

# 15 Cognitive Robot Navigation

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## 15.1 Introduction

Navigation in cognitive robotics has been strongly influenced by studies on navigation in animals. During the later decades of the twentieth century, researchers have focused on studying rats' spatial learning and memory in mazes to help understand the idea of spatial cognition for other species, including humans. The fact that a rat can reach flexible target locations effortlessly in complex mazes inspired scientists to determine that the flexible movement behavior is dependent on an inner map formed in the brain. This inner map can reflect the spatial and geometric relations between animals and surroundings. Furthermore, by observing animals' behaviors, including rats, bats, and more, researchers obtained two important findings: 1) animals were able to successfully return home even when put into a seldomly visited place; 2) animals looked for shortcuts. The two findings could verify that the inner map made it possible for the evaluation of relative positions and navigation from the current position to target places.

By behavioral observation and psychological analysis, researchers started to study animals' spatial cognition (Tolman 1948) and to understand the spatial cognition abilities in complex environments. The concept of a "cognitive map," proposed by Tolman in 1948, has been widely considered to possibly provide the basis for spatial memory and navigation. In order to reveal how animals construct cognitive maps of environments, studies in arthropods found the existence of a highly effective path integration mechanism depending on directional heading and distance computations. Then important discoveries about spatial cells in mammals demonstrated that the path integration mechanism completed by some brain regions is necessary to form inner cognitive maps. These maps represent the topological structures of environments and surrounding landmarks by position coordinates. With the discoveries of place cells (O'Keefe and Dostrovsky 1971), head direction cells (Taube, Muller, and Ranck 1990), and grid cells (Hafting et al. 2005), neuroscientists began to study the mechanisms underlying spatial navigation skills in animals. This research became a milestone of cognitive map and spatial navigation research.

The cognitive map mechanism of animals provides good insight to develop bioinspired models of spatial cognition for robots. Animals can perform simultaneous localization and mapping (SLAM) robustly and effortlessly in daily life. They can also quickly adapt to

new dynamic environments and localize themselves. Based on psychological and neuroscientific studies on animal spatial navigation, researchers have attempted to create bio-inspired map building simulations and make the spatial navigation of mobile robots more flexible and robust (Milford, Wyeth, and Prasser 2004). The target is to make more stable and general intelligent navigation systems for robots to increase the capabilities of autonomy and operation flexibility.

## 15.2 From Psychology to Neuroscience

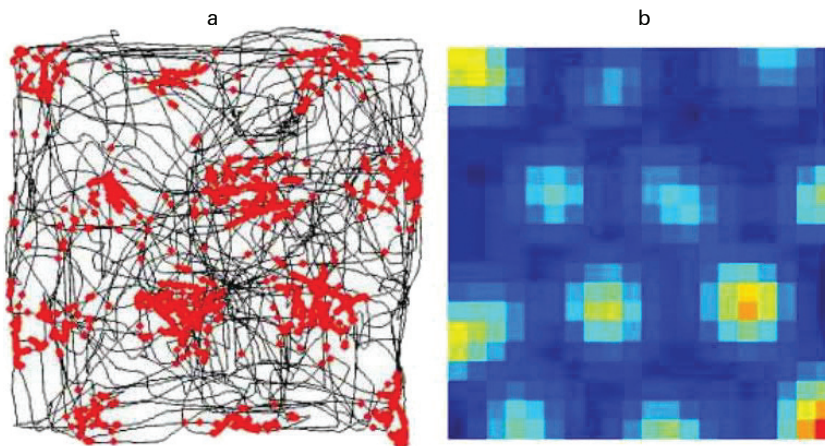
In the 1930s, E. C. Tolman started to research cognitive behavioral psychology by observing rats running in various types of mazes. Experiments showed that rats could plan paths with fewer and fewer mistakes until they finally completed the correct path planning. Thus, Tolman concluded that there is one kind of inner mental knowledge structure in an animal's brain that stores information according to the animal's position in the environment. Tolman (1948) then proposed the concept of the cognitive map in 1948. The key findings of the cognitive map include latent learning and spatial learning. Latent learning means that the rats learn about the structure of the maze without getting a food reward and can quickly plan the optimal path in the maze once food is given. And in the sunburst maze, the rats first learn to plan specific paths according to different rewards. If the planned path is blocked, they can still find an optimal path they have not previously experienced. This ability has been called spatial learning. The cognitive map theory directly sets the stage for studies about how space is represented in the brain.

Neurophysiological experiments have helped to verify Tolman's cognitive map theory and have searched for the neural basis of the cognitive map mechanism in animals' brains from neural structures and cell activities. In one such prominent and successful experiment, Hubel et al. (Hubel and Wiesel 1959, 1977) inserted electrodes into specific brain regions of awake animals and were able to observe and record neurons' activities in the cortex. They demonstrated that animals' specific functional behavior can be understood by neural activity and the interaction between neurons. After that, extensive neuroscience research studies at the neuron and synapse level were carried out to establish the relationship of synaptic physiology and animal behavior. Based on the advancement of neurophysiological experimental techniques, early research on the hippocampal region provided good insight and resulted in widespread agreement that the hippocampus is an important region for encoding and maintaining memories. In another set of studies, neuroscientists were motivated to associate specific individual behaviors with neuronal population activities in specific brain regions, including the hippocampus. A series of studies about special firing patterns of cells in the hippocampus and the surrounding regions were performed to unveil the spatial cognition and navigation mechanism in animal brains. In these studies, when rats moved into controlled maze environments, the activity of a single neuron or a neural population (mainly in the hippocampus and surrounding areas) was recorded through an electrode inserted into a specific brain region. The accumulated experimental results led to the discoveries of multiple types of cells responsible for inner cognitive mapping. Every type of cell shows specific firing patterns for encoding the environment and thus plays an important role in animals' spatial cognition.

Place cells were discovered by O’Keefe and colleagues in the 1970s (O’Keefe and Dostrovsky 1971). These cells are in the hippocampus and fire consistently when a rat is at a particular location in the environment. The firing cell signals recognition of a specific place in an environment, known as the cell’s “place field.” It is suggested that thousands of place cells, covering the surface of any space, act as a mapping system in the hippocampus to create a cognitive map (O’Keefe and Nadel 1978). Each place cell receives two different inputs, one external input about environmental stimuli and external events and an internal input from an inner-path integration system based on its self-motion.

