HOW I LEARNED TO LOVE NORMAL-MODE ROSSBY–HAURWITZ WAVES

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This paper describes the beginning of the author’s interest in normal-mode Rossby–Haurwitz waves and presents some new evidence of them.

During the 1960s several discoveries were made that were of major importance to tropical meteorology. Matsuno (1966) published his theory of waves on an equatorial $\beta$ plane. Yanai at Tokyo University and Wallace at the University of Washington and their colleagues discovered mixed Rossby–gravity waves (Yanai and Maruyama 1966; Maruyama 1967) and Kelvin waves (Wallace and Kousky 1968) in the stratosphere that were predicted by Matsuno’s theory. In addition, they showed how spectral analysis could be used effectively to detect and to describe these and other synoptic-scale tropospheric waves (e.g., Yanai et al. 1968; Wallace and Chang 1969; Wallace 1971).

With this as a background, members of the Synoptic Meteorology Group at the National Center for Atmospheric Research (NCAR) began work with Line Islands Experiment (LIE) data. The LIE (February–April 1967) took place in the equatorial Pacific and was the first NCAR-directed large field experiment. It was designed to take advantage of cloud data provided by the first equatorial synchronous geophysical satellite, the Applications Technology Satellite-1 (ATS-1), launched in December 1966, and to learn about scale interactions in the equatorial region (Zipser 1969, 1970).

Accompanying LIE rawinsonde data also presented an opportunity to repeat the aforementioned important observational studies for a new time period. Results of spectral analyses of the LIE rawinsonde data, determined by the lag-correlation method, were reported at the Symposium on Tropical Meteorology at the University of Hawaii in 1970 (Madden 1970). Figure 1 is a photograph of the meeting participants. A key identifying all meeting attendees appearing in Fig. 1 is included in the online supplemental material (see https://doi.org/10.1175/BAMS-D-17-0293.2).

Results for the lower-stratosphere $v$-wind spectra were consistent with Yanai and Maruyama (1966) and Maruyama (1967), but tropospheric 4–5-day spectral peaks prevalent during April–July 1962 (Yanai et al. 1968) were largely absent during the
LIE. Wallace and Chang (1969) had shown variations in results like this during a 2-yr period that they had examined. It was clear that a study of time variations in a longer record might yield important information. It is difficult to appreciate now, but to gather and process even a 2-yr time series was a major undertaking in the 1960s.

At the same time, Roy Jenne’s Data Support Section at NCAR was in the process of gathering longer time series. Also, Paul Julian, an expert on interpreting spectra (Julian 1971), and a member of the Synoptic Meteorology Group, had an early fast Fourier transform (FFT) code written at NCAR. The relatively recent publication of an FFT algorithm (Cooley and Tukey 1965; Cooley 1987) was another fortunate circumstance. A Fourier transform of an $N$-member series required $N \times N$ complex multiples before the FFT. The FFT needed only $N \times \log(N)$ complex multiples. For a 10-yr record of daily values the computational reduction was near a factor of 100. That was important because the fastest computer available to meteorologists at the time was the National Center for Atmospheric Research’s (NCAR) Control Data Corporation (CDC) 6600, whose clock speed was only 10 MHz, and in 1971 a CDC 7600 with a clock speed of 36 MHz. Today, a typical laptop clock speed is in the gigahertz range.

Upon our return from the Hawaii meeting, Julian and the author embarked on spectral analyses of NCAR’s newly acquired longer records in order to study nonstationary aspects of the LIE results. Instead, very large variations near a 45-day period that are now often referred to as the Madden–Julian oscillation (MJO) were found (Madden and Julian 1971, 1972a). A cross-spectrum analysis between station pressure at Kanton Island (3°S, 172°W) and upper-air data from only six equatorial stations revealed planetary-scale circulation cells in the equatorial plane and eastward movement to be associated with these variations, further demonstrating the diagnostic power of the spectral approach. A summary of the cross-spectrum results plotted in Fig. 6 of Madden and Julian (1972a) led to the schematic (Fig. 16 in Madden and Julian 1972a) that is still used to summarize the oscillation’s features. More on early explorations of the MJO are contained in Lau and Waliser (2005, chapter 1) and Hand (2015).
Cross-spectrum analysis between station pressures in the equatorial zone also provided evidence of a zonal wavenumber 1, westward-propagating wave with a 5-day period (Madden and Julian 1972b). Pole-to-pole plan views of the observed and theoretical “5-day wave” are presented in Madden and Julian (1973, their Fig. 4) and Dickinson (1968, his Fig. 5a), respectively. We learned that this wave had been previously identified and linked to theoretically predicted normal-mode Rossby-Haurwitz waves (NMRHWs) (Eliasen and Machenhauer 1965, 1969).

