3 Embodied, Enactive Education: Conservative versus Radical Approaches

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E-approaches to cognition, which have been developed over recent decades, challenge the mainstream representational-cum-computational approach, offering us an alternative understanding of cognition. Yet fundamental differences in philosophical outlook divide the more conservative and radical branches of the E-family. This chapter introduces the core assumptions of E-approaches to cognition and details in which ways E-theorists divide into more conversative and more radical camps.

Bracketing questions about how to decide between these options and other challenges to E-approaches, this chapter instead focuses on articulating possible practical outcomes for educators should they come to accept either of these E-approaches to cognition. Taking an imaginative leap, this chapter asks the following question: Assuming one has adopted either a more conservative or more radical E-framework, how would that choice matter to one's thinking about educational research and practice?

E-Cognition: The Conservative-Radical Spectrum

When it comes to thinking about mind and cognition, "E" is for embodied, enactive, ecological, embedded, extended, or extensive. Under the E-umbrella one finds many different and diverse approaches for thinking about the nature of mind and cognition; certainly not all of these approaches are in perfect agreement. For this reason, it is perhaps best to think of E-approaches as forming a family—a family in which some members get along better with others, and, as in some families, some members do not get along with certain others at all.[1] Yet even although—at least to date—there has yet to emerge one E to rule them all, it is fair to say that even if this family of views is not established in the sense of being fully unified, it is undeniably an emerging force that must be reckoned with by mainstream Western philosophy of mind and cognitive science.

A longstanding tradition in Western thought regards the mind as fundamentally distinct from the body. This is still very much the dominant view in mainstream analytic philosophy of mind and cognitive science, which tends to accept, by default, that the primary work of minds is to represent the world and to reason about it by manipulating said representations. In its classic cognitivist guise, the core assumption of the mainstream representational-cum-computational theories of mind is that intelligence resides wholly and solely inside us in the form of brain-based, information-driven processes.

New evidence puts this mainstream cognitivism under pressure in ways that cannot be ignored, not even by those most wedded to its mindset. Goldman (2012) directs our attention to a large swathe of empirical findings that provide "substantial evidence in support of the pervasive occurrence of embodied cognition" (Goldman, 2012, p. 80). On this long list of E-friendly experimental findings, we find evidence for the use of circuits associated with motor control functions in higher-level language comprehension tasks (Pulvermuller, 2005); the reuse of motor control circuits for memory (Casasanto & Dijkstra, 2010); the reuse of circuits that mediate spatial cognition for a variety of higher-order cognitive tasks (e.g., the use of spatial cognition for numerical cognition; Andres et al., 2007; Hubbard et al., 2005); mirroring phenomena, including not only motor mirroring but also the mirroring of emotions and sensations (Keysers et al., 2010; Rizzolatti et al., 1996; Rizzolatti & Sinigaglia, 2010); and sensitivity to the perceiver's own bodily states when estimating properties of the distal environment (Proffitt, 2008).

Focusing on empirical results of direct relevance to educational research, Shapiro and Stolz (2019) reach a similar conclusion, reporting that:

> *Recent findings* from research literature on learning and cognition from a diverse array of discipline areas, such as philosophy, psychology, linguistics, neuroscience, and computer science, *have contributed to the view that traditional cognitivist accounts of the mind should be challenged because they exclude the close relationship that exists* between *mind and body that is more profound than initially considered.* (p. 20, emphases added)

One way or another, any credible theory of mind must accommodate these kinds of empirical findings that reveal cognition is—in some centrally important respects—connected, and sensitive, to facts of embodiment.

There is a spectrum of possible ways to accommodate these findings in the theoretical space. At the conservative end of the spectrum, we find adjusted accounts of cognition that seek to make only minimal revisions to classically cognitivist views of cognition (Alsmith & de Vignemont, 2012; Gallese & Sinigaglia, 2011; Goldman, 2012). These conservative E-accounts of cognition (or CEC for short) attempt to accommodate recent findings about the role of embodiment in cognition while still conceiving of cognition as wholly representational and entirely brain bound. Theories of the CEC stripe posit mental representations with special formats that represent features of the body, holding that representations of this special kind play a much larger and more fundamental role in cognition than was previously supposed. Importantly, though advertised as E-cognition theories, accounts of this kind assume that the real work of cognition is still done essentially by manipulating mental representations in the brain.