In the 1980s, Ranck (1984) observed strong directional tuning when cellular activity was recorded from the pre- and parasubiculum regions. These direction-tuned cells were also discovered in other brain regions (Taube 2007), such as the medial entorhinal cortex (MEC; Sargolini et al. 2006). These cells respond to an animal’s head direction and are called head direction cells. They only fire when the rat’s head is at specific orientations. All orientations are represented by the head direction cell population. About thirty years after the discovery of the place cell, grid cells were discovered in the entorhinal cortex (EC) by Edvard I. Moser (Fyhn et al. 2004). Grid cells show the properties most like place cells but have multiple firing fields (figure 15.1)—that is, they can fire in a metrically regular way on the whole surface of a given environment. The firing fields of these cells have been demonstrated to be in a hexagonal pattern. In fact, a single grid cell will fire when the rat is located at any of the vertices of a tessellating hexagonal pattern. Grid cell firing appears to be a signal used for measuring displacement distances and direction—in other words, a “metric.” Grid cells differ from each other in grid spacing, phase, and orientation (Hafting et al. 2005; Fyhn et al. 2004), and the spacing of grid cells increases along the dorsal-ventral axis of the EC (Brun et al. 2008).

In the same parahippocampal brain regions are additional cells, called border cells, related to spatial mapping. The border cells can achieve responses when the animal is near a boundary of the local environment (Solstad et al. 2008; Savelli, Yoganarasimha, and Knierim 2008). Boundary-related cells have also been recorded in the subiculum, which



**Figure 15.1**

(a) The path on which the rat traveled in a square maze and the firing of a grid cell. (b) The firing rate of the grid cell at each place. *Source:* Moser and Moser 2007.

indirectly links the feedback from CA1 to the MEC, the presubiculum and parasubiculum (Lever et al. 2009).

Neuroscientific experiments show us a number of neural representations of the inner cognitive map. They might have innate connection circuitries and together could constitute a metric navigation system: head direction cells are responsible for direction tuning; grid cells play the important roles in path integration; border cells are used for evaluating vicinity to boundaries, and place cells are taken as the place representation. The discovery of spatial cells made the cognitive map theory more dominant in spatial cognition research.

## 15.3 Computational Theories on Robot Spatial Cognition

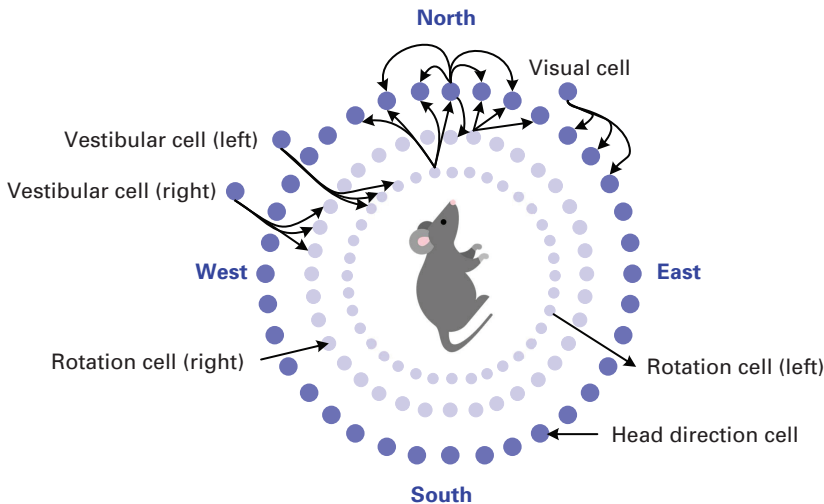
### 15.3.1 Path Integration

Path integration means to estimate positions and plan paths to targets via the continuous integration of movement cues such as directional heading and distance over the whole path. Inspired by animal behaviors, head direction cells are responsible for orientation tuning, grid cells can execute path integration, and place cells contribute to representing places. In order to build cognitive maps, outputs from head direction cells are first considered as the input signals for grid cells, then place cells and grid cells provide a population-encoding method for path integration. Most researchers have reached a consensus on this topic, but a few important questions still rise: How do we simulate the direction-tuning characteristic of head direction cells? How do we provide grid-cell-encoding methods for path integration? How do we represent place cells using grid cells?

#### Direction tuning

Information processing in biological systems is generally considered to be nonlinear dynamic and can be implemented by neural networks. Stable, persistent activity has been thought important for neural computation. Amit (1989) suggested that persistent neural activity in biological networks is a result of dynamical attractors in the state space of recurrent biological networks. This study resulted in the increasing popularity of using attractor networks in neuroscientific simulation and biologically inspired system building. In addition, there was evidence that many brain areas act as attractor networks (Wills et al. 2005), including the hippocampus and the entorhinal cortex. Because of the association with the ability to continuously track changing stimuli in certain brain regions, continuous attractor dynamics are widely used for brain mechanism simulation (McNaughton et al. 1996; Trappenberg 2002).

Simulations of head direction cells can be organized in a ring attractor and modeled as a one-dimensional continuous attractor network (CAN). In this network, the head's angular velocity (inner signals provided by other brain regions) is integrated for head direction representation updating. The rat's turning range ( $360^\circ$ ) is evenly discretized into  $N$  parts that correspond to  $N$  neurons, and each neuron has weighted connections to others, as shown in figure 15.2 (Skaggs et al. 1994). The connection strength decreases with increasing distance between neurons and active neurons, and then only one direction is focused at each time point. Activity in one part of the ring is initialized by visual input from visual



**Figure 15.2**  
One-dimensional CAN modeling head direction cell. *Source:* Skaggs et al. 1994.

cells. When the animal turns its head, sensory inputs (mainly from the vestibular region) can detect the change that activates rotation cells and cause activity bumps to move in the appropriate direction around the ring, keeping the representation concordant with the real head direction (Calton et al. 2008; Knierim and Zhang 2012).

### Path integration

Currently, some proposed computational models of grid cells include oscillatory interference (OI) models (Burgess 2008; Zilli and Hasselmo 2010) and CAN models (Fuhs and Touretzky 2006; Burak and Fiete 2009). In OI models, the grid pattern arises from several oscillators with slightly different frequencies around the theta frequency (Blair, Welday, and Zhang 2007; Burgess, Barry, and O’Keefe 2007). The key requirement is that the frequency is modulated by the animal’s velocity. Under appropriate conditions, the beat frequencies of the interference patterns cause a cell to reach its firing threshold whenever the rat is at the grid vertex. The key assumptions of oscillatory models have been experimentally challenged because theta oscillations have not been observed in fruit bats (Yartsev, Witter, and Ulanovsky 2011) and macaque monkeys (Killian, Jutras, and Buffalo 2012), despite robust grid cell activity having been recorded in both species.

