APPENDIX D

Proof of Theorem 8.1

In this appendix we prove

Theorem 8.1 *If (8.3)–(8.9) hold, then*

$$\lim_{m,n\to\infty} \mathrm{Pr}_n^m(\mathrm{RSP}) \;=\; \lim_{m,n\to\infty} \mathrm{Pr}_n^m(\mathrm{RABD}) \;=\; 1.$$

Proof: One first verifies by a routine induction on $m + n$ that $p_n^m(H)$ is a monotonously nondecreasing function on σ for $p \in \{\mathrm{Pr}, \mathfrak{m}, \mathfrak{s}, \mathfrak{R}, \mathfrak{S}\}$ and $H \in \{\mathrm{RSP}, \mathrm{RABD}\}$. Note that because all these functions are bounded, they converge, and so their limits exist. Further observe that it is a direct consequence of (8.1), the rule for determining probabilities conditional on uncertain evidence, that, for any A, if (1) $\mathrm{Pr}(\mathrm{RSP} \mid E/e) \geqslant \mathrm{Pr}(\mathrm{RSP})$ and (2) $\mathfrak{m}(A \mid E/e) \geqslant \mathfrak{s}(A \mid E/e)$, then

$$\text{(D.1)}\quad \mathrm{Pr}(A \mid E/e) \;\geqslant\; \mathrm{Pr}(\mathrm{RSP})\,\mathfrak{m}(A \mid E/e) \;+\; \mathrm{Pr}(\neg\mathrm{RSP})\,\mathfrak{s}(A \mid E/e).$$

It follows from (D.1) and assumptions (8.6) and (8.7) that, for all $n, m \in \mathbb{N}$

$$\text{(D.2)}\quad \mathrm{Pr}_{n-1}^m(\mathrm{RABD} \mid E_n/e_n) \;\geqslant$$
$$\mathrm{Pr}_{n-1}^m(\mathrm{RSP})\,\mathfrak{m}_{n-1}^m(\mathrm{RABD} \mid E_n/e_n) + \mathrm{Pr}_{n-1}^m(\neg\mathrm{RSP})\,\mathfrak{s}_{n-1}^m(\mathrm{RABD} \mid E_n/e_n).$$

Next note that, by (8.2), (8.5), and (8.7), we have for some $k \geqslant 1$ that

$$\mathrm{Pr}_{n-1}^m(\mathrm{RSP}) \;\geqslant\; \mathrm{Pr}_{n-k}^{m-1}(\mathrm{RABD} \mid E'_m)\,\mathfrak{R}_{n-k}^{m-1}(\mathrm{RSP} \mid E'_m) +$$
$$\mathrm{Pr}_{n-k}^{m-1}(\neg\mathrm{RABD} \mid E'_m)\,\mathfrak{S}_{n-k}^{m-1}(\mathrm{RSP} \mid E'_m).$$

Together with (D.2), this yields

$$\begin{aligned}&\mathrm{Pr}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) \;\geqslant\\ &\Big[\mathrm{Pr}^{m-1}_{n-k}(\mathrm{RABD} \mid E'_m)\,\mathfrak{R}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m) +\\ &\qquad \mathrm{Pr}^{m-1}_{n-k}(\neg\mathrm{RABD} \mid E'_m)\,\mathfrak{S}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m)\Big] \times\\ &\qquad \mathfrak{m}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) \;+\; \mathrm{Pr}^{m}_{n-1}(\neg\mathrm{RSP})\,\mathfrak{s}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n).\end{aligned}$$

Expanding yields

$$\begin{aligned}&\mathrm{Pr}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) \;\geqslant\\ &\quad \mathrm{Pr}^{m-1}_{n-k}(\mathrm{RABD} \mid E'_m)\,\mathfrak{R}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) +\\ &\quad \mathrm{Pr}^{m-1}_{n-k}(\neg\mathrm{RABD} \mid E'_m)\,\mathfrak{S}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) +\\ &\qquad \mathrm{Pr}^{m}_{n-1}(\neg\mathrm{RSP})\,\mathfrak{s}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n).\end{aligned}$$

Rearranging then yields

$$\begin{aligned}\text{(D.3)}\quad &\mathrm{Pr}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) -\\ &\mathrm{Pr}^{m-1}_{n-k}(\mathrm{RABD} \mid E'_m)\,\mathfrak{R}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) \;\geqslant\\ &\quad \mathrm{Pr}^{m-1}_{n-k}(\neg\mathrm{RABD} \mid E'_m)\,\mathfrak{S}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) +\\ &\qquad \mathrm{Pr}^{m}_{n-1}(\neg\mathrm{RSP})\,\mathfrak{s}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n).\end{aligned}$$

Now note that, by the limit rule for subtraction

$$\begin{aligned}\text{(D.4)}\quad &\lim_{m,n\to\infty}\Big[\mathrm{Pr}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) -\\ &\mathrm{Pr}^{m-1}_{n-k}(\mathrm{RABD} \mid E'_m)\,\mathfrak{R}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n)\Big] \;=\\ &\quad \lim_{m,n\to\infty}\mathrm{Pr}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n) \;-\\ &\lim_{m,n\to\infty}\Big[\mathrm{Pr}^{m-1}_{n-k}(\mathrm{RABD} \mid E'_m)\,\mathfrak{R}^{m-1}_{n-k}(\mathrm{RSP} \mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\mathrm{RABD} \mid E_n/e_n)\Big],\end{aligned}$$

and that, by the limit rule for products

(D.5)

$$\lim_{m,n\to\infty}\left[\Pr^{m-1}_{n-k}(\text{RABD}\mid E'_m)\,\mathfrak{R}^{m-1}_{n-k}(\text{RSP}\mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\text{RABD}\mid E_n/e_n)\right] = \lim_{m,n\to\infty}\Pr^{m-1}_{n-k}(\text{RABD}\mid E'_m) \times \lim_{m,n\to\infty}\mathfrak{R}^{m-1}_{n-k}(\text{RSP}\mid E'_m) \times \lim_{m,n\to\infty}\mathfrak{m}^{m}_{n-1}(\text{RABD}\mid E_n/e_n).$$