Slightly more daring CEC theories assume that special kinds of action-oriented and sometimes extraneural representations play a part in cognitive activity, helping to drive and steer dynamic and extended cognitive processes (Clark, 1997, 2008b, 2016). Action-oriented representations are hypothesized to be content-bearing states or processes whose functional role is to indicate the presence of, and to sometimes "stand in" for, states of affairs in order to guide and direct specific kinds of action. What makes action-oriented representations interestingly different from the classic cognitivist conception of representations is that the vehicles of the former are not assumed to be always neural and brain bound. Rather, it is assumed that cognitive vehicles and processes can, at least in some cases, reach across brain, body, and environment. CEC approaches of this slightly less conservative stripe are able to put appropriate emphasis on "the profound contributions that embodiment and embedding make" (Clark, 2008a, p. 45).

At the more revolutionary end of the spectrum we find E-approaches that seek to replace classic cognitivist assumptions, root and branch (for a discussion, see Shapiro, 2011). The most radical E-accounts of cognition (or REC for short) characterize cognition, constitutively, as a kind of organismic activity that occurs in the form of sensitive interactions stretching across the brain, body, and environment (Di Paolo et al., 2017; Gallagher, 2005, 2017; Hutto & Myin, 2013, 2017; Thompson, 2007).

The distinguishing feature of REC accounts is their full-fledged opposition to the mainstream view that cognition essentially involves the collection and transformation of information in order to represent the world. Seeking to move away from the idea that the work of minds is always that of representing and computing, these approaches fundamentally challenge accounts of cognition that "take representation as their central notion" (Varela et al., 1991, p. 172).

The radical arm of the E-cognition movement began to be taken seriously by contemporary Western philosophers of mind and cognitive science in the early 1990s, as a consequence of the publication of a landmark book: *The Embodied Mind* by Varela et al. (1991). One major source of inspiration for radicals within the E-family comes from Buddhist thought and philosophy, as introduced in Varela et al. (1991).[2] There have been fruitful conversations between

Buddhist and Western traditions of philosophy of mind precisely because, although both have a dedicated interest, each for the most part approaches these topics from very different angles. This is most evident if one compares Buddhist thinking with the tenets of mainstream classic cognitivism. Simply put, these schools of thought think radically differently, and think radically different things about thinking.

It is not just Buddhism but other ancient Asian traditions of thought as well that embrace something akin to REC approaches to the mind. For example, Ilundáin-Agurruza (2016) has explored points of connection and overlap between radical enactivism and Japanese *dō*—practices that nurture self-cultivation, emotional attunement, and highly skilled performance (e.g., *kendo*—way of the sword). The most discerning reflections on expert performance, which are still used to inform these practices, regard it as requiring a state of mind literally "no mind"—a Zen expression meaning the mind without mind, known as *mushin* in Japanese and *wuxin* (無心) in Chinese.

For example, Slote (2015) claimed that Asian conceptions of mind can serve to correct the "exceedingly intellectualistic" tendencies of Western thought. It would be a mistake, however, to contrast East Asian with Western philosophy in an undifferentiated, wholesale manner. Such an exaggerated contrast misses important nuances. For one thing, this would wrongly depict Western philosophy as being entirely homogenous with respect to the conceptions of mind and cognition that it embraces. There are strands within Western thinking—such as the phenomenological and American pragmatist traditions of thought—that also lend support and succor to REC approaches. It is no accident that Varela et al. (1991) align their project with that of classic thinkers in the phenomenological tradition, including Husserl (1931/1988), Merleau-Ponty (1945/1962), and Sartre (1943/1956). Many contemporary E-theorists continued that work, renovating ideas from the phenomenological tradition and connecting them directly with current theorizing in the cognitive sciences (Gallagher, 2005, Gallagher & Zahavi, 2008).

The same goes for the American pragmatist tradition. Thus, as Gallagher and Lindgren (2015) observe, the pioneers of REC approaches "could have easily drawn on the work of John Dewey and other pragmatists. Indeed, long before Varela et al. (1991), Dewey (1896) clearly characterized what has become known as enactivism" (p. 392, see also Dewey, 1922). REC approaches gain further support from other traditions and frameworks of a more scientific bent, such as ecological psychology (Gibson, 1979), developments in robotics (Brooks, 1991), and dynamical systems theory (Beer, 1998; Thelan & Smith, 1994).

