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Movement Matters

How Embodied Cognition Informs Teaching and Learning

Edited by: Sheila L. Macrine, Jennifer M.B. Fugate

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10 Manipulatives and Mathematics Learning: The Roles of Perceptual and Interactive Features

Andrea Marquardt Donovan and Martha W. Alibali

Imagine a child working with a balance scale as they are learning about the concept of equality. They look at a symbolic equation and count out blocks to represent the numerical values on each side of the equation. They then place the blocks into the pans on either side of the balance scale and look to see whether the scale balances. Does the presence of the blocks and the balance scale influence the child's understanding of the concept of equality? Do the physical and interactive features of the blocks and the balance scale invite certain actions, which in turn lead to better (or worse) understanding?

A wealth of research has investigated the effects of concrete manipulatives on student performance, learning, and achievement. Several studies have revealed benefits of using concrete objects in a range of tasks. For example, Carraher et al. (1985) found that children were more successful in solving arithmetic problems with real-world objects than solving comparable, symbolically presented problems. They argued that the objects activated real-world knowledge, leading to more accurate performance. Glenberg et al. (2004) found improved reading comprehension and memory when children modeled story actions using physical objects. Indeed, a recent meta-analysis found an overall positive effect of manipulatives on student learning outcomes (Carbonneau et al., 2013). Teachers also endorse the benefits of manipulatives: Moyer (2001) found that teachers believed that learners were more motivated when students used manipulatives, and Moch (2001) found that students expressed positive perceptions about mathematics when they used manipulatives.

By contrast, several studies have revealed challenges or inconsistent benefits of using manipulatives. For example, Donovan et al. (2016) compared students learning to solve mathematical equivalence problems (e.g., $3+4=5+\_\_$) in lessons that used three different types of manipulatives and in a control condition that involved symbolic problems only. There were no benefits of using manipulatives for problem-solving performance, though there were benefits for conceptual understanding of equality. Furthermore, small differences in the features of manipulatives can moderate their effects (Petersen & McNeil, 2013). Finally, the effects of manipulatives also vary depending on characteristics of the instruction and on methodological features of the research (Carbonneau et al., 2013).

These mixed findings have sparked much debate, both among researchers and among teachers. Indeed, middle school teachers sometimes report viewing manipulatives as "fun" but as not reflecting "real math" (Moyer, 2001).

In this chapter, we review research on how perceptual and interactive features of manipulatives afford actions and on how those actions connect to target concepts. We acknowledge there are many other factors that may influence the effectiveness of manipulatives, including features of the instruction (e.g., Carbonneau & Marley 2015), children's prior experience with the manipulatives (e.g., Mayer, 2003), and the ways in which the manipulatives are introduced (Donovan & Alibali, 2021). In this chapter, we focus on characteristics of the manipulatives themselves, specifically the perceptual and interactive features of manipulatives and the *affordances,* or possibilities for action, they offer. We argue that considering manipulatives in terms of affordances can provide new insights into the varying effectiveness of manipulatives in different contexts. We close by discussing implications for the design of lessons that use manipulatives for math instruction.

For the purpose of this chapter, the term "manipulatives" refers to physical objects that can be touched and moved with the hands during problem solving and learning. Some example manipulatives include blocks, chips, Dienes blocks, Geotiles, balance scales, paper clips, popsicle sticks, and beanbags. A growing body of work focuses on computer-based, virtual manipulatives (Moyer-Packenham & Westenskow, 2013; Stull et al., 2013; Suh & Moyer, 2007), which hold promise because technology offers unique affordances for action. However, in this chapter, we focus on manipulatives as objects that can be physically manipulated with the hands.

Manipulatives vary along many dimensions, and some of these variations have implications for how learners perceive and interact with the manipulatives. In the following sections, we consider the perceptual and interactive features of manipulatives in turn.

Perceptual Features of Manipulatives

Perceptual features of manipulatives include features such as color, shape, pattern, visual complexity, degree of perceptual detail, and so on (Willingham, 2017). Objects used as manipulatives vary in their perceptual richness, with some objects being perceptually bland with simple shapes and plain colors, and other objects being perceptually rich with bright colors, unique shapes, and a high degree of perceptual detail. For example, two types of manipulatives that are currently marketed to teachers as useful for counting tasks include simple, bland chips and rich, detailed “bug counters,” which are multicolored plastic bugs (grasshoppers, beetles, dragonflies, etc.) thought to “capture students’ interest in counting activities” (Learning Resources 2021). Both can be used for counting, but is one more effective than the other?