Given assumptions (8.8) and (8.9), (D.5) simplifies to

(D.6)

$$\lim_{m,n\to\infty}\left[\Pr^{m-1}_{n-k}(\text{RABD}\mid E'_m)\,\mathfrak{R}^{m-1}_{n-k}(\text{RSP}\mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\text{RABD}\mid E_n/e_n)\right] = \lim_{m,n\to\infty}\Pr^{m-1}_{n-k}(\text{RABD}\mid E'_m).$$

And from equations (D.3)–(D.6) we obtain

(D.7)
$$\lim_{m,n\to\infty}\left[\Pr^{m}_{n-1}(\text{RABD}\mid E_n/e_n) - \Pr^{m-1}_{n-k}(\text{RABD}\mid E'_m)\right] \geqslant \lim_{m,n\to\infty}\left[\Pr^{m-1}_{n-k}(\neg\text{RABD}\mid E'_m)\,\mathfrak{S}^{m-1}_{n-k}(\text{RSP}\mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\text{RABD}\mid E_n/e_n) + \Pr^{m}_{n-1}(\neg\text{RSP})\,\mathfrak{s}^{m}_{n-1}(\text{RABD}\mid E_n/e_n)\right].$$

Because $\Pr^m_n(\text{RABD})$ converges, we have

(D.8)
$$\lim_{m,n\to\infty}\left[\Pr^{m}_{n-1}(\text{RABD}\mid E_n/e_n) - \Pr^{m-1}_{n-k}(\text{RABD}\mid E'_m)\right] = 0.$$

From (D.7) and (D.8) it follows that

$$\lim_{m,n\to\infty}\left[\Pr^{m-1}_{n-k}(\neg\text{RABD}\mid E'_m)\,\mathfrak{S}^{m-1}_{n-k}(\text{RSP}\mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\text{RABD}\mid E_n/e_n) + \Pr^{m}_{n-1}(\neg\text{RSP})\,\mathfrak{s}^{m}_{n-1}(\text{RABD}\mid E_n/e_n)\right] = 0.$$

Hence it must be that both

(D.9)

$$\lim_{m,n\to\infty}\left[\Pr^{m-1}_{n-k}(\neg\text{RABD}\mid E'_m)\,\mathfrak{S}^{m-1}_{n-k}(\text{RSP}\mid E'_m)\,\mathfrak{m}^{m}_{n-1}(\text{RABD}\mid E_n/e_n)\right] = 0$$

and

$$\text{(D.10)} \qquad \lim_{m,n\to\infty}\left[\Pr^{m}_{n-1}(\neg\text{RSP})\,\mathfrak{s}^{m}_{n-1}(\text{RABD}\mid E_n/e_n)\right] = 0.$$

Because, first, by (8.9) it holds that $\lim_{m,n\to\infty}\mathfrak{m}^{m}_{n-1}(\text{RABD}\mid E_n/e_n) = 1$; second, $\mathfrak{S}^{m-1}_{n-k}(\text{RSP}\mid E'_m)$ is monotonously nondecreasing; and third, by (8.3) it holds that $\mathfrak{S}^{0}_{0}(\text{RSP}) > 0$, we have $\lim_{m,n\to\infty}\mathfrak{S}^{m-1}_{n-k}(\text{RSP}\mid E'_m) \neq 0$. Therefore, (D.9) can only be the case if

$$\lim_{m,n\to\infty}\Pr^{m-1}_{n-k}(\neg\text{RABD}\mid E'_m) = 0$$

and so

$$\lim_{m,n\to\infty}\Pr^{m-1}_{n-k}(\text{RABD}\mid E'_m) = 1,$$

that is, because $\Pr^{m}_{n}(\text{RABD})$ is monotonously nondecreasing,

$$\text{(D.11)} \qquad \lim_{m,n\to\infty}\Pr^{m}_{n}(\text{RABD}) = 1.$$

And it follows from (D.10) and that as $\mathfrak{s}^{m}_{n-1}(\text{RABD}\mid E_n/e_n)$ is monotonously nondecreasing and $\mathfrak{s}^{0}_{0}(\text{RABD}) > 0$, $\lim_{m,n\to\infty}\mathfrak{s}^{m}_{n-1}(\text{RABD}\mid E_n/e_n) \neq 0$, that

$$\lim_{m,n\to\infty}\Pr^{m}_{n-1}(\neg\text{RSP}) = 0$$

and hence that

$$\lim_{m,n\to\infty}\Pr^{m}_{n-1}(\text{RSP}) = 1,$$

that is,

$$\text{(D.12)} \qquad \lim_{m,n\to\infty}\Pr^{m}_{n}(\text{RSP}) = 1.$$

Finally, from (D.11) and (D.12) we get

$$\lim_{m,n\to\infty}\Pr^{m}_{n}(\text{RABD}) = \lim_{m,n\to\infty}\Pr^{m}_{n}(\text{RSP}) = 1,$$

which had to be proved. ■