What follows is some new evidence for the existence of NMRHWs in observations. First, results of Kasahara (1980) and Kasahara and Puri (1981) are related to provide theoretical expectations for NMRHWs. Then, results of spectral analyses of vorticity fields from a 3-yr record (2010–12) of European Centre for Medium-Range Weather Forecasts interim reanalysis (ERA-Interim) data determined by readily available NCAR Command Language (NCL) routines are presented. Both theoretical predictions and observations of wave periods are shown for comparison.¹

THEORETICAL PREDICTIONS OF NORMAL-MODE ROSSBY–HAURWITZ WAVES. Rossby et al. (1939) isolated the basic dynamics that control an important class of the normal modes: the “waves of the second class,” or, in his words, “planetary waves.” Haurwitz (1940a,b) extended Rossby’s treatment thus the reference “normal-mode Rossby–Haurwitz waves.” The theory of normal modes in the ocean and atmosphere has a long history dating back to the formulation of the Laplace tidal equation in the early nineteenth century. The theory has been reviewed by Holton (1975) and many others. It suffices to say here that atmospheric NMRHWs can be described as external waves (oscillations in phase in the vertical) with a slight amplitude growth in the vertical [see Kasahara and Puri (1981); their Fig. 2 for an equivalent depth of 9,570 m], and sums of associated Legendre polynomials, or Hough functions in the horizontal.

For the largest-scale NMRHWs, the leading terms of Hough sums of associated Legendre polynomials are reasonable approximations to the full sums for the meridional structures (Golitsyn and Dikii 1966). For comparison, Fig. 2 presents the latitude dependence of predicted NMRHW vorticity fields depicted by the appropriate Hough function (solid lines) and that depicted by a single associated Legendre polynomial (dotted

¹ The new work reported here was first prepared for the 2015 Modes Workshop at NCAR (Zagar et al. 2016). Aside from the fact that results are based on a new dataset (ERA-Interim data) and a new variable (vorticity fields), their summary contained in Fig. 6 points to wave modes that have already been identified in references cited in the review articles mentioned in the text. Nevertheless, it is hoped that the remarkable correspondence between observations and theory presented here in one place will be of general interest. The background information was added for the 2016 Ed Zipser Symposium at the 32nd Conference on Hurricanes and Tropical Meteorology in San Juan, Puerto Rico, to show how LIE data collected under Zipser’s direction as Chief Scientist led the author to an interest in NMRHWs.
lines), the approximation used here. The indices $s$ and $n$ are the zonal wave-number and meridional index, respectively. Included here and in Fig. 3 (see also Fig. 6 to follow) are mixed Rossby–gravity waves in the $n = 0$ column, whose dispersion properties for the largest zonal scales are “mixed” between westward inertia–gravity waves and NMRHWs. A figure, based on the Swarztrauber and Kasahara (1985) code, showing the latitudinal dependence of NMRHW zonal wind, meridional wind, and geopotential is included in the online supplemental material (see https://doi.org/10.1175/BAMS-D-17-0293.2).

The largest-scale modes also have discrete predicted frequencies in a global shallow-water model linearized about zonally averaged (symmetric) background winds. Figure 3, adapted from Kasahara (1980), shows expected wave periods in days assuming average December–February (DJF) 500-hPa winds. Kasahara includes results for background winds from other seasons and the predicted wave periods for most modes differ from DJF ones by less than 10%. The largest difference of 17% (11.18 vs 13.55 days) is between DJF and June–August (JJA) for the $s = 4$, $n = 3$ mode.

Baer (1972) has suggested a two-dimensional index $l$ ($l = n + s$) as a measure of horizontal scale. Constant $l$ values lie along diagonals from the bottom left to the top right in Figs. 2 and 3 (as well as in Fig. 6). We will use $l$ as an approximate indicator of what modes are most likely to be discernible in the observations. For example, Fig. 3 suggests we might find NMRHWs with discrete frequencies for $l \leq 6$ or maybe 7 ($s = 4$, $n = 3$ mode).

**Fig. 3.** Predicted periods (days) of the discrete modes with DJF winds found by Kasahara (1980, his Table 1) in a global shallow-water model. Rows are zonal wavenumbers $s$ and columns are meridional indices $n$ of the corresponding Hough functions.

