The Working Mind

Meaning and Mental Attention in Human Development

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8 Process Analysis and Mental Task Analysis: Foundations

We outline a method of mental-process task analysis, describing its epistemological foundations. This metasubjective task analysis (MTA) models mental qualitative processes that individuals would have to use when solving tasks according to given strategies. This is a useful way to find the effective processing complexity of tasks. The analyses are constructed using schemes, concretely interpreted within the task, and the subjects' brain resources or hidden operators, as formulated by our theory. These general principles of mental-process task analysis could be applied (using suitable organismic-causal general models or theories) to any sort of task. Our focus is to model macro-processes in problem-solving, learning, and developmental tasks.

> Whenever a *system of conditions* is given that can be realized in different contents, there we can hold to the form of the system itself as an *invariant*, undisturbed by the differences of the contexts, and develop its laws deductively.
>
> —Cassirer, 1953, p. 40

> I have searched in vain for a system that allows one to systematically identify and catalog the elements of a cognitive task that ratchet up its complexity. Such a system would have practical applications as well.
>
> —Gottfredson, 2016, p. 216

Unlike physical scientists, psychologists find it difficult to distinguish between two sorts of equally necessary (and complementary) theories: descriptive-empirical (meta-empiricist) theories and their models or statements versus causal-organismic (meta-subjective) theories and their models/statements. Epistemologically, this is the difference between an empirical modeling done from an observer's perspective (from "outside" the subject-matter or the organism—a *visible process*, as Merleau-Ponty, 1968, would call it) versus a theoretical modeling done from the perspective of "within" the subject matter or functional organism itself (the *invisible process* of Merleau-Ponty). Only the latter can lead to causal-organismic modeling. Here, *"cause"* should be understood not as a single classic cause, but as an organismic one, that is, a constellation of organismic causal determinants, simple or complex, that together overdetermine the empirical manifestations. In every case both approaches are needed, because a descriptive-empirical or meta-empiricist moment is functionally essential (Pascual-Leone & Johnson, 2017).

Congruence between causal-organismic and descriptive-empirical models is the ultimate criterion of a theory's worth. Perhaps for this reason, in human science a valid observer's description of processes often is seen as a causal explanation and not just the description that it is. For instance, it is often expected in developmental research that fine-grained sequential delineations of descriptive steps of children's development are causal accounts of this development. This is not so, however. Proper causal-organismic theories and models must have distinct organismic-causal accounts that are metasubjective models from within the processing subject. We call them metasubjective because they hypothesize causal determinants as working from within the organism and distinct from the descriptive-empirical aspects they attempt to explain (Pascual-Leone & Johnson, 2005, 2017). For example, well-known developmental sequences in math tasks, expressing what children understand and solve at different age levels (developmental stages), are not causally explained just by appealing to working memory or to task difficulty. To causally explain them, we need metasubjective ("from within") models of the subjective processes intuitively explaining the task solution. Such models may include constructs like schemes and working memory or mental/executive attention (e.g., Matt or *M*-capacity) and so forth, processes that can be qualitatively appraised and measured. All these organismic causal factors must be empirically distinct from the task's descriptive process (e.g., from the schemes' application sequence) they help to explain.

Organismic-causal (metasubjective) models can appraise complexity within stipulated acts of mental (sensorimotor or symbolic) processing. In psychology to do so one must adopt a processing complexity viewpoint analogous to that of Gell-Mann (1994). For Gell-Mann, the complexity of processing involves effective (as opposed to crude) complexity. He defines *crude complexity* as follows: "The length of the shortest message that will describe a system, a given level of coarse graining, to someone at a distance, employing language, knowledge, and understanding that both parties share (and know they share) beforehand" (Gell-Mann, 1994, p. 34). This definition is related to the complexity of algorithmic-information content, often called *algorithmic randomness*. Algorithmic-information content is largest for random strings. However, randomness is not what we mean by complexity in rational analysis (or psychology). Instead Gell-Mann (1994, p. 50) defines *effective complexity* with this remark: "In fact it is just the nonrandom aspects of a system or a string that contribute to its effective complexity, which can be roughly characterized as the length of a concise description of the *regularities* of that system or string."

Crude complexity refers to the length of a concise description of the whole system (or string) that includes its random features and not regularities alone. For Gell-Mann (1994, p. 50), effective complexity is "related to the description of regularities of a system by a complex adaptive system [such as a person] that is observing it." Like Piaget, Gell-Mann understands by regularities the relevant probabilistic (often functional) invariants that emerge in the context of repeatable situations (Ullmo, 1958, 1967) relevant for an adaptive system, such as human producers or observers. *Invariant* meaningful aspects are preserved over repetitions even when incidental random aspects vary. In effective complexity, one appraises the power (perhaps estimated by the number of distinct characteristics essential or relevant for the task, i.e., task regularities) of a set of invariants. This concept of probabilistic invariants includes *coordinating relations*—all essential constituents that produce meaning and relevance of a message or agency/praxis. Piaget and developmental constructivists often call such experiential (often nonverbal) invariants schemes or schemas, which are relative to given situations, to agency/praxis, and to human intelligence.

Mathematicians and logicians regard math or logic as being generic explicit abstractions of this sort of invariants, which (relative to a given activity) express general principles of rational procedure with "ideographic symbols and in a form which exhibits the connection of these principles one with another" (to use words of C. I. Lewis, 1960, p. 1). A major implication of these definitions is that effective complexity is not only objective but also the basis for *metasubjective modeling,* that is, a formulation or formula from within the process itself, as synthesized by a human analyst. An implicit or explicit process/task analysis is involved in such modeling.

General causal (e.g., physical, chemical, organismic) theories in science can be applied to concrete situations only when using specific theoretical models that incorporate the task constraints (the resistances of the Real). Local models in classic physical science (e.g., those that instantiate calculus in particular situations) illustrate this sort of implicit task analysis. The equivalent in human science is *process task analysis*, often implicit and unexplicated. Process task analysis modeling tends to be qualitative and descriptive in human science. It becomes explicit and organismically explanatory when the subject's own task activity is modeled from within, particularly if key semantic-pragmatic, spatial, and often temporal constraints of the task are retained. We call this method *metasubjective task analysis* (MTA). Such analysis models functional infrastructure of a given strategy relative to a task.

Effective complexity is relative to the user's goals and context of use, the user's own criteria, and the intended agency/praxis. MTA helps formulating the task processing and effective complexity. We have informally illustrated this method in previous chapters and introduced some notation. In this chapter, we explain and justify it.

Construction of MTA models, with their rational reconstructions of task-relevant processes within the subject, assumes (at least implicitly) an *organismic model* of the subject-agent and his or her goals, together with a model of the task's *situation*. With these initial conditions in mind, analysts infer (doing what Peirce called abductions) the processes and effective complexity of a performance strategy that could in principle solve the task when implemented. This is *specific operative modeling*, from within the person, of processes that may cause the performance. Such rational reconstructions can be called *mereological models*, because they usually intend implicit or explicit experiential thinking (often nonverbal, semantic-pragmatic, spatial and temporal), actions, mental acts, or operations dealing with mental/physical objects[1]; this is not verbal-propositional thought or utterances. Note that situations can be seen as objects that the subject's agency/praxis attempts to control, understand, or modify. In MTA, we assume processes and logical effective complexity of a given strategy, highlighting the manifold psychological sequence involved. When explicitly done, this task analysis uses graphic or symbolic representations of distal objects or entities and represents how mental operations (operative schemes) apply to change relevant objects and anticipate or cause emergence of a performance.

Analysis of Conservation of Substance

Some tasks, such as Piaget's conservation tasks or adult versions of conservation such as the wine and water problem (see chapters 6 and 7), can help to illustrate similarities between analysis-of-figures in classic geometry (Hintikka & Remes, 1974) and analysis-of-schemes done in MTA. Because metasubjective analysis models a task solution from the perspective of a given strategy, schemes (information-bearing processes for task solution) are important. A common problem of task analyses is demarcation of schemes that inform the task. Piaget (Piaget & Morf, 1958, p. 86) defined a scheme as "an organized set of reactions that can be transferred from one situation [a task] to another by the assimilation of the second to the first." As explained in chapter 5, this functional construct can be defined as the (brain) coordination of three components: (1) a *functional component (fc)* that formulates gist and goals of the scheme, often providing expectancies that anticipate what could result from its application; (2) a *releasing component (rc)* that carries conditions under which this scheme can apply; and (3) an *effecting component (ec)* that produces effects that follow from the scheme's application and can also recognize when the scheme should terminate this application (this is a terminal or stop subcomponent, important in recursive schemes). Expectancies and terminal subcomponents show that schemes are minimal but very active functional systems that can recursively apply (schemes of schemes of schemes) and form functional/adaptive hierarchies.

