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8 Seeing Is Achieving: The Importance of Fingers, Touch, and Visual Thinking to Mathematics Learners

Jo Boaler

Sometimes there is a research finding that is such a surprise—conveying a result that is so far from the realms of what seems possible—it causes cognitive dissonance. The extent to which finger knowledge predicts a person's achievement in mathematics is one of those areas of research. This result becomes easier to understand when it is placed within the broader research literature emerging from neuroscience, showing that our brain processes mathematics with the help of two visual brain pathways. This chapter shares some of the compelling and important new research around visual and physical experiences with mathematics as well as some activities and resources that teachers and parents may use to help students benefit from this important knowledge.

Most people think of the mind and the body as completely separate entities, with the mind holding knowledge and abstractions, and the body passively taking ideas from the mind to the physical world. But embodied cognition researchers point out that many of our mathematical concepts are held in visual and sensory motor memories.

Embodied cognition researchers note the ways in which people posture, gaze, gesture, point, and manipulate their writing tools as evidence that mathematical ideas are represented (in part) in the motor and perceptual areas of the brain (Nemirovsky et al., 2012). Indeed, people often draw shapes in the air, using space around us to "spread out" our ideas. For example, we may decide that one side of a table represents an idea and point back to it when we want to refer to that idea, even though there is nothing actually there—just our previous motions designating the space (Alibali & Nathan, 2012). Researchers have concluded that the body is an intrinsic part of cognition: the parts of our brain that control perception and the movement of our bodies are also involved in knowledge representation (Hall & Nemirovsky, 2012). It is fairly well known that knowledge of dance or sport is held in sensory motor areas of our brain, yet many would be surprised to learn that mathematics knowledge is also held in sensory motor memories.

Finger Perception and Mathematics Understanding

Ilaria Berteletti and James R. Booth (2015) showed that the "somatosensory finger area" of the brain, an area thought to represent finger sensation, helps with finger representation of ideas even when fingers are not being used. The researchers found that when eight- and thirteen-year-old students were given complex subtraction problems, the somatosensory finger area of their brain was activated, even though the students did not use their fingers. The researchers also found that this finger representation area was involved to a greater extent with more complex problems that involved higher numbers and more manipulation.

Evidence from both behavioral and neuroscience studies shows that when people receive training on ways to perceive and represent their own fingers, they develop better representations of their fingers, also known as "finger perception," which leads to higher mathematics achievement (Gracia-Bafalluy & Noël, 2008; Ladda et al., 2014). Researchers found that when six-year-old children improved the quality of their finger perception, they improved in arithmetic knowledge, particularly subitizing (the ability to recognize a number in a set without counting), counting, and number ordering.

In an eight-week experimental intervention, Stanford researchers found that first-grade students who used a robotic device that helped develop finger perception, with students using fingers to choose answers to mathematics problems and receiving haptic feedback in their fingers, improved their mathematics achievement to a greater extent than a similar group of students working on the same questions with a computer (Martinez et al., in press).

Penner-Wilger & Anderson (2013) found that even university students' finger perception predicted their scores on calculation tests. She also found that finger perception in grade 1 predicted students' achievement on number comparison and estimation in grade 2 (Penner-Wilger et al., 2009). Researchers assessed whether children had a good awareness of their fingers by touching the finger of a child—held under a desk or book so the child could not see which finger was being touched—then asking them which finger was being touched.

There is clear agreement among neuroscientists that the development of finger perception is important for mathematics achievement, yet debate exists about why this is the case. As neuroscientist Brian Butterworth has pointed out, "without the ability to attach number representation to the neural representation of fingers and hands . . . numbers themselves will never have a normal representation in the brain" (1999, pp. 249–250).

