16 Spectral Pitch in Chords

The timbre of a musical sound depends on the pitch pattern created by its audible partials—its spectral-pitch pattern. *In a musical chord, that pattern comprises audible harmonics of chord tones (spectral pitches). A simple back-of-an-envelope analysis shows that typical musical chords evoke spectral pitches that are not octave equivalent with chord tones. For example, a C-major triad CEG [047] evokes spectral pitches at B [L] (3rd harmonic of E; 5th of G) and D [2] (3rd harmonic of G; 7th of E; 9th of C). In composition or performance, these tones may be added to the chord to create a new chord of higher cardinality (more pitch classes). The new chords are Cmaj7 or Cadd9, and the change in timbre is relatively small because the new tone corresponds to a salient spectral pitch that is already present in the triad. Any chord can be analyzed and extended in this way; for example, CDF [025] has audible partials at G and A, which can be added without changing the chord's basic character.*

The Audible Spectrum of a Chord

The amplitude spectrum of a musical chord typically comprises a large number of partials (pure-tone components). Some are inaudible because they mask each other. The audible partials are heard as spectral pitches.

These pitches are interesting for harmony theory if some of them represent pitch classes that differ from the notes. A C-major triad includes audible spectral pitches at B (3rd harmonic of E, 5th harmonic of G) and D (3rd harmonic of G, 7th of E, 9th of C). A C-minor triad includes audible spectral pitches at B♭ (3rd harmonic of E♭, 7th of C) and D (3rd harmonic of G, 9th of C). These spectral pitches can explain why chords such as maj7, add9, and maj9 in jazz are common substitutes for major triads and collectively described as "major." Adding B and D to a C-major triad (to create a major 7th and 9th) does not change the "major" sound very much, because the chord already includes audible tones at B and D. The same idea can explain why the chords m7, m-add9 and m9 can act as sustitutes for minor triads.

Some pop styles from the 1970s and 1980s featured diatonic 7th chords including Δ7 (I7, IV7), m7 (ii7, iii7, vi7), and diatonic varieties of 7sus such as V11 (IV/V) or V13 (IV7/V, the first chord of "Y.M.C.A.," released in 1968 by Village People). Proponents of this style include Burt Bacharach, Carly Simon, and the Carpenters, and relevant genres include Motown (e.g., Marvin Gaye) and disco (e.g., the Bee Gees). The music sounds consonant, despite the frequent harmonic 2nd and 7th intervals, including psychoacoustically rough major 7th intervals (and their inversions, minor 2nds) in Δ7 chords. The consonant feeling (despite the roughness) may be due to constancy of scalar context if most tones are confined to a simple major diatonic scale.

Ear-training courses train musicians to recognize the notes that were played to create a given chord—which harmonic complex tones the sound physically contains. But a musical chord evokes pitches that are not notated, which is one reason why ear training can be difficult. To find out which tones music listeners (from concertgoers to music students struggling with ear training) actually hear in musical chords, we first need to check which partials (pure-tone components) are present and audible (as spectral pitches) in the chord's spectrum. On that basis, we can ask which virtual pitches are heard. In a first step, this chapter focuses on spectral pitches, although virtual pitches tend to be more salient in both music and everyday life.

Major Triad in Close Root Position

We start with the familiar C-major triad in root position and close position in the middle register, C4E4G4. The first ten harmonics of each tone (*n* = 1–10) are listed in table 16.1.

Table 16.1
Spectrum of a C-major triad in close position and middle register.

n	1	2	3	4	5	6	7	8	9	10
C4	C4	C5	G5	C6	E6	G6	B♭6	C7	D7	E7
E4	E4	E5	B5	E6	G♯6	B6	D7	E7	F♯7	G♯7
G4	G4	G5	D6	G6	B6	D7	F7	G7	A7	B7

The top row (*n*) is the harmonic number. The left column lists the note names. In the body of the table are the first ten partials of each tone.

Note that some harmonics are mistuned relative to 12-EDO. In particular, the 7th harmonic is about 1/3 semitone flatter than an equally tempered minor-7th interval. This discrepancy lies within the auditory system's range of tolerance when recognizing harmonic patterns (B. C. J. Moore et al. 1986).

Combining the three rows of the table into one, we obtain the following overall spectrum:

C4 E4 G4 C5 E5 G5 B5 C6 D6 E6 G6 G♯6 B♭6 B6 C7 D7 E7 F7 F♯7 G7 G♯7 A7 B7

The underlined partials are common to two of the three tones. For example, G5 is both the 3rd harmonic of C4 and the 2nd harmonic of G4. The double-underlined partial (D7) is common to all three tones. In musical renditions of a chord, we might expect doubled partials to be more audible on average.

