Comments on “Analogies of Ocean/Atmosphere Rotating Fluid Dynamics with Gyroscopes: Teaching Opportunities”

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We welcome the interesting recent paper by Haine and Cherian (2013, hereafter HC13), which introduced a physical analogy between the motion of a mechanical gyroscope and two geophysical flow phenomena that occur in the rotating shallow-water (RSW) equations: namely, geostrophic flow and inertial oscillations. HC13 found, for instance, that geostrophy corresponds to gyroscope precession and that inertial oscillations correspond to gyroscope nutation. They also pointed to other examples of physical analogies, including those between the celebrated Lorenz (1963) low-order model (LOM) of thermal convection and phenomena occurring in lasers, the dynamics of a thermohaline loop oscillator, and El Niño–Southern Oscillation (HC13, p. 682). HC13 did not mention, however, that an earlier LOM of barotropic flow (Lorenz 1960) established an analogy between atmospheric dynamics and gyroscopes (Obukhov and Dolzhansky 1975). Besides, the formal correspondence between the theories of fluid mechanics and of rigid-body mechanics, based on Lie algebras, was revealed by Arnol’d (1989). Finally, systems of equations having the form of several coupled such gyroscopes, as models for fluid flow, had been discussed even earlier by A. N. Kolmogorov during a 1958 seminar in Moscow (see Arnol’d 1991) and by A. M. Obukhov and collaborators in a series of papers beginning with Obukhov (1969).

In this Comment we wish to expand the discussion of HC13. Note first that the Lorenz (1963) model was shown (Gluhovsky 1982) to be equivalent to the simplest Volterra gyrostat in a forced, dissipative regime. A gyrostat is a system of bodies whose relative motion does not alter the mass distribution of the system: for example, a gyroscope attached to an axisymmetric rotor; in a Volterra (1899) gyrostat, the rotor spins at a constant angular velocity relative to its “carrier” body (Wittenburg 2008).

Second, the Euler gyroscope in the presence of gravity is described by the Euler–Poisson equations (Leimanis 1965). Geostrophic models thereof were considered by Gluhovsky and Dolzhansky (1980) in a study of convection within a rotating ellipsoid under horizontally nonuniform heating. When the gravity acts along the axis of rotation, the ordinary gyroscope (Euler) equations result. When there is an angle between the axis of rotation and the direction of gravity, a three-model chaotic system with three nonlinear terms was obtained, which in the case of a symmetric ellipsoid proved equivalent to the Lorenz (1963) model.

Third, the use of LOMs has an established tradition in the study of fluid dynamics and atmospheric flows. Extending the analogy between hydrodynamic LOMs and gyroscopes to include LOMs in the form of coupled gyrostats ensures that energy is conserved in the limit of no dissipation and forcing (Gluhovsky and Tong 1999). This fundamental property of the original partial differential equations is not necessarily retained in general LOMs, which may lead to unphysical behaviors (e.g., Thiffeault and Horton 1996; Gluhovsky and Tong 1999). LOMs for RSW systems (Lorenz 1986; Bokhove and Shepherd 1996) can also be presented as coupled gyrostats, as well as LOMs found in other applications, including shell models of turbulence and Hamiltonian LOMs in geophysical fluid dynamics (Gluhovsky and Tong 1999; Gluhovsky et al. 2002; Gluhovsky 2006; Tong and Gluhovsky 2002, 2008).
The analogy has been further extended by Lakshmivarahan and Wang (2008a,b) and Wang and Lakshmivarahan (2009) to include feedback torques existing in some LOMs. Tong (2009) presents a broader discussion of gyrostat analogies in physics. LOMs remain an active area of research in fluid dynamics (e.g., Bihlo and Staufer 2011; Weidauer et al. 2011; Weiss 2011; Gotoda et al. 2013), providing ongoing opportunities for modelers to use coupled gyroscopes and gyrostats to build energy-conserving LOMs of geophysical flows.

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REFERENCES