Fundamental differences in philosophical outlook clearly divide the more conservative and radical branches of the E-family. Yet despite this, when taken as a whole, those on both sides of the divide agree that "the emerging interdisciplinary research agenda of embodied cognition contains fertile ground whose surface has, to date, merely been scratched" (Shapiro & Stolz, 2019, p. 21).

E-Lessons for Educators

Exploring and developing E-approaches is undeniably important for understanding cognition. That being the case, it follows that education research needs to take serious stock of these developments because questions of how to educate cannot be kept apart from the best thinking about how we think and learn.[3] The next section touches on other empirical findings in the E-cognition domain that lend credence to Shapiro and Stolz's (2019) claim that "the emerging research agenda of embodied cognition has much to offer educational practitioners, researchers, and/or policy-makers" (p. 34). Notably, despite their evident optimism about the value of E-approaches for education, these authors are cautious about how swiftly and easily this work will be taken up by educationalists.

Citing the alleged "newness" of E-approaches to cognition, Shapiro and Stolz propose that their encouragement of teachers to acquaint themselves with such research "ought best to be construed as a challenge and a clarion call" (2019, p. 33). Although it is true that E-approaches will likely have an uphill battle in gaining acceptance from those working in mainstream educational theory and practice in the West, the anticipated struggle cannot be put down to the "newness" of E-approaches to cognition.[4] Rather the true source of intellectual resistance to such views derives from the fact that classic cognitivist conceptions of cognition not only dominate much Western philosophy of mind and cognitive science but also infuse and inform the great bulk of ordinary and professional thought inside and outside the academy in the West.[5]

For our purposes, let's bracket the question of how to deal with the philosophical barriers that may, for some, block the acceptance of E-approaches. Focusing more directly on possible practical outcomes, the next section takes an imaginative leap and asks a different kind of *how* question: Assuming one has adopted either a more conservative or more radical E-framework, how would that choice matter to one's thinking about educational research and practice?[6]

Conservative and Radical Thinking about Education

E-thinking about cognition creates new possibilities to consider for those in the business of improving education practices. As Shapiro and Stolz (2019) observe, "there is considerable potential for further research and enough existing literature to suggest new ways to think about instruction and classroom design" (p. 34).

As noted in the previous section, one way or another, researchers must take seriously the empirical findings that reveal the extent to which cognition is sensitive to E-factors. Yet how one understands the relevance of those findings and how they might shape educational theory and practice is nonaccidentally tied to one's philosophical outlook about cognition and where it sits on the conservative-radical spectrum.

Exemplary Embodied Learning Techniques

To get a sense of the importance these outlooks can have to thinking about education, it is useful to consider some high-profile cases. There have been recent experimental attempts to explore the possible advantages of using enactive metaphors for educational gain. Unlike the standard use of so-called disembodied or static metaphors (those that map a source onto a target domain by means of words, diagrams, and models), enactive metaphors involve the learner in full-bodied active engagements—embodied engagements that require learners to move "in a prescribed way or play-acting a specified process" (Gallagher & Lindgren, 2015, p. 398).

Exemplifying the way such enactive metaphors might be used in the domain of teaching science, Gallagher and Lindgren (2015) cite a case in which students are asked to "metaphorically identify with an asteroid and act out its movement in a planetary system in order to learn from their own kinesthetic feedback about the principles of gravity" (p. 398) (see also Megowan-Romanowitz, chapter 11 in this volume; Vierya & Vierya, chapter 14 in this volume).[7]

There is also longstanding research into the potential that gesturing has for improving mathematical education and performance. For over two decades, philosophers and cognitive scientists have labored to understand the implications of Susan Goldin-Meadow's discovery of a correlation between gesture and enhanced mathematical performance (Church & Goldin-Meadow, 1986; Goldin-Meadow et al, 1999, 2001; McNeill, 1992; see also Schenck et al., chapter 9 in this volume). These findings are of special import when supported by recent research, such as that conducted by Wagner-Cook et al. (2017) that shows it is gestures themselves and not their accompanying nonverbal behaviors that facilitate mathematical learning.