One of the recommendations from neuroscientists is that schools focus on *finger discrimination*—developing students' abilities to distinguish between different fingers. The researchers not only have pointed out the importance of

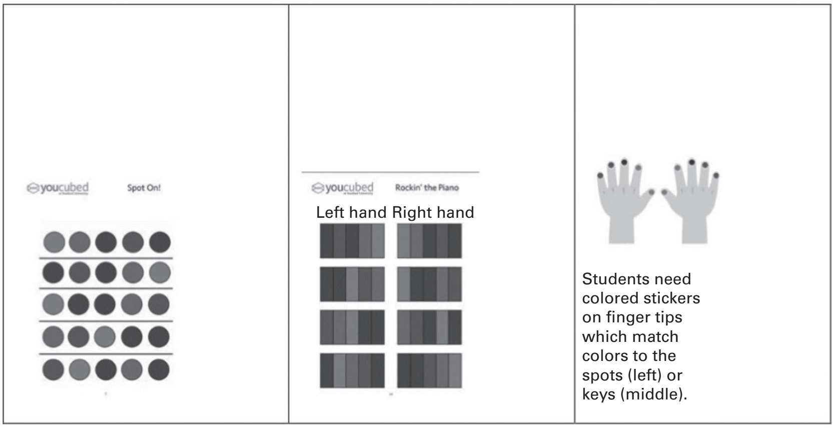


Figure 8.1
Some different examples of finger activities on Youcubed.org.

number counting on fingers for brain development and future mathematics success, but they also advocate that schools help students discriminate between their fingers. This seems particularly significant because neither schools nor curriculum have traditionally paid attention to this kind of finger-based work. Instead, many teachers have been led to believe that finger use is babyish and to be moved past as quickly as possible (see Boaler, 2019). Fingers may be a student's most useful visual aid—critical to mathematical understanding and brain development—that endures well into adulthood. Similarly, the presence of good finger perception among musicians is now thought to be an important part of the reason that musicians often display higher mathematical understanding (see Beilock, 2015). Neuroscientists recommend that fingers be regarded as the *functional link* between numerical quantities and their symbolic representation, and an external support for learning arithmetic problems.

We have drawn upon such knowledge to provide adapted exercises to train children in finger perception. The Youcubed Team, a Stanford center dedicated to giving research-based mathematics resources to teachers and parents, has used such knowledge to create engaging classroom and home activities for young students, which are provided free on Youcubed.org (see figure 8.1; see also the resources section).

The Multidimensional Nature of Understanding

The research on the importance of finger understanding becomes less surprising when it is placed within other research on the visual pathways, which are important for mathematical work and understanding.

Our brains are made up of "distributed networks," and when we handle knowledge, different areas of the brain are activated and communicate with each other. When we work on mathematics in particular, brain activity is distributed between many different networks, which include two visual pathways: the ventral and dorsal visual pathways (see figure 8.2). Neuroimaging has shown that even when people work on a number calculation such as 12×25, with symbolic digits (12 and 25), our mathematical thinking is grounded in visual processing.

A widely distributed brain network underpins the mental processing of mathematics knowledge (Menon, 2014), which includes dynamic communication between the brain systems for memory, control, and detection and the *visual processing regions of the brain*. The dorsal visual pathway has reliably been shown to be involved when both children and adults work on mathematics tasks (see figure 8.2). This area of the brain particularly comes into play when students consider visual or spatial representations of quantity, such as a number line (figure 8.3).

Number line knowledge has been shown in cognitive studies to be particularly important for the development of numerical knowledge and a precursor of children's academic success (Siegler & Booth, 2004; Hubbard et al., 2005;

