Shrinkage estimation of dispersion in Negative Binomial models for RNA-seq experiments with small sample size

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ABSTRACT

Motivation: RNA-seq experiments produce digital counts of reads that are affected by both biological and technical variation. To distinguish the systematic changes in expression between conditions from noise, the counts are frequently modeled by the NegativeBinomial distribution. However, in experiments with small sample size, the per-gene estimates of the dispersion parameter are unreliable.

Method: We propose a simple and effective approach for estimating the dispersions. First, we obtain the initial estimates for each gene using the method of moments. Second, the estimates are regularized, i.e. shrunk towards a common value that minimizes the average squared difference between the initial estimates and the shrinkage estimates. The approach does not require extra modeling assumptions, is easy to compute and is compatible with the exact test of differential expression.

Results: We evaluated the proposed approach using 10 simulated and experimental datasets and compared its performance with that of currently popular packages edgeR, DESeq, baySeq, BBSeq and SAMseq. For these datasets, sSeq performed favorably for experiments with small sample size in sensitivity, specificity and computational time.

Availability: http://www.stat.purdue.edu/~ovitek/Software.html and Bioconductor.

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Supplementary information: Supplementary data are available at Bioinformatics online.

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1 INTRODUCTION

Whole transcriptome shotgun sequencing (RNA-seq) technology (Mardis, 2008; Metzker, 2009; Wang et al., 2009) quantifies gene expression in biological samples in counts of transcript reads mapped to the genes. Accurate and comprehensive, it has made a major impact on genomic research (Garber et al., 2011; Oshlack et al., 2010; Pepke et al., 2009). One common goal of RNA-seq experiments is to detect differentially expressed genes, i.e. genes for which the counts of reads change between conditions more systematically than as expected by random chance (Oshlack et al., 2010). Statistical methods for detecting differentially expressed genes must reflect the experimental design and appropriately account for the stochastic variation.

Moreover, many RNA-seq experiments serve as high-throughput screens of a small number of samples with the goal of subsequent experimental validation. Therefore, the analysis must handle a relatively small number of biological replicates.

A variety of statistical methods and software has recently been proposed for detecting differentially expressed genes. These include DESeq (Anders and Huber, 2010), edgeR (Robinson and Smyth, 2007; Robinson et al., 2010), baySeq (Hardcastle and Kelly, 2010), SAMseq (Li and Tibshirani, 2011), BBSeq(Zhou et al., 2011). We briefly overview these methods. We further propose a direct and effective approach for characterizing the variation in the counts of reads, which improves the sensitivity and specificity of detecting differentially expressed genes for experiments with small sample size. We support this approach with an open-source R-based software package sSeq.

2 BACKGROUND

2.1 The Negative Binomial distribution

The input to the statistical analysis is a set of discrete counts of reads in each experimental run. Although the counts can be modeled with the one-parameter Poisson or Geometric distributions (Auer and Doerge, 2011; Li et al., 2011; Marioni et al., 2008), it is often advantageous to use the two-parameter NegativeBinomial distribution (Anders and Huber, 2010; Hardcastle and Kelly, 2010; Robinson and Smyth, 2007; Robinson et al., 2010). This distribution is more general and flexible and can be viewed as a generalization of both Poisson and Geometric distributions (Supplementary Materials). We focus on the Negative Binomial distribution in what follows.

Denote \( X_{gij} \) as the random variable that expresses the counts of reads mapped to gene \( g \) in sample \( i \) in condition \( j \) (where \( i = 1, \ldots, n_i \); \( j = 1, \ldots, n_j \); \( n_i \) and \( n_j \) are small, e.g. 1–4). We consider the following parametrization:

\[
X_{gij} \sim NB(\mu_{gij}, \phi_g), \quad \text{where } \mu_{gij} \geq 0, \phi_g \geq 0, \text{ such that } \quad E[X_{gij}] = \mu_{gij}, \quad \text{Var}[X_{gij}] = \mu_{gij} + \mu_{gij}^2 \phi_g (\text{denoted } V_g)
\]

The dispersion parameter \( \phi_g \) determines the extent to which the variance \( V_g \) exceeds the expected value \( \mu_{gij} \) (Cameron and Trivedi, 1998; McCullagh and Nelder, 1989).
2.2 Motivation for the proposed approach

The estimation of $\text{Var}(X_{gij})$ is the main focus of this work and is based on the following considerations.

