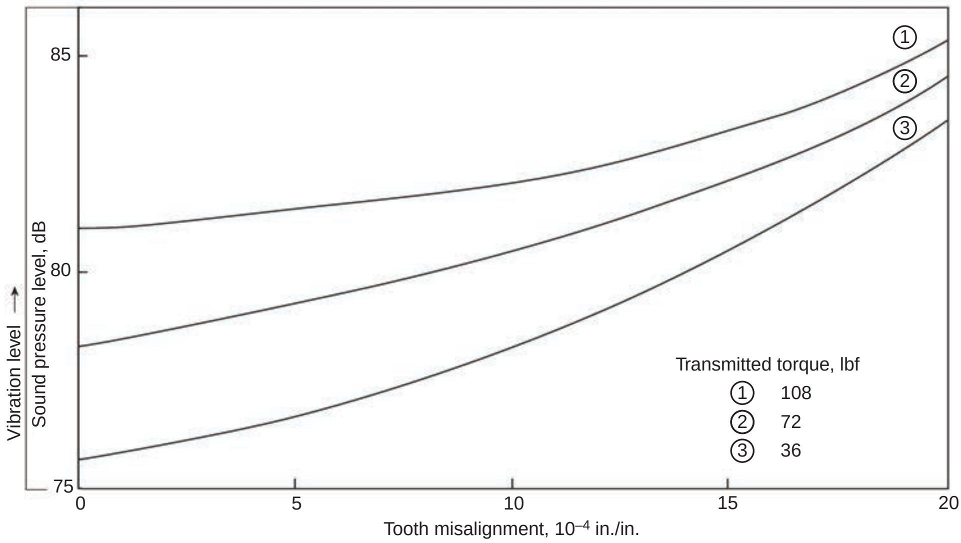


Fig. 1.15 Influence of tooth misalignment on noise/vibration

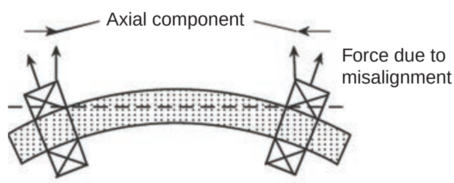


Fig. 1.16 Bent/misaligned shaft with high axial vibration

Looseness produces a directional vibration, a characteristic that makes it easy to distinguish from rotational defects such as unbalance. A technique that works well for detecting and analyzing looseness is to make vibration measurements at several points on the gearbox. Vibration levels are the highest in the direction and vicinity of the looseness.

The directionality that accompanies looseness produces vibrations that vary significantly with the mounting orientation of the transducer used for testing. In contrast to unbalance, which produces similar responses in the horizontal and vertical directions, looseness is characterized by a large vertical component and a much smaller horizontal component.

Installation

A major consideration when installing a gearbox is the alignment of the output shaft relative to the connected equipment. It is also important to ensure that mounting pads are flat to prevent the housing from twisting when the mounting bolts are tightened.

Gearbox Vibrations Induced by Driver and Driven Equipment and Coupling

Gearbox vibrations induced by the driver and driven equipment are influenced by rotational speeds as well as misalignment. This calls for an understanding of the operating characteristics of different types of driven equipment.

Consider, for example, an electric power generator. In addition to the mechanical unbalance of the generator rotor, electrical unbalance may also be present. Electrical unbalance produces magnetic fields in the stator and rotor of the generator, giving rise to electromagnetic forces. If these forces are not balanced, the radial component can induce vibrations at twice the line frequency.

To minimize electric-generator-induced vibrations, a generator may be selected according to design requirements. Choices are limited, however, because the structural characteristics of the generator are fixed and its vibrational responses cannot be altered without a major change in design. In such situations it is logical to select a mechanical coupling with high damping properties to reduce the transmissibility of vibrations between the generator and gearbox.

Gear Tooth Geometry Errors

Above any other factor in play, gear tooth geometry errors have the greatest impact on gearbox vibration at high speeds. These errors are of five types, two based on tooth profile, the others based on tooth spacing or pitch:

- *Involute (profile) error:* deviation from an ideal involute
- *Lead (parallelism) error:* deviation in the tooth surface along the axial direction of the gear
- *Pitch error:* deviation of the angular position of a tooth relative to an adjacent tooth as measured and expressed by linear distance at the pitch circle diameter
- *Accumulated pitch variation:* deviation in the angular position of a tooth relative to an arbitrary initial tooth
- *Runout:* amount of eccentricity between the pitch circle of a gear and the centerline of the shaft on which it is mounted

Involute error causes loss of conjugate action, resulting in nonuniform velocity transfer between meshing gears. Velocity variation due to an imperfect involute (Fig. 1.17) creates a dynamic load on gear teeth that causes them to vibrate. Lead error, on the other hand, increases load intensity, which further raises vibration levels. Pitch and runout errors modulate the contact ratio of mating gear teeth, thereby exciting vibrations in the gearbox.

Inspection of Gear Quality

In practice, there are two ways to determine gear quality:

- Inspect each gear tooth for geometry errors
- Obtain a composite measurement using a master gear

Inspection of Individual Error

To inspect the geometry of each gear tooth, special equipment is required. The following sections describe the instruments and methods used to measure involute profile and lead errors. The assessment of other errors can be done using calipers suggested by AGMA.

Involute Error Measurement

Involute error is typically inspected using an involute checker that measures variations between an actual tooth profile and a true geometric involute. A true involute tooth profile, with no measurable deviations, is plotted as a straight line on a chart or grid. Figure 1.18 illustrates a true

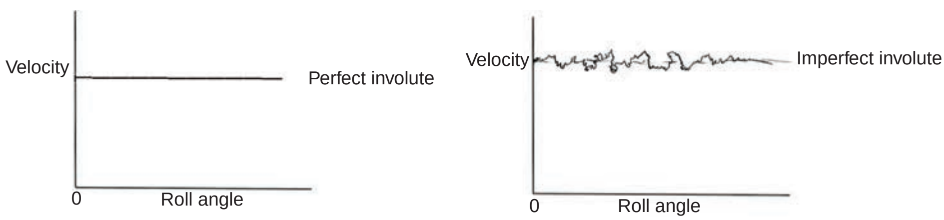


Fig. 1.17 Variation of velocity transfer

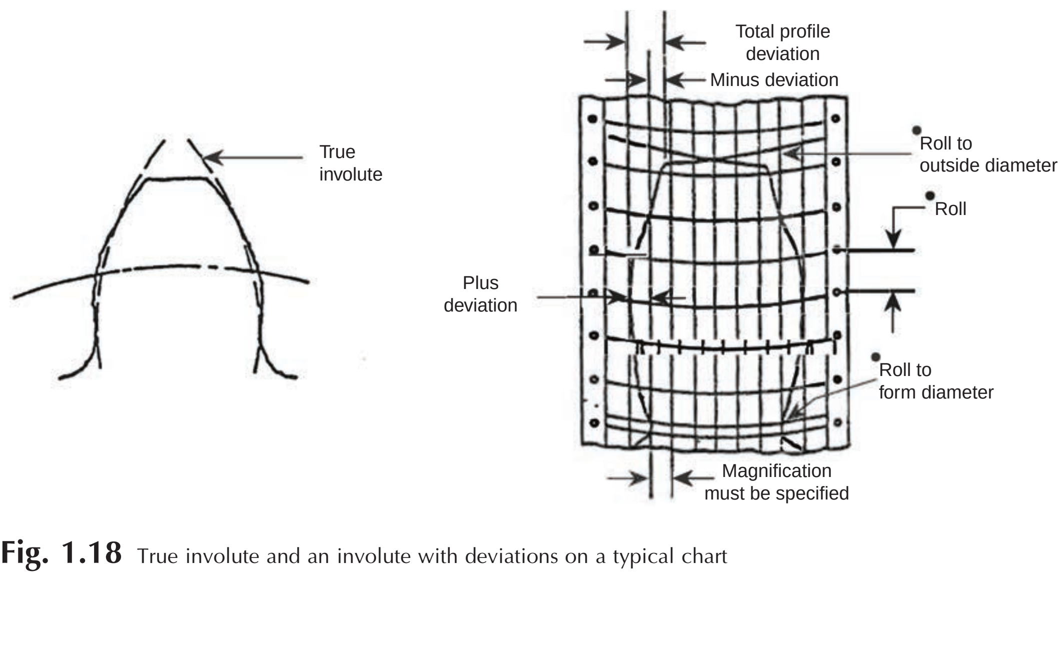


Fig. 1.18 True involute and an involute with deviations on a typical chart

involute in comparison to a tooth profile with measured deviations. Excess material on the tooth profile is indicated by a *plus* deviation, while insufficient material is indicated by a *minus* deviation (Fig. 1.19). Involute error can have a significant influence on vibration as shown in Fig. 1.20.