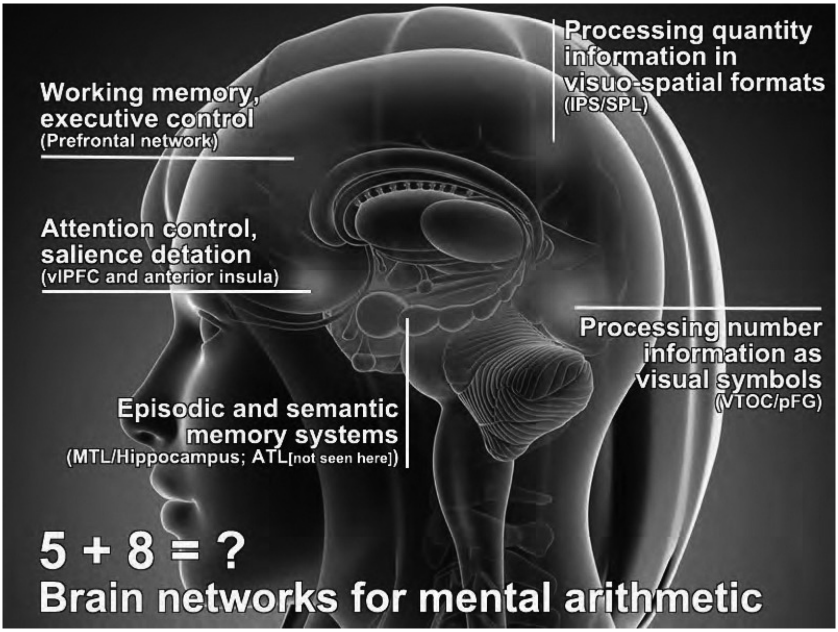


Figure 8.2
Brain networks for mental math. Source: Lang Chen.

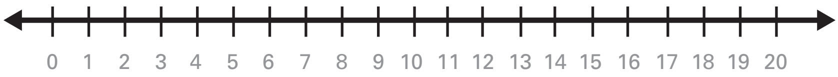


Figure 8.3
A number line.

Kucian et al., 2011; Schneider et al., 2009). Not only are mathematics learners helped by seeing mathematical ideas visually, but also an important part of brain development comes when different brain pathways communicate with each other. When a student sees a problem numerically, and they also depict the idea visually, communication occurs between brain pathways. Researchers even found that students from low socioeconomic backgrounds were achieving at the same levels as students from higher socioeconomic backgrounds after just four fifteen-minute sessions of using a number line (Siegler & Ramani, 2008).

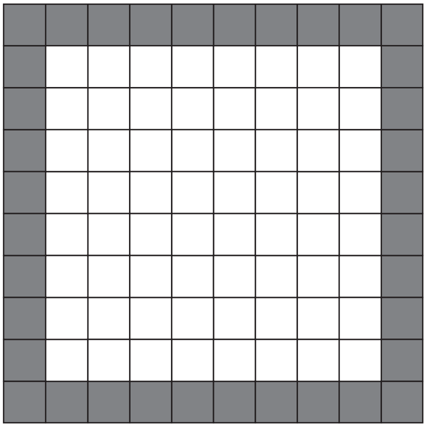
The frontal networks, the medial temporal lobe, and, importantly, the hippocampus are also important brain areas within the "mathematics" network (see figure 8.2). In a recent study, when regular people were compared with particularly successful "trailblazing" people, it was found that the successful people had more communication between brain pathways (see Boaler, 2019a). This suggests that our students should experience mathematics in a more multidimensional way, with multiple opportunities to see and experience mathematics in different ways—through numbers, but also through touch, seeing, drawing, building, and writing in words.

Classroom Examples

In the following section, I describe how the Youcubed Team has developed a middle-school summer mathematics experience for students. Students in this program spend approximately thirty hours, or eighteen lessons, experiencing mathematical ideas visually and creatively.

In our own teaching of the summer camp, we found that students increased their achievement on standardized tests by the equivalent of 2.8 years of school, after eighteen lessons (see also Boaler, 2019b). At the end of the teaching, the students described their experiences as transforming their views of mathematics and, importantly, their own potential.

As part of our summer teaching we taught algebra as a visual subject as well as a numerical and symbolic one. Algebra classes are often dedicated to students rearranging symbols, and students approach important mathematical concepts such as functions through numbers and symbols without any visual understandings. In one activity, for example, we asked students to look briefly at a border



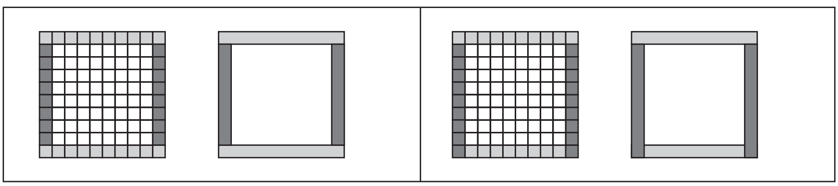


Figure 8.4
An example of an activity from YouCubed. Note the difference in the patterns and . The students thought about the number of squares in the border in many different ways, which they described at first numerically, then though words, and then algebraically.

around a square, and we asked them to work out how many squares were in the border without counting them (figures 8.4 and 8.5) (see also Boaler & Humphreys, 2005). The students' different ways of seeing were a resource for developing the students' understanding of functional relationships and algebraic equivalence.

