

POTENTIAL NUMERICAL TECHNIQUES AND CHALLENGES FOR ATMOSPHERIC MODELING

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The Workshop of Applied Mathematics and Computation (AMCA) 2018 follows a tradition of meetings about every two years since 2001 with different names: SRNWP, MOW, and now AMCA (see appendix for acronyms). Presentations covered theoretical work and validation of new numerical schemes with toy models but also practical experience with real applications in nonhydrostatic models, including real data tests with verification. Now data assimilation (DA), along with artificial intelligence, is a subject of the conferences. Currently an important topic is the creation of a new model generation to be used with exaFLOP computers. This implies new demands on scalability, where different numerical approaches support to different degrees. As nonhydrostatic resolutions will become possible on the globe, different grids covering the sphere need to be investigated.

NUMERICAL TECHNIQUES FOR ATMOSPHERIC MODELS. Numerical schemes and the computational grid (mesh) are two important factors for the improvement of numerical weather prediction (NWP) models.

Numerical approaches. Currently, traditional approaches, including the finite difference method (FDM), finite volume method (FVM), spectral method (SM), and finite element method (FEM), are still the main candidates for most NWP codes, such as Eta, GAMIL, GRAPES_YY, LM-z (FDM), MPAS (FVM), SAMIL (SM), and Fluidity (FEM). Some traditional methods are utilized in NWP models: explicitly quadratic conserving schemes

WORKSHOP OF APPLIED MATHEMATICS AND COMPUTATION IN THE ATMOSPHERE 2018

WHAT: Scientists in atmospheric modeling met to discuss the application of new numerical techniques and algorithms that could be used in numerical forecasting, climate modeling, and research into numerical modeling of the atmosphere.

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with second-order accuracy in GAMIL, semi-implicit semi-Lagrangian scheme for advection with additional arrangement for cross-boundary transport, and fixing the integrated surface pressure to achieve global mass conservation in GRAPES_YY.

Another numerical approach is the spectral element method (SEM) used in the HOMME and NUMA models. This is the only local Galerkin method (LGM) presently close to operational application. The presentations by Steppeler (Climate Service Center) and J. Li [Institute of Atmospheric Physics (IAP), Chinese Academy of Sciences (CAS)] showed new ways to construct a family of continuous Galerkin methods, from which two members, o2o3 and o3o3, are investigated. *onom* methods may be considered as generalizations of the concept of SEM where *n* and *m* are the order of polynomials representing the fields and the fluxes. *onom* allows the use of regular collocation grids, while the classical SEM schemes, based on quadrature approximation, require

the irregular Gauss–Lobatto grid. This property is considered important for the construction of interfaces to physical parameterizations. An important property of LGMs using second- or third-degree polynomials is the grid sparseness, which is currently not used with SEM. However, sparseness was standard for square grids with the older FEMs. In 3D, sparseness has the potential to save a considerable amount of computer time. In their lectures, J. Li and Steppeler presented advection results for hexagonal grids in a 2D plane. The projected savings in 3D from sparseness, the physics interface, and time stepping can reach a factor of 30. Apart from different orders of the basis functions, different continuity properties, such as continuity of derivatives, potentially lead to schemes with interesting properties.

Horizontal grid (mesh). For modeling the global atmosphere, the latitude–longitude grid is a classical choice for the horizontal grid on the sphere. Dong (IAP, CAS) provided a review of studies related to stability at the poles, which is still an inevitable problem and has currently been tackled by several numerical treatments (such as the weighted equi-area coordinate, dissipation, and more elaborate Coriolis

force scheme). In numerical modeling of flows around the pole, the time step can be enlarged to a practical value. Due to the pole problem and inefficiency of massively parallel computing for a latitude–longitude grid, the quasi-uniform grids—such as the Yin–Yang (GRAPES_YY), the icosahedral (ICON), and centroidal Voronoi (MPAS) grids—are becoming attractive. Peng (Chinese Academy of Meteorological Sciences) demonstrated that the Yin–Yang grid is capable of avoiding the strict limitation of Courant–Friedrichs–Lewy condition over high latitudes in a latitude–longitude system and less grid points, and longer time steps make the model more efficient than the original latitude–longitude grid model of GRAPES_YY. Even though the new model focuses on the medium-range weather forecast, numerical results of the long-term run confirm the capability of intraseasonal simulation. However, the Yin–Yang grid as well as other polyhedral grids may face the problem of grid imprinting caused by inhomogeneous order of approximation and the nonuniformity (nonaligned) of meshes. Skamarock (National Center for Atmospheric Research) presented the MPAS model, based on mainly hexagons in an unstructured grid, which is derived from the icosahedron. It became clear that with few exceptions the cubed sphere or icosahedral grids are the method of choice for new model developments, replacing the SM and the latitude–longitude grids. Classical staggered Arakawa C or B/E grids are still used. Examples were presented in the lectures by Dong (GAMIL), Mesinger (Serbian Academy of Sciences and Arts) (Eta), and Skamarock (MPAS).