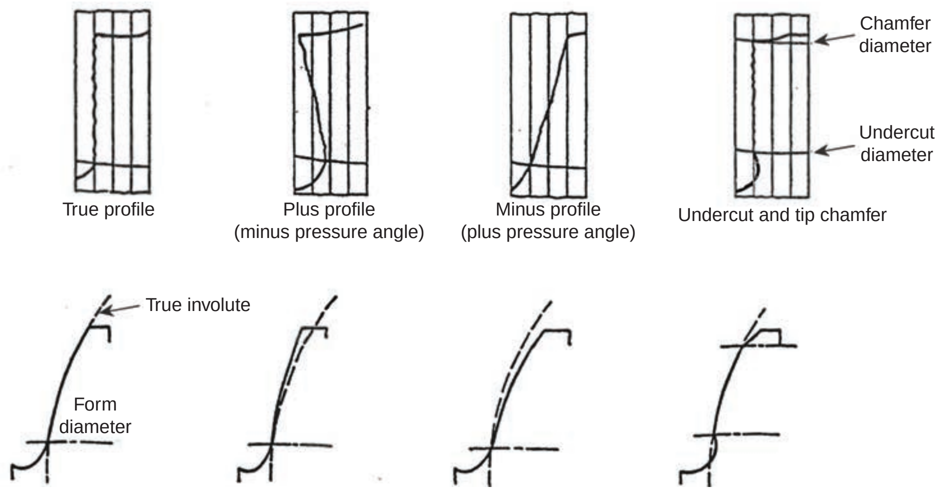


Fig. 1.19 Typical tooth profile tolerance charts

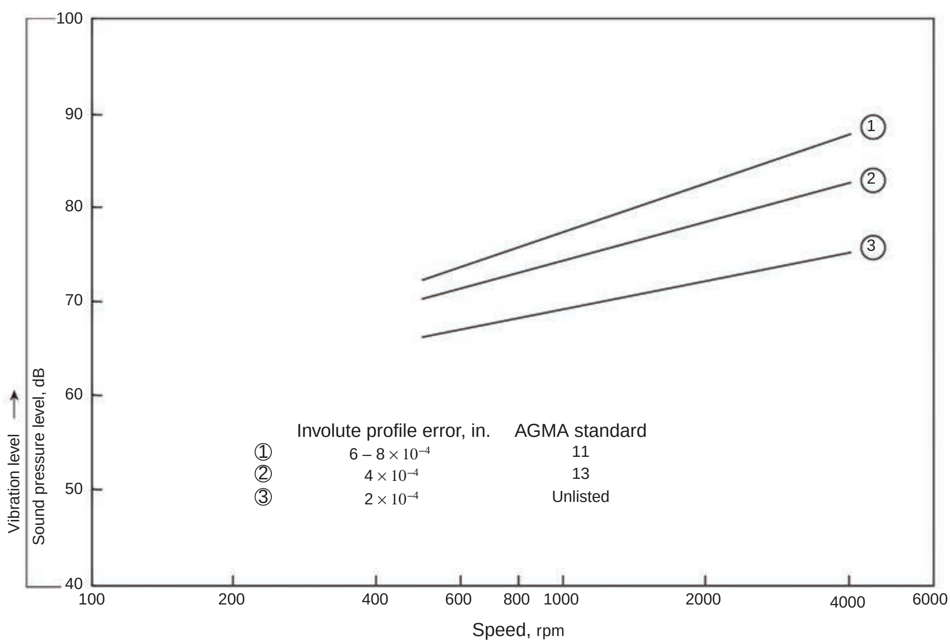


Fig. 1.20 Influence of profile error on noise/vibration

In addition to manufacturing errors, thermal deformation of teeth during gear mesh may also cause a profile to deviate, producing an interference between the teeth of mating gears. For a well-lubricated gearbox, the thermal deformation is expected to be small; thus, it has a negligible effect on profile deviation. Even so, it is beneficial to modify the involute profile with some tooth-tip relief, as discussed later in this chapter.

Involute Profile Quality

Because a true involute profile is difficult to manufacture, allowable tolerances that are applicable for different-quality gears are recommended by AGMA (Tables 1.1 to 1.3). These standards are based on kinematic and deflection analysis of gear teeth and years of field experience. An example of a gear tooth profile chart based on AGMA standards is shown in Fig. 1.21. The profile tolerance in the chart varies with the quality level of gears in accordance with the standard. For

Table 1.1 Run-out tolerance on coarse pitch tooth geometry for high-speed gears
Tolerances in ten thousandths of an inch

AGMA quality number	Normal diametral pitch	Run-out tolerance										Lead tolerance				
		Pitch diameter, in.										Face width, in.				
		¾	1½	3	6	12	25	50	100	200	400	1 and less	2	3	4	5
8	½	...	...	...	...	146.5	174.5	205.8	242.7	286.3	337.6	5	8	11	13	16
	1	...	...	...	88.8	104.8	124.8	147.2	173.6	204.7	241.4					
	2	...	...	53.9	63.5	74.9	89.2	105.2	124.1	146.3	172.6					
	4	...	32.7	38.5	45.4	53.6	63.8	75.2	88.7	104.6	123.4					
	8	19.8	23.3	27.5	32.5	38.3	45.6	53.8	63.4	74.8	88.2					
	12	16.3	19.2	22.6	26.7	31.5	37.5	44.2	52.1	61.5	72.5					
	20	12.7	15.0	17.7	20.8	24.6	29.3	34.5	40.7	48.0	56.6					
9	½	...	...	...	...	104.7	124.7	147.0	173.4	204.5	241.2	4	7	9	11	13
	1	...	...	...	63.5	74.8	89.1	105.1	124.0	146.2	172.4					
	2	...	...	38.5	45.4	53.5	63.7	75.2	88.6	104.5	123.3					
	4	...	23.3	27.5	32.4	38.3	45.6	53.7	63.4	74.7	88.1					
	8	14.1	16.7	19.7	23.2	27.4	32.6	38.4	45.3	53.4	63.0					
	12	11.6	13.7	16.2	19.1	22.5	26.8	31.6	37.2	43.9	51.8					
	20	9.1	10.7	12.6	14.9	17.6	20.9	24.7	29.1	34.3	40.4					
10	½	...	...	...	...	74.8	89.0	105.0	123.8	146.1	172.3	3	5	7	9	10
	1	...	...	...	45.3	53.5	63.7	75.1	88.5	104.4	123.2					
	2	...	...	27.5	32.4	38.2	45.5	53.7	63.3	74.7	88.1					
	4	...	16.7	19.6	23.2	27.3	32.5	38.4	45.3	53.4	63.0					
	8	10.1	11.9	14.0	16.6	19.5	23.3	27.4	32.4	38.2	45.0					
	12	8.3	9.8	11.5	13.6	16.1	19.1	22.6	26.6	31.4	37.0					
	20	6.5	7.6	9.0	10.6	12.5	14.9	17.6	20.8	24.5	28.9					
11	½	...	...	...	...	53.4	63.6	75.0	88.5	104.3	123.0	3	4	6	7	8
	1	...	...	...	32.4	38.2	45.5	53.6	63.2	74.6	88.0					
	2	...	...	19.6	23.1	27.3	32.5	38.3	45.2	53.3	62.9					
	4	...	11.9	14.0	16.6	19.5	23.2	27.4	32.3	38.1	45.0					
	8	7.2	8.5	10.0	11.8	14.0	16.6	19.6	23.1	27.3	32.2					
	12	5.9	7.0	8.2	9.7	11.5	13.7	16.1	19.0	22.4	26.4					
	20	4.6	5.5	6.4	7.6	9.0	10.7	12.6	14.8	17.5	20.6					
12	½	...	...	...	...	38.1	45.4	5.36	63.2	74.5	87.9	2	3	5	6	7
	1	...	...	...	23.1	27.3	32.5	38.3	45.2	53.3	62.8					
	2	...	...	14.0	16.5	19.5	23.2	27.4	32.3	38.1	44.9					
	4	...	8.5	10.0	11.8	13.9	16.6	19.6	23.1	27.2	32.1					
	8	5.2	6.1	7.2	8.5	10.0	11.9	14.0	16.5	19.5	23.0					
	12	4.2	5.0	5.9	6.9	8.2	9.8	11.5	13.6	16.0	18.9					
	20	3.3	3.9	4.6	5.4	6.4	7.6	9.0	10.6	12.5	14.7					

Table 1.2 Pitch tolerance on coarse pitch tooth geometry for high-speed gears

Tolerances in ten thousandths of an inch

AGMA quality number	Normal diametral pitch	Pitch tolerance									Lead tolerance				
		Pitch diameter, in.									Face width, in.				
		¾	1½	3	6	12	25	50	100	200	1 and less	2	3	4	5
8	½	···	···	···	···	19.0	21.7	24.5	27.7	31.3	5	8	11	13	16
	1	···	···	···	14.4	16.3	18.6	21.0	23.7	28.8					
	2	···	···	10.9	12.3	14.0	15.9	18.0	20.3	23.6					
	4	···	8.3	9.3	10.6	11.9	13.6	15.4	17.4	19.7					
	8	6.3	7.1	8.0	9.0	10.2	11.7	13.2	14.9	16.8					
	12	5.7	6.5	7.3	8.3	9.3	10.6	12.0	13.6	15.4					
	20	5.1	5.8	6.5	7.4	8.3	9.5	10.7	12.1	13.7					
9	½	···	···	···	···	13.4	15.3	17.3	19.5	22.1	4	7	9	11	13
	1	···	···	···	10.2	11.5	13.1	14.8	16.7	18.9					
	2	···	···	7.7	8.7	9.8	11.2	12.7	14.3	16.2					
	4	···	5.8	6.6	7.4	8.4	9.6	10.8	12.2	13.8					
	8	4.4	5.0	5.6	6.4	7.2	8.2	9.3	10.5	11.9					
	12	4.0	4.6	5.1	5.8	6.6	7.5	8.5	9.6	10.8					
	20	3.6	4.1	4.6	5.2	5.9	6.7	7.6	8.5	9.7					
10	½	···	···	···	···	9.4	10.8	12.2	13.7	15.5	3	5	7	9	10
	1	···	···	···	7.2	8.1	9.2	10.4	11.8	13.3					
	2	···	···	5.4	6.1	6.9	7.9	8.9	10.1	11.4					
	4	···	4.1	4.6	5.2	5.9	6.7	7.6	8.6	9.8					
	8	3.1	3.5	4.0	4.5	5.1	5.8	6.5	7.4	8.3					
	12	2.8	3.2	3.6	4.1	4.6	5.3	6.0	6.7	7.6					
	20	2.5	2.9	3.2	3.7	4.1	4.7	5.3	6.0	6.8					
11	½	···	···	···	···	6.6	7.6	8.6	9.7	10.9	3	4	6	7	8
	1	···	···	···	5.0	5.7	6.5	7.3	8.3	0.4					
	2	···	···	3.8	4.3	4.9	5.6	6.3	7.1	8.0					
	4	···	2.9	3.3	3.7	4.2	4.8	5.4	6.1	8.9					
	8	2.2	2.5	2.8	3.2	3.6	4.1	4.6	5.2	8.9					
	12	2.0	2.3	2.6	2.9	3.3	3.7	4.2	4.7	8.4					
	20	1.8	2.0	2.3	2.6	2.9	3.3	3.7	4.2	4.8					
12	½	···	···	···	···	4.7	5.3	6.0	6.8	7.7	2	3	5	6	7
	1	···	···	···	3.5	4.0	4.6	5.2	5.8	6.6					
	2	···	···	2.7	3.0	3.4	3.9	4.4	5.0	5.6					
	4	···	2.0	2.3	2.6	2.9	3.3	3.8	4.3	4.8					
	8	1.5	1.7	2.0	2.2	2.5	2.9	3.2	3.7	4.1					
	12	1.4	1.6	1.8	2.0	2.3	2.6	3.0	3.3	3.8					
	20	1.3	1.4	1.6	1.8	2.0	2.3	2.6	3.0	3.4					