In a different lesson we asked the students to consider distance-time graphs, an area of mathematics that is notoriously challenging even for college students (Clement, 1989). We invited the students to learn about distance, time, and velocity by physically walking the line of a distance-time graph, using a motion sensor that tracked their movement. The students stunned district visitors when a girl gave a perfect explanation of the graphing of velocity, rejecting a common misconception that is held by millions of students. When the students explained the concept, they gestured with their hands to show the movement, again showing that their understanding of the concept was held in sensorimotor memories.

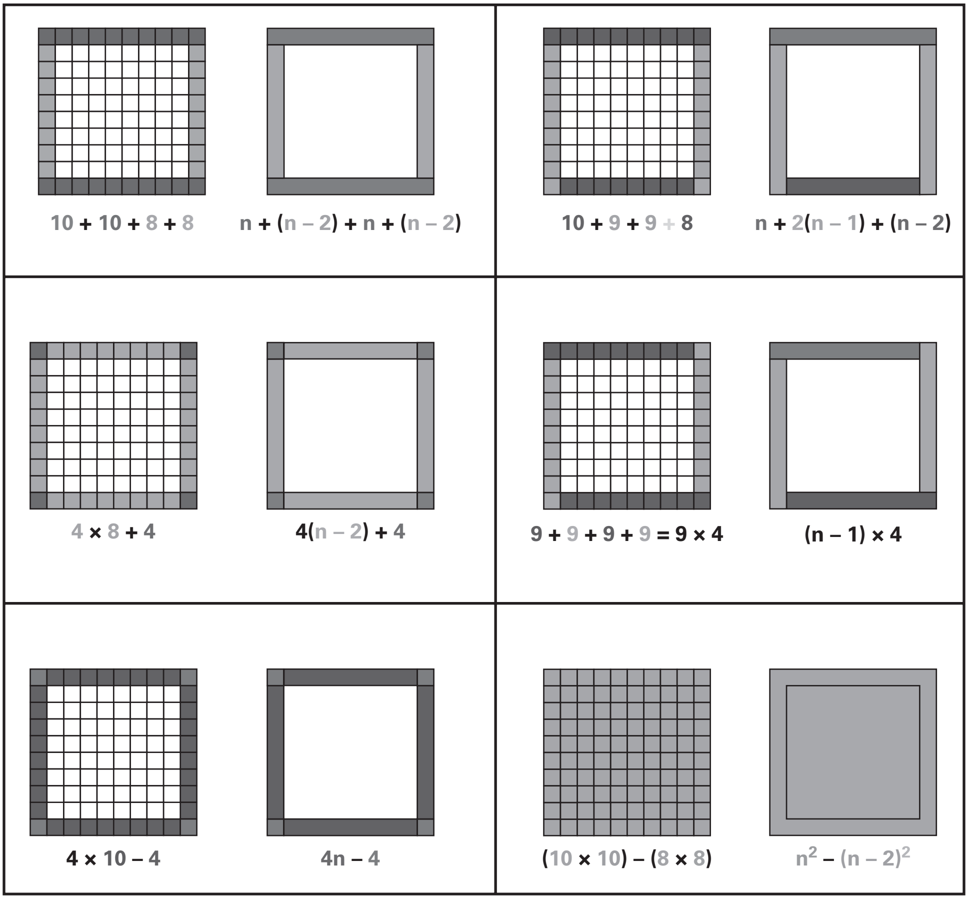


Figure 8.5
A continuation of the activity in figure 8.4 from Youcubed.

To engage students in productive visual thinking, our students were asked, at regular intervals, how they *saw* mathematical ideas, and they were asked to draw what they were seeing. One of the students' reflections at the end of one of our mathematics camps was

> It's like the way the way our schools did it, it's like very black and white. And the way people do it here [in summer camp], it's like very colorful, very bright. You have very different varieties you're looking at. You can look at it one way, turn your head, and all of a sudden you see a whole different picture.