Vertical grid (mesh). The candidates for vertical discretization being discussed were the cut-cell grid and terrain-following grid.

Gadian (National Centre for Atmospheric Science, University of Leeds) gave a review of the development of the cut-cell method and Mesinger’s lecture also treated the representation of orography by cut cells. The representation of orography in Mesinger’s Eta models now has changed to use piecewise linear spline mountains. Therefore, the problems of flow separation and lack of convergence are no longer represented in the cut-cell Eta formulation. Mesinger and Steppeler showed the results of improved forecasts using cut cells. Steppeler reported a systematic improvement of root-mean-square error of temperature on 1-day forecasts using 100 cases and a sensitivity study indicating the sensitivity in the 10-day range for the cyclonic scale, which was not present in the 7-day range.

Y. Li (IAP, CAS) presented a comparison of advection errors in the BTF, HTF, and OTF coordinates.

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In Y. Li's work, skewness (for small values) is demonstrated to be a primary criterion on reducing the advection errors among the criteria for grid quality. The angle between the velocity and the vertical layers (θ) has a smaller effect than the grid quality on reducing advection errors. The variation of the advection errors in the OTF coordinate consistently decreases and then increases according to increasing θ if the terrain is steep. However, the time-step restrictions at the peak of the terrain and the lack of resolution in valleys appear to be the most serious flaws in OTF coordinates, which may be tackled by the unstructured reduced OTF grid. Furthermore, the possible effect on gravity waves remains uncertain.

3D anisotropic adaptive unstructured mesh. Anisotropic adaptive unstructured meshes have been widely used in atmospheric transport models. It has the advantage of accounting for the crucially important process of advection and convection, which is responsible for driving biomass-burning plumes and cloud formation. Failure to resolve all relevant scales in model fully or accurately leads to models that are valid only within empirically defined ranges. Pain (Imperial College London) presented a 3D anisotropic adaptive mesh model (Fluidity) based on discontinuous Galerkin method, which has the capability of accurately representing air pollutant transport processes and predicting indoor and outdoor air flows. Zheng (Institute of Urban Environment, CAS) presented a pioneering air pollution model that makes use of advanced anisotropic h - r adaptive mesh and discretization numerical techniques. The model has been developed and applied to simulate pollution released from over 100 coal-fired power plants across 55 cities in China, including Beijing. Existing air quality models typically use static-structured grids combined with a local nesting technique. The advantage of the anisotropic h - r adaptive model is the ability to adapt the mesh according to the evolving pollutant distribution and flow features. Fluidity, on the other hand, included the parameterizations of cloud microphysics, nonlocal planetary boundary layer, and turbulence leading to simulations of advection over steep terrain, convection, the separation of supercells, and the diurnal variation of the planetary boundary layer. The development of Fluidity is investigated, but it is not near operational application. There still exists a need for developing the anisotropic adaptive mesh atmospheric model: 1) effective algorithms for choosing the mesh refinement criteria; 2) high computational efficiency and computational stability for long-term integration; 3) special physical parameterizations and

test cases for the dynamical core of adaptive meshes; and 4) reducing grid imprinting in tetrahedral adaptive meshes.

DATA ASSIMILATION. DA aims to incorporate observational data into numerical models. Tao (IAP, CAS) developed a new assimilation scheme based on the nonlinear forcing singular vector (NFSV) approach where the tendency error is added into the tendency equation. The NFSV-related assimilation was applied into a new El Niño–Southern Oscillation (ENSO) forecast system that significantly improved the ENSO prediction skills. However, there are two main problems in DA: large data dimensional size and complexity. To overcome these issues, recently, reduced-order modeling (ROM) and machine-learning technologies have been introduced to DA. The unique combination of DA, ROM, and machine learning enables us to link predicting modeling with the data and provide rapid real-time predictions. Fang (Imperial College London) demonstrated the capability of machine-learning-based ROM in a 3D simulation of power plant plumes over a large region in China where, in comparison to the high-fidelity model, the central processing unit cost is reduced by a factor of up to five orders of magnitude while reasonable accuracy remains. This work is at the front of data-centric modeling. Furthermore, the introduction of adaptive meshes into DA and ROM can enable the use of high-resolution meshes around the observation points, thus reducing the representativeness errors of observations and improving the accuracy of modeling results. However, this induces a challenge in the implementation of DA and ROM due to the different length size of ensembles/snapshots at different time levels.