(1) A naïve approach is to estimate $\text{Var}(X_{gij})$ using the method of moments (i.e. the per-gene sample variance). However, it is highly variable in experiments with a small sample size (Croarkin and Tobias, 2006).

(2) RNA-seq experiments simultaneously quantify the expression of many genes. The genes share aspects of biological and technical variation, and therefore a combination of the gene-specific estimates and of consensus estimates can yield better estimates of variation. Such approaches are increasingly popular with RNA-seq experiments (Anders and Huber, 2010; Robinson and Smyth, 2007).

(3) The variance of the Negative Binomial distribution is a known function of the expected value $\mu_g$ and of the dispersion $\phi_g$. Therefore, an accurate estimation of the dispersion (e.g. by combining the gene-specific and consensus estimates, without explicitly modeling its relationship to $\mu_g$) can lead to an accurate estimation of the variance while preserving the mean–variance relationship.

(4) Finally, constraints of throughput sample availability or cost may restrict the number of biological replicates. Although experiments with little or no biological replication have poor reproducibility and are undesirable, such under-replicated screens are the only practical option in some situations (Malo et al., 2006; Markowetz, 2010). They can only detect large changes in expression and require an extensive downstream validation with complementary low-throughput experiments and large sample size. To detect differentially expressed genes, we assume that the majority of the genes are not differentially expressed, and that for these genes, the samples from all conditions can be viewed as biological replicates (Robinson and Oshlack, 2010; Robinson et al., 2010). Under this assumption, a consensus estimate of dispersion helps to improve the accuracy of gene-specific estimates of variation.

Our main concern is in how to (i) accurately define the consensus estimate of dispersion and (ii) accurately combine the gene-specific estimates of dispersion with the consensus estimate.

2.3 Existing approaches for RNA-seq experiments

Among the existing methods, edgeR (Robinson and Smyth, 2007; Robinson et al., 2010), DESeq (Anders and Huber, 2010) and baySeq (Hardcastle and Kelly, 2010) assume the Negative Binomial distribution, and SAMseq (Li and Tibshirani, 2011) and BBSeq (Zhou et al., 2011) use other flexible models. The approaches have been extensively evaluated (Soneson and Delorenzi, 2013) and are broadly used. Hardcastle and Kelly (2010) found that the performance of DESeq, edgeR and baySeq is superior to that of DESeq (Wang et al., 2010), Li and Tibshirani (2011) found that SAMseq improves on PoissonSeq (Li et al., 2011). We briefly overview these approaches in the historical order. Table 1 summarizes the discussion.
Table 1. Existing and proposed approaches for differential analysis of RNA-seq experiments with two conditions

<table>
<thead>
<tr>
<th>Probability model</th>
<th>Estimation of dispersion</th>
<th>Testing</th>
<th>n = 1</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) sSeq (proposed)</td>
<td>$X_{ij} \sim NB(s_{ij}, \mu_i, \phi_i)$</td>
<td>$\hat{\phi}_{ij}^{sSeq} = \xi (1 - \delta \hat{\phi}_i^{MM})$, where $\xi$ is a common dispersion and $\delta$ is a weight</td>
<td>$H_0: \mu_{ij} = \mu_{j*}$</td>
<td>Yes</td>
</tr>
<tr>
<td>(b) edgeR (Robinson and Smyth, 2008)</td>
<td>$X_{ij} \sim NB(m_{ij}^pE_{ij}, \phi_y)$</td>
<td>$\hat{\phi}<em>{ij}^{edgeR} = \frac{N</em>{ij} - \mu_{ij}^pE_{ij}}{\hat{\phi}_{ij}^2}$</td>
<td>$H_0: \mu_{ij} = \mu_{j*}$</td>
<td>Yes</td>
</tr>
<tr>
<td>(c) DESeq (Anders and Huber, 2010)</td>
<td>$X_{ij} \sim NB(s_{ij}, \mu_i, \phi_y)$</td>
<td>$\hat{\phi}<em>{ij}^{DESeq} = \left( V</em>{ij} - \hat{\mu}<em>{ij}^pE</em>{ij} \right) / \hat{\mu}_{ij}^2$</td>
<td>$H_0: \mu_{ij} = \mu_{j*}$</td>
<td>Yes</td>
</tr>
<tr>
<td>(d) baySeq (Hardcastle and Kelly, 2010)</td>
<td>$X_{ij} \sim NB(N_i^pE_i, \phi_y)$</td>
<td>Empirical priors on sets of parameters</td>
<td>$H_0: \mu_{ij} = \mu_{j*}$</td>
<td>Yes</td>
</tr>
<tr>
<td>(e) BBSeq (Zhou et al., 2011)</td>
<td>$X_{ij} \sim Binom(p_i, N_i)$</td>
<td>$p_i^p \sim Beta, logitE_{ij} = ZB_i$, $V(p_i) = E(p_i)(1 - E(p_i))\phi_y$</td>
<td>$H_0: \beta = 0$ Wald test</td>
<td>Yes</td>
</tr>
<tr>
<td>(f) SAMseq (Li and Tibshirani, 2011)</td>
<td>Non-parametric</td>
<td></td>
<td>$H_0$: same distributions A and B</td>
<td>No</td>
</tr>
</tbody>
</table>