When a scheme is proposed in MTA as component of a strategy, the assumption is made that this scheme is sufficiently learned or overlearned to function as a unit in its activation (activation by input, other schemes, or hidden operators). The plausibility of such an assumption should be evaluated rationally by means of a method of psychogenetic reconstructions (**Rec**). This method has four analytical phases:

(**Rec1**) The first phase is a *concrete scheme definition* (instantiation) as a functional semantic-pragmatic system (i.e., *fc*, *rc*, and *ec*). (**Rec2**) The second phase is the schemes' *ecological evaluation* to back the assumption that it is already in the subject's repertoire (i.e., long-term memory). Does the subject's usual expectable concrete environment offer sufficiently frequent opportunities for practicing and thus learning such scheme? (**Rec3**) The third phase involves *motivational/operative evaluation* to assess whether the subject's activities, motives, and interests may have led to learning this scheme in the past. (**Rec4**) The fourth phase, *psychogenetic evaluation,* appraises whether children of this age or younger, from the same population, in view of their known actual performance in suitable tasks (or their known age-bound mental attentional and learning capacities), could have acquired the scheme(s) in question. This fourth phase explicates Piaget's tacit psychogenetic assumption: if younger or peer-age children can be said to have acquired a certain scheme, then older (or peer-age) children of the same sociocultural background also should have it. A scheme that has passed three of these four evaluations is an acceptable scheme for metasubjective task analysis.

Pascual-Leone, Goodman, Ammon, and Subelman (1978) illustrated schemes, their dimensionality, and their codetermination of a performance by using Piaget's conservation of substance task. It may be useful to begin here with such a well-known task. In one of Piaget's versions, children were given two balls of Plasticine (**A** and **B**) and asked to make believe they were food or candy (see figure 2.1 and the upper half of figure 8.1 for a schematic). One ball was red and the other blue. Children were asked whether the two balls had the same amount. If they said "no," changes were made until the children accepted the balls as equivalent (e.g., if the balls were food to be eaten, they would be equally filling). Thus **B = A** represents a higher relational scheme stating that the balls as food amounts were practically equivalent. The tester then transformed one of the balls into a sausage, and children were asked whether now the ball and sausage had the same amount. Young children often would claim (even though nothing had been added or removed) that now they did not have the same amount, thinking that

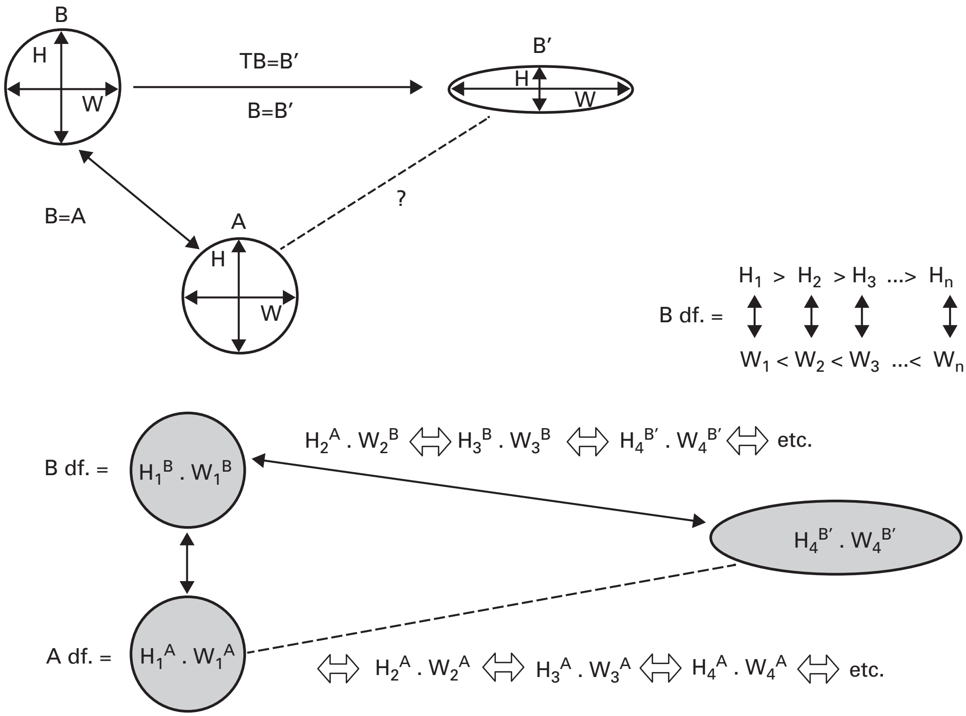

Figure 8.1
Piaget's structural model of conservation.

the sausage (**B′**) had more because it was longer (some children may claim that **A** had more because it is thicker).

A and **B** as object-schemes can be defined as purely experiential coordinations of three components, *fc, rc,* and *ec*. Assuming an experiential, nonlinguistic knowledge, these object schemes could be described as follows: **A** could be [*fc*: a ball of certain size presumed to be food, brought to the game by the tester, and so forth; *rc*: a round patch of red color with features of a manipulable substance and perhaps other features; and *ec*: if held it will not be heavy, will be soft and yielding]. Likewise, for the scheme **B=A** [*fc*: if the two objects were food and we each would eat one of them, each of us would have had as much food to eat as the other; *rc*: both have same size, same shape, although one is red and other blue; *ec*: the two balls are the same in amount]. Notice that other symbols in this subjective analysis, which stand for schemes, could be similarly formulated. For the sake of clarity, task analysts should be prepared to formulate explicitly any scheme used in the analysis if it raises any doubt in their or others' mind.

We call **T** the actual transformation of **B** into **B′** (the ball **B** transformed into sausage **B′**). This is a transformation in which no matter was added, lost, or taken away; for us, **T** is an *identity transformation* relative to amount. We call **TBB′** the child's concrete memory-scheme for this change (a complex experiential scheme that expresses a temporally ordered set of events—an experienced sequence, which we call a *fluent* or *temporal scheme*). Such clear memory is important, because data on children show that 5- or 6-year-olds already recognize the transformation **TBB′** as preserving the original amount (preschoolers know experientially that this particular **T** was, in our terms, an identity transformation).[2] As a result, **B** is seen by the child as having the same amount as **B′** (we represent this idea by scheme **B = B′**). The mental-attentional demand (*M*-demand) for the identity task that uses only one ball, **B**, and transforms it, is just two symbolic schemes: **TB** and **B′** to be held in mind simultaneously, via mental attention (M-demand $= e + 2$); this is accessible to 5- or 6-year-olds.

Notice that both conservation of equivalence and conservation of identity involve verbal questions about future or past events (e.g., transformations of shape do not affect amount of substance), and so the tasks are not sensorimotor but symbolic. Executive processes (boosted by the "e" of Mk-demand $= e + 2$) must be used. In symbolic tasks executive schemes are activated, we believe, by the sensorimotor *Me*-capacity, which in turn is driven by affects and personal schemes. Because 7-year-olds already have an experiential scheme of substance related to identity transformations, we can interpret the upper part of figure 8.1 as symbolically representing the working problem-solving mind (*M*-centration and dynamic synthesis) of a child facing the conservation of equivalence task. It represents the moment when **A** has already been compared to **B**, leading the child to conclude **B = A**, and **B** has been transformed into **B′**, leading the child to infer spontaneously the scheme **TB = B′**. At this point, object **B** has already disappeared, perceptually transformed into **B′**.

Such mental representation of the child is not just perceptual but symbolic. If we use, for brevity, algebraic terms that children do not have, their intuitive problem can be described by the formula: **A = B & TB = B′ → A ? B′** (i.e., Does **A** have equal amount as **B′**?). This algebraic transcription of the upper diagram in figure 8.1 symbolically exhibits the problem-solving conflict of children. They know that **A = B** and **B = B′**, but what can they do with this information? The theory of schemes alone (and this is the only theory that Piaget and other constructivist-learning developmentalists have) could not explain how a 7- or 8-year-old solves this problem, unless we unrealistically assume that these children possess a scheme for the logical rule of transitivity (i.e., if $A = B$ & $B = B' \rightarrow A = B'$). However, such logical rule could not be innate because of its abstractness and cannot be learned without prior experience. Piaget had a different, mistaken, constructivist-learning explanation of how children solve this problem. We shall discuss his explanation later on, to illustrate the structuralist method of analysis that Piaget used (not unlike that of current constructivist-learning theoreticians).

Although preschoolers lack logical structures of transitivity, children after 7 years of age can answer this question correctly, when they have constructed the mental representation of the problem symbolized in figure 8.1. They solve the task, we claim, by using a neo-Gestaltist "field" factor heuristic (of *F*-operator). This factor induces the inference that when two of the relations connecting the three objects (the two balls and the sausage) are equivalence relations, then the third relation that connects **A** and **B′** "should" (likely will) also be an equivalence. This "should" inference (possibly an instance of Peirce's abduction) coordinates, using minimal effective complexity, the active schemes within this mental set. Such is the brain dynamics that Berthoz (2012) has called "simplexity." Indeed, if the child's representation of this problem were called the stimulus and the answer to the question called the response, then this "simplexity" (classically known as Gestaltist "simplicity" principle, but also known as S-R compatibility, Proctor & Reeve, 1990) applies. It induces the inference or response that **A**=**B′**. In our theory, this principle expresses the *F*-factor, which is related to cortical lateral inhibition in the brain, as discussed by Edelman (1987). Only after this correct response has been heuristically (not logically!) evoked, the child may (with much practice) acquire the constructivist structural system or psycho-logical *grouping* (Piaget's groupement) that Piaget proposed for solving this and other problems. Constructivist learning does occur, as Piaget believed, but it follows and cannot precede the solution of this problem (see chapter 5). Notice that this sort of *F*-facilitated "simplicity" of reasoning also appears in a common implicit inference: "the friends of my friends are my friends," often conceptualized in psychology by means of balance and social congruence theories (e.g., Zajonc, 1960). Piaget interpreted this sort of inference using his "equilibration" principle (Pascual-Leone, 2012a), a dialectical and abductive, but not a formal-logical, way of solving problems.