A grid cell model with a single grid scale based on CAN models was proposed (Burak and Fiete 2009) to perform path integration with noise-free velocity inputs. Based on different spacing in grid cells, a grid cell model with multiple grid scales is required for the path integration. In this case, neurons are often arranged in a two-dimensional neural sheet. Recurrent connectivity among neurons with global inhibition leads to random patterns of population activity that spontaneously merge into organized “bumps” of grid cell population activity. A response from the grid cell can be obtained by accumulating the firing activity of a single neuron over a full trajectory. The most remarkable progress in the field (Burak and Fiete 2009) has been to accurately integrate velocity inputs into grid cell models.

### From grid cell to place cell

Functionally, path integration can be accomplished by grid cells driven by the rat's moving velocities in particular directions. Anatomically, the majority of the principal cells in layers II and III of the MEC have grid properties (Sargolini et al. 2006), and there is a strong projection from the MEC to the hippocampus. Therefore, place cell activities might have been, at least partially, generated in response to stimulation from grid cells. As the size and spacing of grid patterns increase from small in the dorsal MEC to large in the ventral MEC (Fyhn et al. 2004; Hafting et al. 2005), it is believed that the input for place cells comes from a combination of several grid cells. Linear combinations of grid fields are generally used for generating firing fields of place cells (O'Keefe and Burgess 2005; Hafting et al. 2005; McNaughton et al. 2006; Solstad, Moser, and Einvoll 2006). Although other mechanisms, such as feedback inhibition of place cells, can achieve similar results (Monaco and Abbott 2011), linear-combination-based models are easy to implement and widely employed. As each place cell receives a subset of grid cells as its input afferent, a learning algorithm is required to do selection. Since Hebbian learning is commonly accepted as a biologically plausible theory for synaptic adaptation, it was chosen to determine the connection weights between the place cell and grid cell input (Hu et al. 2016). Furthermore, the learning performances of different variations of Hebbian learning have also been compared, and potential mechanisms to improve the learning process have been discussed.

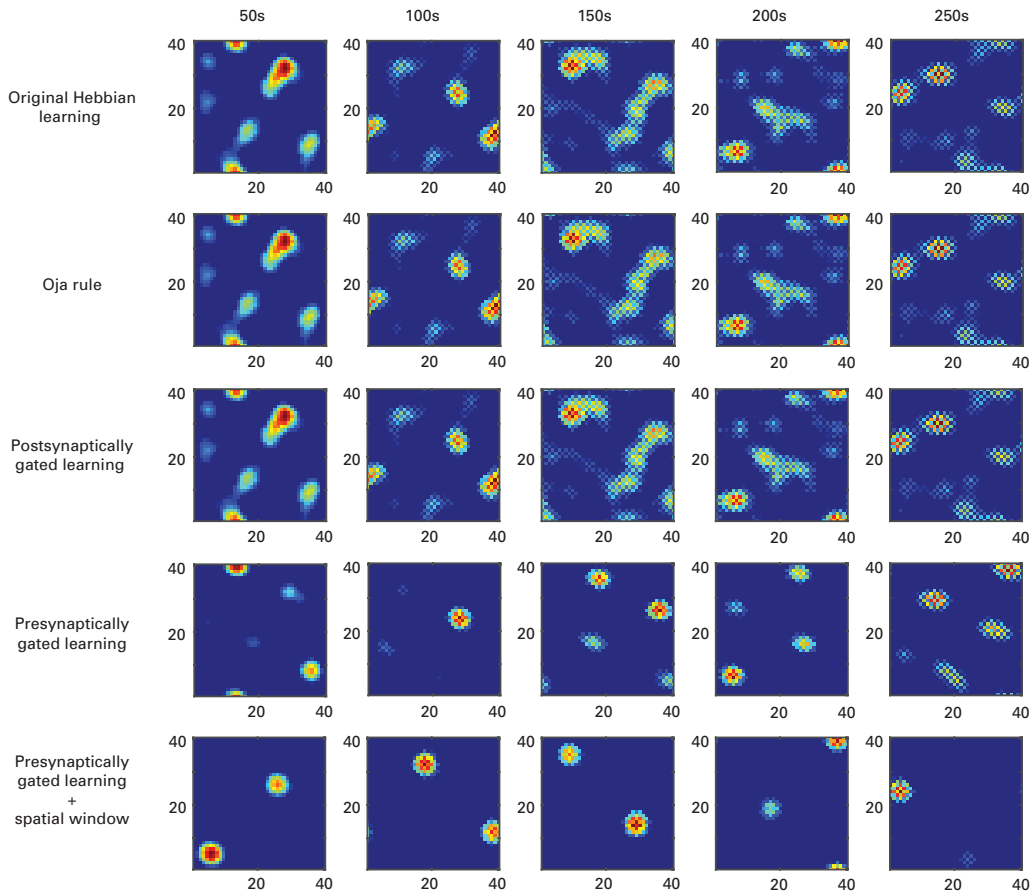
Different learning rules have been tested, and the experimental results are shown in figure 15.3 (Hu et al. 2016). The presynaptically gated learning shows better results with fewer bumps. The gated input stimulation removes the enhancement of unnecessary inputs from grid cells. The introduction of a spatial-learning window weakens stimulation from unnecessary afferents and enhances certain inputs that contribute most to place cells. Hebbian learning refines the place cell activity to fewer bumps, but place cells tend to have more and more bumps during learning without a mechanism to prevent multiple bumps. Therefore, a circle-shaped learning window is applied to the learning process so the number of bumps can be reduced. As shown in figure 15.3, only two bumps are left with the help of the spatial-learning window. To further explore the effect of learning, synaptic weights are examined from grid cells to place cells after learning. Initially, synaptic weights from grid cells to place cells are randomized with a normal distribution. As learning proceeds, synaptic weights from contributing grid cells to corresponding receiving place cells are enhanced. After learning, each place cell is expected to be strongly connected to a subset of grid cells.

One should notice that the current network structure has been simplified. In the current setting, grid cells in the same neuron sheet share the same synaptic weights as place cells. One should notice that stimulation from grid cells provides only partial information for place cells. Other sensory information, such as visual, auditory, and olfactory signals, may also affect the learning process.