(a) $s_{ij}$ is the size factor for sample $j$ in condition $i$ as defined in (Anders and Huber, 2010). $\mu_i$ is the expected normalized expression of gene $g$ for a sample in condition $i$. $\hat{\phi}_i^{MM}$ is the per-gene dispersion estimate using the method of moments in Equation (6).
(b) $m_{ij}$ is the ‘effective’ library size. $p_i^p$ is the probability that a read in $i$ maps to gene $g$. *Up to v2.4.6.
(c) $\phi_y$ is gene- and condition-specific dispersion. $\hat{\mu}_j$ and $\hat{V}_j$ can be estimated by the method of moments or by the Cox-Reid corrected Maximum Likelihood.
(d) $N_i$ is the size of the library $i$ from condition $j$. $V_i$ is as in (d).
(e) $\beta_i$ is as in (b).

Calculating pseudocounts of reads that would have been obtained with libraries of equal size and an iterative computational optimization. For experiments with few replicates, the estimates tend to be discrete values (Lloyd-Smith, 2007; Piegorsch, 1990; Toft et al., 2006). For experiments with many replicates, edgeR specifies a generalized linear model. As conditional likelihoods cannot be easily extended to this case, these are further approximated by adjusted profile likelihoods (McCarthy et al., 2012).

DESeq starts by estimating per-gene means and variances of the normalized counts in each gene and condition by the methods of moments. Next, it re-estimates them by fitting the postulated relationship between the expected values and the variances. The estimates of dispersion can be back-calculated from the estimates of variance as shown in row (c) of Table 1. For experiments without replication, DESeq assumes that the majority of the genes are not differentially expressed, and combines the samples across conditions to estimate the variance. The same strategy is used with the generalized linear models.

The baySeq relies on an iterative estimation of the relative gene expression and of the dispersion. Given an initial partition of the libraries into subsets and an initial estimate of the relative gene expression, it estimates the dispersion using the quasi-likelihood approach. Given the estimates of dispersion, it re-estimates the relative gene expression by maximizing the integrated likelihood. This is repeated for different partitions of the libraries.

BBSeq estimates the dispersion using maximum likelihood for the free model. For the constrained model, it uses the estimates from the free model for all the genes, fits the postulated relationship to the mean and re-estimates the dispersions. SAMseq sidesteps the need to estimate the dispersion by using a fully non-parametric approach.

Testing. For the Negative Binomial model, edgeR tests the null hypothesis $H_0: \mu_{ij} = \mu_{j*}$ and DESeq $H_0: \mu_{ij} = \mu_{j*}$ separately for each gene. Both edgeR and DESeq use the exact test, which is free from asymptotic arguments and is therefore preferred. The test statistic for a gene is the total (normalized) count of reads in all the replicates of a condition. The P-value is the probability of the normalized read counts per group such that under $H_0$ their probability is same or lower than the probability of the observed counts, conditional on the total counts equal to the observed. With the generalized linear models, edgeR and DESeq use the asymptotic likelihood ratio or Wald tests.

baySeq ranks the genes by their posterior probabilities of differential expression. BBSeq tests the coefficient of the linear predictor (i.e. condition) in the generalized linear model with the asymptotic Wald test. SAMseq uses a resampling strategy to estimate the distribution of the test statistic and the P-values.