TEST CASES OF SHALLOW-WATER EQUATIONS. The test case of Rossby–Haurwitz waves is one for the dynamical core of global general circulation models. The results are from the nondivergent barotropic vorticity equation together with a constant free-surface height and the balance equation including the vorticity, zonal velocity, and free surface height. Shamir and Paldor (Hebrew University of Jerusalem) proposed a new test case based on linearized SWEs (LSWEs), which is similar to the Rossby–Haurwitz one. In this case, the height and horizontal velocity fields are obtained from the fully coupled, temporal third-order SWEs. Dong demonstrated that GAMIL can simulate more than 4 years with the new LSWE test case while only over 100 days in the traditional Rossby–Haurwitz case. Shamir and Paldor also suggested four alternative error measures instead of the

traditional spherical L_2 errors for the reason that the L_2 error measure can be misleading for zonally propagating (periodic) wave solutions over a finite predefined time interval (Shamir and Paldor 2016).

INFORMATICS PROBLEMS. *High performance computing on graphics computer systems.* Warburton (Virginia Polytechnic Institute and State University) reported on his work to implement a number of currently discussed numerical procedures, such as classic fourth-order differencing or SEMs of different orders on graphics computers. A range of time-stepping procedures was explored. As compared to a standard implementation using compilers, a dedicated effort is able to make a program faster by a factor of up to 20. A problem with this approach is that there is no easy way to transport such programs to new computer generations. Therefore, there is a constant danger of losing a large investment of manpower at any time.

High performance computing on ice-sheet model. Leng (Academy of Mathematics and System Science, CAS) developed a 3D full Stokes ice-sheet model based on the parallel adaptive FEM toolbox parallel hierarchical grid with rapid development. According to Pattyn et al. (2008), this full Stokes ice-sheet model is more accurate than the reduced ice-dynamics models and suits the next generation of the Earth system simulation featuring high resolution and peta-scale computation. The model has been employed to simulate the flow dynamics of the Greenland ice sheet where up to 17 million unknowns and 1,024 processors are used in the computation and 55% weak scalability efficiency is achieved. There are still some challenges for ice-sheet modeling—for example, the current numerical scheme in full Stokes model is apparently not as accurate as a first-order reduced model on a low-resolution grid—and therefore more advanced numerical skills need to be introduced to deal with the accuracy problem.

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APPENDIX: ACRONYMS

AMCA	Applied Mathematics and Computation in the Atmosphere 2018
BTF	basic terrain-following
CAS	Chinese Academy of Sciences, China

DA	data assimilation
ENSO	El Niño–Southern Oscillation
FDM	finite difference method
FEM	finite element method
FVM	finite volume method
GAMIL	Grid-Point Atmospheric Model of IAP LASG, China
GRAPES_YY	Global/Regional Assimilation and Prediction Enhanced System on the Yin-Yang grid, China
HOMME	High-Order Method Modeling Environment, United States
HTF	hybrid terrain-following
IAP	Institute of Atmospheric Physics, CAS, China
ICON	Icosahedral Non-Hydrostatic Model, Germany
IUE	Institute of Urban Environment, CAS, China
LASG	State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics
LGM	local Galerkin method
LM	Lokal Modell of Deutscher Wetterdienst, Germany
LSWE	linearized shallow-water equation
MOW	Mathematics of the Weather
MPAS	Model for Prediction Across Scales, United States
NFSV	nonlinear forcing singular vector
NUMA	Non-Hydrostatic Unified Model of the Atmosphere, United States
NWP	numerical weather prediction
OTF	orthogonal terrain-following
ROM	reduced-order modeling
SAMIL	Spectral Atmospheric Model of IAP LASG, China
SEM	spectral element method
SM	spectral method
SRNWP	short-range numerical weather prediction
SWE	shallow-water equation

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