Aspects of joint coordination are preserved during pointing in persons with post-stroke hemiparesis

Darcy S. Reisman 2 and John P. Scholz 1,2

1 Department of Physical Therapy and 2 Biomechanics and Movement Science Graduate Program, University of Delaware, Newark, USA

Correspondence to: J. P. Scholz, Department of Physical Therapy, University of Delaware, Newark, DE 19716 USA
E-mail: jpscholz@udel.edu

Summary

Understanding the fundamental deficits that underlie abnormal reaching movements in persons with hemiparesis is important to the development of rehabilitation approaches for these persons. The purpose of the present study was to investigate whether, and to what extent, persons with hemiparesis retain the ability to exploit motor abundance to coordinate their joint motions to control the hand’s path during reaching—a general feature of joint coordination in age-matched control persons. Eight subjects with mild to moderate right hemiparesis following a stroke and seven age and gender matched control subjects performed pointing movements with each arm individually to targets in the contralateral and ipsilateral workspaces. Ten joint motions and characteristics of hand movement were measured over multiple repetitions. The variance (across trials) of joint combinations was partitioned into two components at every point in the hand’s trajectory; joint variance that led to a consistent hand position and joint variance that led to an inconsistent hand position from trial to trial. All participants were able to limit joint configurations that would have led to trial-to-trial variance of the hand’s path, while using a range of joint configurations consistent with a stable hand path. These results demonstrate that persons with mild to moderate hemiparesis and no measured sensory or perceptual deficits utilized available motor abundance to stabilize performance variables that were important to successful completion of a reaching task, just as was found for age-matched control persons. A principal components analysis of joint angle variance across several points in the movement cycle demonstrated that the persons with hemiparesis showed different patterns of joint couplings during reaching compared with the age-matched control persons; this was particularly evident for the more impaired individuals. These findings were related to the two features of synergy as proposed by other workers. It is suggested that these results support the idea that, while subjects with mild-to-moderate hemiparesis demonstrate differences in the feature of a synergy related to the specific patterns of joint coupling, they retain the feature of a synergy that is related to error compensation.

Keywords: pointing; motor control; coordination; redundancy; stroke

Abbreviations: CL = contralateral; GEV = goal-equivalent variance; IP = ipsilateral; NGEV = non goal-equivalent variance; PC = principal component; PCA = principal components analysis; UCM = uncontrolled manifold

Introduction

Individuals with hemiparesis following a stroke often have difficulty performing a variety of reaching and pointing movements (Levin, 1996; Cirstea and Levin, 2000; Michaelsen et al., 2001; Kamper et al., 2002; Levin et al., 2002). Specifically, it has been found that individuals with hemiparesis demonstrate an increased curvature and variability of the hand’s path and endpoint velocity profiles with multiple peaks, when reaching with the hemiparetic arm (Levin, 1996; Archambault et al., 1999; Cirstea and Levin, 2000; Kamper et al., 2002). Evidence has been provided that difficulty controlling the hand’s movement during reaching is related to abnormal torque production (Dewald et al., 2001) and to disrupted interjoint coordination (Levin, 1996; Cirstea and Levin, 2000; Michaelsen et al., 2001) in the hemiparetic limb. Related to these findings is the long-held belief in the rehabilitation sciences that persons with hemiparesis following a stroke demonstrate abnormal, stereotypic movement synergies that limit their ability to coordinate their joints in flexible and adaptive patterns, thus limiting their ability to perform many functional movements (Brunnstrom, 1970; Bobath, 1990). While there is little experimental evidence to support this view, there is evidence to support the clinical
observation that interjoint coordination appears to be disrupted in persons with hemiparesis (Levin, 1996; Michaelsen et al., 2001). However, these studies have examined only a limited number of the actual joint combinations available during functional reaching, focusing on combinations of two joint motions, such as shoulder horizontal abduction and elbow extension (Levin, 1996; Michaelsen et al., 2001). This work has increased our understanding of how persons with hemiparesis combine joint motions in comparison toagematched control individuals. For example, it has been found that while age-matched control persons utilized primarily elbow joint motion to reach a target in the ipsilateral workspace, persons with hemiparesis utilized both shoulder and elbow motion, suggesting that they had difficulty uncoupling shoulder and elbow joint motion during ipsilateral reaching (Levin, 1996). In addition, persons with hemiparesis demonstrated an increased variability in combining shoulder and elbow joint motions compared with age-matched control persons (Levin, 1996; Cirstea and Levin, 2000). Taken at face value, these results suggest significant alterations in the joint coordination underlying the control of the hand’s motion. On the other hand, control of the hand’s motion in reaching and other similar tasks is achieved by the coordination of up to 10 joint motions, if one considers scapular motion. Thus, a complete understanding of coordination deficits in persons with hemiparesis has been limited by the examination of a restricted number of joint combinations.

Because of the large number of joints and muscles comprising the human arm, an infinite number of solutions to coordinating these ‘degrees-of-freedom’ are possible when reaching from a given initial arm configuration to a given spatial location. On any given performance, however, a particular sequence of joint combinations is used. How the choice of a given sequence is made from the infinite possibilities has been of interest to movement scientists since Bernstein (1967) first articulated the problem. Numerous solutions have been proposed—often but not always in the form of cost constraints—that purportedly solve the ‘problem’ (Hollerbach and Suh, 1987; Uno et al., 1989; Cruse et al., 1993; Feldman and Levin 1993; Rosenbaum et al., 1999). Nonetheless, if a unique solution were chosen based on such constraints, one could expect to observe the same pattern of joint coordination each time for performance under a given set of instructions, initial arm configuration and final target location, and with no change in other conditions of performance. Instead, recent studies (Scholz and Schöner 1999; Scholz, et al., 2000, 2001, 2002) of a variety of motor tasks, including reaching and pointing, have reported that healthy young subjects typically use a range of the available joint combinations to accomplish the task when the initial arm configuration, target location and instructions are controlled (Tseng et al., 2002, 2003). This range of solutions is not random. Rather, the variety of joint combinations observed over repetitions all lead to stable values of important performance variables such as the hand’s path during reaching. Thus, it seems unlikely that the observed use of multiple task-specific coordination solutions can be attributed to mechanical factors alone. Instead, the results likely reflect the operation of a particular style of CNS control expressed through an interaction with the arm’s dynamics. A model of such a control scheme that accounts for basic features of reaching movements and the relevant experimental results has recently been developed based on the concept of a virtual trajectory (e.g. Feldman, 1986) that is implemented as a dynamical system (Martin and Schöner, unpublished data). Thus, we prefer the use of the term ‘abundant’ rather than the typically used term ‘redundant’ to emphasize the positive aspects of having multiple potential solutions in terms of flexibility of control (Gelfand and Latash, 1998).