The perceptual characteristics of manipulatives may influence the ways that learners engage with the manipulatives. Perceptually rich manipulatives may engage learners and stimulate exploration because they draw attention with bold colors, interesting shapes, or compelling details. Some support for this idea was found by Petersen and McNeil (2013) in research on preschool children’s counting performance. When the objects to be counted were unfamiliar, children displayed better performance with perceptually rich objects than with perceptually bland ones. For familiar objects (such as toy animals), however, perceptual richness actually hindered children’s performance. Other studies have also suggested that perceptually rich manipulatives are more likely to elicit irrelevant or off-task behavior (e.g., Uttal et al., 2013). Perceptual details may be distracting for learners, and they may evoke or activate knowledge that is irrelevant to the task at hand.

Maria Montessori (1964), one of the first women to put the education of children into the public eye, would not be the least bit surprised. During her quest to establish an educational environment in the tenements of Rome, she expressed a very different intuition from many educators. Montessori believed that didactic materials should be made from the most natural of substances available, and that careful thought should be given to each object being placed into the children’s learning environment. Montessori believed that each feature of any learning material in the classroom should have a specific purpose and should have no extraneous purpose, so as not to distract from the connection between the material and the concept. In her view, learning materials should be designed for learning and not for visual pleasure.

In line with Montessori’s intuition, several studies have demonstrated that perceptually bland manipulatives enhance performance, relative to perceptually rich ones. For example, McNeil et al. (2009) investigated the effects of perceptually rich and bland manipulatives on children’s abilities to solve story problems about money. They found that students who used perceptually bland coins and bills performed better on the story problems than those who used perceptually rich materials that looked like “real” money. However, they also found that the errors children made in the perceptually rich condition were less likely to be conceptual errors that reflected fundamental misunderstandings of the problems.

Another study that revealed differential benefits for rich and bland manipulatives focused on preschoolers' understanding of numerical inequalities using objects as counters (Carbonneau & Marley, 2015). In this study, the realistic manipulatives were green toy frogs, and the bland manipulatives were simple green circles. Although the type of manipulatives used did not influence participants' abilities to apply procedures to solve the problems, participants who used rich manipulatives displayed less knowledge about the underlying structure of the problems than those who used the bland manipulatives. Participants who used rich manipulatives, however, outperformed those who used bland manipulatives on transfer items, which involved comparing quantities using a number line. The different findings for performance and transfer suggest that rich and bland manipulatives may support different aspects of children's learning.

Manipulatives with different perceptual features may influence learning in different ways. The mixed findings in the literature suggest that the goals of lessons need to be carefully considered and that manipulatives should be selected based on those goals. For example, if the goal of a lesson involving money is to build arithmetic skills, then perceptually bland materials might be the best choice. If the goal is to promote foundational understanding of currency and change making, then perceptually rich bills and coins might be the better choice. To our knowledge, guiding principles for choosing manipulatives to suit differing educational objectives have not yet been explored. Considering this distinction in future research on manipulatives could lend clarity to the debate.

Interacting with Manipulatives

Perceptual characteristics of manipulatives may be important, not in and of themselves, but because of the differing actions that they invite. So rather than making a blanket statement that perceptually rich or bland manipulatives are a "better" choice for student learning, a reframing of the question may be needed. What sorts of actions do specific manipulatives afford? James Gibson (1979) argued that *affordances* are the inherent qualities of an environment that allow particular actions on the part of a particular organism within that environment:

> The affordances of the environment are what it offers the animal, what it provides or furnishes, either for good or ill. The verb *to afford* is found in the dictionary, the noun *affordance* is not. I have made it up. I mean by it something that refers to both the environment and the animal in a way that no existing term does. It implies the complementarity of the animal and the environment. (p. 127)

In the Carbonneau and Marley (2015) study, the green frogs may have had *affordances* that that green chips did not. The green frogs may have afforded actions such as making the frogs "jump" or holding a single frog in one's hand. The chips, by contrast, may have afforded stacking or holding many in one's hand at once. Did these differing affordances prompt children to perform different actions that may have helped or hindered their performance on the experimental tasks? Or did the green frogs and chips each relate to the concepts being taught—underpinnings of mathematical equality and inequality—in different ways?