### 15.3.2 Cognitive Map Building

Evidence has revealed that rats can correct accumulative movement errors in path integration when they meet salient landmarks (McNaughton et al. 2006). When a rat returns to a familiar environment, the path integrator should be reset to adjust to the perceived environment (Moser, Kropff, and Moser 2008). However, it remains unclear how the brain





**Figure 15.3**

Neural activities of place cells with different learning algorithms. *Source:* Hu et al. 2016.

senses and transforms external sensory inputs into an internal cognitive map (Burak and Fiete 2009). The cognitive map theory has continuously inspired important advances in robotic mapping and navigation. The multimodal integration of visual place cells and grid cells has been proposed to enhance robot localization (Cuperlier, Quoy, and Gaussier 2007; Jauffret et al. 2012).

Milford et al. made significant progress in emulating the spatial navigation ability of the hippocampal system by building a semimetric topological map in a very large area (Milford and Wyeth 2008, 2010). In their work, the core model, RatSLAM, has been demonstrated to build maps simultaneously in large and complex environments. It emulates the rat's spatial-encoding behavior using three key components: the pose cells that are analogous to the rodent's conjunctive grid cells, the local view cells that provide the interface to the robot's sensors in place of the rodent's perceptual system, and the experience map that functionally replaces place cells. Each local view cell is associated with a distinct visual scene of the environment and activated when the robot sees that scene. A CAN is built for pose cells to encode the estimate of the robot's pose. Each pose cell is connected to proximal cells by excitatory and inhibitory connections with wrapping across

all six faces of the network. Intermediate layers in the  $xy$  plane are not shown. The network connectivity leads to clusters of active cells known as activity packets. Active local view and pose cells drive the creation of experience nodes in the experience map, a semimetric graphic representation of visited places in the environment and their interconnectivity. RatSLAM is an attempt to build a practical robotic system to take advantage of the spatial navigation mechanism highlighted by studies of the rat brain. It can perform well for some challenging problems in robotic navigation. The maps based on RatSLAM are less accurate than those of traditional SLAM systems, but its flexibility can help to cope with noisy input, deal with a changing environment, and accommodate increasing complexity. The cognitive map building mechanism in bioinspired SLAM will create a new generation of lightweight and low-cost mapping and navigation systems to be deployed in the robotic navigation field for real and large environments.

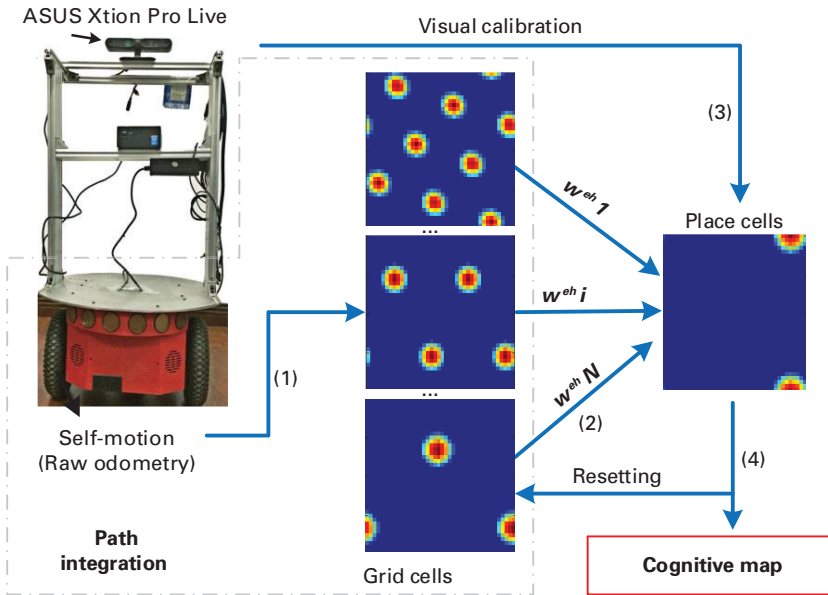
In 2013, Steckel and Peremans (2013) proposed a biomimetic navigation model named BatSLAM, which can solve simultaneous localization and map tasks with a biomimetic sonar mounted on a mobile robot. They analyzed the performance of the proposed robotic implementation operating in the real world and concluded that the biomimetic navigation model operating on the information from the biomimetic sonar can allow an autonomous agent to map unmodified environments efficiently and consistently. This showed that consistent topological maps with semimetric properties can be constructed using only motor commands and biomimetic sonar “fingerprints.” Furthermore, if these sonar “fingerprints” are sufficiently informative, there is no requirement for further interpretation of them in terms of discrete objects positioned in the environment.

In 2015, Silveira et al. (2015) presented a new bioinspired algorithm for underwater SLAM called DolphinSLAM, which extended the successful previous RatSLAM approach from 2D ground vehicles to 3D underwater environments. The proposed model uses a neural network model to localize and deal with low-resolution monocular images and imaging sonar data, in contrast to other available navigation systems that focus on probabilistic methods and occupancy grids. The model is composed of six modules: the preprocessing module, the local view recognition module, the motion detection module, the 3D place cells network module, and an experience map module. It has the particular advantage of being an appearance-based navigation system that can work well with low-resolution sonar and visual image data, in contrast to other available navigation underwater systems that focus on probabilistic methods.

Together with sensory-information processing, grid and place cells are considered to afford animals with an innate sense of the world around them. Inspired by the path integration mechanism of grid cells, Yuan et al. (2015) proposed a cognitive map model (figure 15.4), simulating grid and place cells for path integration and place representation. Visual cues are used for the error correction and cell population activity, resetting when loop closures are detected. Depth information in visual cues is invariant to lighting conditions and makes some similar indoor scenes more distinguishable. A comparison between image profiles is performed for each pair of incoming RGB and depth frames for loop closure and new scene detection. More details can be found in Tian et al. (2013).

In this work the cognitive map contains a set of spatial coordinates that the robot has experienced in its past travels. The robot’s spatial coordinates are calculated from place cell population activities, which are generated from a subset of grid cell population activities. Nodes in the cognitive map are constructed by associating the major peak of the place





**Figure 15.4**  
The system architecture of a cognitive map building model. *Source:* Yuan et al. 2015.

cell population activities with corresponding visual cues and locations being denoted as visual experiences. Algorithm 1, below, shows the cognitive map building process. The incoming visual inputs are compared with past visual experiences. If the latest input matches the previous visual experience, it is considered a familiar scene that the robot has previously seen. The status of the grid cell population activities and the place cell population activities is then reset to the previous matched visual experiences. The current visual input and the matched visual experience are merged into the same experience. Otherwise, a new visual experience is created. Once a loop closure is detected, the map will be adjusted to the recalled visual experiences.