These recent studies of motor abundance have been based on the uncontrolled manifold (UCM) approach. This approach studies the structure of motor coordination by partitioning the variance (across trials) of the motor elements (e.g. joint combinations) into two components at every point in the hand’s trajectory: (i) variance of the elements that leads to a consistent value of an important performance variable (e.g. hand position); and (ii) variance that leads to variations in the value of the performance variable at each point in the movement from trial to trial. The finding noted above, that subjects typically use motor abundance to repeatedly achieve a given hand position when reaching under identical conditions, may reflect a general property of the neural control of movement (e.g. Scholz et al., 2001; Domkin et al. 2002; Reisman et al., 2002a,b), and reflects the utilization of interjoint variability compensation. In fact, Latash et al. (2003) have suggested that this pattern of error compensation among joints represents an important feature that is commonly ignored in formal definitions of a synergy (Turvey, 1990). Specifically, they suggest that any synergy should be defined by two major features: (i) a sharing pattern among motor elements that is observed, on average, as stable relations among individual elements; and (ii) error compensation among motor elements such that the variability of individual elements is coordinated to minimize the effects of spontaneous fluctuations of these elements on the value of important performance variables (motor abundance).

In the context of joint coordination, the first feature of a synergy is analogous to what is commonly identified to indicate the existence of a synergy, that is, positive covariations among joint motions. In the case of persons with hemiparesis, we often refer to this feature of synergy as a ‘pathological synergy’ because the joint couplings observed in these persons often are limited and inflexible (Brunnstrom, 1970; Bobath, 1990), although it can be argued that they are more appropriately referred to as ‘atypical’, reflecting the patient’s attempt to recruit whatever degrees-of-freedom are available to accomplish the task (Carr and Shepherd, 2000). Differences between age-matched control persons and those with hemiparesis with regard to the second feature of synergy have not been investigated. However, given the existing evidence that persons with hemiparesis may have difficulty uncoupling particular joint motions, it might be expected that
such individuals would show limited evidence of the error compensation component of a synergy during pointing.

The purpose of the present study was to test the hypothesis that the two components of a synergy, as proposed by Latash et al. (2003), could be identified in persons with hemiparesis following stroke and that these individuals would demonstrate differences in both aspects of synergy compared with age-matched control subjects. Two different analytical approaches were utilized to investigate each component of synergy.

We evaluated the pattern of joint couplings involved in this pointing task by performing a principal component analysis (PCA). PCA attempts to identify a smaller set of uncorrelated linear combinations of the original variables such that the set of new variables or principal components (PCs) captures most of the information of the original variables (Dunteman, 1989). For example, the joint angles that contribute to movement of the hand in space can be studied to determine if all joint angles are linked in a single functional synergy or if there is evidence for multiple synergic combinations of the joint motions. Furthermore, by examining how much of the variability of each joint angle is explained by each new variable (PC), information about the coupling between joint angles can be uncovered. For example, if, at a given point in the movement cycle, a large proportion of the variance of shoulder flexion and elbow extension are explained by the same PC, these joint angles can be considered to be coupled at that point in the movement. Through the analysis of the results of the PCA, we expected to find that persons with hemiparesis demonstrate a different pattern of joint coupling compared with age-matched control subjects.

We evaluated the error compensation component of synergy through the UCM approach as described above. Through the analysis of the results from this approach, we expected to find that persons with hemiparesis demonstrate a limited ability to exploit motor abundance during pointing.

Methods

Participants

Eight participants (ages = 60.6 ± 4.7 years) with right hemiparesis following a stroke and eight neurologically healthy individuals (ages = 60.7 ± 4.6 years) were recruited for this study. All subjects gave written consent to participate in the study, which was approved by the Human Subjects Review Committee of the University of Delaware, before participating in the experiments. Participants with hemiparesis sustained their first unilateral stroke involving the left hemisphere and were recruited as a sample of convenience through referrals from local physical therapy clinics and stroke support groups. Healthy control participants were age- and gender-matched, and had no musculoskeletal problems affecting either arm. One age-matched control subject’s data were unavailable due to a technical problem during data collection; thus data from only seven control subjects are presented. All subjects were right-hand dominant. Differences in motor behaviour of the dominant and non-dominant arm have been found in previous studies (Tseng et al., 2002); therefore, only subjects with hemiparesis of their dominant arm (right) were studied in this initial research.

Clinical evaluation

Prior to the main experiment, all subjects with hemiparesis participated in a series of clinical tests. Arm impairment was evaluated by the arm portion of the modified Fugl-Meyer scale (Lindmark and Hamrin, 1988). This test has been shown to be reliable and was chosen because of its greater focus on combining different joint movements in different arm motions and its greater item discrimination than the original Fugl-Meyer scale (Lindmark and Hamrin, 1988). The sensation, proprioception and pain assessment portions of the Fugl-Meyer (Fugl-Meyer et al., 1975) were also completed in order to exclude potential participants with deficits in these areas. Potential participants were also excluded if they were unable to follow simple commands or demonstrated visual or perceptual deficits on the motor-free visual-perceptual test. Finally, potential participants were excluded if they demonstrated a cerebellar lesion on an MRI or CT scan of the brain. Demographic and clinical data are summarized in Table 1.

Pointing task

Participants sat at a table in a high back chair that completely supported the trunk. The chair was pushed snugly into the
movement terminated away from the centre of the target. Participants were instructed to note their degree of error and try to improve the accuracy of subsequent trials. Twenty trials were completed in random blocks of five to each target. The tasks were completed first with the right (hemiparetic) arm, then the targets were repositioned and the same procedure was completed with the left arm.