The aural relevance of these partials is limited by psychoacoustic masking. Whereas most or all of the listed partials are probably separately audible when the chord tones are sounded alone, some become inaudible when the chord tones sound together. A pure tone (such as any one of the partials) becomes inaudible if it is close in frequency to a pure tone of higher amplitude. As a rough rule of thumb, if two partials lie a semitone apart, the weaker of the two is masked by the stronger, making the weaker inaudible. For that reason, partials with higher harmonic numbers are more likely to be inaudible: the intervals between them are smaller.

Following these principles, we can delete some of the partials in the chord C4E4G4 because they probably mask each other, crossing them out like this:

C4 E4 G4 C5 E5 G5 B5 ~~C6 D6~~ E6 G6 ~~G♯6 B♭6~~ B6 ~~C7~~ D7 E7 ~~F7 F♯7 G7 G♯7 A7 B7~~

Here, I have assumed that doubled (underlined) tones have higher amplitude than tones that are not doubled. For example, since B5 is doubled, it is likely to mask C6—but not vice versa. Therefore, C6 is likely to be inaudible. Note also that the highest partials in the list have been deleted. That is because, in a real musical chord, the spectrum typically continues toward higher frequencies, with smaller intervals between the partials. At some point, no more partials are audible.

We might reasonably expect the following partials to be audible in a typical musical example of the chord C4E4G4. We could call it the audible spectrum; Terhardt called it the spectral-pitch pattern:

C4 E4 G4 C5 E5 G5 B5 E6 G6 B6 D7 E7

The described procedure is inherently approximate. Each tone in a musical chord can be played by a different instrument and have a different amplitude envelope. Many other aspects of the stimulus can vary in complex ways. There can be subtle variations in attack or release that affect the relative salience of pitches within the chord according to principles of auditory scene analysis. The tones can be more or less loud, and the partials within each tone can be more or less audible.

Within these limitations, we can nevertheless approach the question systematically. The following procedure, inspired by the psychoacoustics of masking, can be applied to any chord:

1. List ten harmonics of each tone, relative to 12-EDO.
2. Create a single ordered series of partials. If a partial is doubled (a harmonic of two different fundamentals), underline it. If three fundamentals, double-underline it.
3. Delete every tone that is not doubled and lies a semitone from an upper or lower neighbor.
4. Delete the highest pitch in the list unless it is doubled.

At the end of this procedure, the lower partials should be generally audible, and the upper partials should only be audible if doubled. Uncertainty can arise if a tone is not doubled and lies midway between two tones four semitones apart, such that the interval is two semitones on both sides. In a typical complex tone, harmonics up to about the 10th are audible (the interval between the 9th and 10th is two semitones).

The audible spectrum of the chord C4E4G4, obtained according to this procedure, includes audible partials at B5, B6, and D7. That is consistent with the timbral similarity of major triads and Cmaj7, Cadd9, and Cmaj9 chords.

Preliminary Experiment

The predicted tones have been shown to be audible (aurally relevant) in a preliminary experiment (Parncutt, Engel, et al., 2023 submitted). In each trial, listeners heard a chord (duration 300 ms) followed by 300 ms of silence and then the same chord with one partial removed, and they were asked to rate similarity (cf. Terhardt 1998, sound example 17). Here are the tested chords and tentative results:

- [047] (major triad) in root position and both inversions: in all three cases, the chord CEG evoked B and D.
- [037] (minor triad) in root position and both inversions: in all three cases, CE♭G evoked B♭ (and perhaps D or F).
- [036] (diminished triad): CE♭G♭ evoked B♭ and D♭.
- [048] (augmented triad): CEG♯ evoked F♯.
- [025]: CDF evoked G and A.
- [035]: CE♭F evoked G and B♭.
- [047T] (Mm7): CEGB♭ evoked D and B♮.

The extra pitch classes that were evoked by these trichords were often tones that can be added to create tetrachords. But given other psychoacoustic reasons for adding particular tones (minimizing roughness, maximizing harmonicity), the listed tones do not necessarily predict the most common tetrachords.

The findings for the major triad can explain why major-7th and major-9th intervals are often added. The general idea is this: if the sound changes relatively little when a given tone is added, that addition is more likely to be made. Musicians are not aware that these partials are present in a major triad (the effect is not a result of conscious theorizing). Instead, they are sensitive to how the sound of a chord changes when a tone is added. The principle is not confined to jazz; the same process applied in the eighteenth and nineteenth centuries as different passing tones were heard against major triads.

Virtual pitches may also predict added tones. It is common to add a major 6th to a major triad, coinciding with the virtual pitch at A (see chapter 15). There is also a virtual pitch at F, but that tone is added less often, being a semitone above chord tone E (EF sounds quite rough). Despite the semitone, triadically or diatonically oriented clusters that include [0457] (an add4 chord) have become popular in modernist choral arrangements. In the choral works of Eric Whitacre, multiple traditional dissonances and their resolutions are often sung simultaneously to create new cluster sonorities (e.g., *When David Heard*; A. Larson 2006). The present theory offers a promising new approach for the analysis of such sonorities.

