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Erratum: "A Numerical Scheme for Anisotropic Reactive Nonlinear Viscoelasticity" [ASME J. Biomech Eng., 2023, 145(1), p. 011004; DOI: 10.1115/1.4054983]

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In this erratum, we correct a mistake in a subcomponent of the numerical algorithm proposed in our recent study for modeling anisotropic reactive nonlinear viscoelasticity (doi:10.1115/1.4054983), for the special case where multiple weak bond families may be recruited with loading. This correction overcomes a nonphysical response noted under uni-axial cyclical loading.

1 Introduction

In our recently published study [1], we reported on a numerical scheme for the framework of reactive viscoelasticity initially presented in our prior theoretical report [2]. We also extended that original theoretical framework to incorporate multiple families of weak bonds, which break and reform into a stress-free state under loading, to be optionally recruited with increasing amount of loading (Sec. 2.3 of the paper [1]). We explained that this weak-bond recruitment could be easily incorporated into the existing framework via Eq. (2.31) of the paper. We provided a pseudo-code that incorporated Eq. (2.31) in Algorithm 2 of the paper. Finally, we illustrated the application of weak-bond recruitment in the curve-fitting results of Fig. 9 of the paper, which analyzed the response of the tissue to consecutive ramp-and-hold displacement profiles.

Upon release of the code in the open-source finite element software FEBio [3], we got feedback from a user (see Acknowledgments below) that this algorithm produced nonphysical responses when a tissue is subjected to uni-axial prescribed cyclical displacement with constant amplitude, such as load amplification along the loading direction, instead of the expected load attenuation. Here, we present a correction to Eq. (2.31), and an update to Algorithm 2 and Fig. 9, which overcome this original limitation.

2 Correction

The strain energy density of a reactive viscoelastic material was given in Eq. (2.1) of the paper. To account for weak-bond recruitment, we replace the original Eq. (2.31), which had proposed a modification to the function $f^u(t^v)$ used in the calculation of the bond mass fraction $w^u(t)$ in Eq. (2.2), with a modification of Eq. (2.1),

$$\Psi_r(\mathbf{F}(t)) = \Psi_r^e(\mathbf{F}(t)) + \sum_u w^u(t) F(\Xi(t)) \Psi_0^a(\mathbf{F}^u(t)) \qquad (2.31)$$

where Ξ is the strain measure described in Eq. (2.29) of the paper, and $F(\Xi)$ represents the enhancement in weak bond mass fraction relative to the zero strain state, as illustrated in the example constitutive model of Eq. (2.30). The main difference between this corrected model and the original model is that the new model accounts for the enhancement $F(\Xi(t))$ in the weak bond mass fraction based on the strain measure Ξ evaluated at the current time t. The original formulation employed the maximum strain measure achieved in the strain history of the material, up until the first time-step t^v when weak bonds from generation u started breaking.

This formulation adjusts the availability of weak bonds as the strain varies; in particular, it accommodates cyclical strains.

Algorithm 2. The `Function Update` is called at each iteration of the nonlinear solver until convergence is achieved at the current time t_{n+1}. The `Function` Ψ_r is called when evaluating the mixture free energy density at each iteration of the nonlinear solver. This function serves as a template for the similar calculations of the stress $\boldsymbol{\sigma}$ and elasticity $\mathcal{C}$ as described in Eq. (2.20) of Sec. 2.1.3. When $u = 0$ in this function, $u - 1$ is equivalent to the superscripted $-\infty$ in the text, see Eq. (2.23). These functions depend on those presented in Algorithm 1.

```
Function update
    Let m =number of generations
    If (m = 0) Or (t_{n+1} > t^{m-1})
    If NewGeneration is True
      Push t_{n+1} ≡ t^m onto t^{u+1} stack
      Push U(t_{n+1}) onto U^{u+1} stack
      //Implement Sec. 2.3
      Evaluate Ξ(U(t_{n+1})) using Eq. (2.29)
      Push F(Ξ(U(t_{n+1})))
      onto F^{u+1} stack
      Push f^{m-1}(t^m) =
      ReformingBondMassFraction
      onto f^{u+1} stack
      Call CullGenerations
  Else if t_{n+1} = t^m
    Update previously pushed
    U(t^m), F(Ξ(U(t^m))) and f^{m-1}(t^m)
Function Ψ_r
    Let m =number of generations
    If m = 0 Return Ψ_r = Ψ_r^e(F(t_{n+1})) + Ψ_0^a(F(t_{n+1}))
    Let Ψ_r = Ψ_r^e(F(t_{n+1}))
    For u = 0 to m − 1
      Evaluate w^{u-1}(t_{n+1}) = f^{u-1}(t^u) g(F(t^u), t_{n+1} − t^u)
      Evaluate F^{u-1}(t_{n+1}) = F(t_{n+1}) · U^{-1}(t^{u-1})
      Evaluate Ψ_r += w^{u-1}(t_{n+1}) F(Ξ(U(t_{n+1}))) Ψ_0^a(F^{u-1}(t_{n+1}))
    Return Ψ_r
```