3 METHODS

The proposed approach combines aspects of the existing approaches, but is simpler, requires fewer assumptions and streamlines the computation. It is summarized in Table 1a. More details regarding the method and its implementation in sSeq are in Supplementary Materials.

Probability model. The model for the counts $X_{ij}$ of gene $g = 1, \ldots, G$, replicate $j = 1, \ldots, n_i$ and condition $i = A, B$ is

$$X_{ij} \sim NB(\mu_{ij}, s_{ij}, \phi_i/s_{ij})$$

such that
\[
E[X_{gj}] = \mu_g s_{ij}, \text{ and } \text{Var}[X_{gj}] = \mu_g s_{ij} + \mu_g^2\theta_g s_{ij}
\]

We follow edgeR, DESeq and baySeq by specifying a Negative Binomial distribution. As in experiments with a small sample size, it may be difficult to distinguish the true dependency of dispersions on expected values from artifacts of random variation, the model specifies free gene-specific dispersion parameters \( \phi_g \). As the initial versions of edgeR, we specify a common dispersion across conditions, i.e. \( \phi_g = \phi_{gb} \equiv \phi \). As a consequence, the counts of differentially expressed genes have different variances in each condition.

We follow DESeq in normalizing the counts by the size factor \( s_{ij} \). However, in the proposed normalization, the size factor affects not only the expected value but also the dispersion. Equation (4) shows that under this assumption the size factor linearly scales both \( E[X_{gj}] \) and \( \text{Var}[X_{gj}] \). Such linear scaling is consistent with the technical variation in RNA-seq experiments, which can be characterized by the Poisson distribution (Marioni et al., 2008). As typical size factors are close to 1, the proposed model has little practical difference from the model in DESeq. However, as shown in Supplementary Section 2, it allows us to directly conduct the exact test and contributes to the accuracy of the results.

**Estimation of dispersion.** Similarly to DESeq, we start by estimating the dispersion parameters by the methods of moments. A conservative estimate of the per-gene variance in experiments with a small sample size is obtained by pooling the samples across conditions, i.e.

\[
\hat{\nu}_g = \frac{\sum_i \sum_j (X_{gij}/\hat{\beta}_g)^2}{\sum_i n_i - 1}, \quad \text{with } \hat{\mu}_g = \frac{\sum_i \sum_j X_{gij}}{\sum_i n_i}
\]

and \( g = 1, \ldots, G \). The estimate of dispersion \( \hat{\phi}^{\text{MM}}_g \) is then calculated from Equation (1), and negative values are truncated at zero.

\[
\hat{\phi}^{\text{MM}}_g = \max\left(0, \frac{\sum_i n_i \hat{\nu}_g - \hat{\mu}_g \sum_i \frac{1}{\hat{\beta}_g}}{\hat{\mu}_g^2 \sum_i \frac{1}{\hat{\beta}_g}}\right)
\]

Unfortunately, in experiments with small sample size, \( \hat{\phi}^{\text{MM}}_g \) is unsatisfactory owing to high variance (Bowman, 1984; Clark and Perry, 1989; Willson et al., 1984). Next, we improve the statistical properties of these estimates by introducing shrinkage.

Stein (1956) showed that when we estimate the expected values of three or more independent Normal random variables with known constant variance, shrinking the per-dimension estimates toward a target value \( \xi \) produces biased estimates, but reduces the mean squared error (MSE) for all choices of \( \xi \). The shrinkage estimator by James and Stein (1961) (Lehmann and Casella, 1998; Richards, 1999) implements this strategy. More recently, Hansen extended the approach of James and Stein with a generalized shrinkage estimator, (Hansen, 2008). Hansen’s shrinkage can produce biased estimates, but reduces the mean squared error (MSE) for numeric stability. It is shown that

\[
\text{ASD}(\xi) = \frac{\text{constant}}{\sum_{g=1}^{G} (\hat{\phi}^{\text{MM}}_g - \xi)^2}
\]