**Algorithm 1. The Cognitive Map Building Algorithm**

Input: Raw odometry data from wheel encoders and visual images from the RGB-D sensor (1)

Output: Cognitive map

Begin: Calculate grid cell population activities (2)

    Calculate place cell population activities (3)

    Obtain one major peak of place cell population activities

    Perform visual profile comparison (4)

*if* the incoming visual input matches the previous visual experiences

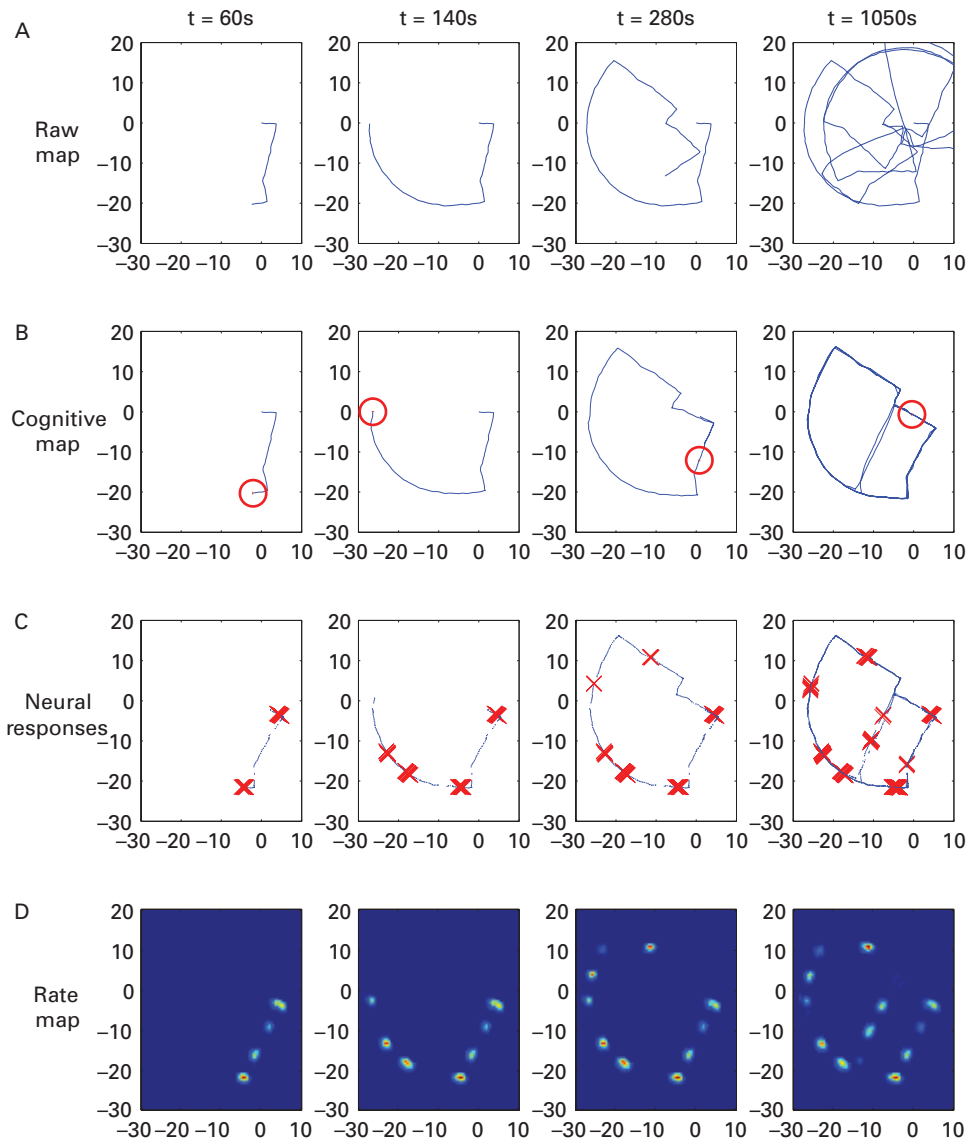
*then* perform resetting and map correction (6)

*else* create a new visual experience (5)

*end if*

End

A cognitive map for a large office environment of  $35\text{ m} \times 35\text{ m}$  on a mobile robot was built to validate the effectiveness of the proposed model (for more details of the parameter setting and platforms, see Yuan et al. [2015]). Figure 15.5 demonstrates the experimental results. Row A shows the dead-reckoning map obtained from the robot odometry. Obviously, this map cannot represent the environment properly. Row B shows the cognitive map based on the proposed computational model. With visual inputs, the system can successfully perform loop closure detection and correct the odometry drift. Finally, it generates a cognitive map that encodes both topological and metric information. In Row C, the



**Figure 15.5**  
Neural responses in the map building process. *Source:* Yuan et al. 2015.

blue dotted line shows the real trajectory traveled by the robot, and the red crosses indicate the firing locations of the grid cell located at (20, 20) in the twenty-first layer of the neural sheets. Row D shows the performance of maps corresponding to different rates. To generate the rate map, a spatial smoothing algorithm with a Gaussian kernel, as described in Hafting et al. (2005), is adopted with a bin size of  $0.5 \text{ m} \times 0.5 \text{ m}$ .

It is a significant challenge to build robust SLAM systems in dynamical large-scale environments. Inspired by findings in the entorhinal-hippocampal neuronal circuits, Zeng and Si (2017) proposed a cognitive mapping model that includes continuous attractor networks of head-direction cells and conjunctive grid cells to integrate velocity information by conjunctive encodings of space and movement. Visual inputs from the local view cells in the model provide feedback cues to correct drifting errors of the attractors caused by the noisy velocity inputs. The key components of the proposed model include head direction (HD) cells, conjunctive grid cells, and local view cells. Both HD cells and conjunctive grid cells are modeled by continuous attractor networks that operate on the same principles. The conjunctive representations of space and movement allow the networks to reach stable states for all movement conditions. And the proposed model is robust in building a coherent semimetric topological map of the entire urban area using a monocular camera, even though the image inputs contain various changes caused by different light conditions and terrains.