Table 1.3 Profile tolerance on coarse pitch tooth geometry for high-speed gears

Tolerances in ten thousandths of an inch

AGMA quality number	Normal diametral pitch	Profile tolerance										Lead tolerance				
		Pitch diameter, in.										Face width, in.				
		¾	1½	3	6	12	25	50	100	200	400	1 and less	2	3	4	5
8	½	···	···	···	···	42.6	47.7	53.1	59.1	65.7	73.1	5	8	11	13	16
	1	···	···	···	28.3	31.5	35.3	39.3	43.7	48.6	54.1					
	2	···	···	18.8	21.0	23.3	26.1	29.0	32.3	36.0	40.0					
	4	···	12.5	13.9	15.5	17.2	19.3	21.5	23.9	26.6	29.6					
	8	8.3	9.3	10.3	11.5	12.8	14.3	15.9	17.7	19.7	21.9					
	12	7.0	7.8	8.6	9.6	10.7	12.0	13.3	14.8	16.6	18.4					
	20	5.6	6.2	6.9	7.7	8.6	9.6	10.7	11.9	13.2	14.7					
9	½	···	···	···	···	30.4	34.1	37.9	42.2	46.9	52.2	4	7	9	11	13
	1	···	···	···	20.2	22.5	25.2	28.1	31.2	34.7	38.6					
	2	···	···	13.5	15.0	16.7	18.6	20.7	23.1	25.7	28.6					
	4	···	8.9	10.0	11.1	12.3	13.8	15.3	17.1	19.0	21.1					
	8	5.9	6.6	7.4	8.2	9.1	10.2	11.4	12.6	14.1	15.6					
	12	5.0	5.5	6.2	6.9	7.6	8.6	9.5	10.6	11.8	13.1					
	20	4.0	4.4	4.9	5.5	6.1	6.8	7.6	8.5	9.4	10.5					
10	½	···	···	···	···	21.7	24.3	27.1	30.1	33.5	37.3	3	5	7	9	10
	1	···	···	···	14.5	16.1	18.0	20.0	22.3	24.8	27.6					
	2	···	···	9.6	10.7	11.9	13.3	14.8	16.5	18.3	20.4					
	4	···	6.4	7.1	7.9	8.8	9.9	11.0	12.2	13.6	16.1					

(continued)

Table 1.3 (Continued)

AGMA quality number	Normal diametral pitch	Profile tolerance — Pitch diameter, in. ¾	1½	3	6	12	25	50	100	200	400	Lead tolerance — Face width, in. 1 and less	2	3	4	5
	8	4.2	4.7	5.3	5.9	6.5	7.3	8.1	9.0	10.0	11.2					
	12	3.6	4.0	4.4	4.8	5.5	8.1	6.8	7.6	8.4	9.4					
	20	2.9	3.2	3.5	3.9	4.4	4.9	5.4	6.1	8.7	7.5					
11	½	...	...	...	...	15.5	17.4	19.3	21.5	24.0	26.7	3	4	6	7	8
	1	...	...	...	10.3	11.5	12.9	14.3	15.9	17.7	19.7					
	2	...	...	8.9	7.6	8.5	9.5	10.6	11.8	13.1	14.6					
	4	...	4.6	5.1	5.6	6.3	7.0	7.8	8.7	9.7	10.8					
	8	3.0	3.4	3.8	4.2	4.6	5.2	5.8	6.4	7.2	8.0					
	12	2.5	2.8	3.1	3.5	3.9	4.4	4.9	5.4	6.0	6.7					
	20	2.0	2.3	2.5	2.8	3.1	3.5	3.9	4.3	4.8	5.4					
12	½	...	...	...	...	11.1	12.4	13.8	15.4	17.1	19.0	2	3	5	6	7
	1	...	...	...	7.4	8.2	9.2	10.2	11.4	12.7	14.1					
	2	...	...	4.9	5.5	6.1	6.8	7.6	8.4	9.4	10.4					
	4	...	3.3	3.6	4.0	4.5	5.0	5.6	6.2	6.9	7.7					
	8	2.2	2.4	2.7	3.0	3.3	3.7	4.1	4.6	5.1	5.7					
	12	1.8	2.0	2.2	2.5	2.8	3.1	3.5	3.9	4.3	4.8					
	20	1.5	1.6	1.8	2.0	2.2	2.5	2.8	3.1	3.4	3.8					

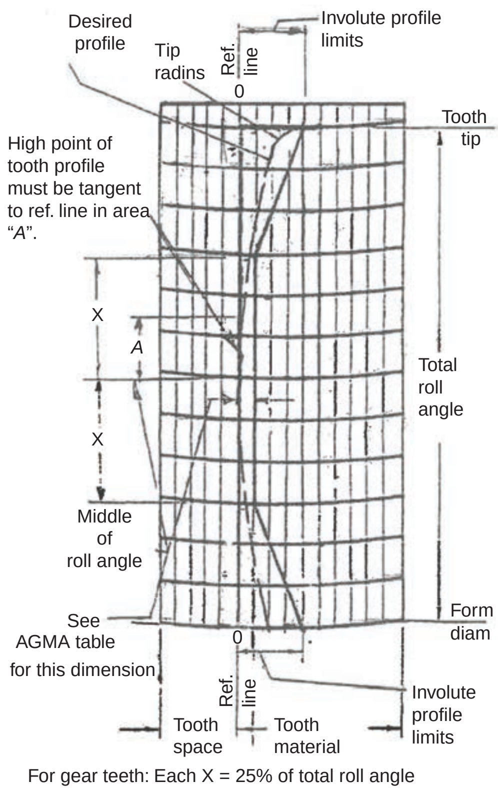


Fig. 1.21 Involute profile for gears with speeds between 610 and 1220 m/min (2000 and 4000 ft/min)

higher-quality gears, the allowable profile error diminishes. Figure 1.22 shows how vibration levels vary with speed and load intensity for a profile error of 0.051 mm (0.002 in.). The gear tooth geometry standards work well for gears running at or below 1220 m/min (4000 ft/min).

At higher speeds, profile tolerance standards are either not available or not helpful. A typical involute profile for gears designed to run at 220 m/min (4000 ft/min) and above is illustrated in Fig. 1.23. Similar to the profile for low-speed gears, some tip relief is provided to avoid interference between deflected meshing teeth under load. However, as several investigators have shown, maintaining an unmodified profile over a region of one base pitch centered on the pitch diameter of the gear maximizes vibration reduction.

Occasionally, high-speed gears, even when manufactured with similar equipment and processes, exhibit uncommon undulations on the gear

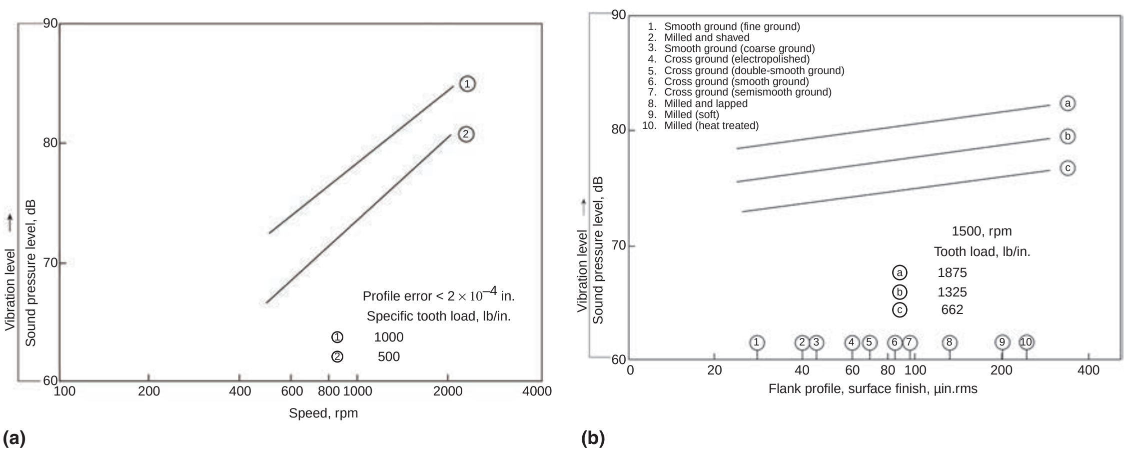


Fig. 1.22 Noise/vibration influence based on (a) speed, (b) flank profile surface finish

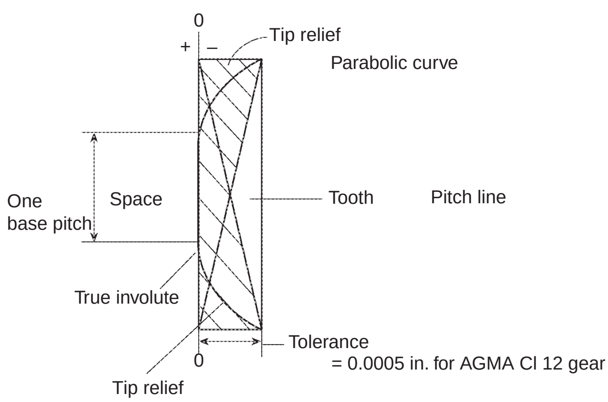


Fig. 1.23 Involute profile for gear speed above 1220 m/min (4000 ft/min)

tooth profile as shown in Fig. 1.24. Although these profiles fit in the tolerance zone depicted in Fig. 1.23, the gears fail to maintain constant velocity transfer, which can significantly increase vibration levels in high-speed gearboxes.