**Fig. 4.** Hodograph of the $s = 1$, $n = 3$ projection (16-day wave) for 19 days during early 2010. Only 0000 UTC observations are plotted. Corresponding Greenwich, 90°W, and 90°E longitudes are indicated. The asymmetric propagation about 0.0 suggests a time-mean projection centered near 90°E.

**OBSERVATIONS.** Early evidence of NMRHWs is summarized in Madden (1979), Ahlquist (1982), and Salby (1984). For an inventory of published evidence from 1982 to 2007, see Table 5 in Madden (2007). More recently, evidence of NMRHWs has been found in a number of variables, for example, in tropical winds (Hendon and Wheeler 2008) and in observations of the very high atmosphere (Sassi et al. 2012). Data used in the present analyses are four-times-per-day (0000, 0600, 1200, and 1800 UTC) vorticity fields.

To approximate the vertical structure of predicted waves, the vorticity data were averaged in the vertical from 850 to 100 hPa. This approximation captures the in-phase nature but not Kasahara and Puri’s (1981) predicted amplitude growth with height. Next,
vertically averaged vorticity fields were projected onto the single associated Legendre polynomials shown by the dotted lines in Fig. 2 (using NCL routine shagC) to approximate the Hough horizontal structures. The 5-day wave is, for example, the \( s = 1, n = 1 \) projection.

Figure 4 shows a hodograph of the behavior of the \( s = 1, n = 3 \) projection, a frequently observed NMRHW also known as the “16-day wave,” for a selected 19-day period. Regular westward propagation is evident. The vertical and horizontal filtering has served to isolate this NMRHW with no need for time filtering. If westward propagation is a regular feature, we expect the cross spectra between the real and imaginary spatial coefficients of the \( s = 1, n = 3 \) projections to show coherence near Kasahara’s corresponding predicted period of 18.39 days and the real should lead the imaginary coefficient by one-quarter cycle. Hayashi’s (1971) “westward variance” reflects these expectations in a single variable.

Westward variance is estimated by \( \frac{1}{4}[S(f) + C(f) + 2Q(f)] \) [Eq. (4-4) in Hayashi (1971)], where \( S(f) \) and \( C(f) \) are the power spectra of the sine and cosine spatial coefficients, respectively, and \( Q(f) \) is the quadrature spectrum between them. Spectra and quadrature spectra were determined by the NCL routine specxy_anal. Twenty-nine periodogram estimates were averaged to provide smooth spectra.

Since the time series are each four observations per day for 3 years there are 4,384 observations and 2,192 periodogram estimates between 0.0 and 2 cycles per day (cpd). The resulting frequency resolution is close to 0.026 cpd, and the number of degrees of freedom is, to first approximation, 58 \((29 \times 2)\). The 5% sampling limit of a null hypothesis of zero coherence is about 0.10 (Julian 1975).

Figure 5 shows the westward variance of the \( s = 1, n = 3 \) projections. Coherence squares between real and imaginary spatial coefficients are also plotted in Fig. 5, along with the predicted period (18.39 days) from Kasahara (1980) (listed in Fig. 3 here). Figure 6 shows results similar to Fig. 5 for all of the modes. In Fig. 6, the predicted frequency for \( s = 5, n = 0 \) is from Longuet-Higgins (1968, his Fig. 6) for a 10-km equivalent depth. Its corresponding period is shown as “3.2 days.” The Longuet–Higgins result is without background winds, but for the four Rossby–gravity \((n = 0)\) waves Kasahara (1980) examined, the largest difference between no-wind and DJF wind cases was only 13% \((s = 4, n = 0 \text{ mode with } 2.56 \text{ days for no wind and } 2.90 \text{ days for DJF winds})\), so we can expect the wind effect on the \( s = 5, n = 0 \) mode to be small.
Because of the results from the already-mentioned modeling and observational studies, it is not surprising that most of the largest-scale modes (in this case \( l \leq 5 \)) show relative maxima westward variance close to the predicted frequencies. Modes with \( l = 5 \) lay along the diagonal from lower left (\( s = 5, n = 0 \)) to upper right (\( s = 1, n = 4 \)). Modes with small relative maxima in westward variance near predicted frequencies (e.g., \( s = 4 \) and \( n = 0, s = 3 \) and \( n = 2 \)) have large relative maxima in coherence there, and phase angles (not shown) with the real leading the imaginary spatial coefficient by one-quarter cycle, consistent with westward propagation. In fact, where coherence is high for \( l \leq 5 \) modes, all corresponding phase angles support westward propagation. On the contrary, for \( l > 5 \) high coherence is accompanied by phase angles with the imaginary leading the real coefficient by one-quarter cycle, consistent with eastward propagation. For three of the Rossby–gravity waves, coherence is near 1.0, implying that nearly all of the variability of one spatial coefficient can be explained by the other.