Alibali and Nathan (2012) have begun to investigate the educational value that may be conferred by the use of different kinds of gestures. This is important since the gestures under scrutiny in educational contexts are not merely those in the familiar playbook used for conventional communication. Rather, they include pointing gestures, iconic gestures (using body parts, say, one's fingers to create a circle), and metaphoric gestures (such as using one's arms to create circular motions indicating "repetition") (see also Marquardt Donovan & Alibali, chapter 10 in this volume). In an experiment involving college students who were asked to prove a mathematical conjecture, Walkington et al. (2014) discovered that students who used dynamic gestures (compared with those who used no gestures or only static, depictive gestures of an iconic sort) were more successful, helping them achieve the correct outcome 63.6 percent of the time (see also Schenck et al., chapter 9 in this volume).

Understanding Embodied, Enactive Learning

These exciting findings raise deeper philosophical questions: Does such embodied activity convey information or content directly to the centers of cognition by bodily routes?[8] Or does it simply lighten the cognitive load, freeing up our centers for cognition to do their work better and quicker? Or is it a way of directly getting a grip on the relevant concepts? Might such embodied activity in of itself constitute direct cognitive gains?

These results can be thought of, most cautiously and conservatively, as showing that embodied activity correlates with certain educational benefits. Or, a bit more bravely, that it is causally producing said benefits. Or, much more radically, that it is actually constitutive of such benefits. It is likely that one's tendency to regard this evidence through a more conservative or more radical lens will correlate with the philosophical framework one adopts for thinking about cognition.

When it comes to thinking about cognition, those at the most conservative end of the spectrum will be inclined to interpret these findings as revealing that embodied activity noncognitively scaffolds tasks by reducing their cognitive load and freeing up properly cognitive resources. Even those who are a bit less conservative in their views about cognition will only be inclined to think that embodied activity at best shapes or contributes to cognition indirectly. Thus, they might think such activity makes a difference: to noncognitive aspects of cognitive processes, or to the way relevant information is formatted or encoded, or by supplying additional or different kinds of information.

There have been recent explanatory attempts, very much in the conservative vein, to understand how engaged activity of the sort Gallagher and Lindgren (2015) describe as "enacting metaphors" might boost educational performance. Kontra et al. (2015) proposed that the learning of scientific concepts such as torque and angular momentum is "aided by activation of sensorimotor brain systems that *add kinetic detail and meaning* to students' thinking" (p. 1, emphasis added). Similarly, Hayes and Kraemer (2017) have speculated that we will understand these educational gains once we understand "how *body-centered* *information, as computed in sensorimotor brain regions* and visuomotor association cortex can form a useful foundation upon which to build an understanding of abstract scientific concepts" (p. 1, emphasis added).

Church and Goldin-Meadow's (1986) also provide a conservative explanation to account for the discordance and concordance that can arise between gestures and speech acts. In describing this approach, Shapiro and Stolz (2019) write,

> In these cases, *the body becomes a conveyer of information* that *might be used to supplement or replace the information* provided by symbolic constructions of the sort more standardly associated with educational instruction, that is, words or writing on a board. (p. 29, emphases added)

It is easy to see a similar CEC line of thinking at work when Shapiro and Stolz (2019) suggest that it may be that certain kinds of gestures might be merely indicating a student's underlying conceptual understanding, or lack thereof. For example, as they put it, it may be that those "who display static gestures are *merely* signaling an existing . . . conceptual misunderstanding" (p. 32).

Those who adopt a REC framework put a very different spin on the evidence, making room for the possibility that nonsymbolic, nonconceptual embodied activity is constitutive, and not merely indicative, of certain kind of competence or knowledge. As such, embodied interactions with specific kinds of phenomena would qualify as varieties of knowledge and competence in their own right.

In thinking of certain embodied activities as constitutively intelligent, REC accounts can tap into a longstanding philosophical tradition in which intelligent performances are not explained in terms of "underlying, rationalising knowledge enabling the competence" (Wright, 2007, p. 498). Intelligent embodied engagements can be conceived of as structured doings that "make up a structured pattern of dynamic, bodily interaction with the environment that *exhibits* intelligence" (Hasselberger, 2018, p. 455, emphasis added).