Animals such as birds and bats possess superlative navigation capabilities, robustly navigating over vast three-dimensional environments and leveraging an internal neural representation of space combined with external sensory cues and self-motion cues. Yu et al. (2019) presented a novel, neuroinspired 4-DOF (degrees of freedom) SLAM system named NeuroSLAM for mapping and localization in large, real-world three-dimensional environments that integrated with a vision system that provides external visual cues and self-motion cues. In this model, the robot's state of a 4-DOF pose ( $x, y, z, \text{yaw}$ ) in 3D environments is represented by the activity in the 3D grid cell network and the multilayered head direction cell network, conjunctively. The conjunctive pose cell network performs path integration on the basis of the self-motion cues and performs calibration based on the local visual cues. The approaches to the creation and relaxation of the multilayered graphic experience map are based on the combination of local view cells, conjunctive pose cells, and 3D visual odometry. The 3D multilayered experience map generated by the NeuroSLAM system can be learned and generated when the robot visits unknown environments. It can also be incrementally maintained and updated based on the learning and recalling mechanism. The 3D spatial experience nodes represent a 4-DOF pose in a specific 3D location, and the links contain distances and directions between nodes. This metric and topology information can be used for 3D path planning and guidance control in 3D environments. It is likely that map maintenance routines could also be deployed to ensure long-term map stability as well as computation and storage viability.

The computational mechanisms of mammalian brains in integrating different sensory modalities under uncertainty for navigation are enlightening for robot navigation. Zeng et al. (2020) proposed a concise yet biologically plausible model integrating visual and vestibular inputs, NeuroBayesSLAM, based on spatial cognitive mechanisms of mammalian brains to solve the SLAM problem. The proposed model successfully built coherent cognitive maps in both large-scale outdoor and small indoor environments. In the model,

the pose of the robot is encoded separately by two subnetworks—namely, a head direction network for angle representation and a grid cell network for position representation, using the similar neural codes of head direction cells and grid cells observed in mammalian brains. The neural codes in each of the subnetworks are updated in a Bayesian manner by a population of integrator cells for vestibular cue integration, as well as a population of calibration cells for visual cue calibration. The conflict between the vestibular cue and visual cue is resolved by the competitive dynamics between the two populations. The model successfully builds semimetric topological maps and self-localizes in outdoor and indoor environments with different characteristics, achieving a performance comparable to previous neurobiologically inspired navigation systems but with much less computation complexity. The proposed multisensory integration method constitutes a concise yet robust and biologically plausible method for robot navigation in large environments. The model provides a viable Bayesian mechanism for multisensory integration that may pertain to other neural subsystems beyond spatial cognition.

One should note that in most experiments (Burak and Fiete 2009; Zilli and Hasselmo 2010), velocity inputs are extracted from ground-truth trajectories. However, for animals or autonomous mobile robots, accumulated errors are inevitable. In the above model, velocity inputs were extracted from idiothetic wheel encoders to drive CAN-based grid cell population activities, and accumulated errors exist in raw odometry data. Together with visual cues for loop-closure detection and map correction, the model can produce an accurate representation of the environment and contributes to developing, innovative robotic spatial cognition approaches (Huang, Tang, and Tian 2014; Milford and Wyeth 2010), showing the potential for machines that mimic more complex activity in the brain.

### 15.3.3 Cognitive Navigation

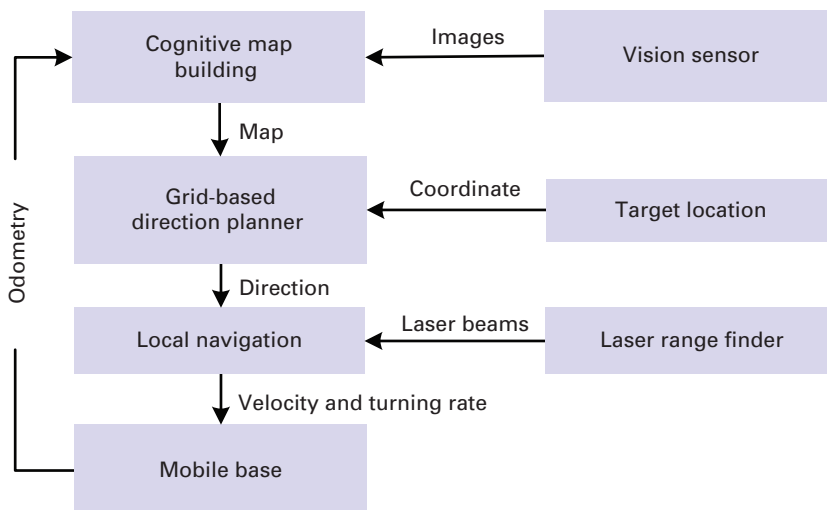
Humans and animals have an instinctual ability to navigate freely in environments. However, it is a challenging task to endow a robot with this ability, as a robot needs to be integrated with several functional mechanisms, such as scene understanding, mapping, self-localization, obstacle avoidance, dead reckoning and path planning (Brooks 1999; Thrun 2002). Discoveries of spatial cells and the development of the cognitive map theory motivate researchers to use biologically plausible principles for acquiring, storing, and maintaining spatial knowledge and to explore biologically inspired navigation strategies for robots.

The use of “directions” as guidance has been raised in several studies. Méndez (2012) presented a spatial conceptual map framework to transfer cognitive human navigation behaviors to an artificial agent, which can generate route directions similar to those created by humans. This conceptual map was modeled as three levels of interconnected graphs to simulate human spatial reasoning. However, this navigation system was only tested in a simulation environment. A method for modeling environments from a route perspective was discussed in Saiki et al. (2011). The route perspective is defined as a mental tour of an environment, which is represented by a person when they are walking around the area. When describing an environment in this perspective, the terms regarding relative directions such as left and right are used. Another perspective is known as a survey perspective, which describes an environment from a top view where routes and landmarks are known in advance.

A navigation strategy considering both the route and survey perspectives, called direction-driven navigation, was presented by Shim et al. (2014). The directions extracted from a cognitive map denote the use of the survey perspective, while the execution of the directions by a mobile robot in a real environment implies the involvement of the route perspective. When traveling to a target destination, the robot is guided by a direction-driven behavior, such as following the directional guidance from someone else or from GPS, instead of closely following a global or local path.

The system architecture of the proposed navigation system is presented in figure 15.6, consisting of three main components: cognitive map building, a grid-based direction planner, and multilayered asymmetrical local navigation. The proposed grid-based direction planner (as global planner) and multilayered asymmetrical local navigation (as local planner) construct the direction-driven navigation system. The global planner plans a global path connecting its current location and the final goal destination. The local planner creates a local path, connecting the current location to a local goal destination, which follows the global path closely. Initially, images are captured by a vision sensor, and odometry is obtained from the mobile base. They are assisted by a CAN (McNaughton et al. 2006), which constructs the cognitive map of the environment.