**DISCUSSION.** Data studied here suggest discrete modes only for \( l \leq 5 \). Kasahara’s (1980) results summarized in Fig. 3 predict the existence of discrete modes for \( l = 6 (s = 2, n = 4; s = 3, n = 3; \text{and } s = 4, n = 2) \) and for \( l = 7 (s = 4, n = 3) \). There have been reports of observed modes with \( l = 6 \) in the literature. The \( l = 6 \) modes (\( s = 2, n = 4 \) and \( s = 4, n = 2 \)) were noted by Ahlquist (1982) and Madden (2007). The \( l = 6 \) mode (\( s = 3, n = 3 \)) was noted by Lindzen et al. (1984). Previously mentioned reviews contain more references concerning these three modes. While we offer \( l = 5 \) as a typical smallest spatial scale that will allow NMRHWs, that limit will vary depending, presumably, on the changing background flow. Further, there can always be ambiguity about identifying traveling disturbances as NMRHWs. For example, the Branstator–Kushnir wave (Branstator 1987; Kushnir 1987) is similar in structure to the \( s = 1, n = 3 \) mode and in period to the \( s = 1, n = 4 \) mode, but Branstator and Held (1995) indicate it is likely an unstable mode rather than neutral NMRHWs. Interestingly, they used a mode tracking method to learn which NMRHWs are most likely to survive in a nondivergent barotropic vorticity model linearized about wavy (asymmetric) November–March, 300-hPa flows. They found that modes likely to maintain a reasonable resemblance to the no-wind NMRHWs all have \( l \leq 5 \) (their Table 2).

It is important to stress that evidences for NMRHWs are not necessarily dependent on reanalysis products. NMRHWs were seen in data that preceded objective analysis (e.g., Kubota and Iida 1954; Deland 1964; and some other references above).

![Fig. 6. As in Fig. 5, but for all projections. Predicted periods (days) from Kasahara (1980) and Fig. 3 here are indicated by the dashed vertical line and the accompanying number. Predicted period for \( s = 5, n = 0 \) from Longuet-Higgins (1968, his Fig. 6) for a 10-km equivalent depth. Highest frequency plotted is 1cpd in column 1, 0.5 cpd in column 2, and 0.25 cpd in columns 3–5. Bandwidth (bw) is indicated (top row). Thin (thick) line is the westward variance (coherence squared).](http://journals.ametsoc.org/bams/article-pdf/100/3/503/4828984/bams-d-17-0293_1.pdf)
More recently, they have been seen in new measuring systems (e.g., Rüfenacht et al. 2016).

The existence of waves similar to those predicted by Kasahara's linear theory is interesting in itself, but beyond an academic interest why should we care? NMRHWs have been identified in observations for more than 60 years, and even the first barotropic forecast was plagued by a poorly handled NMRHW (Charney and Eliassen 1949). Further, it has been argued that they may play a role in large-scale circulation, fluctuating eddy heat transports, blocking, and even motions of Earth’s poles, and, yet, during all this time the notion that NMRHWs may be of practical importance has not taken hold.

The ease with which NMRHWs can now be isolated using readily available software may allow researchers to be more convincing in defining important practical consequences of NMRHWs.

For example, the role that they may play in local weather needs further exploration. Already, there is considerable evidence that, despite its small associated convergence, the 5-day wave affects tropical convection (Burpee 1976; Patel 2001; King et al. 2015, 2017). King et al. (2015) argue that it is not large-scale convergence associated with the 5-day wave but rather the interaction of its associated wind field with local terrain that is the defining influence. Careful studies of other modes may reveal additional important effects on local weather.

**SUMMARY.** Members of the Synoptic Meteorology Section at NCAR were looking at LIE data after Matsuno, Yanai, and colleagues and after Wallace and colleagues had opened doors to the theory and diagnoses of tropical waves. Following their lead, work on the LIE led to analyses of long time series with Julian. This, in turn, led to the MJO and a beginning interest in NMRHWs by the author. Here, expectations of NMRHW were outlined based on the theory and modeling of Kasahara (1980) and Kasahara and Puri (1981). ERA-Interim data were analyzed and the largest scale modes with \( n + s = l \leq 5 \) are identified. We think it is an important advance that routines are readily available to isolate NMRHWs. Here, NCL routines were used to provide rough approximations, but code that provides perfect fidelity to the Kasahara–Puri predictions is now available (Zagar et al. 2015).

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