Data collection
A six camera VICON (Oxford Metrics, Oxford, UK) motion measurement and analysis system was used to record motion at 120 Hz. Rigid, retro-reflective marker arrays, each containing four markers (Fig. 1), were placed atop the subject’s shoulder girdle just medial to the acromioclavicular joint and on the upper and lower arm. Subjects wore a rigid hand splint with four retro-reflective markers attached, one each at the tip and back end of the pointer. The hand splint completely covered the palmar and most of the dorsal surface of the hand, and was shaped so that the fingers were in the standard functional resting position. The different marker arrays were used to track the motion of the scapula, arm, forearm and hand during reaching. During an initial static trial, joint markers were also placed on the sternoclavicular joint, at the shoulder joint (one finger’s breadth below the middle of the acromion process of the clavicle), at the elbow joint on the medial and lateral epicondyles of the humerus, and at the wrist joint on the ulnar and radial styloid processes. These served to locate the joints with respect to the marker arrays. In addition to retro-reflective markers placed on the body, data from markers placed at the starting position and at each target location were captured. These data were used to create a local coordinate system whose x-axis was aligned with the axis from the starting position to the target and whose y- and z-axes were orthogonal (following the right-hand rule). Data rotated into this coordinate system were used to calculate movement onset and offset (see Data reduction section) and test hypotheses regarding control of hand path extent (x-axis) and direction (y- and z-axes).

Data reduction
The markers were identified in each camera view, labelled off line and transformed to their three-dimensional coordinates using the VICON motion system software. The marker coordinates were filtered with a bi-directional, second order Butterworth low-pass filter (5 Hz) in Matlab™ (The Mathworks Inc, Natick, MA, USA). Movement onset (termination) were determined as the time when the velocity profiles of the marker coordinates of the pointer marker exceeded (returned) to 1% of the maximum value from the time of peak velocity. Specifically, an automatic algorithm determined the time of peak velocity of the terminal pointer marker and then searched backward and forward in the velocity trajectory for the first instance when the velocity exceeded (returned) to 1% of the maximum. In this way,
small oscillations at the beginning and end of the movement were not included. All automatic picks were then verified visually by the user and compared with the position trace of the pointer to ensure accuracy. Joint angles were calculated for 3 degrees of freedom at the scapula and shoulder, and for 2 degrees of freedom at the elbow and wrist. Joint angles were obtained using the method outlined by Söderkvist and Wedin (1993). Details are described elsewhere (Scholz et al., 2000; Tseng et al., 2002). The joint angles were normalized for each trial individually to 100% of the movement using a cubic spline interpolation applied by custom software in Matlab™.

Conceptual framework—the uncontrolled manifold

A major purpose of this study was to characterize differences in the features of a motor synergy between age-matched control persons and those with hemiparesis. One important feature of a synergy is reflected in the use of motor abundance in coordinating the joints of the arm to control important performance variables necessary to accomplish the task of pointing to different parts of the workspace. A method has been developed and utilized extensively (Scholz and Schöner, 1999; Scholz et al., 2000, 2001, 2002; Reisman et al., 2002a,b; Tseng et al., 2002) that allows determination of the range of goal-equivalent joint combinations used to provide for a stable sequence of values of different performance variables. The mathematical details of this method are described in the Appendix. In this section, our goal is to provide a conceptual definition of this method and to define terminology that will be used throughout the following sections.

Redundancy and manifolds. Many joint angle combinations could be adopted to achieve a particular value of a performance variable such as the position of the hand. A performance variable is defined as a variable whose control is important for successful completion of the task. Here, control is defined with respect to the concept of stability, or of the persistence of a particular value in the face of phasic perturbations (Schöner, 1995). Consider, for example, keeping your finger on the button of a doorbell while wiggling your arm around. All joint combinations performed during the wiggle are, obviously, consistent with the same position of the finger. An important question is whether, on repeated presses of the button, the same combination of joint angles are used each time or whether a range of the wiggles is used and, if so, how large is that range? This possibility is illustrated with an example in Fig. 2A. In this example, three different combinations of the shoulder, elbow and wrist joint angles (J1,J2,J3) can achieve the terminal position of the hand (H1), assuming we are considering only the control of hand extent.

It is possible to obtain mathematically a linear estimate of all possible joint angle combinations that could be used to achieve a particular value of a performance variable. These combinations lie on a multi-dimensional surface in joint space referred to as a manifold. It is important to note that these manifolds are calculated in joint space and not in Euclidean space. In our example of positioning the hand in a plane, the joint angle combinations J1, J2 and J3 of Fig. 2A lie on a manifold depicted in Fig. 2C, which represents the two-dimensional terminal hand position value H1. In this simple example, the ‘manifold’ is a line in joint space. In contrast, a different terminal position of the hand, such as H2 in Fig. 2B, is achieved by a different set of joint angle combinations (J4 and J5) and is represented by a different manifold in Fig. 2C. Thus, joint angle combinations lying on this second manifold are consistent with the position H2 and inconsistent with the position H1. These two manifolds represent all the possible combinations of joint angles that could achieve a hand position with value H1 and H2. J1–J3 represent three different joint angle combinations that are consistent with the manifold for hand position H1. However, it is important to note that only one joint angle combination that could achieve hand position H1 is actually necessary for calculation of the manifold representing H1 (please see Appendix for details).

If the particular terminal hand position, H1 was necessary to complete the task successfully, then keeping this position stable over multiple task repetitions could be considered a goal of the task of the motor system. Consequently, all joint

![Fig. 2 (A) Three different joint angle configurations that would achieve the same hand position, H1. (B) Two different joint angle configurations that would achieve hand position H2. (C) The uncontrolled manifolds in joint space corresponding to hand positions H1 and H2.](https://academic.oup.com/brain/article-abstract/126/11/2510/403800)
angle combinations used by the subject that achieve this position of the hand (J1, J2 and J3 in our example) could be considered goal-equivalent solutions to joint coordination. Thus, variations of joint angle combinations, across repetitions of the task, that lead to a consistent position of the hand, will be considered goal-equivalent variance (GEV). Note, however, it is theoretically possible to use only one joint combination repeatedly to achieve the same hand position so that the use of goal-equivalent joint combinations is not, a priori, a requirement. Alternatively, observed variations in joint angle combinations across repetitions that lead to variable positions of the hand, e.g. slightly different than H1 (e.g. joint combination J4 and J5 in our example) will be referred to as non goal-equivalent variance (NGEV). Although we have been discussing the partitioning of joint variance at the terminal hand position, this partitioning could be performed for the hand position at any point in the movement cycle, assuming that the person is attempting to meter out a consistent hand trajectory from trial to trial. To the extent that this assumption is false, NGEV will be larger than GEV.

The mathematical procedure used to estimate GEV and NGEV is described in more detail in the Appendix and by Scholz et al. (2000).