In fact, in all of the studies reviewed thus far, different materials afforded differing actions—different ways of interacting with the manipulatives. We use the term *interactive features* to refer to features of manipulatives that may influence how learners interact with them, such as their size, weight, and their ability to be picked up and handled. Learners' history of actions on the objects and the conventional ways in which the objects are handled or used may matter as well. For example, it may be challenging for learners to use a toothbrush as a measuring device because the usual way in which a toothbrush is used—for brushing teeth—is both conventional and well-practiced. Rather than making general claims that some manipulatives are better or worse for student learning, we must consider the interactive features that particular manipulatives have and the actions those features afford.

Manipulatives with different degrees of perceptual richness may also have different interactive features. Rich perceptual features may invite learners to explore the objects manually in the first place. Realistic manipulatives (such as the toy frogs) may invite learners to interact with the objects in ways that are shaped by learners' prior knowledge and experience with the objects (in this example, the toy frogs) or the things they represent (in this example, real frogs).

At the same time, realistic features may be detrimental to learning with manipulatives because real objects afford specific actions that may not be relevant to the target concept. For example, crayons may not be well suited to being counters because they invite drawing rather than being counted. By contrast, bland manipulatives may allow children to look beyond their knowledge of the objects themselves and to act on the objects in ways that relate to the concept being taught. Pouw et al. (2014) argue that perceptual features invite particular actions on manipulatives, thus "embedding" learners' cognitive activities in the objects. In this sense, the varying perceptual features of manipulatives and the actions they afford may lead to differing experiences with manipulatives—a two-way flow between the features of the manipulatives and the actions of the learner.

How Perceptual and Interactive Features Support Learning with Manipulatives

When people see objects, they automatically activate plans for actions that they could perform on or with those objects. Neuropsychological evidence suggests that when people view objects with strong associations to possible actions, such as tools, they experience activation of premotor and motor areas (e.g., Grafton et al., 1997). Moreover, people experience greater motor activation when viewing objects that are readily manipulated (such as an apple) than when viewing objects that are not easily manipulated (such as a traffic light) (e.g., Gerlach et al., 2002). These findings suggest that when people see particular objects, they perceive affordances for actions on those objects.

This general principle can be applied to manipulatives, as well. Affordances for action may be set in motion when the manipulatives are perceived or may be activated when the manipulatives are handled. The affordances of the manipulatives may invite or constrain particular sorts of actions and explorations. Different manipulatives may prompt differing actions, such as touching, grasping, rotating, stacking, pushing, lifting, and so forth. When learners lack relevant knowledge, strategies, or teacher guidance to inform their actions, the affordances of the manipulatives themselves can inform their actions (Manches & O'Malley, 2016).

Manipulatives that afford actions that closely align with the concept being taught could contribute to student success. In some cases, merely touching the manipulatives may be enough to boost performance, as in preschool children's counting (Alibali & DiRusso, 1999). In this case, the target concept of counting was closely tied to sequential touching, and sequential touching enhanced children's counting performance.

Other research on action and cognition suggests that when learners produce actions that align well with the target concepts, they make greater progress than if they make irrelevant or conflicting actions (e.g., Nathan et al., 2014; Thomas & Lleras, 2009). This same principle may hold for actions with manipulatives, as well. Depending on how closely the actions with the manipulatives align with concept being taught, learners may have a more or less aligned physical experience of the concept.

One recent study compared students' learning from varying manipulatives that afforded different sorts of actions. Donovan, Alibali and Waters (2016) taught elementary school students the concept of mathematical equivalence with three different types of manipulatives, each of which afforded different sorts of actions. One group received a lesson and practice with a balance scale, another group received a lesson and practice with Lego blocks, and another group received a lesson and practice with a set of buckets and beanbags. Each of the manipulatives was intended to support students in understanding equivalence, but each afforded very different actions. In the pan balance group, students used their dominant hand to place the cubes into the pans, and they relied on visual cues to support the connection between the pan balance and the equation. In the Lego blocks group, children used both hands to stack the blocks into two towers that represented the values on the two sides of the equation. In this condition, children also relied on visual cues to support the connection between the heights of the Lego towers and the values on each side of the equation. In the buckets and beanbags group, students engaged both hands as they "became" the balance scale with their bodies, by picking up the buckets with both hands and holding them out to their sides with the beanbags placed in them. Thus, each of the three manipulatives afforded very different actions.