Figure 1a visualizes the functional form of \( \text{ASD}(\xi) \) for a simulation in Section 4 and shows that the tail of the curve flattens for large \( \xi \). Therefore, we can also minimize the bias by minimizing \( \xi \) while enforcing the constraint that \( \text{ASD}(\xi) \) is comparably small. In practice, sSeq estimates \( \hat{\xi} \) by calculating the slope of \( \text{ASD}(\xi) \) and setting

\[
\hat{\xi} = \text{argmin}_{\xi} \left(-\epsilon < \text{slope}(\text{ASD}(\xi)) < 0\right)
\]

for a small constant \( \epsilon \) such as \( \epsilon = 0.05 \). The selected value is shown by the vertical line in Figure 1a.

Figure 1b illustrates the fact that the proposed shrinkage estimator is a linear transformation of \( \hat{\phi}^{\text{MM}}_g \). The slope of the transformation is \( (1 - \delta)/(1 - \xi) \), and the fixed point is the shrinkage target \( \xi \). The shrinkage increases the per-gene estimates of dispersion that are smaller than \( \xi \) and decreases the values that are larger than \( \xi \). From our experience with multiple datasets, \( \xi \) is often around the 95.5th quantile of \( \hat{\phi}^{\text{MM}}_g \). In other words, it biases the majority of the estimates towards conservative values.

The proposed estimate of dispersion has analogies in methods developed for other high-throughput technologies. For example, it is similar in spirit to the moderated variance estimator in the package Limma (Smyth, 2004, 2005), which is also a linear combination of per-gene and consensus estimates.

**Testing.** We follow edgeR and DESeq by testing \( H_0 : \mu_{gij} = \mu_{gb} \) per gene with the exact test. The test statistic is \( X_{gij}^2 \), i.e. the sum of the read counts in each condition. Under \( H_0 \), the \( P \)-values are calculated with respect to the reference distribution \( X_{gij}^2 \sim NB\left(\sum_j s_{ij} \cdot \mu_{ij} \cdot \phi_g/\sum_j s_{ij}\right) \) and are adjusted to control the false discovery rate (FDR) (Benjamini and Hochberg, 1995). See Supplementary Materials for more details.

**Extensions to experiments with complex designs.** The proposed approach can be extended to pairwise comparisons of conditions in experiments with more complex designs without recurring to a generalized linear model and without additional assumptions. See Supplementary Materials for details.

**4 DATASETS**

We evaluated the proposed approach using 10 simulated and experimental datasets. The first five datasets had an external
genes. A subset of the genes from four of the libraries was assayed were sequenced with the Illumina platform, resulting in 19 580 from Stratagene human universal reference RNA. The libraries from Ambion human brain reference RNA against two libraries Quality Control (MAQC) consortium, comparing three libraries in the past (Arikawa et al., 2008; Bullard et al., 2010; Patterson et al., 2006). Griffith et al. (2010) compared fluorouracil (5-FU)-resistant human colorectal cancer cell lines MIP101 against their non-resistant counterpart MIP/5-FU24. One library from each condition was quantified with the paired-end Illumina platform, resulting in 27 145 genes. In all, 197 of these genes from the same samples were assayed by quantitative PCR. We used 12 truly differential genes and 19 truly non-differentially expressed genes as determined by quantitative PCR as the ‘gold standard’ for method comparison.

Brooks et al. (2011) compared untreated cells of Drosophila melanogaster against cells cultured in presence of Pasilla, the homologue of the mammalian Nova-1 and Nova-2 protein. Two biological samples per condition were sequenced with the paired-end Illumina platform, resulting in 14 470 genes.

Sultan et al. (2008) (Frazee et al., 2011) compared two biological replicates of human cell lines Ramos B and HEK293T with the Illumina platform, yielding 6 573 643 uniquely aligned reads.

Bottomly et al. (2011) (Frazee et al., 2011) compared brain tissues of two inbred mouse strains, C57BL/6J (B6) and DBA/2J (D2), using the Illumina platform. The analysis of 10 and 11 biological samples per condition resulted in 13 932 genes.