Data analysis

Dependent variables

The two components of joint configuration variance, GEV and NGEV, determined according to the procedure outlined above, were used to evaluate the extent to which goal-equivalent patterns of joint coordination were used to achieve stability of performance variables during the reaching tasks. As discussed above, GEV and NGEV are determined with respect to particular performance variables, which for these experiments were: (i) the position of the hand along its path as defined by a vector from the starting position to the target (hand path extent); and (ii) hand movement direction, which is defined as movement in directions orthogonal to hand path extent. Two additional dependent measures were calculated: (i) the actual variance of hand path extent and direction; and (ii) the average resultant error as measured by the resultant distance from the pointer tip to the centre of the target at movement termination.

Independent variables.

The independent variables were: (i) arm used to reach (L or R); (ii) target direction (CL or IP); and (iii) group (stroke or control).

Within-group and between-group comparisons were made with a four-factor mixed design repeated measures analysis of variance ANOVA (group × arm × target × component of joint configuration variance). Similar comparisons were made for performance variable variance in a separate three-factor mixed design repeated measures analysis ANOVA. Because of a lack of homogeneity between groups on some of the dependant variables, a natural log transformation was completed on all data before statistics were computed. A non-parametric test (Mann–Whitney U test) was used when comparing the patient sub-groups on absolute error because of the small sample size (see below).

The statistical tests for joint configuration and performance variable variance were performed on average data corresponding to four selected phases of the reaching movement: (i) the initial phase of movement of the hand, representing 10–25% of the movement trajectory (referred to as ‘early’); (ii) the period just before to just after the hand’s peak velocity, representing 26–45% of the trajectory (referred to as ‘middle’); (iii) the deceleration phase of reaching, representing

| Table 2 Movement times for the right and left arm of all subjects |
|----------------------|------------------|------------------|------------------|
| Subjects with hemiparesis | Arm | Movement time (s), mean ± SD | Age-matched control subjects | Movement time (s), mean ± SD |
|----------------------|------------------|------------------|------------------|
| S1 | Right | 0.7979 ± 0.0745 | C1 | 0.5167 ± 0.0573 |
| | Left | 0.3821 ± 0.0300 | | 0.4221 ± 0.0396 |
| S2 | Right | 0.7167 ± 0.1436 | C2 | 0.6404 ± 0.0787 |
| | Left | 0.8250 ± 0.0910 | | 0.6711 ± 0.1020 |
| S3 | Right | 0.9546 ± 0.0691 | C3 | 0.5969 ± 0.0381 |
| | Left | 0.7333 ± 0.0651 | | 0.6604 ± 0.0608 |
| S4 | Right | 0.8167 ± 0.0651 | C4 | 0.7083 ± 0.0388 |
| | Left | 0.5654 ± 0.0795 | | 0.6579 ± 0.0963 |
| S5 | Right | 1.7864 ± 0.2766 | C5 | 0.8667 ± 0.0647 |
| | Left | 0.9206 ± 0.1110 | | 0.7996 ± 0.0540 |
| S6 | Right | 2.689 ± 0.7300 | C6 | 0.8785 ± 0.0913 |
| | Left | 0.7397 ± 0.0548 | | 0.7579 ± 0.0872 |
| S7 | Right | 0.9596 ± 0.2188 | C7 | 0.6544 ± 0.0338 |
| | Left | 0.7467 ± 0.1326 | | 0.7301 ± 0.0431 |
| S8 | Right | 1.188 ± 0.4095 | | 0.7979 ± 0.0451 |
| | Left | 0.6754 ± 0.1073 | | 0.6544 ± 0.0431 |
70–90% of the movement trajectory (referred to as ‘late’); and (iv) at movement termination (100% of the movement trajectory).

Because we were interested in understanding the influence of the amount of motor impairment on the variables of interest, data for the subjects with hemiparesis were divided into two groups based on the following criteria: (i) a subject’s score on the arm portion (excluding the hand) of the modified Fugl-Meyer; and (ii) on their movement time and movement time variability (see Table 2). Based on both these criteria, four subjects were identified as more impaired and four as less impaired. It is important to note that the magnitude of both a subject’s movement time and modified Fugl-Meyer score were considered in this determination. This became particularly important when a subject’s score on one of these two variables was close to the score of subjects in the other group. For example, while subject S5 (included in the more impaired group) had a modified Fugl-Meyer score that was only one point different than the lowest score of any subject in the less impaired group, her movement time was substantially slower and more variable than the movement time of the subjects in the less impaired group, leading to her inclusion in the more impaired group. When differences on the dependent measures were found for these two groups, the data are presented separately for each group. Because of the small sample size when comparing the groups of more and less impaired subjects to the control group, a non-parametric test (Mann-Whitney U) was used (for comparisons of absolute error, see above).

PCA

We evaluated the preferred relationships between the joint motions involved in this pointing task by performing a PCA. In the present experiments, 10 joint angles that contribute to movement of the hand in space were studied to determine if all joint angles were linked in a single functional synergy or if there was evidence for multiple synergic combinations of the joint motions. For example, one can imagine that there might be separate synergies related to, say, hand transport and to final adjustment of hand position at the target. The PCA was performed across trials for a group of subjects for seven points in the movement cycle. The analysis was performed across trials for a group, rather than an individual subject, because the reliability of PCA depends upon using a large number of trials to perform the analysis and the number of trials available for an individual was limited. As was discussed earlier in the Conceptual framework section, joint configuration variability was partitioned for each point in the movement cycle. In order to be consistent with this analysis, we computed the principal components at a point in time, rather than across time as is often performed (Mah et al., 1994; Alexandrov et al. 1998; St-Onge and Feldman, 2003). The analysis was performed in the following manner and all analyses were performed using Matlab™. First, the covariance matrix of 10 joint angles across all trials was obtained at seven different points spread throughout the movement. Then, the eigenvectors (weighted contributions of each angle to each PC) and eigenvalues (variance of each PC) were obtained. The matrix of eigenvectors was then rescaled [eigenvectors *sqrt(eigenvalues)] to obtain factor loadings (Dunteman, 1989). These loadings provide an indication of the extent to which each of the original joint angles covaries with each of the resulting PCs. The relative size of the weights or loadings of a particular joint angle on a given PC should be interpreted with caution: joint angles with larger excursions (e.g. elbow) will naturally contribute more to the variance than angles with less motion (e.g. wrist). The proportion of variance explained by each eigenvector or PC
was obtained as the eigenvalue for that PC divided by the sum of the eigenvalues for all 10 PCs. We based the choice of how many (new) independent variables were adequate to explain the joint angle variations across trials at a given point in the reaching movement on the criterion that the included PCs should account for 90% of the total variance. In addition, we determined the percent of variance of each individual joint angle that was explained by each of the resulting PCs following their identification based on the 90% criterion.