Donovan et al. found that some forms of action were more helpful than other forms for fostering conceptual understanding of equivalence. Many children in the bucket-and-beanbags condition and the blocks condition demonstrated a relational understanding of the equal sign at posttest, but very few of the children in the pan-balance and control conditions did so. In the buckets-and-beanbags condition, children simulated a balance scale with their own bodies; that is, they used their bodies to physically experience or "feel" the concept of equivalence by holding the buckets and beanbags of equal weight in their two hands. The buckets and beanbags afforded actions that the other materials did not—specifically, being lifted with two hands by the child. By "becoming" the balance scale themselves, the children felt the identical weights of the buckets and performed the same action with both hands—both experiences that highlighted the idea of *sameness,* which is central to the concept of mathematical equivalence. In the blocks condition, children created two block towers and then engaged in visual comparison of quantities by looking back and forth between the two towers. Both in construction and in visual comparison, the *sameness* of the towers' heights was highly salient. By engaging the body in actions that were readily aligned with the very concept to be learned, children could take advantage of additional sensory and perceptual input that was not available in the other conditions.

Martin (2009) has suggested that when a learner is "stuck" on an idea, action might also help spark a new idea. According to this account, actions can help learners develop new interpretations of concepts. When representing equations with buckets and beanbags, the action of lifting a bucket in each hand and the experience of feeling the same amount of weight in each hand may have helped learners to develop a new interpretation of equality. Indeed, one of the participants in the buckets-and-beanbags condition continued to talk about the buckets and beanbags on the posttest, when they were no longer available for use (Donovan et al., 2016). For example, in trying to solve the symbolically presented problem $5+7=4+\_\_$, the child said, "Can I just pretend I have the bucket? Because, um, the bucket I'm pretending to have, has . . . four bean-bags in one and five in the other. I added one more to the, um, other, the four one, that would equal five . . ."

Other research also supports the view that actions on manipulatives can make learners more open to new ways of thinking about concepts. In one study that focused on understanding of fraction division (Sidney & Alibali, 2017), fifth- and sixth-grade students were asked to represent a series of arithmetic expressions with whole numbers and fractions using plastic fraction bars. Each individual bar represented one whole, but the bars could be broken into different numbers of fractional pieces; for example, some of the bars could be split into two pieces to represent halves, others could be split into three pieces to represent thirds, and so on. To model $12 \div 3$ (a whole number division expression), a child might place 12 bars on the table and then divide them into three groups of four bars each. Similarly, to model $12 \div 1/3$ (a fraction division expression), a child might place twelve bars on the table and then divide each bar into thirds, yielding thirty-six pieces.

The primary focus of the study was on participants' abilities to successfully model fraction division, and specifically on whether particular sequences of items differentially supported them in doing so. Participants who modeled *whole number division* just prior to fraction division were more successful than participants who modeled *fraction multiplication* prior to fraction division—presumably because the relevant actions were well aligned for whole number division and fraction division. These findings suggest that the action of representing whole number division as "forming groups of a particular size" supported learners in conceptualizing fraction division as "forming groups of a particular (fractional) size." Thus, participants' actions with the manipulatives appear to have influenced their understanding of the target concept of fraction division.

Depending on the actions afforded by particular manipulatives, children may engage with manipulatives in ways that spark new ideas or that make them open to thinking about concepts in new ways. Thus, by inviting particular actions, manipulatives may aid learners in exploring conceptual spaces in fruitful ways.

Connecting Manipulatives to Concepts

The focus of this chapter thus far has been on how perceptual and interactive features of objects afford actions. It is also critical, however, to consider how both objects and actions relate to concepts. For manipulatives to be effective, learners need to be able to map from the concept to the manipulatives, from the manipulatives to the action, and from their actions on the manipulatives back to the target concept. The ease with which learners can make these mappings is sometimes referred to as the *transparency* of the mappings. Is the target concept "obvious" in the object itself? Is it highlighted in the actions afforded by the objects? Are the actions that are afforded by the objects the important ones for learning the concept? Researchers have distinguished two contributors to transparency: (1) the mapping from the physical object itself to the concept (which is sometimes referred to as *epistemic fidelity*; see Roschelle, 1994), and (2) the mapping from action to the concept.