Lead Error of Gear Tooth

The lead of a gear tooth is defined as the axial advance of a helix over a 360° rotation of the gear, as shown in Fig. 1.25. Lead tolerance is the

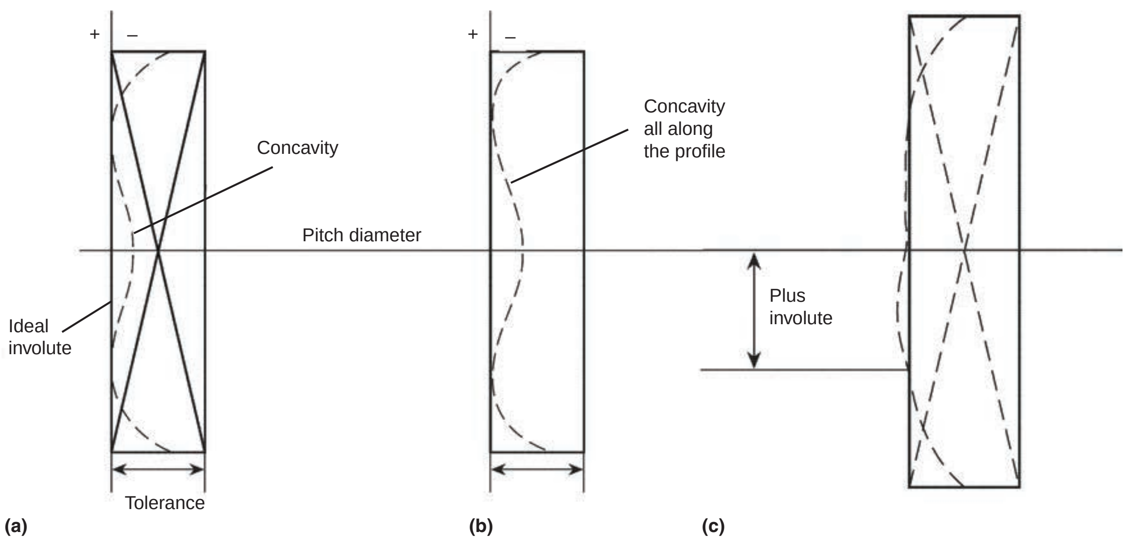


Fig. 1.24 Unacceptable involute profiles for high-speed gears

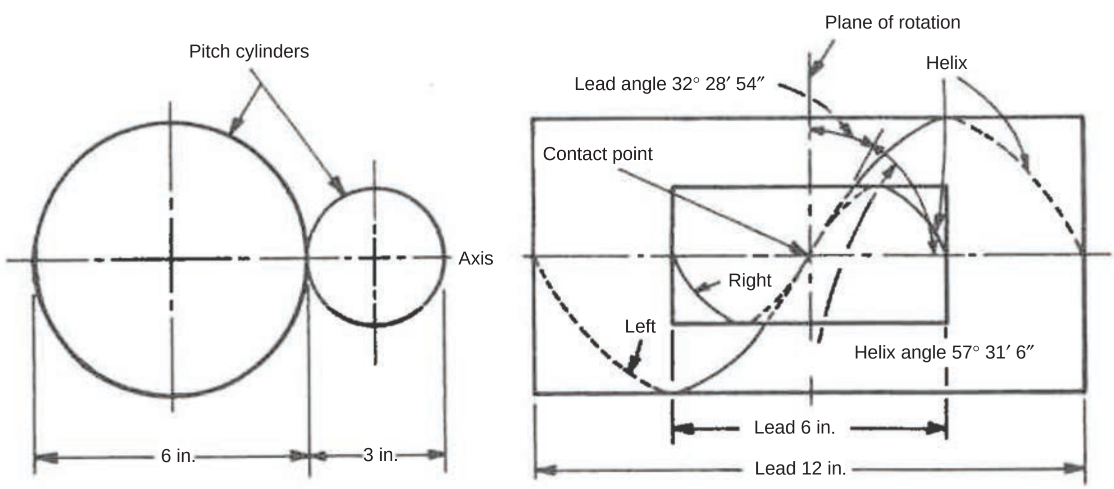


Fig. 1.25 Diagram showing difference between *lead, lead angle,* and *helix angle.*

total allowable variation between the actual (measured) and specified lead. Controlling gear tooth lead is necessary to ensure adequate tooth contact across the face width of meshing gears.

Lead is measured on a lead-checking instrument that translates a probe along the tooth surface, parallel to the axis, while the gear rotates in a timed cycle. At the start of the measurement, the probe is positioned normal to the tooth surface at or near the pitch cylinder. Lead tolerances are interpreted as shown in Fig. 1.26.

The primary motivation for low lead error is reduced load intensity on the tooth surface and a corresponding reduction in vibration. To further reduce load intensity, AGMA recommends a certain amount of crowning of the tooth surface as shown in Fig. 1.27. Such modification allows the load to be distributed over a larger tooth surface area centered along the face width. Tip relief near the tooth edges (Fig. 1.28) may be added as well to eliminate edge loading of the tooth.

In field installations, a visual inspection of the tooth contact area will show whether or not lead error measurement is required. A contact area

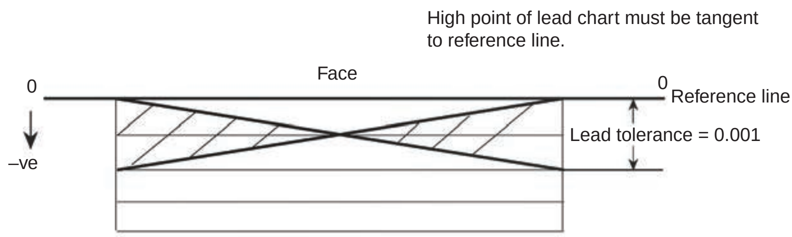


Fig. 1.26 Allowable lead tolerances

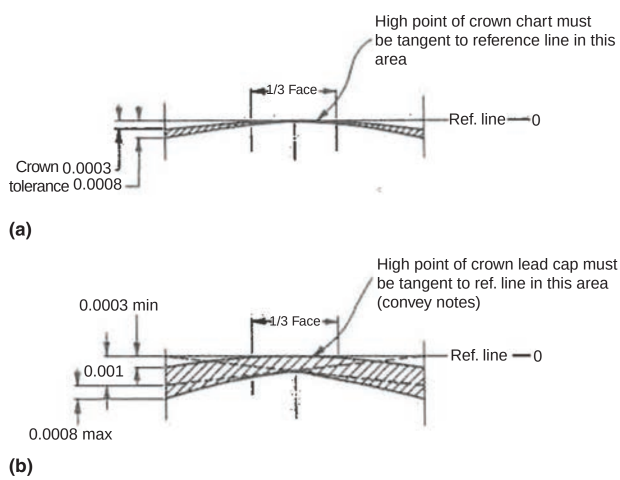


Fig. 1.27 Lead tolerances for crowned teeth. (a) Crown tolerance only. (b) Total tolerance band crown lead

of approximately 80% centrally located on the tooth surface is taken as an acceptance criterion. In cases where the contact area is less than 80%, it is advisable to inspect the gear with lead checking equipment.

Tooth-to-Tooth (Pitch) Error

Figure 1.29(a) illustrates the difference between tooth-to-tooth and accumulated pitch error, both of which affect gear meshing and load