Results

Movement time

Movement times for reaching with the right and left arms of subjects in both groups are shown in Table 2. Movement times are generally longer for the more impaired persons with hemiparesis compared with the less impaired or age-matched control persons.

Joint motions

Typical angle-angle plots for an age-matched control person and two persons with less and more moderate impairment associated with hemiparesis are presented in Fig. 3A and Fig. 3B, respectively. These figures demonstrate that both persons with hemiparesis show a decrease in the total range of motion utilized at a joint and this was particularly noticeable for elbow extension. The reader is reminded that the distance reached by the subjects was scaled to their functional reach length and, therefore, differed between individuals, particularly between patients and controls. The greatest limitations in joint range of motion were observed for the more impaired subject with
hemiparesis (Fig. 3B). Furthermore, the pattern of coordination between the shoulder and elbow was different for the persons with hemiparesis compared with the age-matched control individuals (Fig. 3).

**Performance variable variance**

The results for the variance of hand path extent (consistency of position along the vector from starting position to target) and direction (consistency of position orthogonal to that vector) are shown in Fig. 4. For persons with right hemiparesis and their age-matched control subjects, the variance of hand path extent in the middle of the movement and at movement termination was greater than the variance of hand path direction, when collapsing across arm used to point and the target location \(F(1,13) = 151.33, P < 0.0001; F(1,13) = 133.73, P < 0.0001\), respectively, Fig. 4. Examining the results across performance variables, when pointing with the right arm, the subjects with hemiparesis demonstrated greater variance of the hand’s path in the middle, late and terminal phases of pointing compared with the age-matched control subjects, while the opposite was true when the subjects pointed with the left arm [arm × group interactions: \(F(1,13) = 7.71, P < 0.05; F(1,13) = 23.92, P < 0.0001; F(1,13) = 7.48, P < 0.05\), respectively; Fig. 4].

**Average absolute pointing error**

The results for average absolute pointing error are presented separately for the two groups of subjects with hemiparesis (less and more impaired, see Methods) because there were differences in absolute error for these two groups when pointing with the hemiparetic limb (Fig. 5). The less impaired group of subjects with hemiparesis demonstrated no significant difference in this measure compared with the age-matched control subjects when pointing to either target with the right (hemiparetic) arm, despite the generally larger absolute error for the less impaired group \([P = 0.109 (CL); P = 0.412 (IP)];\) Fig. 5). In contrast, when the more impaired subjects with hemiparesis pointed to the either target with the right arm, they demonstrated a larger absolute error than the age-matched control subjects \([P < 0.05 (CL), P < 0.01 (IP)];\) Fig. 5). The individual differences in absolute error when pointing with the right arm are presented in Fig. 6. Subject S8, who was one of the most impaired subjects, had the highest absolute error of all subjects, and there was some overlap between the absolute error of our more and less impaired groupings. There were no differences in average absolute error between groups when subjects pointed to either target with the left arm (Fig. 5).
Joint configuration variance
Subjects in both groups used goal-equivalent solutions throughout most of the movement. During all phases of the movement GEV was greater than NGEV, when collapsing across group (stroke or control), target location (CL or IP), arm used to point (right or left), or performance variable analysed (hand path extent or direction) [Early: $F(1,13) = 190.02, P < 0.0001$; Middle: $F(1,13) = 167.1, P < 0.0001$; Late: $F(1,13) = 300.13, P < 0.0001$, $F(1,13) = 337.47, P < 0.0001$; Figs 7 and 8).

The pattern of joint configuration variance did not differ between groups
When testing for differences in the strength of this effect (i.e. GEV > NGEV) between groups for target location, performance variable or arm used to point, no significant differences were found in any of the movement phases [$P = 0.972$ (early); $P = 0.258$ (middle), $P = 0.481$ (late); $P = 0.559$ (termination); Figs 7 and 8]. There were individual differences in the magnitudes of GEV and NGEV. Examples of those differences for the right arm pointing to the ipsilateral target at movement termination are presented in Fig. 9. Note, for example, that patient S5 had much higher overall joint configuration variance than did the other subjects although most of this variance was goal-equivalent. Both S2 and S8 exhibited some of the highest levels of NGEV, even though they fell at separate ends of the impairment continuum.

PCA
The results for the PCA are presented separately for the two groups of subjects with hemipareses (less and more
Fig. 8  Goal-equivalent (GEV, open bars) and non goal-equivalent variance (NGEV, hatched bars) underlying control of hand path extent (A) and (B) and direction (C) and (D) for the left arm pointing to a contralateral (A) and (C) or ipsilateral (B) and (D) target. Results are shown for the early, middle, late and termination (E,M,L,T) portions of the movement, respectively. Results for the persons with hemiparesis are shown on the left of each plot and results for the age-matched control persons are shown on the right. Data represent the average over subjects in each group with error bars representing ±SEM.

Fig. 9  An example of individual differences in goal-equivalent (GEV, open bars) and non goal-equivalent variance (NGEV, hatched bars) underlying control of hand path extent (A) and direction (B) for the right arm pointing to an ipsilateral target. Results are shown for individual subjects for the termination of hand movement. Results for the persons with hemiparesis are shown on the left of each plot and results for the age-matched control persons are shown on the right.
impaired) compared with the age-matched controls. As was mentioned in the Introduction, the results from a PCA can be used to determine if the joint angles involved in the movement are linked in a single functional synergy or if there is evidence for multiple synergic combinations of the joint motions. For example, if only one PC is required to explain a large proportion of the joint angle variance in the group of subjects with hemiparesis, this would indicate that the joint motions are tightly coupled and may indicate limited flexibility in joint couplings, as would be expected if subjects demonstrated ‘pathologic movement synergies’. Furthermore, by examining how much of the variability of each joint angle is explained by each principal component, information about the coupling between particular joint angles can be uncovered. Persons with hemiparesis are reported clinically to produce stereotyped movement patterns, i.e. to have difficulty producing a variety of couplings between the joints. The results of the PCA provide a means to explore such deficits quantitatively.

In the more impaired individuals, only two PCs were typically required to account for 90% of the total joint variance, particularly late in the movement. In contrast, three or more PCs were typically required to explain the same amount of variance in the less impaired and age-matched control individuals (Fig. 10A). This result indicates that the more impaired persons with hemiparesis utilized fewer synergic combinations of joint motions compared with the less impaired subjects with hemiparesis and the age-matched control subjects. The less impaired individuals did differ from the age-matched control persons, however, in that the variance explained was typically spread somewhat more evenly across PCs in the age-matched control persons (Fig. 10B).