3 Results

The results corresponding to Fig. 9 of the original paper become updated when employing this new formulation as follows.

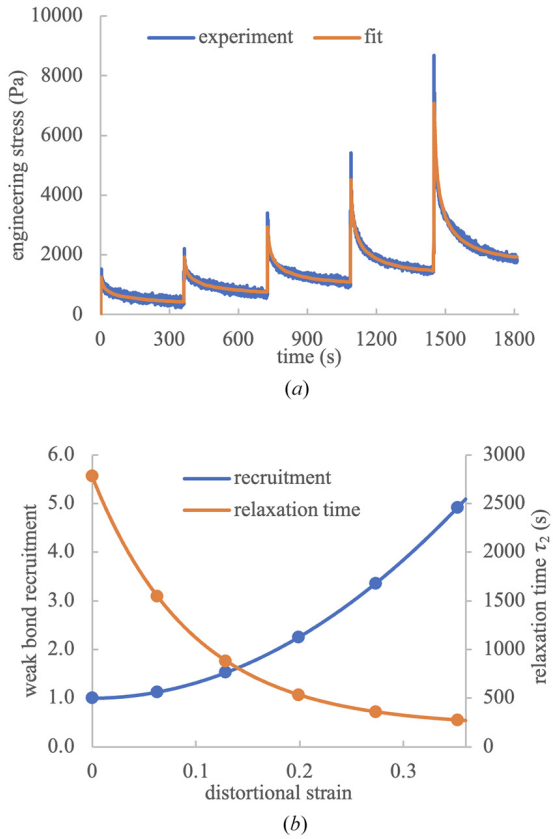

Fig. 9 Curve-fitting of nucleus pulposus unconfined compression stress-relaxation response to data from Ref. [28]. (*a*) Experimental data and reactive viscoelasticity curvefit of the compressive engineering stress, for five consecutive increases of 5% in the compressive engineering strain. (*b*) Dependence of weak bond recruitment F and relaxation time τ_2 on distortional strain K_2. Symbols represent the values of K_2 corresponding to each engineering strain increment in the stress-relaxation response.

We refitted the experimental data to this model to extract the material parameters μ^e and μ^a (shear moduli of the strong and weak bonds); τ_{10}, τ_{20}, τ_{21}, and s_2 for the reduced relaxation function; and μ and Ξ_0 for the recruitment function. This eight-parameter fit produced $\mu^e = 0.40\,\text{kPa}$, $\mu^a = 1.79\,\text{kPa}$, $\tau_{10} = 0.244\,\text{s}$, $\tau_{20} = 205\,\text{s}$, $\tau_{21} = 2575\,\text{s}$, $s_2 = 0.0096$, $\mu = 30.0$, and $\Xi_0 = 0.971$. The nonlinear regression coefficient between the fit and data was $R^2 = 0.991$, and the fit is shown in Fig. 9(*a*). To better understand the strain-dependence of $\tau_2(K_2)$ and $F(K_2)$ for this fit, these functions are shown in Fig. 9(*b*), calculated using τ_{20}, τ_{21}, and s_2 for $\tau_2(K_2)$ and μ, Ξ_0 and α for $F(K_2)$.

The most significant difference we found in the curve-fit of the experimental data of Fig. 9, when compared to the original publication, was the observation of a smoother decrease with increasing strain, in the relaxation time τ_2 of the Malkin-distortion reduced relaxation function (Fig. 9(*b*)).

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Funding Data

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Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

Disclaimer

The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health.

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