How to Do Metasubjective Task Analysis (MTA)

An illustration of objective task analysis (tacitly, also of subjective analysis) is available in Polya's (1973) heuristic method for mathematical reasoning. In part inspired by Polya's methods, MTA investigates heuristically how to solve a problem or task by first finding a strategy (the idea of a plan), which is the first necessary step of our method. In the words of Polya (1973), this is done as follows:

> [W]e shall distinguish four phases of the work: First we have to *understand* the problem; we have to see clearly what is required. Second, we have to see how the various items are connected, how the unknown is linked to the data, in order to obtain the idea of the solution, to make a *plan*. Third we *carry out* our plan. Fourth, we *look back* at the completed solution, we review and discuss it. … It is generally useless to carry out details without having seen the main connection, or having made a sort of *plan*. Many mistakes can be avoided if, carrying out his plan, the student *checks each step*. Some of the best effects may be lost if the student fails to reexamine and to *reconsider* the completed solution. … The student should also be able to point out the principal parts of the problem, the unknown, the data, the conditions. … *What is the unknown? What are the data? What is the condition? … Is it possible to satisfy the condition?* (pp. 5–7)

Then Polya (1973, p. 7) recommends the student/analyst to represent (“*draw a figure*” of) the data and unknown and design a suitable notation.

Whereas Polya’s method suggests a heuristic for conducting an objective/subjective logical analysis of effective complexity and how to solve the problem in question, MTA methods attempt to formulate models of how psychological organismic schemes and hidden operators of participants could actually achieve what Polya talks about. The following questions may be raised in MTA to translate Polya’s own questions: What is the task? What does the subject have to do or solve (this is the “unknown”)? Which are the schemes elicited or needed by the task situation? Which schemes in the subject’s repertoire are related or necessary for the task solution (this corresponds to the “data” of Polya)? How do these effective-complexity schemes relate to one another according to the chosen organismic general model (e.g., the TCO)? How are schemes affected and changed in their functioning by organismic hidden operators/principles elicited locally within the task situation and ongoing plan (this corresponds to Polya’s “condition”)? Can schemes activated by the situation, or retrieved from memory by an executive strategy, suffice to solve the task if aided by elicited hidden operators/principles (this is Polya’s question: “Is it possible to satisfy the condition?”)?

Notice that the “condition” of Polya tacitly (or explicitly) includes schemes and hidden operators/principles not directly elicited by the situation but contributed by subjects’ willfully using their cognitive resources and following the executive strategy. This is related to what Hintikka and Remes (1974) called “auxiliary constructions” in classic geometry. Notice further that in MTA one also has to recognize automatically elicited schemes (i. e., shadow schemes) and strategies that are irrelevant or misleading for the task at hand. One may have to task-analyze and understand them as well, because control of these automatic irrelevant schemes (and their related automatic hidden-operator applications) can impose constraints on the strategy’s effective complexity.

Because human organisms tend to follow a mini-max principle (minimal complexity that maximizes adaptation) we seek to represent only the essential task processes (the minimal effective complexity that could solve the task). Thus, an important first principle of metasubjective task analysis (**MTA1**) is that organisms that process information and perform *take as much error as they can afford*. That is to say: for as long as the intended results are adequate enough for the goals pursued, the task's performance may not be improved; consequently, task analysis should adopt the simplest and least precise executive strategy that still meets the intended goals. An explicit formulation of these supposed goals (the tacit or explicit strategy/plan) of the subject is important, and these goals should be assumed to be as simple as they can be to still satisfy strategy and task constraints. Secondly (**MTA2**), because task analysis always and only bears on a given, chosen strategy for a given task, one should take up first the *most likely strategy for the intended population of subjects*. Then, if other strategies appear that we should investigate, other MTAs on the new strategies can be done.

Given a strategy, analysts can infer or derive the predicted or postdicted performance by coordinating four sources of constraint in subjects' performance. (**Source 1**) First is the *structure of the situation* in which the performance is produced, including objects, instructions, expectations, emotions, habits or automatisms, and other structural elements, which could affect performance within the situation. In this way an objective/subjective model of the situation can be formulated. (**Source 2**) Second is a *general* (explicit as a functional totality) *model* of the subject's psychological organism, that is, a modeling "from within" of the processes that enable subjects to cope with the task, given the chosen strategy. This involves what we may call a *metasubject*—an idealized working model of the subjects' general processing formulated "from within" to explain how they cope with task problems. Our metasubjective models tend to have three sorts of processes: schemes, hidden operators, and hidden principles. The latter express the most general regulations or "functional axioms" of the psychological organism (or metasubject). The three most important hidden principles for metasubjective task analysis, because they must be assumed by the analyst, are the principle of Schemes' Overdetermination of Performance (*SOP*), the principle of Equilibration (*EQU*), and the principle of Schemes Inhibition and Decay (*SID*) (Pascual-Leone et al., 1978; Pascual-Leone & Goodman, 1979).

The *SOP* principle says that performances (perceptions, imaginal representations, actions, mental operations) are coproduced at any moment by the currently dominant set of most highly activated schemes within the subject's internal field of activation (in long-term memory). The *EQU* principle says that the psychological organism (or metasubject) tacitly pursues, and changes toward, three concurrent tacit goals: (a) maximize internal consistency among its activated schemes; (b) maximize adaptation (functional payoff) in its dealing with the environment; and (c) minimize internal complexity within and among its coactivated schemes. The *SID* principle says that activated schemes are subject to various sorts of inhibition: (a) *automatic inhibition* (our I_{au}-operator), caused by interference from other dominant, currently applied, contradictory, and competing schemes; (b) *effortful inhibition* (i.e., mental-attention driven, our I_e-operator) willfully applied to competing/contradictory ongoing schemes; (c) automatic *habituation* of the schemes in question (e.g., scheme attenuation via habit-making, most apparent in perception, also attributed by us to the I_{au}-operator); and (d) *decay* propensity of schemes' activation (forgetting, attributed to the I_{au}-operator) resulting from time of inactivity or repeated inhibition because of other competing or contradictory, shadow, or task-irrelevant schemes. Schemes and hidden operators, along with these few organismic principles, formulate, explain, and model effective complexity of a given task, analyzed from the perspective of the metasubject (Pascual-Leone & Goodman, 1979; Pascual-Leone & Johnson, 2005).

(**Source 3**) The third source of constraint in subjects' performance are the *plausible motives* (affective goals) *and motivational factors*, that **Source 1** could elicit from **Source 2**, in terms of potential actions or reactions. (**Source 4**) The final source is a detailed observer's description of *the performance to be explained* (which corresponds to Polya's "unknown" in a predictive task analysis). The performance to be produced must be inferred if it is unknown. Some sort of objective-subjective problem solving, equivalent to Polya's, must be achieved before our subjective-metasubjective analysis can be done on it.

Two different sorts of task analysis have different requirements on the strength of **Source 2** (i.e., the theoretical model of the metasubject). In *postdictive task analysis* there is an already completed performance to be task-analyzed. This task analysis is a generic metasubjective operative model of what the subjects may have attended to or done (within their mind or body actions) to produce the performance. In this sort of analysis our working model of the subject could be quite weak, because performance is known. A key outcome of this analysis often is differentiation and strengthening (via abduction) of our general working model of a metasubject. In contrast, in *predictive task analysis*, where performance is not known, the subject's working model (**Source 2**) has to be well differentiated, and the context (**Source 1**), motivation (**Source 3**), and the coordinating task analysis itself must be carefully done, if we are to infer the performance (**Source 4**) with plausibility.

Dialectical Steps in Metasubjective Task Analysis

There are three equally important nested sorts of task analysis that constitute different and dialectically complementary moments; progress in any of them aids and benefits from completion of the others. Although analysts could work concurrently with all these moments and change from one to another as convenient, they all must be completed (they constitute a dialectical triplet). These three nested analytical moments are called *objective*, *subjective* (subjective 1 & 2), and *metasubjective*, or organismic-process, analysis (Pascual-Leone & Johnson, 1991, 2011, 2017).

Objective Analysis

Objective analysis is a method that psychology shares with all sciences. Here, one describes objects, procedures, and relations known about the actual situation, task, subject, and performance. This method is a common practice familiar to scientists. It can be described as some form of liberalized descriptive or phenomenological analysis coupled to a rational semantic-pragmatic analysis of relevant causal (or organismic-causal) processes, which fully contextualize the situations. This constructive analysis uses results from empirical science when possible. The analyst may begin by asking a few guiding questions: what do we have to model? That is, what is the intended (internal or external) performance? What does the subject have to do in this task (i.e., what is the strategy or procedure chosen)? What aspects of the empirical situation could affect (facilitate or hinder) the subject's task—either intended internal process or external performance? What is the effective complexity (or functional infrastructure) of the task? That is, which are the essential or necessary skills (in concrete acts and percepts) needed to produce the performance?

This is the first essential moment in MTA. One does not describe at this stage natural organismic units of processing (such as schemes) nor refer to the brain constraints and maturational capacities (the brain's "functional hardware" of hidden operators and principles) that produce the internal/external performance.