One key to understanding transparency may be exploring what makes individuals believe two things are the "same." Some similarity relations may seem natural or obvious, in the sense that there is an easily apprehended connection from the object or action to the target concept. For example, there is a natural, easily apprehended connection between the action of sequential touching and the concept of counting. Other similarity relations may require instructional support that highlights corresponding features of the object or action and the target concept, using common labels or gestures, spatial alignment, or making explicit mappings, such as instructional analogies. The need for such supports may be greater if the connections are less transparent.

We argue that, in order for manipulatives to be beneficial for learning, the connections from the manipulatives and/or the actions performed to the target concept need to either be transparent or be supported via instruction. Considering these connections may help educators choose what manipulatives to use and how to instruct learners in using them. Thus, we argue that, in deciding whether and how to use a given manipulative, educators need to first identify the target concept, then (1) consider the goals of the lesson; (2) consider how the manipulatives under consideration relate to the target concept; (3) consider what actions the manipulatives afford; and (4) consider how those actions relate to the target concept.

Manipulatives to Concept

To enhance the likelihood that manipulatives are beneficial, they should physically align with the target concepts to the greatest extent possible. One construct that has been invoked to capture this alignment is the idea of *epistemic fidelity* (e.g., Meira, 1998; Roschelle, 1990; Wenger, 1987). Epistemic fidelity can be defined as "the strength and breadth of the analogical mapping of the physical material to the [mathematical] domain" (Stacey et al., 2001, p. 200).

Representations that have high epistemic fidelity are easily mapped to the target domain because the analogical mappings are strong and deep. If the mapping from the manipulatives to the concept is perfectly transparent, learners should be readily able to use the manipulatives to build their understanding. For example, in their study of reading comprehension, Glenberg and colleagues (2004) found that children who manipulated objects that physically resembled the people and objects described in a story performed better than children who simply reread the story. The manipulatives that the children used mapped transparently to the characters and objects in the story, so the links from the manipulatives to the target concepts were easily apprehended by the learners. If the connection from the manipulatives to the concept is less transparent, then learners may not benefit from the manipulatives as intended. For example, if the objects in the study by Glenberg and colleagues had not looked like the characters and objects in the story—that is, if they had looked like other characters and objects—participants might not have performed as well.

Kamii et al. (2001) made a related argument to support their view that a balance scale is *not* a useful manipulative for teaching children about addition. They argued that addition is a mental operation in which two values are combined to make a higher-order value. Importantly, the two original values remain part of the larger value; for example, in $3+2=5$, the 3 and the 2 remain "in" the 5. This part/whole structure is not reflected in the balance scale, where one side might be used to represent $3+2$ and the other side to represent 5. In this respect, the balance scale does *not* have epistemic fidelity with the operation of addition. In the view of Kamii and colleagues, mathematical relationships are not well represented by the physical phenomenon of balancing sides, so they recommend against the balance scale as a tool for teaching arithmetic.

As another example, Stacey et al. (2001) compared two different materials for teaching decimals—one that they deemed to have greater epistemic fidelity and one that they deemed to have less epistemic fidelity. Specifically, they compared learners' understanding of decimal concepts after lessons that involved linear arithmetic blocks, which represent quantity in terms of *length,* or multibase arithmetic blocks, which represent quantity in terms of *volume.* Stacey and colleagues argued that length connects more transparently to number than volume. Indeed, they found that the linear arithmetic blocks promoted greater learning of the target decimal concepts than the multibase arithmetic blocks, as well as more active engagement and deeper discussion of the concepts.

Stacey and colleagues were quick to point out that the effectiveness of manipulatives cannot be predicted by epistemic fidelity alone, because the two types of blocks also differed in other ways. Though different learning materials can have varying levels of epistemic fidelity, it is also possible that transparency is made by the learner from their actions (Meira, 1998). From this perspective, transparency may derive not only from features of the object itself, but also from the process of acting on or using that object.

Actions to Concept

Some particular actions may align with a target concept more or less than other actions. In some cases, the structure of objects makes certain actions on those objects natural or obvious. Gaver (1991) called these natural affordances "perceptible," meaning that they offer a direct link between the object and the action. If an object naturally affords particular actions, and if these actions are well aligned with the target concepts, learning may be enhanced.