By analyzing the constructed map, the movement directions can easily be extracted in the form of “moving forward,” “turning left,” “turning right,” and “making a U-turn.” The grid-based direction planner provides directional guidance at junctions for guiding the robot to a target location. The robot compares its current visual cues to the templates associated in the cognitive map in order to localize itself. It should be noted that the localization is crucial, as a bad localization may lead the direction planner to give wrong directions. Given a direction, the navigation system executes the corresponding action only when it conforms with real conditions. For example, the robot will not execute the “turning right” instruction when the right junction is not detected.



**Figure 15.6**

System architecture of the direction-driven navigation system. *Source:* Shim et al. 2014.

Otherwise, the proposed multilayered asymmetrical local navigation module is used to control the velocity and turning rate of the robot to guarantee a safe motion such as obstacle avoidance. A laser range finder is used as the sensor input to the local navigation module.

### 15.3.4 Beyond Spatial Navigation

Humans have an innate ability to explore, map, and navigate in unknown environments while simultaneously performing variant tasks. However, current technology is still far from producing a robotic servant to perform daily tasks in unstructured environments. Taking the task of serving milk tea as an example, when one orders a cup of milk tea, a robotic servant needs to understand the environment first before performing a sequence of preparatory actions at specific locations. This is a common task requiring the cognitive map and episodic memory (Buzsáki and Moser 2013), and both components play important roles for humans to perform spatiotemporal tasks. The cognitive map can provide internal spatial representations of the environment, and episodic memory for humans to learn cognitive tasks through self-experiences and then plan the actions accordingly. Biologically, the entorhinal-hippocampal region is necessary for cognitive maps and episodic memory, though it may not be sufficient (Fyhn et al. 2004; Hafting et al. 2005; Tulving and Markowitsch 1998). Functionally, the cognitive map and episodic memory form the main technologies for robotic spatial cognition. Some work has been accomplished in this field (Fleischer et al. 2007; Krichmar et al. 2005). The integration of the cognitive map and episodic memory can make the performance of the robotic system more brain-like. The cognitive map-based SLAM approaches have been successfully applied to mobile robots in real-life environments (Tian et al. 2013; Shim et al. 2014; Yuan et al. 2015). The cognitive map interfering with cognitive memory has been explored by computational modeling and applied to robotic applications (Tang et al. 2017; Hu et al. 2016).

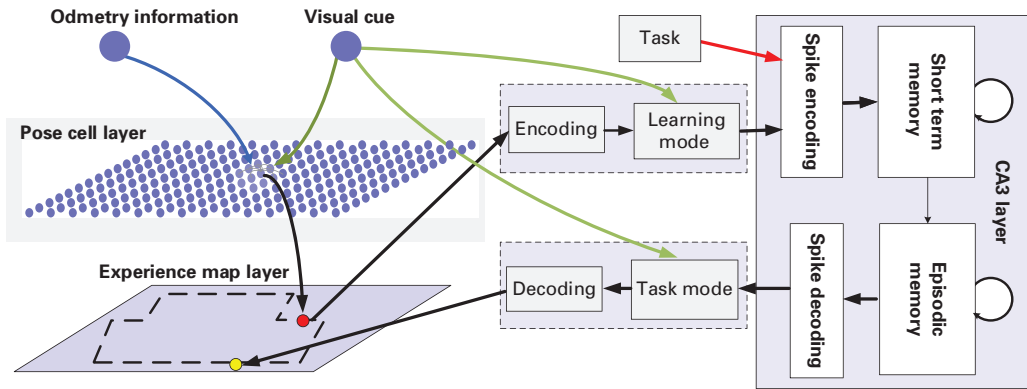
#### Integrating cognitive navigation with episodic memory

Episodic memory endows humans with the ability to respond to salient events in a temporal sequence (Moser, Kropff, and Moser 2008) and recall them sequentially (Tulving and Markowitsch 1998). Though episodic memory has been studied for decades in psychology and neuroscience, recently, researchers have started to build models of episodic memory for intelligent systems. A few studies have developed episodic memory models for cognitive robots using designed data structure to simulate the functionality of episodic memory (Endo 2008; Stachowicz and Kruijff 2012; Jockel, Westhoff, and Zhang 2007). A cognitive memory network plays the role of episodic memory and is involved in navigation through recalling travel experiences, as shown in figure 15.7. This enables a robot to recognize and memorize different locations while storing and recalling the correct sequence to accomplish a task. This system takes advantage of the autoassociation of the memories through neural activities, which can achieve better flexibility and generalization abilities compared to data structure-based models relying on explicit symbolical knowledge programming. The details of the cognitive map can be found in Tang, Yan, and Tan (2018).

#### Episodic memory

As shown in figure 15.7, a dual network model for the CA3 region in the hippocampus is used for encoding and representing episodic memory (Tang, Yan, and Tan 2018). Both networks have synchronized gamma cycles as they share common inhibitory neurons. The





**Figure 15.7**

Overview of system architecture. The system is mainly divided into two parts: the cognitive map and the episodic memory. The pose information is updated by the odometry and visual input. It forms an energy package in the CAN structure. The energy package projects to the experience map, which is then converted to a grid map. In training mode, the task-related location information will be stored in memory. In task mode, the task-related location information is retrieved from memory and used to navigate the robot. *Source:* Tang, Yan, and Tan 2018.

episodic memory network stores the active sequence in its synaptic weights before transferring them to the neocortex. During recall, a cue consisting of the first two items in the desired sequence is presented to the neocortex, which will then reproduce the rest of the stored sequence. The output sequence will only be produced once and will not be repeated. The main steps will be discussed as follows:

1) Storage: the storage of a memory sequence is first demonstrated by introducing seven distinct items in the memory sequence to the CA3 short-term memory network. Each item is introduced to the network at the trough of the theta rhythm, and the network will repeat this value near the peak of each subsequent oscillation. Once the entire sequence has been fully introduced to the short-term memory network, the sequence is presented to the episodic memory network for storage. Here, the sequence is repeated a few times in its entirety until the episodic memory network can learn and store the sequence by updating synaptic weights. Once the storing phase is completed, the amplitude of theta rhythm is reset to zero to stop the function of short-term memory.