The more impaired subjects with hemiparesis demonstrated patterns of joint couplings that were quite different from the less impaired and age-matched control subjects, particularly late in the movement. First, >75% of the variance of shoulder flexion was described by the major PCs for the
less impaired persons, age-matched control subjects. For the more impaired persons, <70% of shoulder flexion variance was described by these PCs (Fig. 11A). This result demonstrates that shoulder flexion was not coupled with other joint motions in the more impaired subjects with hemiparesis, while it was for the less impaired and age-matched control subjects. Secondly, ~80% of the variance of the four distal joint angles (elbow flexion/extension, forearm pronation/supination, wrist flexion/extension and wrist ulnar/radial deviation) was accounted for by the first PC in the more impaired subjects. In contrast, <50% of the variance of these joint angles was accounted for by the first PC for the less impaired and age-matched control subjects (Fig. 11B). This result demonstrates that the more impaired subjects were unable to decouple the distal four joint angles, while this was not the case for the less impaired persons with hemiparesis.

Similar results to those presented above were found when subjects pointed to the ipsilateral target; however, the differences between groups were not as strong as when subjects pointed to the contralateral target.

Discussion

Both persons with hemiparesis and age-matched control persons utilize motor abundance

An important finding of this study was that regardless of movement phase, target location or the arm used to point, persons with hemiparesis demonstrated a relationship between GEV and NGEV, related to the control of both hand path direction and extent, that was similar to the age-matched control persons. Furthermore, the finding that GEV was significantly and fairly consistently higher than NGEV for both groups indicates that all participants were able to limit joint configurations (NGEV) that would have led to inter-trial variance of the hand’s path, while allowing joint configurations that were consistent with a stable hand path. This strategy for utilizing available motor abundance to stabilize performance variables that are important to successful task completion has been a consistent finding in pointing studies performed with healthy young adults (Domkin et al., 2002; Tseng et al., 2002, 2003) and in other arm and postural tasks (Scholz and Schöner, 1999; Scholz et al., 2000, 2001; Reisman et al., 2002a,b). These results from a variety of tasks suggest the existence of a fundamental coordination strategy. As was discussed in the Introduction, Latash et al. (2003) have suggested that the pattern of error compensation among joints (evidenced by GEV) represents an important feature that is commonly ignored in formal definitions of a synergy (Turvey, 1990). Until the current report, differences between age-matched control persons and those with hemiparesis with regard to the error compensation component of a synergy have not been investigated.

The notion of motor abundance is complementary to the notion of motor equivalence is typically used to refer to the ability to achieve a particular goal with the use of multiple effector systems. For example, the ability to write a sentence with the hand on a piece of paper, with the entire arm on a blackboard, or with a pen strapped to the foot (Raibert, 1977). Both Kelso et al. (1984) and Abbs and his collaborators (Cole and Abbs, 1986; Gracco and Abbs, 1986) discussed the results of their work in the context of motor equivalence. Their results may be more properly thought of as examples of motor abundance, however, where abundant degrees of freedom of a given effector system allow for many equivalent combinations of those effectors to achieve the goal of the task. As a result, task success was preserved in the face of unexpected perturbations. For example, perturbation of the lower jaw resulted in adjustments of tongue activity to preserve the /zl/ of the utterance /baez/ (Kelso et al., 1984). In either case, these complementary concepts reflect a motor system with a high level of flexibility.

Persons with hemiparesis demonstrate differences in one feature of synergy compared with age-matched control persons

Utilizing Latash and colleagues’ conceptualization (Latash et al., 2003) of a synergy allows us to characterize the differences in joint coordination observed in persons with hemiparesis compared with age-matched control individuals. The results from the partitioning of joint variance clearly demonstrate that persons with mild to moderate hemiparesis and no perceptual or sensory deficits retain the ability to coordinate their joints during reaching (within arm’s length) to minimize spontaneous changes in the hand’s path. In other words, the error compensation feature of a movement synergy was preserved in this group. In contrast, the results of the PCA suggests that the persons with hemiparesis, particularly the more impaired persons, demonstrated differences with regard to the exact nature of the coupling pattern among the joints when compared with the age-matched control persons (the first feature of a synergy as described by Latash et al., 2003). Several lines of evidence from this study support this conclusion. First, the results of the PCA demonstrated that typically only two PCs were required to account for 90% of the joint angle variance in the more impaired persons, while at least three PCs were required to explain this same amount of variance in the less impaired and age-matched control persons. This result demonstrates that the more impaired persons used fewer synergic joint combinations to accomplish this pointing task. This is consistent with the clinical observation that persons with hemiparesis often demonstrate fewer and less flexible patterns of joint coupling. Secondly, for the more impaired persons, the four most distal joint motions consistently had a large portion of their variance accounted for by PC1, which was not the case for the less impaired and age-matched control persons. This indicates that the more impaired persons were unable to vary the
coupling of these joint motions during reaching and pointing. Once again, this is consistent with clinical observations that persons with hemiparesis often have difficulty isolating joint motions. Taken together, the PCA results confirm the common clinical observation that patients with greater impairments have limited and different patterns of joint couplings compared with the less impaired and age-matched control persons. In accordance with previous conclusions that persons with hemiparesis demonstrate ‘disrupted interjoint coordination’ during reaching (Levin, 1996; Cirstea and Levin, 2000), these results indicate that their joint coordination is different and less flexible in terms of the pattern of joint couplings available, while still retaining a characteristic feature of joint coordination found in age-matched control persons, namely, error compensation between the joints.

The existence of the former deficit does not imply, however, that the limited combinations of joints observed in persons with stroke resulted directly from their brain lesion. As pointed out by Carr and Shepherd (2000), such limitations may reflect instead the patient’s best attempt to organize their available motor elements to achieve some success at the task, given the weakness and inadequate central activation of selective motor elements. Thus, if increases in strength and improvement in the central activation of these selective motor elements can be achieved through practice in functional contexts, the number of synergic components available should increase and lead to greater flexibility of movement organization. This hypothesis requires clinical verification.