Returning to the study of reading comprehension described earlier, Glenberg and colleagues (2004) asked children to physically act out the story with objects. In this case, the objects naturally afforded certain actions, and these actions aligned well with events in the story itself, so the actions enhanced comprehension of the story. Likewise, in the study of mathematical equivalence described earlier (Donovan et al., 2016), the buckets naturally afforded participants placing beanbags inside them and lifting them by their handles. Lifting the buckets allowed participants to feel the weights of the buckets and to experience whether they were the same. The actions with the buckets supported understanding of mathematical equivalence because these actions aligned readily with the target concept of equivalence.

As with the link from manipulatives to the concept, if the "reach" from the action to the concept is too far, the relation between the two may not be apparent to the learner, and the learner may not benefit from acting on the manipulatives. In the study of mathematical equivalence described earlier (Donovan et al., 2016), children who placed cubes on a pan balance did not demonstrate substantial gains in understanding of mathematical equivalence. The action of placing cubes in pans may have been challenging to align with the target concept.

Just having manipulatives present is not enough to evoke actions that align with the concept being learned—and indeed, relevant actions may need to be modeled for the learner. Again, the buckets and beanbags provide a valuable example. In addition to placing and lifting actions, these objects also afford tossing the beanbags into the buckets—and many children choose to engage in such actions when they encounter these materials. Tossing actions do not ordinarily align well with the concept of mathematical equivalence, but depending on how the beanbags are tossed, they may not enhance learning of equivalence, but they may not harm learning of the concept either (see Donovan & Alibali, 2021).

In sum, to ensure that the manipulatives are beneficial, the manipulatives themselves should physically align with the target concepts, and the actions performed on those manipulatives should also physically align with the target concepts. If the object features and relevant actions are not readily connected to the target concepts, support for these mappings may be necessary to increase the likelihood of successful learning.

Implications for the Design of Lessons Using Manipulatives: Promoting Understanding of Relevant Connections

With so many differing types of manipulatives available to teachers, choosing which ones to use and how to use them can be a daunting task. Given the conflicting findings about the effectiveness of manipulatives (e.g., Carbonneau et al., 2013), caution should be used in deciding how and under what circumstances they should be used. We have argued that it is crucial to consider whether learners can appreciate the connections from the manipulatives to the concept to be learned, as well as the connections from the actions afforded by the manipulatives to the concept to be learned.

There may be an optimal structure for manipulatives to be beneficial to learning. We suggest that the most effective manipulatives are objects that offer transparent links to the target concepts and that afford actions that readily align with the target concepts, as well. In our view, transparency can emerge either as a result of the perceptual features of the objects themselves or as a result of their affordances for action. Thus, the best manipulatives are those that can be readily linked to the target concepts based on their perceptual and interactive features.

Conclusion

In this chapter, we have considered perceptual and interactive features of manipulatives and how these features contribute to or detract from student learning. We considered the varying *affordances* for actions that manipulatives possess, and we highlighted the varying ways in which physical objects and actions connect to target concepts. These considerations from perception and action can help to describe, explain, and predict why certain types of manipulatives are more or less effective for learning. Finally, we considered the issue of the transparency of the connections between manipulatives and the concepts to be learned, both in terms of the objects themselves and in terms of the actions afforded by the objects.

An embodied perspective on cognition holds that "cognitive processes are rooted in the actions of the human body in the physical world" (Alibali & Nathan, 2018, p. 75). From this perspective, the body and the motor system are integral to psychological processes (Glenberg, 2010). It may be the case that physical activity in general opens the mind to new ideas, as some authors have suggested (e.g., Have et al., 2018). However, in this chapter, we have argued that manipulatives and the actions they afford can also be a driving force behind changes in thinking.

Indeed, many researchers as well as practitioners advocate that actions with manipulatives can support learners' construction of more advanced conceptual structures (e.g., Fuson et al., 1997). However, the beneficial effects of manipulatives may only be realized over time and with careful planning. We have outlined several considerations that are paramount for deciding whether and how to use a given manipulative: identifying the target concept, considering how the object under consideration relates to the target concept, considering what actions the object affords, and considering how those actions relate to the target concept.

We have argued that an affordances perspective can provide new insights into the body of conflicting findings about the effectiveness of manipulatives. To understand the dynamic processes of learning with manipulatives, a new framework is needed—one that places manipulatives' *perceptual and interactive features* at the center. These features highlight the importance of learners' *actions on manipulatives* and at the same time emphasize the necessity of transparent *connections* to the target concepts—both connections that seem natural and obvious, and ones that can be made with instructional support.

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