The teaching of velocity through movement was clearly powerful for the students, and motion is a helpful resource for students in reaching depth of understanding (see Boaler, 2019a, and Youcubed.org).

When mathematics classrooms focus on numbers, status differences between students often emerge to the detriment of classroom culture and learning. Some students state that work is “easy” or “hard” or announce they have “finished” after racing through a worksheet. But when the same content is taught visually, the status differences that so often beleaguer mathematics classrooms often disappear. Thomas West has noted the equalizing effect of visual work with adults, describing the time that various experts from academic disciplines came together to think visually, showing mutual respect toward each other and to different ideas, in ways that rarely happen when work is numerical (West, 2014). It seems possible that visual mathematics may contribute to equitable outcomes—valuing students’ thinking in different ways as well as encouraging deep engagement.

In my teaching of Stanford undergraduates, I introduce mathematics problems to them saying, “I don’t care about speed. In fact, I am unimpressed by those who finish quickly—that shows you are not thinking deeply. Instead, I would like to see interesting and creative representations of ideas.” After a few lessons the students start to broaden their views of mathematics and begin to create different insightful representations, along with new understandings of ideas.

Conclusion

The evidence I have reviewed—showing the distributed, visual, and physical nature of mathematical understanding—seems particularly significant when considering that mathematics, for most students, is taught as a series of numbers and abstract concepts. It is probably not surprising that so many students feel that mathematics is inaccessible and uninteresting when they are plunged into a world of abstraction and numbers. Most curriculum standards and published textbooks do not invite visual thinking. Many textbooks provide pictures, but they do not invite students to think visually or to draw their own representations of ideas. When textbook and classroom approaches do encourage visual work, it is usually encouraged as a prelude to the development of abstract ideas rather than a tool for seeing and extending mathematical ideas and strengthening important brain networks.

The new knowledge that we have, showing the visual processing of mathematical ideas, may explain the many research studies indicating that the teachers who emphasize visual mathematics and who use well-chosen manipulatives encourage higher achievement for students, not only in elementary school (e.g., Reimer & Moyer, 2005) but also in middle school, high school, and college (Sowell, 1989). Entire volumes from the Mathematical Association of America have been devoted to the encouragement of visual mathematics in college (see, e.g., Zimmermann & Cunningham, 1991). The visual K-12 mathematics lessons created by our team at Youcubed are downloaded and used in every state across the United States and in approximately two-thirds of US schools. In surveys completed by teachers and students, 88 percent of teachers say they would like more of the activities, and 83 percent of students report that the visual activities enhance their learning of mathematics.

Despite the prevalence of the idea that drawing, visualizing, or working with models is something only for young children, some of the most interesting and high-level mathematics is predominantly visual. Mathematician Maryam Mirzakhani contributed important new mathematical ideas though visual mathematics. Visual mathematics can also come from abstract mathematics and can extend the ideas to much higher levels. They can also inspire students and teachers to see mathematics differently—to see the creativity and beauty in mathematics and to understand mathematical ideas.

Years ago, workplace knowledge was based on words and numbers, but our new knowledge of the world is based largely on images that are "rich in content and information" (West, 2014). Most companies now compile large amounts of data, known as "big data," and the fastest growing job of the future is the task of making sense of that data, including seeing data patterns visually. Computer scientists and mathematicians at Stanford and elsewhere now *see* patterns in data that could never have been picked up by numerical techniques (for more detail see https://www.youcubed.org/resource/data-literacy/).

Some scholars note that it will be those who have developed visual thinking that will be at the top of the class in our new high-technology workplaces that increasingly draw on information visualization technologies and techniques in business, technology, art, and data science (West, 2004, p. 17). In our education system it is important not to prioritize any type of learner over others—or even to give the idea that it is productive to take one learning approach and focus upon it. The new neuroscience supports this approach—students should be encouraged to develop mathematical thinking through visuals, numbers, symbols, models, movement, and words and draw the connections between them. This is twenty-first century learning that invites teachers and students to see mathematics as the subject it really is: a beautiful, creative set of connected ideas that empower.

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