Subjective-1 Analysis

Subjective-1 is a more advanced form of analysis. It includes functional-*structuralist* (structural constructivist) modeling, which Piaget's psycho-logic pioneered. Piaget's functional schemes/structures such as (psycho-logical) groupings or the INRC group (of four transformations, i.e., Identity, Negation, Reciprocal, and Correlative; see Beth & Piaget, 1961; Pascual-Leone, Escobar, & Johnson, 2012) illustrate well his form of structuralist modeling. A more recent but analogous form of neo-Piagetian structuralism is the functionalist analysis of relational complexity suggested by Halford and associates (Andrews & Halford, 2011; Halford, Cowan, & Andrews, 2007; Halford, Wilson, Andrews, & Phillips, 2014). Such relational analyses, as Piaget's own, go beyond (while including) objective analysis. They make assumptions about logical/functional structures of the subject's task-solving processes and adopt a perspective that is both constructivist and tacitly "from within" the process. They attempt to capture coordinated task-essential aspects (dimensions of variation) that subjects must master and use in their minds (or externally) to produce the performance.

Both Piaget's psycho-logic and Halford's relational complexity theory state that meaning occurs when a link is formed between or among schemes by way of interrelations (a view that we, Case, Demetriou, Cowan, and many others also share). Chapter 1 discusses Halford's relational complexity in more detail and provides examples, However, Halford's relational complexity or Piaget's psycho-logic are not the only task-relevant sorts of effective complexity. Tasks' mental demand may not increase with relations when they are overlearned. As well, mental demand can accrue also with other not interrelated but relevant relations or pieces of knowledge, such as those that must be kept in mind for later use in a task.

Subjective-1 analysis adds to objective analysis the modeling of processes in terms of organismic units (such as schemes/schemas or structures). Thus, after we have intuitively grasped useful process-structural relations in the task, a first question to ask should be, "Which are the schemes involved in this task?" Analysts then must instantiate these relational structures as schemes of a concrete functional totality that could model the performance (such as Piaget's concrete operative/operational structures, Case's concrete central conceptual structures, or Halford's concrete relations of various complexities). In this manner, we can obtain distinct, independent but coordinated, functional constituents of the task processing. Every constituent should be formulated as a scheme/schema with its three functional components: releasing, effective, and functional.

To achieve such performance decomposition, one adopts a within-the-subject perspective and analyzes what processual information a subject must attend to and coordinate to solve the task. This is the *first heuristic prescription* of MTA. The analyst must express this information deconstructed into organismic units (to repeat: schemes implicitly or explicitly defined in terms of their three components *fc, rc,* and *ec*). The set of necessary (relevant) units for the task solution is part of the effective complexity in the task. These are essential constituents, because without them the intended outcome cannot be attained. *Essential* constituents are not absolute but relative to motives and goals of subjects, relative to the needed task and strategy adopted.

In the analysis we should retain all influential, and not only the essential, schemes for the task and strategy adopted. This is the *second heuristic prescription* of MTA. We should also describe some haunting schemes that are not infrastructural, are nonrelevant, and often are unconscious, because they are in fact like a shadow that can affect performance (we call them shadow schemes). Shadow schemes are important in modeling MTA, whether they strongly facilitate the task-intended performance or strongly interfere with it. Eminent neuroscientists, who do not do any explicit metasubjective analyses (and in theorizing do not use well-defined scheme units) recognize nonetheless the great importance of often unconscious processes as codeterminers of task performance (e.g., Dehaene, 2014). A major problem of structuralist analysis, including relational-complexity analysis, is that these methods fail to analyze relevant shadow/unconscious schemes, and because scheme units are not well defined in these methods, process-analytical contradictions caused by shadow schemes (a frequent cause of misleadingness) often are not detected. To illustrate this important point, we shall now discuss critically the conservation of equivalence model that Piaget developed. We shall present Piaget's as the only example, but similar criticisms can be addressed to many contemporary learning developmentalists and structuralists.

Piaget's Final Structuralist Model of Conservation of Equivalence of Substance The lower half of figure 8.1 presents Piaget's structural model of equivalence conservation as he formulated it, although the figure is ours. The formulas in this part of the figure are our representation of Piaget's own theoretical model. Figure 8.1 represents the mental state of a 7- or 8-year-old child who, after been exposed to the whole task (i.e., the comparison between **A** and **B** to conclude equality, and identity transformation of **B** into **B′**), is confronted with the conservation-problem question: "If I eat this ball of candy (**A**) and you eat this sausage of candy (**B′**), do you think that we will have eaten as much (same amount of) candy?" Young children may reply that they would eat less, because **B′** is much thinner. Alternatively, they may say they would eat more, because **B′** is much longer than **A**.

These responses are induced by two factors: (1) the *F*-factor of simplicity (e.g., because **A** looks to have more in the thickness dimension, it must have more amount) and (2) automatized past-learning experience (things that have more surface tend to have more substance, e.g., an orange has more substance than a clementine). These shadow schemes (unconscious but compelling, often unsuitable habits from past learning) interfere with the problem solution. How then could children ever come to solve a conservation problem by themselves? Piaget thought that this solution comes via constructivist learning: experiential construction of a relational structure that Piaget called a psycho-logical "groupement," that is, the grouping of vicariances (Beth & Piaget, 1961). The term vicarious has an uncommon meaning in Piaget that fits the etymology (from Latin, *vicarious*: substitute). This grouping structure is best explained with the help of three other examples briefly described in table 8.1.

The first example comes from Piaget. Consider the concept (logical class) of European. One way of understanding "European" is as a logical class: a collection or set of people who share an invariant relational characteristic—being distinct national-types within Europe. So European can be described using different alternative facets, all equal or equivalent: for example, the set of Spaniards and of non-Spaniards is equal to the set of French and non-French, which is equal to the set of Polish and non-Polish. Each of these partitions of the collective "persons from Europe" constitutes a qualitative aspect or facet of the European concept, which quantitatively (number of European members) is equal to other facets or partitions of the same collective; this shared invariant corresponds to the people from Europe. Such logical relational invariant (qualitative and the quantitative equality of all vicariances/partitions for this collective) is what Piaget's logic defines as the concept "European." Leaving aside the practical relevance of this definition, this is an instance of Piaget's *grouping of vicariances*: the system of dichotomous partitions of a large set, or collective, such that all partitions are equal in number of members; and all partitions, qualitatively sorted in kinds as facets, are instances of the same invariant concept—"European," in this case.

Another example comes from Frege (1980), who seems to have thought that a number can be characterized as a relational invariant (quantitative identity) defined on a large interrelated collection of numerical units. From this perspective, the number seven would emerge, under various arithmetic operations, as the invariant generated by $3+4 \Leftrightarrow 1+6 \Leftrightarrow 10-3 \Leftrightarrow 21 \div 3 \Leftrightarrow \ldots$ and so on. This relational structure is another example of Piaget's grouping of vicariances, and each of the equal (identity) terms is a facet of the number seven.

Consider a last example. Driving a standard-gear car requires coordinating the foot pedals to get the car moving. This coordination as a relational structure also corresponds to Piaget's grouping of vicariances. If we call Accel_n the graded act of pressing

Table 8.1

Examples of grouping of vicariances

1. **Piaget's logical class** of "European" is the conceptual invariant common to alternative partitions of the set: [Spaniards & Non-Spaniards ⇔ Polish & Non-Polish ⇔ French & Non-French ⇔ … etc.]
2. **Frege's notion of number as relational object/symbol**. For instance, "7" is the quantitative invariant common to an infinite family of arithmetic operations: [$3+4 \Leftrightarrow 1+6 \Leftrightarrow 21/3 \Leftrightarrow \ldots$ etc.]
3. **Pedal coordination in driving a manual-shift car**: [$\text{Accel}_1.\text{Clutch}_1 \Leftrightarrow \text{Accel}_2.\text{Clutch}_2 \Leftrightarrow \text{Accel}_3.\text{Clutch}_3 \Leftrightarrow$ etc.]

(or relaxing) the accelerator pedal (this is a graded series 1, 2, 3, … n); and call Clutch_n the graded relaxing (or pressing) of pressure on the clutch pedal, it is evident that a suitable coordination of these two motor series is necessary to ensure that the car can start moving smoothly, without too much noise or stalling. At each graded moment (1, 2, 3, etc.), actions in the respective series must be adjusted to constitute a proper facet or moment: one where smooth car movement—a coordinated dual series—occurs. This coordination can be interpreted as a grouping of vicariances of the form shown in table 8.1.

These distinct concrete examples of grouping of vicariances should make Piaget's logical model clear. Piaget thought that one such grouping, acquired by coordinating schemes during prior experience with suitable objects (constructivist learning), could explain why children of at least 7 years can solve a conservation of equivalence problem. Our symbolic explanation of Piaget's vicariances model for conservation is represented in the lower part of figure 8.1.

In this figure, the letters *H* and *W* stand, respectively, for the height and width measures of the perceived objects (ball, sausage). Each object, for instance ball **B**, emerges in cognition as a functional invariant constructed by coordination of a grouping of vicariances—not unlike the number seven or the concept of European, just mentioned. Only now the facets are constituted by coordinated graded values, in inverse relation, of height (H) and width (W) of the object. These two series represent the distal object perceptually presented. As H decreases (>) because ball **B** is being rolled, the corresponding W increases (<). All these facets (H × W) are equivalently indexing the distal object, indexing the set of state-configurations the object could adopt under any transformation that preserves amount of substance as an invariant. The transition from one facet to another is any identity transformation (where no matter is added or taken away).