Pitch classes B and D are not consciously perceived within a C-major triad—except perhaps by some analytic-hearing individuals (cf. Schneider and Wengenroth 2009; Seither-Preisler et al. 2007). Nor are the other spectral and virtual pitches, including those corresponding to chord tones, necessarily consciously perceived in typical musical contexts. The point is that pitch classes

B and D are present in the chord, even if they are not noticed. They therefore contribute to the chord's overall character. Adding harmonic complex tones at those pitch classes changes the overall sound of the chord relatively little.

This informal spectral analysis of a simple musical chord illustrates how a theory of chord perception can be applied in music theory pedagogy. Whereas music theory is often speculative, there is little speculation here. There can be no doubt that the predicted audible partials (spectral pitches) of a given chord really are audible and hence psychologically real.

Revisiting Virtual Pitch

Presented with a complex spectrum such as the chord C4E4G4—created with harmonic complex tones—the brain looks for incomplete harmonic series within it, as it does for any sound. The fundamental of the best-fitting spectrum corresponds to the most salient perceived pitch. A harmonic series can be matched to the above audible spectrum at various transpositions with various degrees of success. In this way, Terhardt's model predicts several possible pitches, and they do not always correspond to chord tones.

Take the pitch C3, an octave below the bass. It is not physically present in the chord as a pure tone, but we might expect the chord to imply C3 as a missing fundamental because most lower harmonics of C3 correspond to audible partials in the audible spectrum. The first ten harmonics of C3 are C3 C4 G4 C5 E5 G5 B♭5 C6 D6 E6, and the spectrum of the chord is C4 E4 G4 C5 E5 G5 B5 E6 G6 B6 D7 E7. Comparing these, we see that six harmonics of C3 (C4, G4, C5, E5, G5, and E6) correspond to audible partials in the chord, and four do not (C3, B♭5, C6, and D6). Therefore, C3 is implied. (I am assuming here that partials above the 10th harmonic of C3—i.e., above E6—are not relevant for the perception of C3.)

Also implied is A2, whose first ten harmonics are A2 A3 E4 A4 C♯5 E5 G5 A5 B5 C♯6. Four of these—E4, E5, G5, and B5, with harmonic numbers relative to A2 of 3, 6, 7, and 9—are audible within the spectrum of C4E4G4, making A2 a possible root. Thus, the chord implies a minor-7th chord on A (Am7), which is a common harmonic substitution (e.g., ii^7 instead of IV in the key of G major).

The idea that CEG implies A can also be explained another way. Musicians are free to add any tone to any chord to produce a chord of higher cardinality, but they tend to add tones that minimize dissonance. Adding A to CEG produces a tetrachord with relatively low dissonance. That can explain why the 6 or m7 chord is used so often by Western composers and performers. An analysis of this kind cannot distinguish between two possibilities: the "nature" idea that we perceive A2 directly as a missing fundamental of C4E4G4, or the "nurture" idea that we perceive A2 in this situation because Am7 is so familiar (which in turn is because it is relatively consonant).

The chord C4E4G4 also implies F2. The first ten harmonics of F2 are F2 F3 C4 F4 A4 C5 E♭5 F5 G5 A5. Comparing this list with the chord's audible spectrum, three harmonics of F2—C4, C5, and G5 (harmonic numbers 3, 6, and 9)—are present within the spectrum. If we then add F2 to the chord, we get a more dissonant chord than before. F2C4E4G4 (or C/F) is more dissonant than A2C4E4G4 (or C/A) due to the major 7th between F and E, which can explain why it is used less often.

Table 16.2

Spectrum of a C-minor triad in close position and middle register.

n	1	2	3	4	5	6	7	8	9	10
C4	C4	C5	G5	C6	E6	G6	B♭6	C7	D7	E7
E♭4	E♭4	E♭5	B♭5	E♭6	G6	B♭6	D♭7	E♭7	F7	G7
G4	G4	G5	D6	G6	B6	D7	F7	G7	A7	B7

Minor Triad in Close Root Position

Now consider the C-minor triad shown in table 16.2. Following the same procedure as before, the spectrum is:

C4 E♭4 G4 C5 E♭5 G5 B♭5 C6 D6 E♭6 E6 G6 B♭6 B6 C7 D♭7 D7 E♭7 E7 F7 G7 A7 B7.

Some of these partials can be deleted according to the above rules, as follows:

C4 E♭4 G4 C5 E♭5 G5 B♭5 C6 ~~D6 E♭6 E6~~ G6 B♭6 ~~B6 C7 D♭7~~ D7 ~~E♭7 E7~~ F7 G7 ~~A7 B7~~.