To illustrate the point, consider the innovative work that is being done with mathematics imagery trainers, or MITs (see also Flood et al., chapter 12 in this volume, and Tancredi et al., chapter 13 in this volume).[9] MITs use natural user interface systems that enable children to engage in tasks that initially do not demand any proficiency with mathematical symbols at all, only sensorimotor behaviors such as moving (virtual) objects in order to satisfy some task condition. Once they have solved the set problem, the students are offered mathematical tools to enhance their interactions. The students adopt these tools because they recognize in them potential utilities for enhancing their actions. But in so doing the students shift into quantitative forms of reasoning about their own actions.

These innovative educational devices focus on giving students opportunities for nonsymbolic interactions with mathematical phenomena. MITs have been designed specifically to allow students to "experience first, signify later" (Abrahamson, 2015a, 2015b; see also Hutto et al., 2015; Hutto & Sánchez-García, 2015). Importantly, MITs enable children to engage in tasks that initially do not demand any proficiency with mathematical symbols at all but rely only on their engaging in embodied ways with the interface (e.g., moving virtual objects so as to satisfy specific task conditions).

In other words, MITs allow users to get a nonsymbolic, embodied grip on mathematical phenomena (Abrahamson, 2020). They are designed so that specific and mathematically relevant sensorimotor patterns arise while students use them to solve set tasks, such as keeping a screen green, which can only be achieved if the participant moves their body in conformity to a mathematical rule. Moving in accord with these patterns is novel for the student; as they explore what it takes to solve the task, they develop and demonstrate their competence in mastering the relevant norms in an embodied, enactive manner (for more on MITs, see Tancredi et al., chapter 13 in this volume; for a mathematics imagery trainer for proportion [MITp] schematic map, see Flood et al., chapter 12 in this volume).[10]

The REC slogan with respect to nonsymbolic embodied educational activity of this sort is not "stop thinking and start doing" but "starting thinking by doing." Or, as Dennett (2017) would have it, REC embraces the idea that "competence without comprehension is nature's way" (p. 84). Viewing these phenomena through the REC lens, one might be inclined to follow Glenberg (2008) in concluding that "all of these studies point to the same conclusion: Mathematics is not the cognitive manipulation of abstract symbols by rules" (p. 359). That conclusion, however, does not appear to be supported by the evidence. It is much safer to conclude that mathematics is "not only" the manipulation of abstract symbols by rules.

To provide a complete REC account of mathematical cognition requires explaining how it is possible that symbolically based concepts can be constructed and emerge from nonsymbolic embodied activity—without surrendering the idea that the content of mathematical propositions and the rules of mathematics are strongly objective. A fully satisfying account of mathematical cognition of this kind will need to make sense of its embodied, nonconceptual, nonsymbolic varieties as well as those that are symbol-involving. It will also need to provide workable explanations of how these two forms of mathematical cognition interrelate and interact despite having special features that strongly distinguish them.

In other words, a complete REC account of mathematical cognition needs to accommodate both its nonconceptual, nonsymbolic and symbolic forms.[11] This can be achieved if we assume that basic mathematical performances are not best explained in terms of learners already grasping the content of a set of rules. Instead, following REC, we need to embrace the view that, in general, cognizing is a matter of embodied engagements that enable us to get "a *grip* on the *patterns that matter* for the *interactions that matter*" (Clark, 2016, p. 292).

With this in mind, and taking a leaf out of Malafouris's material engagement theory (2013), we can think of symbols as special objects that we learn to manipulate by means of mastering public practices in accord with special norms and rules. Accordingly, "[mathematical symbols] are not an accomplishment of the [human] brain, they are an opportunity for the [human] brain—that is an opportunity for active material engagement" (Malafouris, 2013, p. 169, with edits). Moreover, the knowledge of how to use such symbols does not come from anything like an instructive prior intention, rather "the [mathematical] intention is constituted, at least partially, by [how we engage with] the [symbols themselves]" (Malafouris, 2013, pp. 173–174, with edits).

A fully detailed and satisfactory theory of mathematical cognition will require detailed accounts of the various forms and norms of mathematical cognition, as well as their origins in practices that emerged in human prehistory and those that now shape individual development and acquisition of mathematical competence (see Hutto & Satne, 2015).

REC approaches do not regard embodied activity of the sort under scrutiny here, whether purely embodied or symbol-involving, as merely instrumental—as serving, for example, only as a different sort of bodily-based supply chain for information that is to be processed by brain-based computations over mental representations.[12]

The Future of Enactive, Embodied Education

The foregoing analysis reveals the potential for fruitful alliances between philosophers working in the domain of E-cognition and educational researchers. The cross-disciplinary work of philosophers and educational scientists and practitioners can be mutually beneficial.