A similar logical grouping of facets could of course be attributed to **A**, as the lower part of figure 8.1 shows. Because the particular facet corresponding to **B′** (i.e., $H_4^{B'} \times W_4^{B'}$) can also be found as a facet in the vicariances grouping of **A** (the H × W pair 4), the child (Piaget would say) can recognize that **A** and **B′** belong to the same distal object (i.e., the same concrete grouping of vicariances) and therefore **A = B′**. However, this elegant explanation of Piaget's model fails, because past learning could not possibly have led to the emergence of such grouping of vicariances for conservation. The reason is that there is no prior objective empirical invariant here, which could induce and guide empirical coordination of the object facets.

This is in sharp contrast with the three examples of vicariances given before. Piaget's grouping for the concept of Europeans does have an independent empirical invariant (i.e., the nationals inhabiting geographic Europe), which can guide relational abstraction of this grouping. There is also an independent empirical invariant for the number seven. By counting the numerical units, or counting objects, we can generate the number seven prior to its emergence as invariant of its grouping of vicariances. The same happens with the pedal-coordination grouping. In contrast, in the case of equivalence conservation there is no prior independent empirical invariant: the equality of substance/matter (as a quantitative invariant) would have to be hypothesized ("invented") to be confirmed after experimentation.

Thus, this conservation grouping is totally abstract and could not have been inferred in prior experience, unless heuristic methods of a different sort lead the subject to guess an equivalence. In conclusion, conservation invariants in Piagetian tasks are not empirically grounded, and the subject must hypothesize them (Peirce's abduction) before they can be empirically discovered and reflectively abstracted; the invariant of conservations is a rationalist fact and is not an empiricist fact (Pascual-Leone & Sparkman, 1980). This learning-paradox problem of Piaget's structuralist models often is shared by other structuralisms.

The abduction process needed to solve the problem is presented in formula **f1**.

$$\{\ldots M[\text{COM.AMOUNT}(A,B'),\ TB=B',\ B=A]\ \ldots\} \Rightarrow [B'=A]_{F,LC} \qquad \textbf{(f1)}$$

In this mental attention model, when children have reached the problem representation indicated, they can coordinate three schemes: (1) an operative scheme that says COMPARE(A, B′); (2) a figurative relational scheme that says TB = B′, where T preserves identity of amount, that is, B = B′; and (3) another figurative scheme that says B = A. However, as mentioned above, there is a problem. Some organismic factor must exist that enables, without a logical rule of transitivity, this global heuristic inference: Because two terms (B = A, TB = B′)[3] have an equality in them, the third term (B′?A) of this formula should also have an equality (B′ = A). Indeed, simplicity *F*-factor should induce such inference, according to our TCO model. If this is the solution, it has also demanded use of hidden operators, such as the *F* and *M* (factors of mental attention) to boost activation of the three essential schemes and synthesize mental representation of figure 8.1. But we are now ahead of our discussion; we introduce hidden operators in task analysis in our third and final moment of task analysis.

Limitations of Subjective-1 Modeling A good subjective task analysis begins by examining relevant objects and procedures, with their coordinating relations. These entities are modeled into schemes (or related constructs) that could have been created by the subject during his or her past history (via constructivist learning, e.g., *C*- or *L*-operators; see chapter 5). Before a scheme is posited for introduction in task analysis, a rational reconstruction must be done to appraise whether all posited schemes could have been acquired by the subjects in their expectable past, given their personal history (as we have just shown, Piaget's schema/structure for conservation—his vicariances model—cannot have been acquired).

At the end of this subjective-1 analysis, researchers should have formulated (and, at best, represented symbolically with suitably compact logical notation) all task-relevant schemes/schemas, and all the shadow (unconscious compelling habit) schemes, that may have an effect (positive or negative) on performance. Clear definition and coordination of these schemes may still be missing, however; and relevant processes may have been modeled top-down holistically as functional-structural totalities for solving tasks (illustrated with Piaget's vicariances model of equivalence conservation). Many contemporary task analyses are done as simple flowcharts, diagrams, theoretical figures, or simple enumeration of relevant processes, without detailed explanatory organismic models. This is generally inadequate. Neither true parallel processing of schemes nor models of real-time unfolding (i.e., step-by-step natural sequencing of task processes) is available in this sort of task model. Also omitted is the relevant "functional hardware" the organism needs to generate performance, as our analysis of Piaget's model shows.

However, there are intermediate methods. The task analyses used by Case and his students (Case, 1992, 1998; Case, Okamoto, Henderson, & McKeough, 1993), for instance, go beyond subjective-1 analysis in that they occasionally include real-time reconstructions of some task-solving processes. They have executive processes and also organismic hardware constraints such as those of working memory (mental/executive attention).

Subjective-2 Analysis

Also called *ultrasubjective analysis*, subjective-2 task analysis goes beyond structuralism and associative relational learning, because its focus is on the real-time deconstructing of tasks and total-task structures of the subject. Subjective-2 analysis deconstructs the task in terms of schemes (whether task-compatible or incompatible) that may be simultaneously coactivated or sequentially coactivated to constitute structures that can together describe a task-relevant functional totality. This temporally sequenced form of analysis coordinates relevant and shadow schemes to explicate task processing and performance. It often begins with a simple question: can the process-task analysis be obtained in steps of (perhaps competing) scheme applications?

Adding this finer level of discrete schemes to the analysis and modeling real-time process force analysts to use a clear and explicit formulation of schemes. This creates the need to differentiate among four distinct and necessary sorts of schemes, which may be intentionally conscious (during their manifestation) or unconscious (e.g., shadow schemes). For this unfolding of sequential process, we distinguish between figurative schemes that stand for logical “objects” (logical “arguments”) and operative schemes. The latter are processor operations that apply on figuratives. Notice that figurative and operative schemes must be regulated in their intertwining to produce the intended processing change. This is done by temporally structured schemes that we call *fluents*. Fluents can function either as figuratives (“objects”) or operatives (functors), depending upon the task circumstances. In our task analyses we call these fluent schemes parameters, when they stipulate (boundary conditions) how to achieve successful application of specific operatives on the given figuratives. We use the term fluent (Pascual-Leone et al., 1978), because this sort of operative-or-figurative scheme expresses or regulates the changing flow of processing and involves expectancies. A simple example of a fluent is the scheme TB = B′ of our model of conservation (see figure 8.1 and accompanying text). In task analysis, parameters often are relational fluent schemes.

Subjective-2 task analysis also requires executive schemes, needed to plan and coordinate into the future action schemes (figuratives, operatives, and parameters) to produce performance. Finally, planned anticipations and preparations are driven by affects/emotions (affective goals, motives) and social/personal schemes with their values, biases, and preferences. These affective and personal schemes in fact regulate and produce complex, often intrinsic, motivation (Pascual-Leone & Johnson, 2004; Pascual-Leone, Pascual-Leone, & Arsalidou, 2015)—the sort needed for cognitive problem solving and cognitive development. These schemes, although important, are assumed, but not directly represented in the usual sorts of cognitive task analysis. However, they can be made the direct focus of analysis if necessary (Antonio Pascual-Leone, Greenberg, & Pascual-Leone, 2009, 2014).

Attention to the interactions between essential/infrastructural schemes and shadow schemes leads analysts to discover error factors or facilitating factors in situations. These usually are caused either by shadow schemes or sensorial-perceptual task features that influence cognitive/perceptual scheme representations. Misleading schemes may force adoption of a more effortful (*M*-demanding) strategy, for example, one that segments the originally intended schemes into relevant subschemes that must be *M*-boosted separately, to bypass the misleading aspects (by opening alternative strategies). In contrast, facilitating schemes could induce suitable cue-generating moves in the solution process that may spare the need for *M*-boosting some task-relevant schemes (e.g., making more salient some cue that activates relevant schemes).

Metasubjective Analysis (MTA)

An MTA can be dimensional (an analysis of task-relevant executive processes), a real-time mental-process reconstruction (which we call an *M*-construction), or a synthesis of both methods. In any case, it is a generic summary representation of real-time organismic processes responsible for a subject's performance. A step in this analysis need not correspond to a single moment or unit of time but may represent a time segment unfoldable into a sequence. In this respect, the steps of a metasubjective analysis are like block units of a detailed flowchart, but better defined and including hidden operators that regulate schemes.

Objective and subjective (1 & 2) methods of analysis do not suffice. As our discussion of these methods and of equivalence conservation have shown, we are forced to refer again and again to hidden operators such as the *F*-, *M*-, *C*-, and *L*-factors, as well as hidden principles (e.g., *SOP, SID, EQU*). These hidden processes are alternative general-theoretical constructs that enable task completion, and they should be represented and carefully studied in MTA.

At this point, if we aim toward a predictive task analysis, it is necessary to adopt an explicitly general metasubject's model of organismic processes that is detailed enough. Analysts may already have from subjective analysis a list of scheme types and their characteristics. As this subjective analysis progressed, they may also have recorded various distinct schemes that appeared necessary for the task, each explicitly defined (perhaps with their functional, releasing, and effecting components). Now analysts need a list of all hidden operators of the organismic theory that could be relevant for the current task analysis. It could be used as a checklist to decide the hidden operators needed for the chosen strategy. Table 7.1 offers one such a list.

