Corrigendum

Higher-dimensional analogs of Châtelet surfaces

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Let $K/k$ be a cyclic Galois extension of fields of degree $n$, and let $P(x) \in k[x]$ be a separable polynomial of degree $dn$ or $dn - 1$. Let $X_0$ be the affine norm hypersurface in $\mathbb{A}^{n+1}_k$ given by

$$N_{K/k}(\vec{z}) = P(x) \neq 0.$$  \hspace{1cm} (1)

In [1, §2], we attempted to construct a smooth proper model $X$ of $X_0$ extending the map $X_0 \to \mathbb{A}^1_k \setminus V(P(x))$ given by $(\vec{z}, x) \mapsto x$ to a map $X \to \mathbb{P}^1_k$. However, [1, Proposition 2.1] is false whenever $n > 2$. In this note, we explain how all statements and proofs of [1] can be rectified using a construction from [2], particularly the following theorem.

**THEOREM 1** [2, Theorem 1]. Let $K/k$ be a cyclic Galois extension of fields of degree $n$, and let $P(x) \in k[x]$ be a separable polynomial of degree $dn$ or $dn - 1$. There exists a smooth proper compactification $X$ of $X_0$, fibered over $\mathbb{P}^1_k = \text{Proj} k[x_0, x_1]$, such that $X \to \mathbb{P}^1_k$ extends the map $X_0 \to \mathbb{A}^1_k$. Furthermore, the generic fiber of $X \to \mathbb{P}^1_k$ is a Severi–Brauer variety, and the degenerate fibers lie over $V(P(x_0/x_1)x_1^{dn})$, and consist of the union of $n$ rational varieties all conjugate under $\text{Gal}(K/k)$.

Replacing the variety $X$ of [1, §2] with the variety $X$ from Theorem 1 immediately rectifies all but one of the statements and proofs of [1, §§3–5] (we return to this exception below). More precisely, the proofs of [1, Proposition 3.1 and Theorem 3.2] depend only on

1. the generic fiber of $X \to \mathbb{P}^1$ being a Severi–Brauer variety and
2. the degenerate fibers of $X \to \mathbb{P}^1$ lying over $V(P(x_0/x_1)x_1^{dn})$ and consisting of the union of $n$ rational varieties, all conjugate under $\text{Gal}(K/k)$.

That these properties hold is exactly the result of Theorem 1. The remaining proofs of statements in [1, §§4–5] rely on [1, Proposition 3.1 and Theorem 3.2] without further mention of the specific geometry of the fibration $X \to \mathbb{P}^1$, except for part of the proof of [1, Theorem 1.3].

To rectify the remaining part of the proof of [1, Theorem 1.3], we must correct the construction of the Châtelet $p$-fold bundle over $\mathbb{P}^1 \times \mathbb{P}^1$ given by $u^p P_\infty(x) + P_0(x)$. To do so, we carry out the same constructions as in [2] over the polynomial rings $k[u, x], k[u^{-1}, x], k[u, x^{-1}]$, and $k[u^{-1}, x^{-1}]$ and glue to construct a bundle over $\mathbb{P}^1 \times \mathbb{P}^1$. More precisely, the proof of [1, Theorem 1.3] requires a smooth compactification of the normic bundle $X_0 \to U \subset \mathbb{A}^2$ given by

$$N_{K/k}(\vec{z}) = u^p P_\infty(x) + P_0(x) \neq 0,$$

where the closure of $V(u^p P_\infty(x) + P_0(x))$ in $\mathbb{P}^1_u \times \mathbb{P}^1_x$ is smooth and of bidegree $(d_1p, d_2p)$ for some positive integers $d_1$ and $d_2$. (In characteristic $p$, we instead consider the normic bundle $X_0$ given by $N_{K/k}(\vec{z}) = u^p P_\infty(x) + u^{p-1} P_\infty(x) + P_0(x) \neq 0$, with the same conditions on the closure of the curve in $\mathbb{P}^1_u \times \mathbb{P}^1_x$.)

Sections 3 and 4 of [2] give a smooth partial compactification

Received 19 November 2014; published online 18 February 2015.

2010 Mathematics Subject Classification 11G35 (primary), 14G05 (secondary).
$X \to \text{Spec } R$ of any normic bundle $X_0 \to D(a) \subset \text{Spec } R$ given by $N_{K/k}(\vec{z}) = a \neq 0$ for any $k$-algebra $R$, as long as $V(a)$ is smooth in $\text{Spec } R$. Thus, we may apply [2, §§3 and 4] to construct relative compactifications over the polynomial rings $k[u, x]$, $k[u^{-1}, x]$, $k[u, x^{-1}]$, and $k[u^{-1}, x^{-1}]$. Then, since the closure of $V(u^pP_\infty(x) + P_0(x))$ (respectively, $V(u^pP_\infty(x) + u^{p-1}P_\infty(x) + P_0(x))$) has bidegree $(d_1p, d_2p)$ for some positive integers $d_1$ and $d_2$, we may glue the relative compactifications as in [2, Lemma 4] to construct a smooth proper model $X \to \mathbb{P}^1 \times \mathbb{P}^1$.

References


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