Fig. 1.28 Tip relief on lead

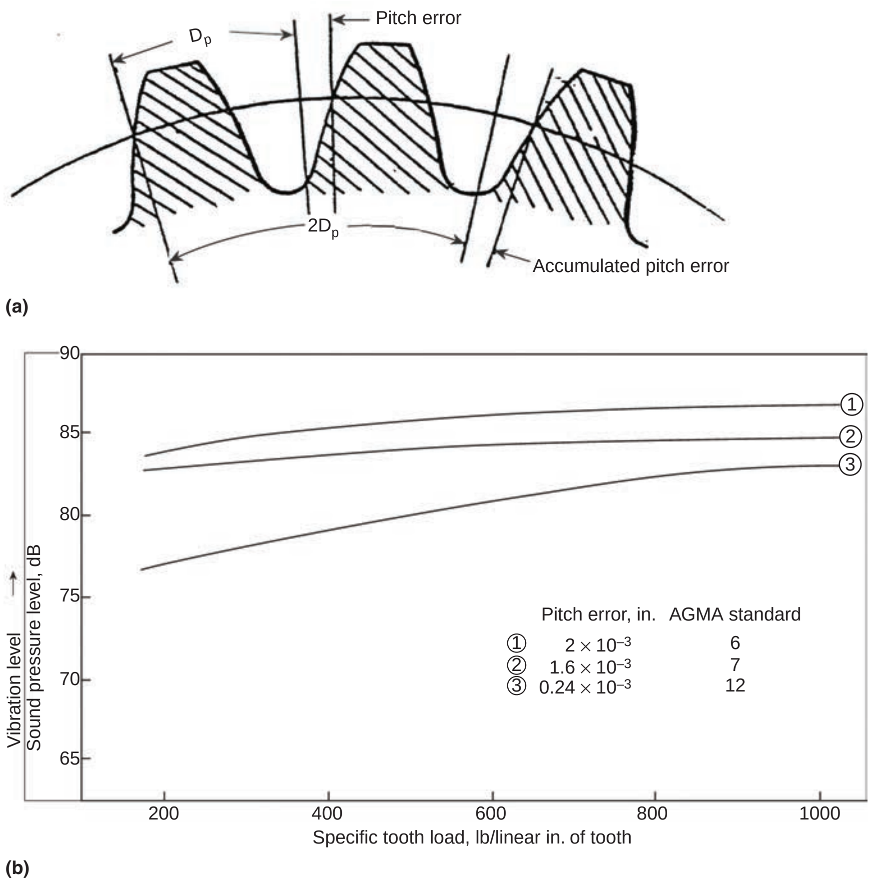


Fig. 1.29 (a) Accumulated pitch error. D_p, design pitch. (b) Influence of pitch error on noise/vibration

intensity. Figure 1.29(b) shows how vibration varies with tooth load in gears with different levels of pitch error, corresponding to AGMA 6-12. The vibration measurements were recorded at a running speed of around 610 m/min (2000 ft/min).

At higher running speeds, tooth-to-tooth pitch error can change the dynamics of tooth meshing, resulting in significant increases in vibration. Consider a 10 MW gas turbine operating at an input speed of 10,000 revolutions per minute (rpm). The number of pinion teeth passing per second may be more than 4000. At that rate, it is not uncommon for some pinion teeth to not even contact the mating gear teeth. In these cases, the angular momentum moves the pinion forward with an impact on the next gear tooth. Several such impacts may occur with every rotation of the pinion, producing shock-induced vibration. To minimize this type of vibration, tooth-to-tooth error for both pinion and gear should be less than 0.05 mm (0.002 in.), as suggested by AGMA.

It is also important to note that a designed tooth-to-tooth error does not remain fixed during the life of a gearbox. Figure 1.30 shows tooth-to-tooth spacing in a gear set measured upon installation and after 6000 h of operation. A corresponding increase in vibration was also measured, which may have been prevented with a gear material with higher surface hardness.

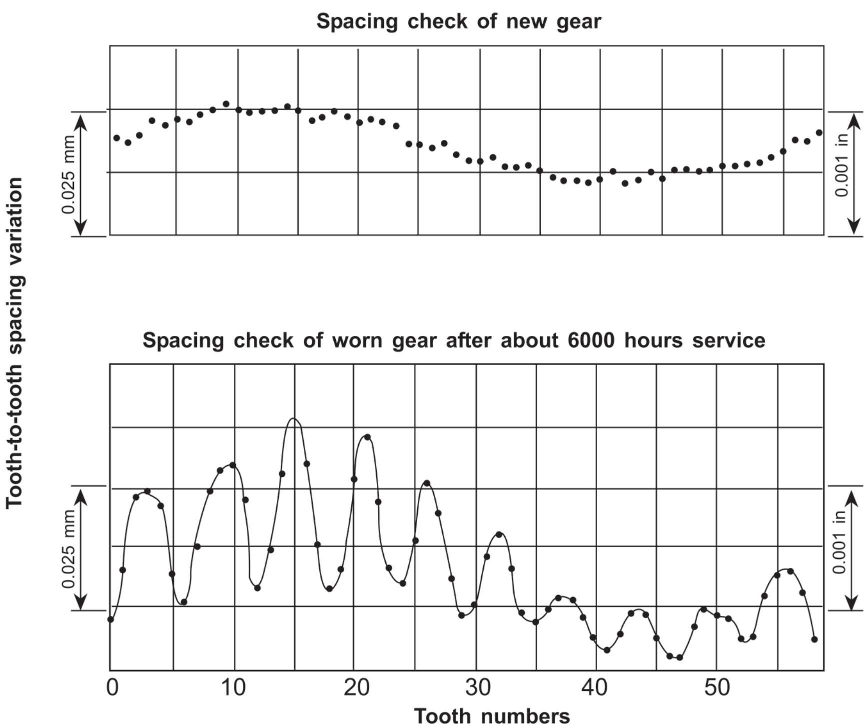


Fig. 1.30 Change of tooth-to-tooth spacing error after service

Composite Measurement

Composite measurements are an alternative to inspecting the geometry of individual gear teeth. As normally implemented, the gear to be inspected runs in tight mesh (no backlash) with a master gear manufactured in accordance with AGMA quality class 14. The measurement procedure is conducted under temperature-controlled conditions using calibrated equipment. Maximum allowable variations in the center distance between gears is specified by AGMA for different quality levels.

The information obtained through composite measurements is sufficient for analyzing and reducing vibration in AGMA quality class 5–8 gears, which are generally used in low-speed gearboxes. For gears intended for use in high-speed gearboxes (AGMA 9–13), composite measurement alone is not sufficient to determine the cause of vibrations. Very often, profile and lead errors must be inspected separately in order to isolate their effects on vibration.

An error chart generated using conventional (double flank) composite measurement data is shown in Fig. 1.31. Double-flank testing is a relatively simple method, but it cannot be used to find any specific gear tooth error. It also requires a tight mesh condition, which is not the actual operating mode for a gear set, and it has been known to produce identical results for gears with different profile errors. To overcome these limitations, a single-flank testing method has been developed that facilitates composite measurement of gear tooth errors.

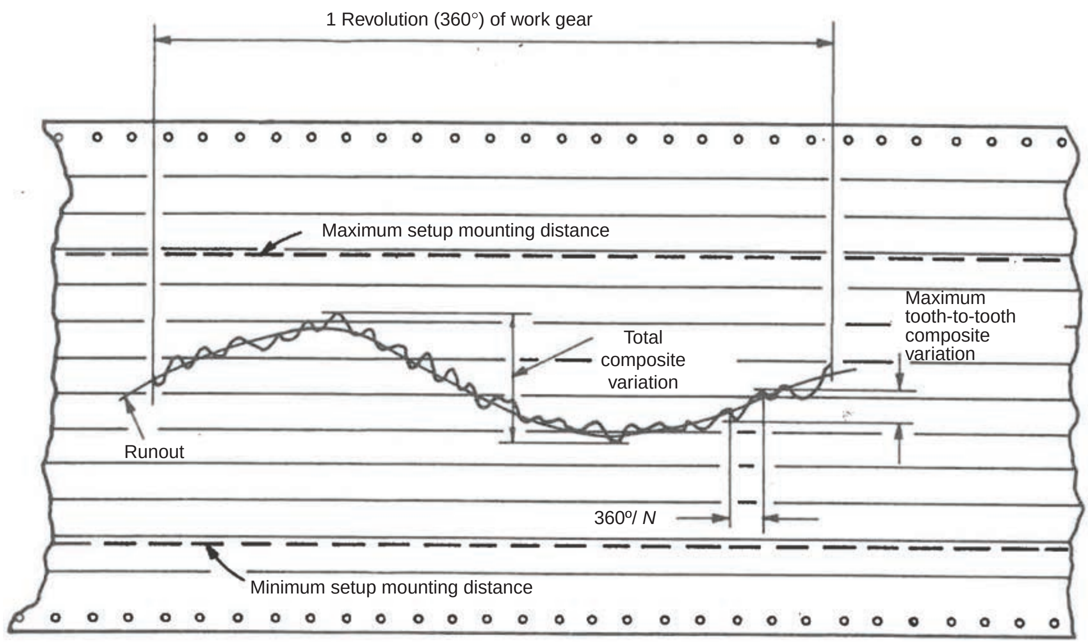


Fig. 1.31 Chart of gear tooth errors of a typical gear when run with a specified gear in a rolling fixture

Transmission Error

The transmission error of a gear tooth is the difference between its actual position and the position it would have if the gear was perfect. Gear tooth geometry errors of different types contribute to transmission error which, by virtue of its composite nature, is well suited for measurement by single-flank testing.

In single-flank testing, the master and work gears roll at their proper center distance with only one flank in contact as their angular positions are measured. The difference between the measured angular position of a given tooth and the position it would occupy in the absence of geometry errors is taken as the transmission error of the tooth. Figure 1.32 presents both double- and single-flank test arrangements.

In the past, gear tooth error measurement by the single-flank method was not used because of the difficulty in resolving small changes in shaft rotation due to minor imperfections in gear teeth. With the development of sophisticated transducers and instrumentation, as shown in Fig. 1.33, this limitation no longer holds.

Transmission Error (Composite)

Transmission error, although not explicitly stated in the definition, is a form of composite error that can be used to analyze several types of gear tooth error. A transmission error chart, shown in Fig. 1.34, contains information on all major gear tooth errors except lead errors.

Similar to double-flank composite errors, higher transmission errors indicate higher vibration levels. In contrast to double-flank measurements, however, there are currently no standards available for the single-flank test and until they are developed, the double-flank measurement standard must be used.