There are other deficits found in persons with hemiparesis that may be related to the differences found in the joint coupling patterns in this study. Results from previous studies revealed that persons with hemiparesis demonstrated abnormal coupling of torques at the shoulder and elbow during isometric force production tasks (Dewald and Beer, 2001; Lum et al., 2003). For example, persons with hemiparesis produced abnormal secondary elbow flexion torques during the generation of maximal shoulder abduction torques (Dewald and Beer, 2001). The production of abnormal secondary torques would give rise to joint coupling patterns in persons with hemiparesis that are not typically observed in age-matched control persons. While it is unclear what aspect(s) of the motor system are responsible for such deficits, it has been suggested that they may reflect changes in descending corticospinal input that results in an increased dependence on brainstem pathways. Since the brainstem pathways have diffuse innervation to neurons at multiple spinal segments, it has been suggested that reliance on brainstem pathways may result in increased coactivation (Dewald and Beer, 2001; Lum et al., 2003).

Similarly, it is unknown what element(s) of the motor system are responsible for the error compensation component of a synergy. Unexpected perturbation of the lower jaw of humans during syllabic production tasks results in compensatory activity in articulators remote from the perturbation site with a response latency of ~20 ms so that the syllable is still produced (Kelso et al., 1984). Studies of the wiping reflex in the spinal frog have shown that the frog is capable of accurately wiping the stimulus off its back or forelimb even if movement of one of the joints is unexpectedly blocked or if a heavy load is placed on one of the distal segments (Berkinblit et al., 1986). Finally, a study of reaching involving trunk motion found that when the trunk was unexpectedly restrained, changes in arm inter-joint coordination that preserved the hand’s motion could be detected within 40 ms of trunk arrest (Rossi et al., 2002). These responses are unlikely to be due to transcortical pathways and suggest that the error compensation feature of a synergy may depend only on spinal and brainstem pathways.

In summary, while it is unclear precisely what element(s) of the motor system are responsible for the production of the two components of synergy, they appear to be differentially affected by stroke—at least in mild-to-moderately impaired individuals. Furthermore, the suggestion that joint sharing patterns may be mediated by higher brain centres—while error compensation may be mediated by elements at the spinal and, perhaps, brainstem— is consistent with the finding that the error compensation component is similar in persons with mild to moderate hemiparesis and age-matched control persons, while there are differences in joint sharing patterns between these groups.

Finally, we wish to emphasize that the effects reported here on motor coordination, obtained through an analysis of kinematic variables, are likely to result from emergent properties of the movement dynamics. That is, the measured effects reflect the mechanical response of the limbs and the resulting influence ofafferentation as well as central control signals. Clarification of the independent contribution of central control variables will require methods to uncover their operation in complex movements as has been done in more simplified tasks (Feldman, 1986; Feldman and Levin, 1993). Complementary methods to analyse EMG activity of many participating muscles for evidence of a smaller set of synergic modes, used in conjunction with the UCM approach, may provide a basis for studying this contribution, as was recently done in an analysis of postural control (Krishnamoorthy et al., 2003).

**More impaired persons with hemiparesis demonstrate greater average absolute error when pointing with the hemiparetic arm**

The more impaired persons with hemiparesis demonstrated greater average absolute error compared with the less impaired and age-matched control persons. This finding is consistent with the results of a previous study that found that persons with the greatest clinical impairment of arm function demonstrated the greatest constant error during reaching (Cirstea and Levin, 2000). However, the results of the previous study were limited to reaching movements made in the contralateral workspace and our results extend those findings to movements made in the ipsilateral workspace.
Furthermore, because trunk movement was restricted during the current study and not in the previous study (Cirstea and Levin, 2000), we are able to conclude that the higher errors of the more impaired persons in our study are related to difficulties in the control of arm movements alone and not related to difficulties coordinating arm and trunk motion. We would like to point out that the NGEV component of joint configuration variability, identified via the UCM approach, does not reflect in any way the amount of absolute error for a reaching task. That is, a subject can have low NGEV and a corresponding consistent terminal hand position that, nonetheless, systematically deviates from the target.

**Persons with hemiparesis demonstrate greater hand path variance when pointing with the hemiparetic arm, but not with the less involved arm**

The variance of the hand’s path was greater throughout the reaching movement for the persons with hemiparesis compared with the age-matched control persons when pointing with the hemiparetic (right) arm, while the opposite was true when pointing with the left arm. The increased variance of the hand’s path for the persons with hemiparesis when pointing with the hemiparetic arm is consistent with a previous report (Archambault et al., 1999). However, the increased hand path variance for the left arm of the age-matched control persons compared with the persons with hemiparesis has not been reported previously. This finding could be related to the fact that while all subjects were right-hand dominant, the subjects with hemiparesis reported using their left hand for at least some tasks they had previously completed with the right hand. The greater experience of the left, non-dominant arm in persons with hemiparesis may have positively influenced their ability to limit hand path variance when pointing with this arm. This argument is consistent with the suggestion that the non-dominant arm is less skillful in motor tasks as simple as pointing and that this may be related to the limited experience of the non-dominant arm (Sainburg and Kalakanis, 2000; Tseng et al., 2002). However, this hypothesis requires further testing.

The results for the variance of hand path direction and extent are qualitatively consistent with and support the results found for NGEV. This is important because trial-to-trial fluctuations of the value of performance variables, such as the hand path direction or extent, result only from the component of joint variance that is non goal-equivalent. The relationship between task-variable variance and NGEV, however, is not a simple one. Not only is this relationship non-linear, as evidenced by the geometric model, but the exact relationship varies with the limb geometry throughout the movement path. Thus, a comparison between variance in the joint space and that in task space requires caution, especially with respect to specific variance magnitudes. Nonetheless, while there were no significant differences between the groups with respect to NGEV; more patients had higher NGEV than their matched control subjects and the time series of NGEV across the entire movement period was qualitatively similar to changes of performance variable variance across the phases of the reach for all groups.

**The influence of target distance**

Often, in studies examining reaching in persons with hemiparesis, the target distance is set at a standard distance that is the same for all subjects, regardless of their functional ability (Levin, 1996; Archambault et al., 1999; Kamper et al., 2002; Reinkensmeyer et al., 2002). In this study, the target distance was scaled for individual subjects based on their functional arm length, which reflected their active range of motion. A few important differences result from these two experimental designs. First, with target distance scaled for each subject, all subjects were able to reach the target, i.e. there was a definitive start and endpoint to the reaching movement. Having this definitive start and endpoint helped subjects reproduce a similar hand path trajectory from trial to trial. This was important because we wanted to limit joint variability that was due to hand path variability introduced by the experimental design. Secondly, scaling target distance based on each subject’s functional reaching ability ensures that the task is similarly challenging for each subject, i.e. subjects use a similar range of their available active range of motion during the movement. This was important in order to eliminate the influence of differences in the task difficulty and in the use of available range of motion on the pattern of joint coordination observed.