In MTA, and after having demarcated relevant schemes for a given strategy in the task, analysts should take this list and ask themselves, as a heuristic, how the various operators and key principles (e.g., overdetermination, inhibition/decay, equilibration, and reflective abstraction) may intervene in the chosen task strategy. To illustrate this checklist heuristic, consider again Piaget's conservation of equivalence task. We have discussed two models for conservation of equivalence. One expresses Piaget's constructivist-learning strategy, which could not have been learned in time and is therefore contradictory. The other presents our own maturational-attention (i.e., *M*-operator) problem-solving or synthesis strategy, which may be correct. Let us follow the list of hidden operators in table 7.1.

A-operator, affective schemes and motives (affective goals), and with it the *LA* (logical-structural learning of affective/personal processes), and *B*-operator (the self-defining, personal or sociocultural schemes of a particular human being) are usually mediated by "good" or "bad" shadow schemes, more or less automatized for cognitive psychosocial tasks. To maximize a child's motivation, these complex affective/personal schemes must modulate cordial, affectively warm interactions between the child and experimenter. In many symbolic problem-solving tasks, like this one, participants often are not intrinsically motivated to solve the problem, but they are indirectly motivated because they like the experimenter and want to answer his or her questions.

C-operator appears in perceptual processes that lead to a representation of proximal/distal objects, such as **A** and **B** or **B′**, often intervening tacitly but decisively. The *F*-operator is the factor imposing a simplicity (a mini-max *F*) bias in perception and cognition. As mentioned, dynamic synthesis of the conclusion **A = B′** is mediated by *F*. To summarize, we illustrate this dynamic synthesis using a logical representation of the equivalence conservation task:

$$\underline{\text{COM:A,B}'}\ (\underline{\text{A[A=B]}}, \underline{\text{B}'[\mathit{TB{=}B}']}, \#\{\text{MIN:Rel.Diff}\}_{F,\,C}) \Rightarrow \text{A} = \text{B}' \qquad \textbf{(f2)}$$

This logical formula is an alternative representation of formula **f1** that carries the solution to the problem. The parentheses (…) in **f2** demarcate schemes on which the operative COM applies. A[A = B] is the distal object **A**; the square bracket enclosure […] signifies that this is a cognitive-intellective (symbolic) distal-object scheme and has as part of its meaning the prior relational conclusion that A = B. The first step in this task was precisely intended to induce in the participant this intellective representation of **A** as a distal object, via automatization based on mental attention (*LCLM*-schemes; see chapter 5 and below). A similar meaning applies to B′[TB = B′], except that here the identity scheme of TB = B′ has the structure of a fluent, the sequence of events that transformed **B** into **B′**.

This structuring would not be possible without effortless participation of two hidden operators and the principle of overdetermination. The first operator contributing to the TB = B′ synthesis is the *T*-operator (number 5 in table 7.1), discussed in chapter 6. The *T*-operator is needed to learn relational sequences that make objects meaningful. When a concrete transformation **T** (e.g., rolling the ball on the table) is actually applied on **B** to progressively turn it into **B′** (a sausage), the *T*-operator in the child's working mind internalizes these sequentially related elements as a relational structure, without need of mental attention. These evolving coactivated schemes are unified into a temporal structure by *T* with the help of *F*-operator and *SOP*, which induce **B** and **B′** to combine as two facets of the same distal object.

The role of mental/executive attention is shown in formula **f2** by underlining of three schemes (one operative and the two figuratives), to signify that *M* is boosting them, which causes the mental operation. *M*-boosting is necessary because the schemes are symbolic and not directly cued (in contrast, **A** and **B′** are perceptually given and nonsymbolic object-schemes—radically different schemes from their symbolic counterparts). The fourth scheme, a parameter (#) does not need to be boosted with *M* because *F* is boosting it; which in the formula's notation is indicated by $\{\ldots\}_{F,\,LC}$. In our method of notation, the braces enclosure indicates that schemes inside do not need to be boosted by the *M*-operator. The subscripts at the end of the enclosure (*F, LC*) show that *F*-operator and *LC*-operator are boosting the enclosed scheme. In **f2**, this fluent scheme is a parameter (indicated by prefix #) that constrains the operative scheme to **MIN**imize **Diff**erences among **Rel**ations interconnecting the various object-schemes (this is the mini-max *F*-tendency described in chapter 6). Such constraint, imposed by *F* (principle of simplicity) is likely to be already automatized (*LC*-learned).

If the four schemes mentioned (three of them boosted by *M*) are highly activated together in the mind (i.e., the focus of attention or centration), *F-SOP* can impose a dynamic synthesis that appear after the arrow (➔ symbolizes *SOP*). This is the conclusion scheme A = B′. Formula **f2** illustrates emergence of a scheme solving the conservation problem. The *I*-operator, promoting mental-attentional (active, effortful) interruption or inhibition, intervenes in the task but only implicitly. It is not represented in the formula, because this would require modeling misleading shadow schemes. To illustrate the competition of the task-relevant schemes in **f2** versus the task's interfering shadow strategy, we now present in formula **f3** the alternative misleading scheme, likely to be automatized in perception:

<u>COM:A</u>^{L1}, ({A}$_{L1}$, <u>B</u>′, #{MAX:Per.Diff}$_{F,\ LC}$) ➔ A > B′ **(f3)**

In this formula subjects try to compare **A** with **B′**, but here the two objects are purely perceptual (proximal objects). Now operative scheme COM already carries (because of perceptual *LC*-learning comparing objects) the reference to **A**, its first proximal object to compare. Thus, during the actual comparison this term ({A}$_{L1}$) is boosted (chunked) by *L1* with COM. However, **B′** has to be explored to appraise its shape to compare amount with **A**. In perception, our cognitive system automatically would minimize complexity by grouping features into objects that MAXimize Differences among these objects. This is the meaning of parameter #{MAX:Per.Diff}$_{F,\ LC}$. Such perceptual mechanism, facilitated by *F* and by *LC*, is a parameter (#) inducing incorrect judgment. Thus, **B′** may be perceived as having less matter than **A**, because **B′** is much thinner, that is, **f3** infers that A > B′.[4]

Because conclusions of the automatic scheme of **f3** contradict the correct emergent scheme of **f2**, an internal activation of mental attention (Matt = <*E, M, I, F*>) must intervene in **f2**, boosting relevant schemes with *M* and inhibiting **f3** with the *I*-operator. All this is monitored by executive schemes (*E*-operator). Such a competitive misleading situation can be expressed in our MTA method as a *strategy-relational formula* that shows how various strategies elicited by the task relate:

$(*E, M, I, LM* [formula **f2**]) <u>**V**</u> ^(*F, LC* [formula **f3**]) **(f4)**

Formula **f4** shows that **f2** and **f3** are in competition prior to the correct response. Strategy **f3**, which is wrong (prefixed ^), will be activated first, because *F* and *LC*, two fast automatic processes, are boosting it. To overcome this initial dominance of **f3**, the correct (prefixed $) strategy **f2** needs executive schemes (*E*-operator) to effortfully inhibit **f3** with *I*-operator and allow **f2** to unfold. Because both strategies share lower-level schemes, an initial dominant activation of **f3** should decrease (competitive interference) probability of subsequent activation and application of **f2**, unless Matt is mobilized by *E*. We call such a state of affairs a misleading situation. We represent this misleadingness with $\underline{V}$, a logical connective of mutual incompatibility.

People differ in the relative weight (propensity, probability of activation) their own hidden operators have in their working mind. *Field-dependent* people's propensity is to have very high weight for *F* and *LC*, with a lower weight for *E*, *M*, and *LM*, whereas the contrary is true for *field-independent* people. Research is congruent with such interpretation (Hederich-Martinez, & Camargo-Uribe, 2016; Pascual-Leone, 1989; Witkin, Dyk, Faterson, Goodenough, & Karp, 1962; Witkin & Goodenough, 1981). We should expect, and it happens, that in tasks exhibiting misleading competition as in formula **f4**, such as conservation tasks or the water and wine problem, field-independent people will do much better, provided that they have enough Matt to synthesize the correct solution (formula **f2**).

Judgment Tasks versus Action/Operative Tasks, and Knowledge Metadomains

Conservation is an example of a *judgment task*. Such tasks involve collecting information regarding schemes from the recent past about relevant events and circumstances—relevant to reach some problem-solving judgment conclusion. Judgment tasks tend to be representational (involving figurative schemes, whether perceptual or relational-conceptual), which often require recognizing/synthesizing relations of actual coexistence among relevant schemes. Note that abstracting undiscovered relations of coexistence from structural totalities (e.g., in space, in rational abstraction, in music) is characteristic of the brain's dorsal pathway activity (Pascual-Leone & Johnson, 2017; see chapter 11), which we formulate as *S*-operator, an operator that facilitates coordinated simultaneous integration of several distinct processes (see chapter 6 and table 7.1).

A common and very different kind of task could be called *temporal-meaning operative integration* (episodic) *task*, because in it one's attention is applied to a sequence or series of specific actions or steps, in proper order, to appraise objects/situations or achieve a given concrete result. Many learning tasks, such as learning to drive, experimental (motor or matching) learning tasks, many language tasks, or music, illustrate in part this sort of action/operative task, which often uses temporally structured (fluent) schemes. Abstraction and coordination of temporal sequences of events, or steps to appraise objects or situations, are essential in these tasks, which often involve use of the brain's temporal lobes (e.g., hippocampus and related areas, lateral temporal lobes). This sort of temporal-meaning structuring of experience often is done using automatic attention and our *T*-operator (see chapter 6 and table 7.1).