The predicted audible spectrum or spectral-pitch pattern is then:

C4 E♭4 G4 C5 E♭5 G5 B♭5 C6 G6 B♭6 D7 F7 G7.

Four spectral pitches in this pattern are not octave equivalent with chord tones. They are B♭5, B♭6, D7, and F7. If we want to create a tetrachord by adding a pitch class to this C-minor trichord while changing the sound as little as possible, we have various options:

- add a minor 7th to make a Cm7 chord,
- add a major 9th to make a Cm-add9,
- add a perfect 4th to make Cm-add4, or
- add two or all three of these tones.

Intuitively, none of these options affects the chord's "minor" character.

The virtual pitches evoked by this chord include A♭ and F. F can be added as a kind of color tone; in the bass, it creates an F9 chord with a missing 3rd. Adding A♭ in the bass changes the root from C to A♭, and the chord becomes major.

The analysis of a minor triad can also be done in an octave-generalized fashion, ignoring the octave registers of the partials. Starting with the A-minor triad ACE, the tone A usually has audible harmonics at chromas A, E, C♯, G, and B; the tone C, at C, G, E, B♭, and D; and the tone E, at E, B, G♯, D, and F♯. Of those partials, the following have pitch classes that are not included among the chord tones: C♯, D, F♯, G, G♯, B, and B♭ (here, underlining denotes pitch classes that are harmonics of two chord tones). Applying the above masking rules, this list can be reduced to D, G, and B. These are tones that can be added to an A-minor triad without changing the sound very much, and the chords that result are Am-add4, Am7, and Am-add9.

What virtual pitches are implied by an A-minor triad in an octave-generalized approach? The chord first evokes virtual pitches at the chord tones: A, C, and E. Of those, A is the most salient, but C is not far behind. The chord also evokes virtual pitches at three missing fundamentals. Starting with the most salient, they are D (because all three tones are harmonics of D), F (because A and C are harmonics of F), and B (because A is a harmonic of B). If we ignore the masking procedure in the previous paragraph and consider all harmonics of all tones up to the 10th, the missing fundamental at D could be associated with partials at D, A, F♯, C and E; the missing fundamental at F, with C, A and G; and the missing fundamental at B, with B, F♯, A, and C♯.

Table 16.3
Spectrum of chord CDF in close position and middle register.

n	1	2	3	4	5	6	7	8	9	10
C4	C4	C5	G5	C6	E6	G6	B♭6	C7	D7	E7
D4	D4	D5	A5	D6	F♯6	A6	C7	D7	E7	F♯7
F4	F4	F5	C6	F6	A6	C7	E♭7	F7	G7	A7

[025] in Close Root Position

As a third example, consider the chord CDF (table 16.3)—a familiar sound that has no name in traditional music theory. The partials of C4D4F4 are:

C4 D4 F4 C5 D5 F5 G5 A5 C6 D6 E6 F6 F♯6 G6 A6 B♭6 C7 D7 E♭7 E7 F7 F♯7 G7 A7.

Applying the above rules, several partials can be deleted:

C4 D4 F4 C5 D5 F5 G5 A5 C6 ~~D6 E6 F6 F♯6 G6~~ A6 ~~B♭6~~ C7 D7 ~~E♭7~~ E7 ~~F7~~ ~~F♯7~~ ~~G7 A7.~~

The predicted audible spectrum is then:

C4 D4 F4 C5 D5 F5 G5 A5 C6 A6 C7 D7 E7.

The spectral pitches that do not correspond to chord tones are G5 A5 A6 E7. The chord implies A, which, if added, makes it an F6 or Dm7 chord, to which an additional E may be added without changing the chord's character. The chord also implies G, making it a Csus-add9 or G7sus chord.

Reflection

We have seen that the audible partials of a musical chord not only affect its timbre and musical function—they also influence what tones can be added to the chord to create a chord of higher cardinality. That is particularly true for trichords, in which a few audible partials can usually be identified whose pitch classes do not correspond to chord tones. It is less true for tetrachords, whose audible partials less often have pitch classes that differ from chord tones because the partials are closer together and mask each other more.

Among pitch classes that do not correspond to chord tones, virtual pitch theory predicts that virtual pitches tend to be more salient than spectral pitches—at least when the chords are perceived spontaneously, without focusing attention on specific pitches (whereas in analytic listening, virtual pitches tend to disappear). For example, virtual pitches at F and A, when implied by a C-major triad, are predicted to be more salient in a musical context (and hence musically more important) than spectral pitches at B and D. There are also effects of voicing. A tone that immediately precedes a chord in a musical passage may attract attention to a nearby audible partial according to the principle of pitch proximity in auditory scene analysis.

Psychoacoustic Foundations of Major-Minor Tonality

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