In this study, trunk motion was restricted and subjects completed the task using only the joints of the scapula and arm. While control of the trunk has been shown to influence the joint coordination of the arm when it is included in the reaching movement of both healthy subjects (Pigeon et al., 2000; Adamovich et al., 2001) and persons with hemiparesis (Archambault et al., 1999; Michaelsen et al., 2001; Levin et al., 2002), we chose to eliminate trunk motion in the current study. This was done to eliminate the influence of problems that persons with hemiparesis often have with trunk control or balance, in order to investigate differences in the coordination of the arm alone. We are currently investigating similar issues in a study of reaching in persons with hemiparesis that includes natural trunk motion to extend and clarify the current results.

**Conclusions**

The results of the present study demonstrated that persons with mild to moderate hemiparesis and no sensory or perceptual deficits utilized available motor abundance to stabilize performance variables that are important to successful completion of a reaching task, just as was found for age-matched control persons. In contrast, the persons with hemiparesis showed different patterns of joint couplings.
during reaching compared with the age-matched control persons; this was particularly evident for the more impaired individuals. These findings were related to the two features of synergy, proposed by Latash et al. (2003). Finally, it was found that while the persons with hemiparesis demonstrated greater hand path variability when reaching with their right (hemiparetic) arm, the age-matched control persons showed greater hand path variability when reaching with their left arm. This finding is probably related to the greater experience of the left arm in persons with hemiparesis compared with the age-matched control persons.

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References
The initial step in estimating GEV and NGEV is to obtain the geometric model relating the performance variable, \( r \), (e.g. the position of the hand) to the joint configuration \( \theta \). This is done using the product of exponentials formula derived by Murray et al. (1994). In our experiment, the state-space configuration for the hypothesis about controlling the position of the CM and hand is composed of 10 angles (three at the scapula and shoulder joints, two at the elbow, flexion-extension and pronation-supination, two at the wrist flexion-extension and abduction-adduction). Small changes in \( r \) are related to changes in \( \theta \) through the Jacobian, which is the matrix of partial derivatives of the performance variable, \( \dot{r} \), with respect to the joint angles, \( \dot{\theta} \). For example, if the performance variable under consideration is the position of the hand, \( \dot{r} \), and \( \dot{\theta} \) is the rotation matrix associated with rotation about the first axis of the first joint contributing to the position of the hand and \( R_\theta \) is the rotation matrix associated with rotation about the last axis of last joint contributing to the position of hand. The variable, \( d_i \), is the distance from the origin of the body referenced coordinate system (sternoclavicular joint marker) to the joint proximal to the hand (wrist) and \( s_h \) is the length from the wrist to the tip of the pointer. \( p_0 \) is the translation vector associated with rotation about the first axis of the first joint contributing to the position of hand (i.e. the first joint proximal to the base frame) and \( p_\theta \) is the translation vector associated with rotation about the last axis of the last joint contributing to the to the position of the hand. The second step is to estimate the linear approximation to the UCM from the geometric model. Because the UCM differs for each value of the performance variable, it is necessary to decide what value to use for the estimation. In reality, both joint configurations and performance variables vary from trial to trial. Based on the assumption that the normalization of movement time has aligned matching states of the underlying joint space across trials, we compute the mean joint configuration, \( \bar{\theta} \), at each percent of the movement. Effectively, the rotation matrix associated with rotation about the first axis of the first joint contributing to the position of the hand and \( R_\theta \) is the rotation matrix associated with rotation about the last axis of last joint contributing to the position of hand. The variable, \( d_i \), is the distance from the origin of the body referenced coordinate system (sternoclavicular joint marker) to the joint proximal to the hand (wrist) and \( s_h \) is the length from the wrist to the tip of the pointer. \( p_0 \) is the translation vector associated with rotation about the first axis of the first joint contributing to the position of hand (i.e. the first joint proximal to the base frame) and \( p_\theta \) is the translation vector associated with rotation about the last axis of the last joint contributing to the to the position of the hand. The second step is to estimate the linear approximation to the UCM from the geometric model. Because the UCM differs for each value of the performance variable, it is necessary to decide what value to use for the estimation. In reality, both joint configurations and performance variables vary from trial to trial. Based on the assumption that the normalization of movement time has aligned matching states of the underlying joint space across trials, we compute the mean joint configuration, \( \bar{\theta} \), at each percent of the movement. Effectively, the

\[
\text{Hand}(r) = R_\theta \cdot R_{\theta_{n-1}} \cdot \ldots \cdot R_{\theta_1}(d_i + s_h) + p_0 + R_\theta \cdot p_{\theta_{n-1}} + \ldots + R_{\theta_1} \cdot p_0
\]

where \( \text{Hand} \) is the three-dimensional position of the hand, \( R_\theta \), is the
value of the performance variable, \( r \), associated with that mean joint configuration is represented by the UCM. Again, continuing with the example for the hand position, the linear approximation to the UCM was obtained from the geometrical model, linearized around the mean joint configuration:

\[
\mathbf{r} = \mathbf{J}(\mathbf{q}) \ast (\mathbf{q}^\ast - \mathbf{q})
\]

Here, \( \mathbf{J} \) is the Jacobian, composed of \( \partial \mathbf{CM}/\partial \mathbf{q}_i \), where \( i = \{ \text{scapular, shoulder, elbow, wrist and joint angles} \} \), obtained at each point in sampled time. The linear approximation of the UCM is then the null-space of the Jacobian (the linear subspace of all deviations from the mean joint configuration that are mapped onto zero by the Jacobian). Using Matlab\textsuperscript{TM} for the numerical computation of the null-space, the actual value of the joint configuration minus the mean joint configuration at each point along the movement path of each trial is decomposed into a component that lies within this null space and a component in its complement. The components of the deviation vector of the joint configuration lying within the UCM and those in its complement are then squared, summed across dimensions of the UCM (i.e. sum of squares) and averaged across all trials, resulting in variance measures. The estimates of variance were then divided by the appropriate number of degrees of freedom. For example, for the hypothesis about controlling the hand position, the joint configuration space is 10-dimensional and the performance variable is three-dimensional. Therefore, the null space has seven dimensions. Thus, variance of the joint configuration within the UCM is divided by seven. The variance perpendicular to the UCM (i.e. the variance that changes the value of the performance variable from its mean value) is divided by three. This normalized variance is reported as variance per degrees of freedom within the UCM (or GEV) and orthogonal to the UCM (NGEV).

**Reference**