Hammer et al. (2010) (Frazee et al., 2011) compared gene expression in rat strains Sprague Dawley and L5 SNL Sprague Dawley 2, at two times (2 weeks and 2 months) in a factorial design. Two distinct biological libraries per condition and per time slot were quantified using the Illumina platform, resulting in 18 635 genes.

Tuch et al. (2010) compared the expression of genes in normal human tissues and in tissues with oral squamous cell carcinoma. The experiment compared pairs of normal and tumor samples

\[ s_{ij} = \begin{cases} \text{Uniform(0.5, 1.7)} & \text{if } i = j \\ \text{Uniform(0, 1)} & \text{otherwise} \end{cases} \]

The experiment compared pairs of normal and tumor samples with the ‘gold standard’ of differential expression, and the last two had experimental designs more complex than a two-group comparison. See Supplementary Materials for more details.

Simulation1, Simulation2 and Simulation3 each generated \( G = 20 000 \) genes in conditions \( A \) and \( B \), \( n_A = n_B = 2 \). Thirty percent of the genes were simulated as differentially expressed. Size factors were sampled from the Uniform distribution \( s_{ij} \sim \text{Uniform}(0.5, 1.7) \).

Simulation1 assumed a constant dispersion parameter across all genes and is favorable to sSeq. Simulation2 assumed that \( \phi_0 \) is a non-linear function of \( \mu \), i.e. \( \phi_0 = 1/(100 + \mu) \) and as such is favorable to DESeq. Simulation3 is most realistic. From the dataset by Bottomly et al. (2011) (Frazee et al., 2011), the largest experimental dataset in this manuscript, we selected a subset of non-differentially expressed genes (as determined by a consensus of sSeq, edgeR and DESeq) and sampled pairs \( \{\mu_{\text{MM}}^i, \phi_{\text{MM}}^i\} \) from this subset as the true parameters for simulating read counts.

MAQC (Shi et al., 2006) is the dataset from the MicroArray Quality Control (MAQC) consortium, comparing three libraries from Ambion human brain reference RNA against two libraries from Stratagene human universal reference RNA. The libraries were sequenced with the Illumina platform, resulting in 19 580 genes. A subset of the genes from four of the libraries was assayed by real-time reverse-transcription PCR (Shi et al., 2006; Zhining et al., 2010). We used the 323 differential genes and 85 non-differentially expressed genes determined by real-time reverse-transcription PCR as the ‘gold standard’. Although the dataset only has technical replicates, it has been used extensively as the benchmark in the past (Arikawa et al., 2008; Bullard et al., 2010; Patterson et al., 2006).
from three patients. The six libraries were sequenced using the SOLiD platform, resulting in 10,453 genes.

5 RESULTS

In addition to sSeq, the following versions of the existing packages were used: edgeR v3.0.8 (January 2013), DESeq v1.10.1 (October 2012), baySeq v1.12.0 (October 2012), BBSeq v1.0 (March 2011), SAMSeq as part of the R package samr v2.0 (June 2011).

For sSeq, all the datasets were analyzed with the exact test, and analyses of the Hammer and the Tuch datasets accounted for their experimental designs. For edgeR and DESeq, the datasets with two-group comparisons were analyzed with the exact test, and Hammer and Tuch datasets were analyzed with the glm-based approaches. For edgeR, the estimateCommonDisp function in an older version of edgeR package (v2.4.6) was used to analyze unreplicated datasets. For DESeq, the option fitType=’local’ was used to estimate the per-group variance. The default parameters were used otherwise. See Supplementary Materials for details and representative R scripts.

5.1 sSeq accurately estimates the variation

As the proposed approach shares most similarities with edgeR and DESeq, we compared their estimates of dispersions and variances in more details. Figure 1c, e and g use the simplest case of Simulation1 to illustrate the estimates by the method of moments and by the three approaches. As expected, $\phi_{g}^{MM}$ have a high variance, which increases with the mean. Also as expected, estimates by $\phi_{g}^{Seq}$ are biased towards larger values but have smaller deviations from the true values as compared to $\phi_{g}^{MM}$. Estimates by the other two methods fit the pattern of $\phi_{g}^{MM}$.