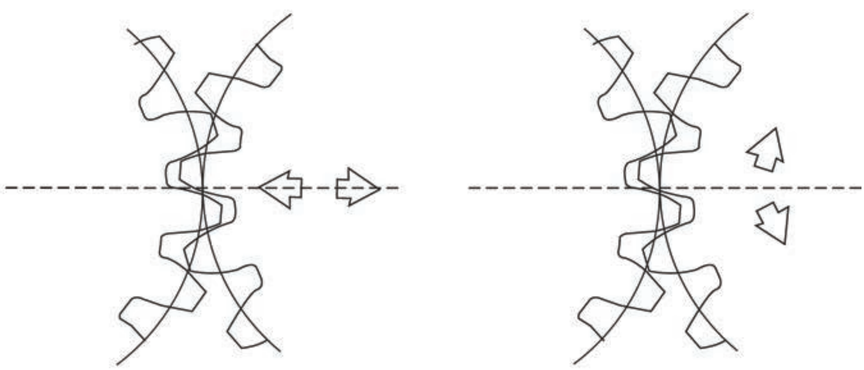


Fig. 1.32 Composite tests, (a) double-flank test measures variation in center distance, and (b) single-flank test measures rotational movements

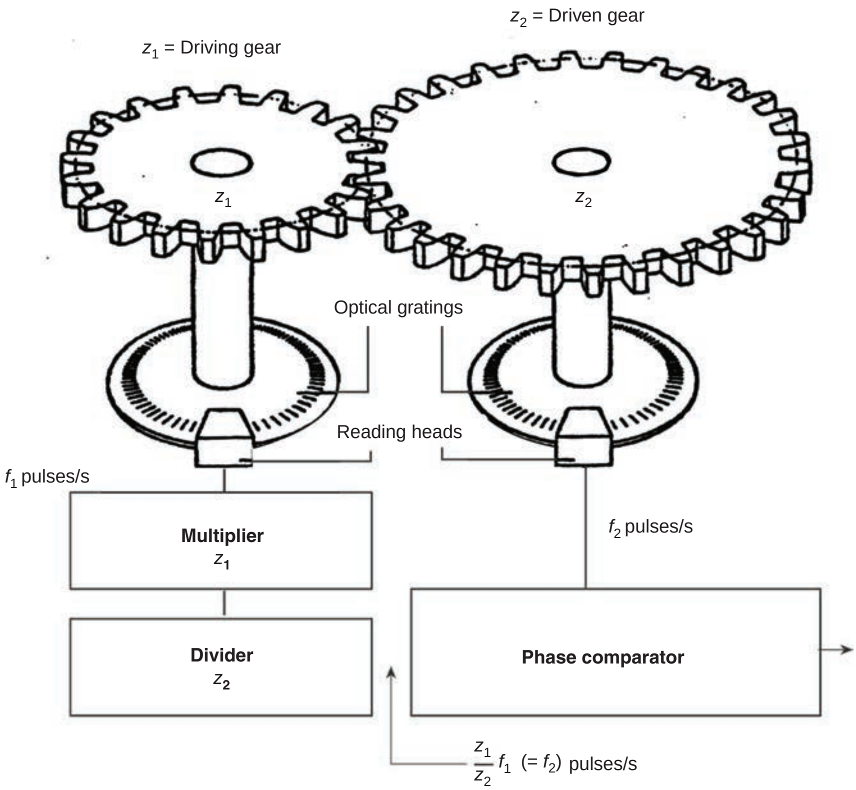


Fig. 1.33 Calculation of transmission error

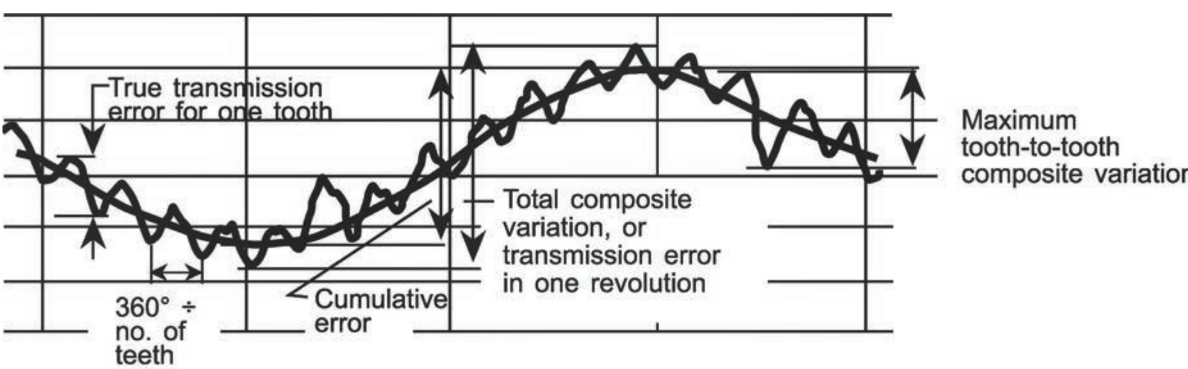


Fig. 1.34 Single-flank composite check

Splines

Splines are used to transmit torque in epicyclic gearboxes. They are similar to gears except for the rolling motion. To minimize vibration, the axis of the spline must be concentric with those of the components they connect. Splines with an involute profile help achieve concentricity and are thus often used.

In general, an involute spline in a high-speed gearbox may be of AGMA quality class 7 or lower. Typical tolerances for involute splines are illustrated in Fig. 1.35. The profile shows some deviation on the "+" side (excess material). Conjugacy between mating splines is not required, however, and thus this type of error does not play a role in controlling vibration.

Tooth Index Variation Form Error

Tooth index variation form error is observed in thin-rimmed internal ring gears used in epicyclic gearboxes. It is due to specific gear blank designs and contributes to low-frequency vibration at high speeds.

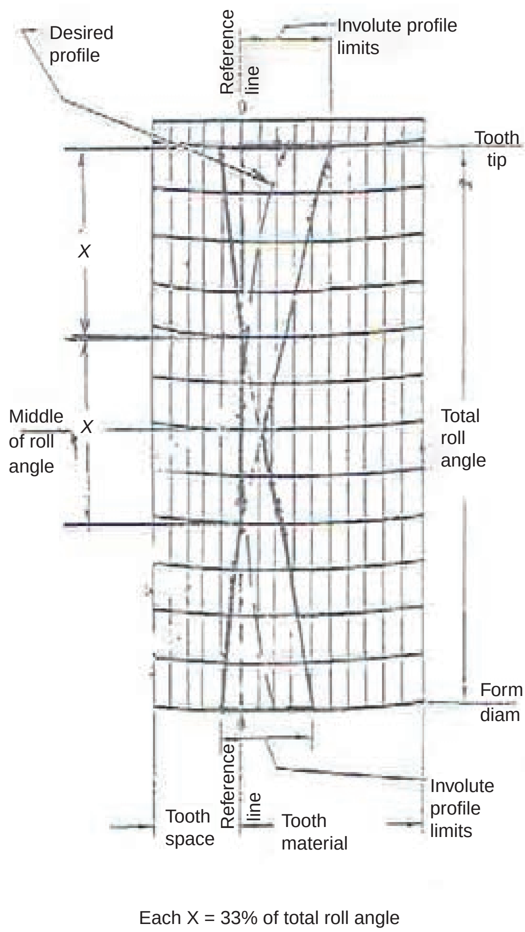


Fig. 1.35 Involute profiles for splines

Index Variation of Teeth

Tooth index variation is the extent to which a gear tooth is out of its true position. Tolerance limits are given in inches of arc length on the pitch circle and are determined by taking measurements through 360° on the pitch circle in both clockwise and counterclockwise directions. The average of these measurements is the index variation of teeth. Taking measurements in both directions eliminates the effect of tooth thickness variation at the pitch circle.

Index Variation Forms

Index variation forms indicate the distribution of index variations along the pitch circle. Index variation is periodic in form characterized by peaks and valleys as illustrated in Fig. 1.36.

Index variation forms, particularly on internal gears, are determined by the kinematics of cutter-machine tool-fixturing systems. Normally, one peak is observed in index variations with equal spacings between the teeth. In the case of a gear with an extremely low ratio of rim thickness to pitch diameter, two or more peaks may appear due to a change in gear-cutting-machine kinematics. Analysis of such form variation and how it affects gearbox vibration is beyond the scope of this book. However, an experimental verification presented in Chapter 3, "Identification of Vibration Sources in a Gearbox" in this book, shows how this error induces vibration in an epicyclic gearbox, which is then transmitted to turbine bearings.

Effect of Rim Thickness on Index Variation Form

In the design of epicyclic gearboxes, the ratio of ring gear rim thickness to pitch diameter is usually kept small for flexibility to facilitate load sharing among planet gears. Such a design, however, is prone to the development of a form error on the index variation of teeth during the gear-cutting operation. If that were to occur, rim thickness would be increased to stabilize the cutting operation.

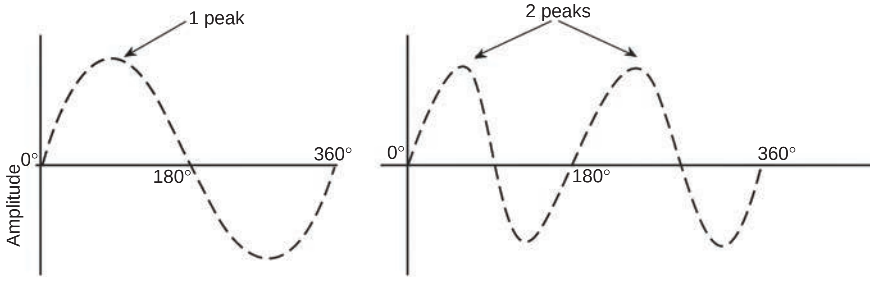


Fig. 1.36 Index variation forms

In external gears, rim thickness is normally high, keeping tooth index variation form error in check, except when gear blanks are designed for reduced weight within the central part of the blank, known as the web.

Balancing of Rotating Components for Vibration Reduction

Rotating components in gearboxes must be precisely balanced to limit vibration at high speeds. Unbalanced gears and shafts, for example, can cause synchronous vibrations that reach extremely high levels. To reduce such vibration, both static and dynamic balancing is performed.