The distinction just made is important in task analysis, because the required modes of processing are different in one sort or another, engaging preferentially different hidden operators (as we suggested, among others, *S*- vs. *T*-operators). To this end, it is important to distinguish between metastrategies of task solution and metadomains where task analyses can be made. *Metastrategies* (or *operative modes of processing*) are functional categories of strategies that differ in operative viewpoint or perspective—adopted because of task characteristics, or subject bias, or bias of the analyst. The distinction between judgment tasks and temporal-meaning operative tasks is an example of metastrategies imposed by task characteristics.

An example of metastrategies adopted because of subjects' individual differences bias is the field-dependence-independence distinction just made. An example of analysts' bias is found in two alternative ways of doing science or problem-solving: by adopting an observer's perspective from outside the subject (i.e., a meta-empiricist perspective) or adopting a perspective from within the subject's own thinking process, which we have called metasubjective perspective. The latter informs our methods of task analysis (Pascual-Leone, 2013, 2019; Pascual-Leone & Johnson, 2017; Pascual-Leone et al., 2015; see previous chapters). There are other types of metastrategies.

Metadomains (or content-abstraction modes of processing) are another important functional category relevant for task analysis. Expanding on categories initiated by Piaget (who contrasted infralogical versus logical versus linguistic), we differentiate among four functionally ordered metadomains. They are ordered, because as knowledge advances the earlier metadomains "force" emergence of the more symbolically abstracted metadomains that follow. This is the (developmental) weakly ordered set of categories that follow:

Metadomains:= infralogical ➔ mereological ➔ logological ➔ linguistic **(f5)**

The *infralogical category* was formulated by Piaget: it refers to purely experiential objects (i.e., transformations and relations felt prior to logical or conceptual characterization) and to sensorimotor *prelogical* (prepredicative) experiencing. The *mereological category* refers to the experiential construction of objects and their coordination or interrelations (e.g., transformations). Such coordination creates, among others, collective classes (collections of actual infralogical objects or entities). Formally formulated by Lesniewski as an alternative logic based on concrete-activity (Mieville &Vernant, 1996), *mereologic*, if psychologically interpreted, becomes a mode of psycho-logical modeling in the infralogical domain, using conceptual (logological) tools for qualitative modeling of activity (praxis, goals, operations, object relations) in the world. This modeling produces mereological entities.

The *logological category* was called logic by Piaget (in the sense of *psycho-logic*; Pascual-Leone et al., 2012). It clarifies from within a child's processing developmental progress of conceptual psycho-logic. Psycho-logic expresses the most general relations in experiential activity. From this perspective, Ferdinand Gonseth (1936/1974) defined logic as the physics of any object whatsoever. For instance, for babies not older than about 6 months, the mother they seek, smile to, and embrace, is an infralogical (emotion arousing and purely experiential) entity. For this baby, mother becomes a mereological entity when the baby is about 10 to 12 months. At this time, mother is an individuated and specific object-person (a subject), who comes to feed and play with the child, who is on call, and at times disappears. Perhaps at 18 to 35 months, with the use of symbolic function and the parents' linguistic communication, logological schemes finally appear in the baby (who then can classify, relate, and organize generically his or her own experiences). When the child's language emerges as an explicit communication tool, its elements are sign representations of mereological and logological entities (including relations) extracted from actual experience.

In formula **f5** the arrow (➔) symbolizes an overdetermination (*SOP*). Metadomains to the left of the arrow force emergence (i.e., are semantic-pragmatic origin, semiotic foundation, and referent) of all metadomains to the arrow's right. Conversely, metadomains to the right progressively abstract, make compact, and refine the effective complexity of the metadomains to the left, which thus become explicit and communicable. Understanding all these functional categories of processing is useful when doing task analysis. MTA often is modeled using linguistic and logological skills of the analyst but usually represents mereological or infralogical schemes of the subject. We give examples below.

In task analysis, it is useful to investigate various metadomains, using distinct methods (i.e., objective, subjective-1, subjective-2, and metasubjective). These analyses can be done in any order or done concurrently (perhaps using various pages to separate methods and metadomains). The analysis is completed when the pages become coordinated and mutually congruent.

Symbolic Notation in Metasubjective Task Analysis (MTA)

Task analysis can be made verbally if it is not too complicated. However, a symbolic notation and representation of the analysis by psycho-logical process formulas helps reasoning and makes results explicit and compact, facilitating awareness of deficiencies and helping in process modeling. In this section, we summarize some notational conventions but give a more detailed list of conventions in the appendix at the end of the book. As with all heuristics, notation and rules should be changed when desirable. The best MTA representations are always intuitive, simple, and clear. Usually, compactness is best. It is useful to omit from MTA formulas aspects or features of little current importance and retain what is essential. Whenever formulas are contrasted (as in formula **f4**) only differential characteristics of the formulas being compared should be given.

One distinct advantage of using standard notation and rules is that, once known to practitioners, they need not be repeated. However, we are repeating much notation in this book to facilitate learning. Readers of previous chapters are familiar with basic conventions (see table 3.2), which we now summarize again. Operative schemes are written in CAPITALS, and figuratives written in lowercase with an optional asterisk (*) prefixed or postfixed. Parameters (adjunct information to regulate application of operative to figuratives) are written with a number-sign prefix (#); and executives, if we should need to highlight them, can be represented in capitals with a prefixed superscripted E (^{E}OPERATIVE). Temporally structured schemes (fluents) are represented in italics (*OPERATIVE*) or with a pre-superscripted *fl*- (e.g., $^{fl\text{-}}$OPERATIVE, $^{fl\text{-}}$figurative). Information that belongs to the meaning of a main symbolic scheme is written inside square brackets to the right of this scheme, for example, *figurative[…], or OPERATIVE[…]. This meaning may be expressed in ordinary language by a relative clause. Schemes that in the formula must be boosted by *M* usually carry an underline (<u>OP</u> (#<u>scheme</u>, *<u>scheme</u>…scheme)) to indicate this boosting; three schemes are being *M*-boosted in this example. Operatives control (and apply on) schemes located to their immediate right inside parentheses; the parentheses mark the scope of application for the operative. Application/performance of operators in a formula, with its line sequence of operations, goes from the rightmost to the leftmost operator, in that order. Usually rightmost operations are more concrete, and the leftmost ones apply on their results. When a formula has several steps, the order of application goes from the first to last step (i.e., from right to left in the formula). Result of a formula's application is written after an arrow (➔), which stands for Schemes' Overdetermination of Performance (*SOP*). Consult the appendix at the end of the book for more details.

Metasubjective Analysis and Analysis of Figures in Geometry

People uninterested in relating this analysis to classic geometry could, on first reading, skip this section. Metasubjective analysis is always intuitive in Kant's sense, that is, it deals with objects and processes—concrete or abstract—in a truly experiential-representational way, adopting the perspective of the subject and often using infralogical or mereological modes. The method of analysis in classic geometry, or in Newton and Galileo (Hintikka & Remes, 1974), is intuitive in this Kantian-sense of representational concreteness. It is also constructivist, within our definition, as Newton's (1974) writing on his experimental philosophy shows. As described by Hintikka and Remes (1974), two dialectically complementary ways (backward/bottom-up and forward/top-down) of analysis exist that in combination produce the "analysis of figures" (geometrical analysis) of ancient Greek philosophers such as Pappus. Pappus' work was recognized by Polya as an influence. Polya is a twentieth-century innovator in heuristic methods for problem solving in mathematics, and his book *How to Solve It* became very popular (Polya, 1973).

In contrast to Polya's, our metasubjective method is process-organismic and qualitative (dealing with schemes), not fully objective or quantitative. Whereas in the Greek geometry (or analysis of figures) the objects given or created as auxiliary constructions represent actual external figures, lines, or objects, in our task analyses these (given or constructed) objects are schemes, regulated by hidden operators or principles. The contrast is striking with regard to *auxiliary constructions*, which in analysis of figures are drawn to become new objects on which geometry proofs can be based. In the case of MTA, invoked auxiliary constructions are organismic schemes and hidden operators posited as activated within the subjects (when they confront a problem-solving situation). The theory of geometry is essential in analysis of figures, whereas an explicit organismic theory of schemes and operators is needed in our analysis. The two complementary (dialectically intertwined) ways for these analyses follow.

The first way or sub-method is the *backward* (abductive, *ascendent*, bottom-up) way, which begins from the given (problem to be explained), in our case the predicted or postdicted performance, and then proceeds to infer, via abduction, causal determinants (in our case organismic causal processes such as needed schemes or operators). The second, *forward* (*descendent,* deductive, top-down) way of synthesis begins with principles, laws, and theorems (in our case, objective task analysis of the situation and a general operative model of the subject/metasubject), to derive in various steps the results to be explained.

A description of Newton's research analysis, provided by Hintikka and Remes, shows clearly these concrete and dialectical intertwining backward-and-forward ways. Hintikka and Remes (1974, p. 110) wrote:

> The Newtonian method may perhaps be schematized as follows:
>
> (i) An analysis of certain situations into its ingredients and factors →
>
> (ii) An examination into the interdependencies between these factors →

(iii) A generalization of the relationships so discovered to all similar situations →

(iv) Deductive applications of these general laws to explain and to predict other situations.