2) Retrieval: in the retrieval phase, the first two items in the memory sequence are presented directly to the neocortex as a retrieval cue. After receiving the cue, the pyramidal cells representing the first two items will fire and transmit the action potentials down through synaptic connections to subsequent memory items. Synaptic inputs from the firing of the first two memory items are sufficient to trigger the firing of the next memory item but insufficient for other items. Next, the cumulative synaptic inputs from the firing of the first three memory items trigger the firing of the fourth memory item. The process continues until the entire sequence has been triggered. Hence, the stored sequence memory is retrieved.

### Exploration and navigation

The proposed architecture is verified based on a mobile robot platform Neco in a laboratory environment and a convention hall environment. The robot is equipped with sonar sensors and laser scanners for obstacle avoidance, maintaining a straight path, and detecting

turns and junctions. Neco is programmed to conduct five types of motion: moving forward, turning 90° right, turning 90° left, turning 180°, and then stopping at intersections and stopping at the end. In the task mode, after decoding the neural signals to grid indexes, the memory in CA3 is converted to a sequence of target locations on a cognitive map. Based on the current and target position, a sequence of motion types from the motion pool is generated to guide the robot from its current position to the target position. The navigation combines egocentric local obstacle avoidance and the allocentric global cognitive map. Local navigation is based on data collected directly from sensors, and global navigation is the path planning inside a self-generated global map.

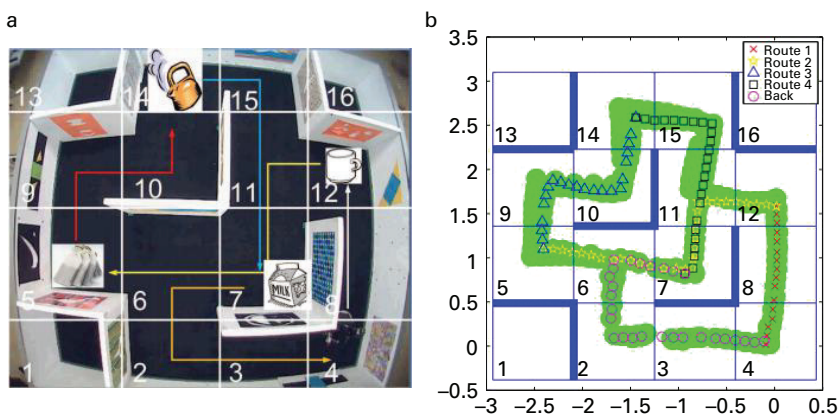
To put the architecture in the real world, a task named “serving milk tea to guest” was performed. We simplified a living-room environment into a 4 m × 4 m maze, as shown in figure 15.8a. The “cup,” “tea,” “hot water,” and “milk” were placed in different locations. The actual experiment with trajectory data is shown in figure 15.8b.

This system offers the capabilities of navigating and mapping in a spatial environment as well as storing and retrieving high-level episodic memories and can be applied to solve high-level service robot tasks. This work would also contribute to developmental robotics by providing a neurophysiological cognitive architecture.

## 15.4 Conclusion

In this chapter, we presented the development history and the state-of-the-art and elementary components of spatial navigation from the bioinspired perspective, mainly focusing on spatial cells, the cognitive map, and navigation. Next we list some valuable future research directions in biologically inspired spatial cognition and navigation as references for readers.

**Multi-map mechanism** Rat studies indicate that the brain hosts multiple cognitive maps representing different subsets of the environment at different times and scales. Maps



**Figure 15.8**

(a) The maze environment for the “serving milk tea to guest” experiment. The required items are placed in different locations in the maze. The arrows indicate the sequence of the action order. (b) The actual experiences trajectory of the mobile robot in the “serving milk tea to guest” task. Task 1 is to get a cup, task 2 is to get the tea, task 3 is to fill the cup with hot water, and task 4 is to add milk.

can be stored and retrieved within a few hundred milliseconds or quickly remapped when environments change or some actions are taken. A future major objective may be to determine how the multiple maps interact with each other and how spatial cells and other factors in the brain contribute to the spatial mapping dynamics. A more comprehensive architecture for space representation and bioinspired navigation will hopefully be developed.

**Spatial memory** By processing inner and environmental signals, the brain can encode and store spatial information for future retrieval. Research on spatial memory is an ongoing topic in the neuroscience and computer science communities. The input signals can be self-movement signals from the vestibular system, visual information, tactile information, and olfactory and auditory cues. Spatial memory can be stored at several levels, including working memory, short-term memory, and long-term memory. The inner cognitive map in the brain and spatial memory can be integrated to help the robot complete very complex cognitive tasks.

**AI and cognitive navigation** The firing patterns of spatial neurons in the brain shed new light on spatial navigation research. Whatever form the cognitive map takes, a broad consensus has emerged that spatial cognition and learning can be achieved through a priori and inherent patterns in the brain. In addition, artificial intelligence (AI) studies demonstrate that these a priori patterns can be obtained through pretraining with large data sets and many learning epochs. So the pretraining design may be an important future research direction. Recent studies, such as the curiosity model and the Bert language model, show us how to design pretraining for a priori structure generation.

**Nonspatial cognitive task** The exploration of spatial cognition provides great inspiration to study many nonspatial cognitive tasks. For example, language reflects a human's ability to use and control signs and can be correlated to spatial cognition: signs correspond to spatial points (or spatial cells), and language corresponds to spatial navigation. If the relationship between spatial cells and navigation can be abstracted as a general cognitive mechanism, maybe we can try to model signs and language from another novel aspect. In an abstracted map with signs, the movement is no longer an action from one point to another in Euclidean space but may be a logical-thinking flow.

## Additional Reading and Resources

- This book is the key publication presenting the hippocampal-based approaches to robot navigation and the RatSLAM approach: Milford, Michael. 2008. *Robot Navigation from Nature: Simultaneous Localisation, Mapping, and Path Planning Based on Hippocampal Models*. Vol. 41. Berlin: Springer Science and Business Media.
- This book gives a comprehensive overview of human spatial navigation: Ekstrom, Arne D., Hugo J. Spiers, Véronique D. Bohbot, and R. Shayna Rosenbaum. 2018. *Human Spatial Navigation*. Princeton, NJ: Princeton University Press.
- This paper provides a recent analysis of the neurobiology of mammal navigation: Poulter, Steven, Tom Hartley, and Colin Lever. 2018. "The Neurobiology of Mammalian Navigation." *Current Biology* 28 (17): R1023–R1042.
- Accessible code for RatSLAM: <https://github.com/davidmball/ratslam>.
- Accessible code for NeuroSLAM: <https://github.com/cognav/NeuroSLAM>.

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