Static balancing is relatively simple and can be done by mounting the part between two centers of a lathe and turning it by hand. Dynamic balancing, on the other hand, requires special equipment. Dynamic balancing procedures improve mass distribution along a shaft or shaft assembly such that it rotates in its bearings with the least amount of centrifugal force. Because centrifugal force varies with speed, a small amount of unbalance may exert significant force at high speeds, causing excessive gearbox vibration.

Unbalance is measured in units of ounce inches (oz in.), gram inches (g in.), or gram millimeters (g mm). Each has a similar interpretation: mass multiplied by its distance from the rotational axis of the shaft equals the radius of the shaft. An unbalance of 100 g in., for example, indicates that the shaft has an excess mass equivalent of 10 g at 10 in. radius, or 20 g at 5 in. radius.

Whether or not a small amount of shaft unbalance should be removed depends on shaft speed as well as the type of gearbox. For example, in a low-speed gearbox, centrifugal force due to unbalance is usually too small to cause gearbox failure. In contrast, a small amount of unbalance in a shaft in a high-speed gearbox can create a large centrifugal force, producing vibration levels that can shorten gearbox life.

In 1940, the International Organization for Standardization (ISO) established standards for both low- and high-speed shafts. These standards are widely used in the gear industry. Because the speed of the rotor and its unbalanced mass cause centrifugal force that excites vibrations, gearboxes are classified based on speed.

For shaft speeds up to 1000 rpm:

$$U_{pp} = \frac{4000W}{N} \quad \text{(Eq 1.5)}$$

For rotor speeds above 1000 rpm:

$$U_{pp} = \frac{4W}{N} \quad \text{(Eq 1.6)}$$

where U_{pp} is the permissible residual unbalance in ounce inches for each correction plane, W is the rotor weight in pounds, and N is the maximum service speed in revolutions per minute.

Equation 1.5 is applied to all low-speed rotors used in industrial gearboxes. Equation 1.6, which applies to high-speed shafts, was developed for balancing components in gearboxes used in turbomachinery. For extremely high-speed shafts (1525 m/min, or 5000 ft/min, and higher), Eq 1.6 is modified to:

$$U_{\mathrm{pp}} = \frac{3W}{N} \qquad \text{(Eq 1.7)}$$

The frequency of vibration for any unbalanced centrifugal force is once per revolution and can be easily identified in a vibration spectrum.

Influence of Critical Speed on Gearbox Vibrations

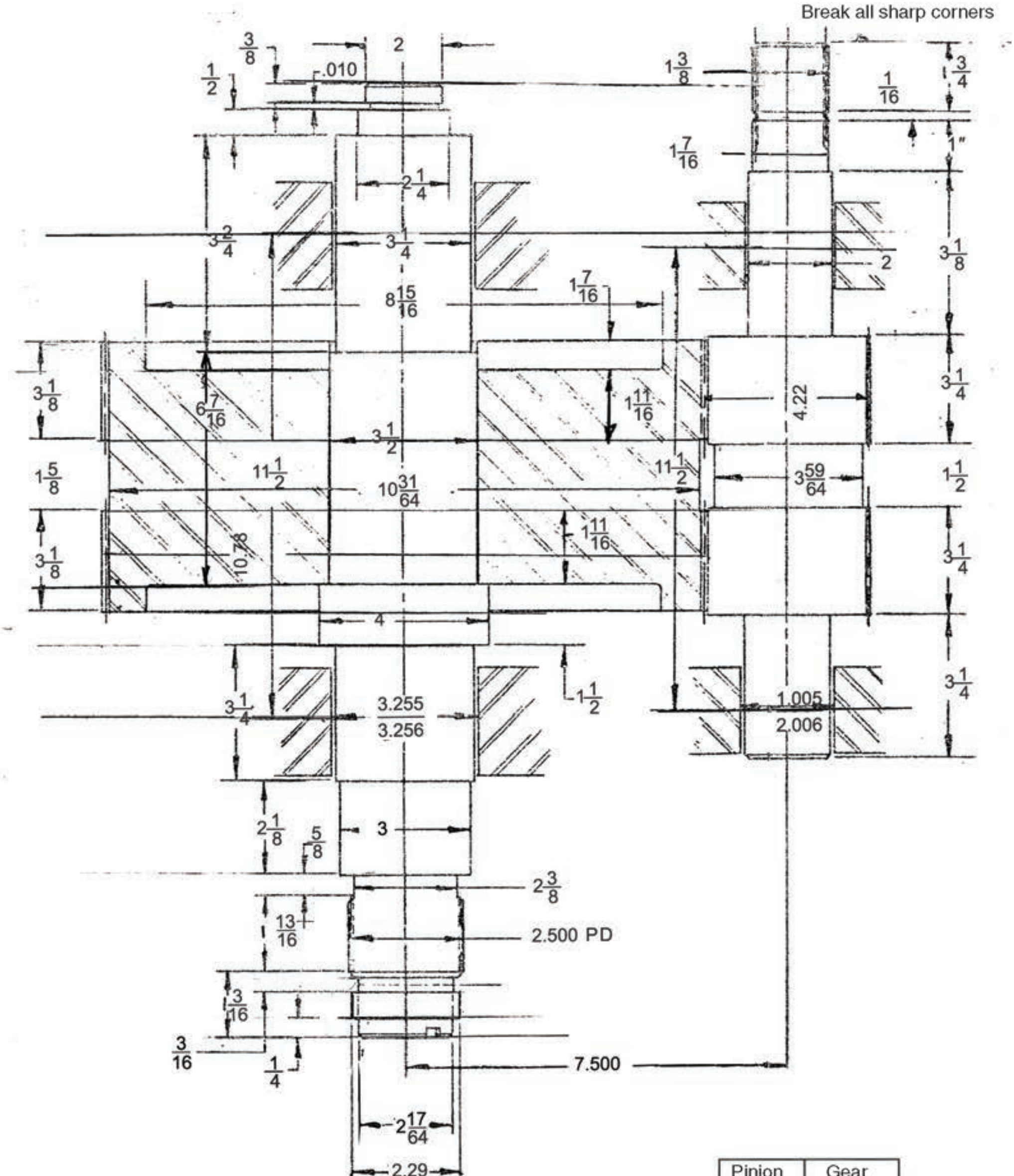
Gearbox shafts are designed to ensure their natural frequencies do not cause resonance with the shaft critical speed. At critical speed, the rotating shaft whirls within the bearing. Whirling of the shaft may cause the bearings to fail. Calculating shaft whirling speed requires an understanding of the unsymmetrical stiffnesses of the shaft and bearing as well as hysteresis damping, gyroscopic forces, and oil friction in the case of sleeve bearings. Whirling may occur in or opposite to the direction of shaft rotation. Whirling speed may or may not be equal to the rotational speed.

To avoid critical speeds, there should be at least a 10% separation between the rotational frequency of the shaft and its natural frequency and harmonics. In low-speed industrial gearboxes, the natural frequency of the shaft is usually much higher than its rotational frequency, thus critical speed is not a concern. However, this is not the case with high-speed gearboxes, where shaft stiffness and the stiffness of support bearings may vary. To calculate the natural frequencies and critical speed of a shaft, a roto-dynamic computer program was developed.

Using this program, critical speed calculations were carried for a single-stage offset parallel gearbox designed for a high-speed turbo generator. The reducer has a speed ratio of 2.558 with an input speed of 18,790 rpm. Figure 1.37 shows the various dimensions of the shafts and their mass elastic data. Masses and inertias are added at the shaft end to account for the coupling supported by splines. The bearings used are sleeve type with axial grooves for oil feed. Analytical models of the pinion and gear shafts are given in Fig. 1.38. Undamped critical speeds obtained for the gear shafts are tabulated as:

	Lateral critical speeds, rpm	
Mode	**High-speed pinion**	**Low-speed gear**
1	24,920	22,770
2	31,120	22,370
3	57,100	68,580

The results show that operating speed does not influence the lateral critical speeds of the gears. The pinion mode shape at various critical speeds is given in Fig. 1.39. At the maximum operating conditions, the

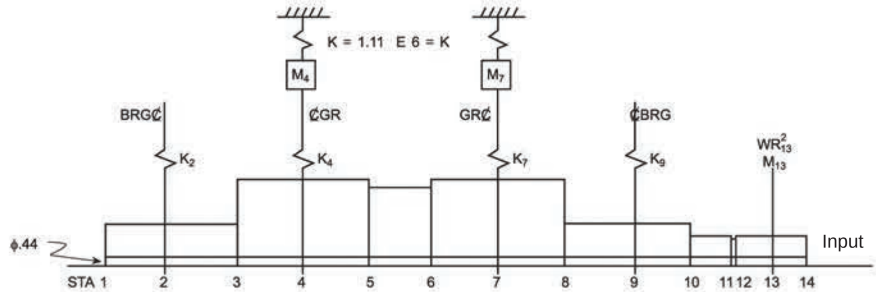


hp	Input, rpm	Output, rpm	Ratio
1400	18,790	7345	2.558

WK² referred to high-speed shaft 3.06 lb-ft²

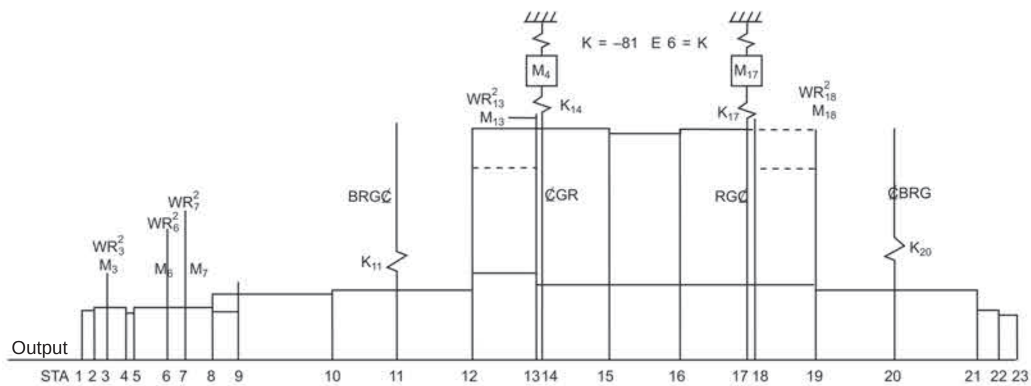
	Pinion	Gear
No.teeth	43	110
Pitch diam (PD)	4.216	10.784
Pitch (Normal	12	
Helix angle	31°47′18″	
Press angle	20°	20°
WK², lb-ft²	49	16.84
W_T	...	185′ lb