It is easy to see that steps (i) and (ii) belong to the backward/ascendent submethod, whereas steps (iii) and (iv) belong to the forward/descendent submethod. Further, step (iii) very often constitutes an instance of abduction, as Peirce formulated this form of inference. An abduction is an "explanation" of results by postulating a theoretical hypothesis as a fact under the presupposition of a general law (here we are paraphrasing Apel, 1995, p. 40; see also Johansen, 1993). This inference may be true or false. It is a hypothesis that requires experimental/formal verification.

The descendent, or forward, way derives the intended performance from the subject's organismic operative model and the particulars of the dynamic situation as elucidated by the ascendent/backward way (which for us includes consideration of shadow schemes, affective goals, and intended cognitive goals of the subject). Eventually the analyst can confirm that the ascendent procedure and the descendent procedure are congruent, which heuristically makes acceptable as a model the metasubjective analysis in question.

To illustrate this intertwined ascendent (often based on abduction) and descendent (often based on deduction) method, let us again use the conservation task of Piaget summarized in figure 8.1. After an operation on balls **A** and **B** to determine their equality in mass, and the transformation (**T**) of ball **B** into **B′**, the child is asked whether there is more amount (more to eat if the pieces of candy were to be eaten) in **A** or **B′**. These two balls are the only task-relevant objects present to the subject when this question is raised. To properly address the issue, the child must somewhat proceed as Polya's method suggested, but referring to relevant schemes and other organismic processes: "What is the unknown?" (i.e., What is the problem issue?); "What are the data?" (i.e., What is it that we know?); "What do we have to do in this task?" A here-and-now simplistic analysis may lead the child to conclude that **A** has more (or less) mass than **B′** because the latter appears perceptually to be much thinner (or much longer) and so could have less (or more) mass. These are *F*-operator's effects when the task has not been analytically explored, and the longer length (or greater thinness) of **B′** has not been noticed.

To correct this error of judgment, caused by the "bad" scheme represented in formula **f3**, participants must go beyond the information given (i.e., ball **A** and "sausage" **B′**) and use a mentally ascendent way from the schemes immediately given (the ball and "sausage") and the problem issue (which has more **A** or **B′**?), to the events that have preceded the current situation. Participants may then remember the no-longer-visible ball **B** transformed into **B′** by hand rolling; a transformation that reminds children, who can solve the problem, of transformations in which nothing is added or taken away, and so quantity is preserved—identity transformations, as we adults call them. With the help of this auxiliary construction (recollected inference), the child may conclude that **B** under an identity transformation (i.e., **TB**) has the same amount as **B′** (**TB = B′**). Another ascendent recollection is the initial comparison between **A** and **B**, which leads the child to conclude (a second auxiliary construction) that the object-schemes involved are equal in amount (**A = B**). With these assumptions in place, **TB = B′** and **A = B**, participants can represent, in a descendent analysis manner, the task as the mental-relational representation of the whole problem illustrated in figure 8.1 above (i.e., [A = B & TB = B′ → A?B′]).

This strategy representation raises the issue, within a descendent analysis, of whether these findings can yield the correct problem solution. For a solution, we need to appeal to other auxiliary constructions (i.e., schemes or hidden operators and principles) possibly existent in the participant. As we pointed out before, the auxiliary elements needed cannot be a learned logical principle of transitivity (if A = B and B = B′, then A = B′). Children have not had life opportunities to learn this logical transitivity rule. It could, however, instead be an innate/maturational *F*-operator (principle of simplicity in the brain), as already discussed. This example and Piaget's own model for conservation illustrate a key function of the descendent analysis: to appraise whether a given auxiliary construction or element in a solution model is likely to be available to participants as scheme or as a dynamic synthesis (a momentary emergent representation).

Let us show that this sort of analysis can be applied to geometry. An example of geometric analysis using auxiliary constructions (instantiated schemes or scheme relations) is given in figure 8.2, modified from Polya (1973, p. 216). As this figure illustrates, the participant is given a triangle BCA and asked to prove the theorem that in any triangle the sum of the three angles is equal to two right angles. With this problem and the present triangle, the unknown is the sum of the three angles a, b, and c. Using an ascending/abductive way of inference to go from the triangle to the final construction, which already appears in figure 8.2, a participant may think of physically bringing together the three angles (this would be an executive strategy or plan). This goal may lead him or her to recall a theorem (an instantiated complex relational scheme available in the repertoire—a mental auxiliary construction previously learned). Such theorem is that alternate angles constituted by a straight line and two parallel straight lines are equal. The participant then proceeds (this is a physical auxiliary construction) to draw the parallel line passing by point A illustrated by figure 8.2. They may then notice that this parallel line creates not only an angle alternate to b (b′—marked with double curved line) but also another similar angle c′ that is alternate to c (another complex, now synthesized, relational scheme). Using a descendent/deductive way of analysis (which evaluates all relations found in the total construction) he or she then notices that b′+c′+a are equal to two right angles, because the three together encompass 180 degrees. This proves the theorem.

If this mathematical proof were to be re-presented as a metasubjective process-model of the participant's mind as he or she solved the problem, we should also consider the hidden general-purpose organismic resources, that is, hidden operators (*M, I, E, F,…*) and principles (e.g., *SOP*) that intervene to make possible this solution discovery. Key to the discovery is the moment when participants have the idea of drawing the parallel line on the point A—a psychological (metasubjective) process that Polya (1973, p. 216) may not have understood, because he introduces figure 8.2 by saying: "Fig 23 [i.e., our figure 8.2] which is an inalienable mental property of most of us, needs little explanation." Let us represent this moment when the participant thinks of implementing the executive plan of physically producing the sum of the three angles (a, b, and c) by drawing a parallel line at A. This moment of the analysis is represented by formula **f6**.

$$M\ [\underline{\text{SUM:a,b,c}}\ (\#\{\underline{\text{b=b}'} \leftarrow \underline{\text{2parallels +1oblique line}}\}, \underline{\text{DRAW}}^{L1}(\{*\underline{\text{parallel}}\}_{L1?}, \#\underline{\text{TO:BC}}))\ ] \Rightarrow \text{Figure 8.2} \qquad \textbf{(f6)}$$

Translating into English, this formula says that the scheme operative seeking the physical sum of angles a, b, and c (i.e., SUM:a,b,c—note that here : symbolizes that SUM is an operative that applies on a, b, and c) has a parameter # stating that when two parallel lines are crossed by one oblique line the two appearing alternative angles, b and b′, are equal. Operative SUM and its parameter # are monitoring the DRAWing of a line parallel to the line BC (we represent the adverb "*to*" as an operative TO that applies on the line BC to create its parallel). The result of this operation induced by SOP and *F* (here symbolized by ➔) is represented in figure 8.2. Note that all these schemes are complex and symbolic. Their meaning is mediated by intuitive/experiential schemes that function as signs (not necessarily verbal) of previously found complex scheme

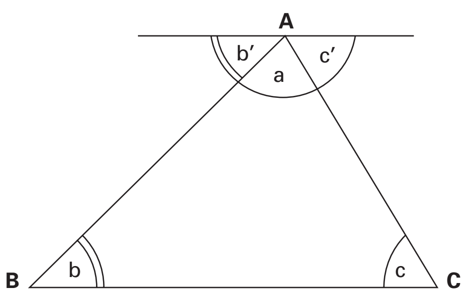

Figure 8.2
Diagram (after Polya, 1973) for proving the theorem regarding the sum of the angles of a triangle.

relations. When participants are practiced in drawing lines, the DRAW operative may perhaps be already *L*-structured (chunked) with the scheme for drawing the line (the superscripted *L1* and $\{\ldots\}_{L1?}$ indicate that this is uncertain). If we count the underlined schemes (there are six—an *M*-demand of $e+6$), our developmental prediction would be that this problem is accessible at the age of 13 to 14 years—unless *L-structuring* has lowered the *M*-demand to $e+5$, accessible to 11- to 12-year-olds.

Final Comments on the Two Modes of Metasubjective Task Analysis

The ascendent/abductive and descendent/deductive mental ways of an effective task analysis are dialectically complementary and in continuous interaction. Together they help to synthesize problem solutions, as we have just illustrated. Our method of task analysis is, in this regard, analogous to an analysis of classic geometry. This is particularly the case when participants must synthesize complex figures or aspects (in our case, schemes at progressively higher levels) to reach the complexity needed for a total proving/processing formula. *Executive MTA* or *executive M-dimensional analysis* is our name for a sort of procedure that uses ascendent/abductive less often than descendent/deductive ways. We illustrated this sort of analysis above with the conservation task and the proof of the sum of angles in a triangle.

Another complementary procedure exists that is mostly step-by-step in an ascendent-abductive manner and can be made as concrete as desirable, by deconstructing and segmenting the often descendent/deductive macrosteps of any executive MTA. This would be a *sequential MTA* or *step M-construction*. Such step-MTA is a sequence, each step representing a moment of process analysis concretely represented. Unlike executive MTA, this sequential MTA may not often require changing levels of processing (decentration or shifting of mental attention during processing of schemes) and may limit itself to recentration (updating of mental-attentional application) when the task does not demand changing levels of abstraction. Sequential step-MTA is initially more intuitive and can be easily unfolded to any degree of refined concreteness by deconstructing steps and unfolding further their sequence. The next chapter uses these foundations to present in a more focused manner our methods of MTA followed by concrete illustrations in various domains.