Philosophers gain from analyzing empirical studies that require them to think differently about the nature of cognition. Educational scientists, practitioners, and policy-makers gain by having a deeper understanding of the different philosophical ways of accommodating such findings. Knowing about those various possibilities can help them when it comes to evaluating educational activities and tools and their potential to improve teaching practice by enabling educators to do things differently.

A positive outcome of such collaborations would be for policy-makers to recognize the potential of implementing paradigms from empirical-oriented philosophy and learning sciences in the service of educational institutions. As the examples and analysis provided the previous sections demonstrate, there is potential to break new ground in educational research, practice, and policy by attending to the available evidence about cognition and considering it through the various E-frameworks we have described (see Abrahamson et al., in press).

The observations of this chapter should encourage philosophers and educators to join forces in investigating and refining our understanding of what E-approaches to cognition have to offer to teaching, and—on that basis—to cooperate in thinking about special educational tools and practices that may someday become mainstays of the regular curriculum.

Notes

1. For a detailed overview of the history and differences between E-approaches and an update on the emerging debates within and beyond this family of views, one could hardly do better than to look at Newen et al.'s *The Oxford Handbook of 4E Cognition* (2018).

2. As Thompson remarks in a recent interview, "I think it's fair to say it was the first book that related Buddhist philosophy to cognitive science, the scientific study of the mind, and the Western philosophy of mind" (Littlefair, 2020).

3. Or, to put the point in conditional, one could say, along with Glenberg (2008), "If embodied approaches to cognition are on the right track, then they should provide key insights into educational processes" (p. 370).

4. The struggle for acceptance that E-approaches are likely to face is highlighted by the mere fact that the entire family of E-views is deemed to have to prove itself against a reigning champion. We can see this assumption at work in the very idea that E-cognition has a kind of "upstart status" (Shapiro & Stolz 2019, p. 33), and that it is thought to be, despite its long history, "still in its infancy" (Shapiro & Stolz, p. 34).

5. Key educational decision-makers are likely to have strong intuitions and deeply held philosophical convictions about the nature of specific domains, such as, say, mathematics, and how these must be taught in light of their more general views about the nature of cognition. Such deep-seated intuitions, though implicit and invisible, can play a powerful and perhaps pivotal role when it comes to evaluating the tenability and plausibility of new teaching methods and practices. For further discussion of how certain philosophies get embedded and infused into the warp and weft of our everyday thinking through our sociocultural practices and institutions, see Hutto (2020).

6. We follow Shapiro and Stolz (2019) in supposing that the question of interest is not "whether embodied cognition might help to inform educational practices, but how" (p. 26).

7. Work on enactive metaphors is in part inspired by Lakoff and Johnson's (1980, 1999) attempt to show that abstract concepts have their roots in metaphors grounded in embodied activity.

8. A standard assumption, as Glenberg (2008) reports, is that "perceptual systems are used to encode the mathematical information, but then the cognitive processes are independent of any perceptual information such as modality of presentation" (Glenberg, 2008, p. 358).

9. The Berkeley-based Embodied Design Research Laboratory began research and design of the MIT devices in 2008, focusing on proportions (Abrahamson et al., 2016). MITs have been implemented successfully to create effective learning opportunities for young children studying challenging

concepts, including area (Shvarts, 2017), the Cartesian coordinate system (Duijzer et al., 2017), and parabolas (Shvarts & Abrahamson, 2019).

10. It would be interesting to conduct quantitative experiments to systematically evaluate what the effects, if any, embodied problem-solving using MITs might have on students' performance in more canonical symbol-based mathematical tasks.

11. Providing such an account would answer sceptics who hold that REC accounts are not capable of giving a full general account of cognition, since mathematics is typically held up as posing the greatest challenge for such approaches to cognition to accommodate (see, e.g, Núñez, 2008).

12. In rejecting the last vestiges of the information-processing framework of classical cognitivism, REC gives a quite different account of the data gleaned from neuropsychology that emphasize action-oriented processes, brain plasticity, and neural 're-use' (Anderson, 2014; Gallagher, 2017).

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