Figure 1d, f and h show that despite the differences in dispersion estimation, the estimates of variance by the three methods fit the pattern of $\phi_{g}^{MM}$ and summarized with ROC curves. Table 2 shows that the proposed approach consistently had a similar or a higher accuracy and summarized with ROC curves. Table 2 shows that the proposed approach consistently had a similar or a higher accuracy and summarized with ROC curves. Table 2 shows that the proposed approach consistently had a similar or a higher accuracy and summarized with ROC curves.

Table 2. Areas under the ROC curves of detecting differentially expressed genes for the datasets with an external ‘gold standard’ while varying the FDR-adjusted P-value or posterior probability cutoff

<table>
<thead>
<tr>
<th>Methods</th>
<th>Simulation1</th>
<th>Simulation2</th>
<th>Simulation3</th>
<th>MAQC Project</th>
<th>Griffith et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_{i} = 1$</td>
<td>$n_{i} = 2$</td>
<td>$n_{i} = 1$</td>
<td>$n_{i} = 1$</td>
<td>$n_{i} = 1$</td>
</tr>
<tr>
<td>Proposed</td>
<td>sSeq</td>
<td>0.947</td>
<td>0.962</td>
<td>0.951</td>
<td>0.967</td>
</tr>
<tr>
<td>Existing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>edgeR</td>
<td>0.918</td>
<td>0.948</td>
<td>0.938</td>
<td>0.951</td>
<td></td>
</tr>
<tr>
<td>DESeq</td>
<td>0.932</td>
<td>0.940</td>
<td>0.937</td>
<td>0.949</td>
<td></td>
</tr>
<tr>
<td>baySeq</td>
<td>0.568</td>
<td>0.711</td>
<td>0.548</td>
<td>0.714</td>
<td></td>
</tr>
<tr>
<td>BBSeq</td>
<td>0.675</td>
<td>0.672</td>
<td>0.669</td>
<td>0.674</td>
<td></td>
</tr>
<tr>
<td>SAMseq</td>
<td>0.964</td>
<td>0.968</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sub-columns are subsets of the data with one randomly selected replicate per condition and the full datasets. Values closer to 1 indicate higher sensitivity and specificity.
Second, comparisons of replicates of a same condition ('AvsA') had no differentially expressed genes. Therefore, methods with higher specificity should have ECDF curves at or below the 45 degree line. For baySeq, we expect similar patterns of the ECDF curves based on the posterior probability cutoff. Figure 2 summarizes the curves for the five datasets. It shows that sSeq produced most consistently the expected pattern and had a similar or a higher accuracy as compared with the existing methods.

The effect of sample size and of size factors on the accuracy was investigated using the three simulated datasets. Supplementary Section 4.4 and 4.5 indicate that sSeq is particularly advantageous for experiments when \( n \leq 4 \).

6 DISCUSSION
In this manuscript, we advocated a model that specifies free per-gene dispersion parameters in the Negative Binomial model for counts of RNA-seq reads. We also advocated a biased estimation of these parameters, which can reduce the variance of the estimates and minimize the overall MSE. Biased estimation is different from specifying a probability model (such as in DESeq) that assumes a true systematic relationship of the true variance and the true mean. It is particularly useful for experiments with a small sample size, where the systematic relationship may be difficult to evaluate. The shrinkage estimates are easy to compute, avoid iterative estimation, minimize the potential for overfitting and do not require extra computation time. They are compatible with the exact test of differential expression. For the datasets in this manuscript, sSeq consistently had a similar or a higher sensitivity and specificity of detecting differential expression than the existing methods. The approach can be generalized to express the dependence of the dispersions on the expected value or on other covariates such as guanine-cytosine (GC) content or Gene Ontology annotations.
sSeq can produce meaningful results in under-replicated RNA-seq screens. However, we stress that RNA-seq screens do not eliminate the biological variation in gene expression Equation (12). As evidenced by Table 2 and Figure 2, the under-replicated screens have lower reproducibility as compared with the replicated studies. Multiple biological replicates are necessary to adequately assess the full extent of the variation in the biological system. Therefore, the under-replicated screens can only be conducted when followed by a rigorous experimental validation with complementary technologies and adequate sample size.

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