Fig. 1.37 Gearbox shaft dimensions and mass elastic data



STA	Distance	Diam. stiff/mass	Stiffness, (K), lb/in.	Mass, (M), lb	WR², lb-in².
1.	0.00	2.00			
2.	1.50	2.00	1.25 E 6		
3.	3.25	4.22			
4	4.875	4.22	2.00 E 6	92.5	
5.	6.500	3.92			
6.	8.000	4.22			
7.	9.625	4.22	2.00 E 6	92.5	
8.	11.250	2.00			
9.	13.000	2.00	1.25 E 6		
10.	14.375	1.44			
11.	15.375	1.25			
12.	15.435	1.44			
13.	16.335	1.44		2.8E	4. 41
14.	17.185	1.44			

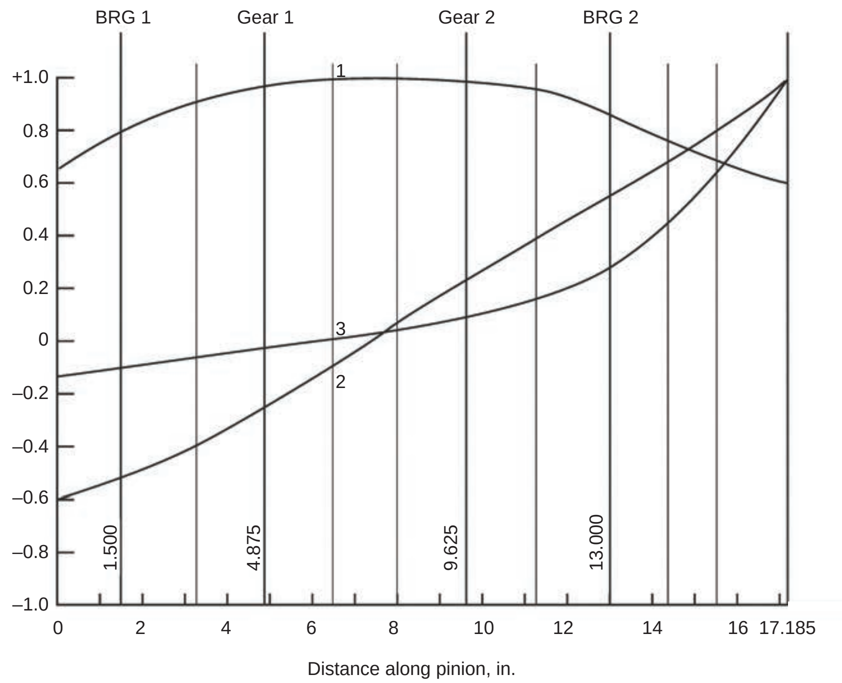
(a)



STA	Distance	Diam. stiff/mass	Stiffness, (K), lb/in.	Mass, (M), lb	WR², lb-in².
1	0.00	2.26			
2	0.25	2.29			
3	0.58	2.29		-0.3	1.99
4	1.00	2.26			
5	1.19	2.50			
6	1.99	2.50		12.14	75.14
7	2.39	2.50		6.04	20.03
8	3.00	2.38			
9	3.62	3.00			
10	5.75	3.25			
11	7.25	3.25	1.43 E6		
12	9.00	4.00			
13	10.50	3.50/10.78		11.6	284.2
14	10.62	3.50/10.78	2.0 E6	18.5	
15	12.19	3.50/10.47			
16	13.81	3.50/10.78			
17	15.37	3.50/10.78	2.0 E6	18.5	
18	15.50	3.50		11.6	284.2
19	16.94	3.25			
20	18.75	3.25	.43 E6		
21	20.69	2.25			
22	21.19	2.00			
23	21.56	2.00			

(b)

Fig. 1.38 Speed-reducer lateral vibration models; (a) pinion, (b) gear



Station	Stiffness, lb/in.		Critical speed, rpm	
BRG 1	1.25	E6	1	24,970
Gear 1	2.0	E6	2	31,120
Gear 2	2.0	E6	3	57,100
BRG 2	1.25	E6		

Fig. 1.39 Pinion mode shape at critical speeds

separation margins are found to be 112% for the pinion and 254% for the gear, which are indicative of an acceptable gearbox design.

Bearing-Induced Vibrations

Gearboxes use both antifriction rolling elements and fluid-film bearings to support gear-mounted shafts. Rolling elements are either balls, rollers, or taper rollers. Fluid-film bearings are either journal or sleeve type.

Ball bearings are commonly used in low-horsepower gearboxes with gears running at low-to-moderate speeds. Roller bearings, on the other hand, are used in high-horsepower, low-speed applications, with taper roller bearings preferred in gearboxes that experience axial thrust. In short-duration aerospace applications, such as rocket-launching equipment, specially manufactured ball bearings have been used for gear shafts running higher than 10,000 rpm.

Vibrations caused by rolling element bearings are due primarily to dimensional irregularities in the bearing elements. They may also be the result of bearing cage deflection under load and bearing misalignment in the housing. Loose bearing cages and races can also contribute to vibration.

Rolling-element-bearing-induced vibrations are the most common cause of low-speed gearbox failure. For minor bearing defects, overall vibration-level changes are virtually undetectable at the early stages of deterioration. Because of their low amplitude, they are often masked by vibrations from other sources in a gearbox. Bearing-induced vibrations are also less dominated by discrete frequencies. Furthermore, with wear of rolling elements, races excite random vibrations that appear similar to noise, making bearing defects even less identifiable on a vibration spectrum.

A practical approach for extending the life of gearboxes with rolling element bearings is regular monitoring of bearing-induced vibrations with high-resolution measuring equipment. It is also advisable to calculate characteristic frequencies due to bearing defects and prepare a chart that can be used to identify and replace faulty bearings. Formulas for such calculations are presented in Chapter 3 in this book.

Fluid-Film-Bearing-Induced Vibrations

High-speed gearbox shafts are generally supported with fluid-film bearings such as babbitted sleeve bearings. These bearings experience inherent instabilities that are not seen with rolling-element bearings. When such instabilities occur with shafts running at or near critical speed, the resulting vibrations can be catastrophic, giving the impression that the failure is due to shaft unbalance.

The basic difference between vibrations due to fluid-film bearing instability and shaft unbalance is that unbalance causes a forced response and occurs at the shaft speed. Bearing instability, on the other hand, causes self-excited vibrations that draw energy into vibratory motion and are relatively independent of the rotational frequency of the shaft. Some high-speed gearboxes designed with multiple tilt-pad sleeve bearings do not experience this type of instability.

Instability due to Oil Whirl and Whip

Deviations from normal hydrodynamic conditions, defined by the attitude angle and eccentricity ratio, are the most common causes of instability in fluid-film-bearing-supported rotors. As shown in Fig. 1.40, the rotor is supported by a thin film of oil. Because of viscous losses in

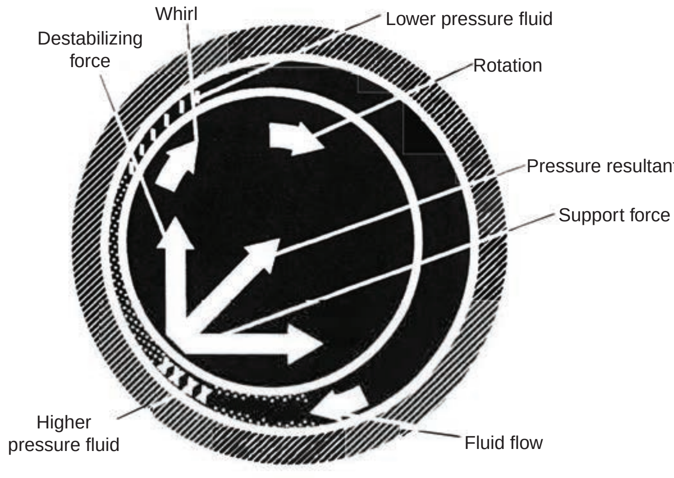


Fig. 1.40 Oil whirl in fluid-film bearing

the oil, the pressure ahead of the point of minimum clearance is lower than behind it. This pressure differential causes a tangential destabilizing force in the direction of rotation that results in a whirl of the rotor at slightly less than half the rotational speed. This type of whirl usually varies from 0.43 to 0.48.

Changes in fluid viscosity due to pressure and external preloads are among the various conditions that can lead to a reduction in the ability of the fluid to support the shaft, causing instability. In some cases, the speed of the shaft can be reduced to eliminate such instability. Sometimes, however, it requires a bearing redesign, such as the use of a pressure dam in sleeve bearings or replacing a conventional sleeve bearing with a tilt-pad type.

Because whirl shows up at approximately half the shaft speed, it causes instability and significant vibration when the shaft reaches twice the critical speed. Whirl must therefore be suppressed if the gearbox is to operate near twice the critical speed.

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