

## Special Issue: Sensitivity Analysis and Uncertainty Quantification

As modeling and simulation of physical phenomena are taking a central role in the design, assessment, and optimization of engineering systems, engineers must often answer questions such as: How well does a mathematical model capture the relevant physical phenomena? What confidence can be placed on simulation results? How far from the nominal design can computational results be extrapolated? What are the impacts of inherent model parameter variability and imprecise measurements on the calculated results? How can a design be made robust to imperfections and uncertainties, with performance and operational safety maximized and cost minimized? Qualitative and quantitative answers to such questions are provided by sensitivity and uncertainty analyses.

The past few decades have witnessed a large amount of research effort in the formulation of global and local sensitivity analyses for various linear and nonlinear, algebraic and differential mathematical models, as well as efficient computational techniques for automatic generation of computer code for computation of derivative information. In parallel, uncertainty quantification methods have been developed in the domains of statistical science and applied mathematics and applied in solving engineering problems. Sensitivity analysis (SA) and uncertainty quantification (UQ) are becoming mainstream tools for assessing the effect of errors stemming from mathematical idealization, model approximation, parameter variation, and uncertain operational conditions. These techniques have been used in model evaluation, reduced-order modeling, data assimilation, design optimization, model verification, and validation.

In this special issue, state-of-the-art developments and applications of SA and UQ to system dynamics problems are highlighted. The special issue contains 14 papers, half of which are about SA techniques and half on UQ methods. While the two topics are often inextricably coupled, we list them in two separate categories.

**Sensitivity Analysis.** The seven papers in this category highlight various techniques for computing model derivative information, including direct and adjoint local SA for time-dependent dynamic systems, global SA using polynomial chaos, and probabilistic analysis through Monte Carlo simulations. They demonstrate the use of SA in various applications, such as vehicle dynamics, aircraft optimization, robotic workspace analysis, and topology optimization.

The paper entitled “Discrete Adjoint Method for the Sensitivity Analysis of Flexible Multibody Systems,” by Callejo, Sonnevill, and Bauchau presents a discrete version of the adjoint method for sensitivity analysis applied to the dynamic simulation of flexible multibody systems. In contrast to the continuous adjoint method, the discrete approach computes the exact adjoint of the discrete forward solution, while leading to a backward algebraic problem which circumvents the need for backward time integration.

The paper “Sensitivity of Lyapunov Exponents in Design Optimization of Nonlinear Dampers,” by Tamer and Masarati uses analytical sensitivity of Lyapunov Characteristic Exponents (LCEs) in the design of nonlinear dampers. Using a discrete QR

method, the proposed approach permits evaluation of the LCE and their analytical sensitivities simultaneously with the underlying nonlinear problem. The method is demonstrated on a parameter identification problem in helicopter ground resonance and landing gear shimmy vibration and is shown to be more accurate and efficient than the alternative of approximating sensitivities through finite differencing.

The paper entitled “Nonintrusive Global Sensitivity Analysis for Linear Systems with Process Noise,” by Nandi and Singh focuses on the global sensitivity analysis of linear systems with time-invariant model parameter uncertainties and driven by stochastic inputs. Polynomial Chaos models without the necessity of evaluating indefinite integrals illustrate that the parameters which do not contribute to the uncertainty of the mean can significantly contribute to the uncertainty of the variances.

In the paper “Direct Sensitivity Analysis of Multibody Systems: A Vehicle Dynamics Benchmark,” by Callejo and Dopico, the authors present verification of the direct differentiation method for optimization of multibody dynamics using two radically different computational techniques, manual and automatic differentiation. The efficiency and ease of implementation of the two approaches are compared on an 18-degree-of-freedom multibody model of a bus.

In the paper “Reliable and Failure-Free Workspaces for Motion Planning Algorithms for Parallel Manipulators Under Geometrical Uncertainties,” by Vieira, Fontes, Beck, and da Silva, failure probabilities and failure maps of a 3RRR manipulator in the workspace configuration are estimated by Monte Carlo simulation, given the kinematic parallel singularities caused by geometrical uncertainty. The failure modes and failure probabilities related to singularities are identified with the inverse of the condition number of the Jacobian matrix. The metrics can be applied to assess the reliability of motion planning.

In their paper “Topology Optimization Under Stress Relaxation Effect Using Internal Element Connectivity Parameterization,” by Takaloozadeh and Yoon, the authors present a topology optimization method, by employing a simplified creep model, to take into account the stress relaxation effect due to temperature variation.

The paper “Topology Optimization of Dynamic Systems Under Uncertain Loads: An  $H_\infty$ -Norm-Based Approach,” by Venini, discusses topology optimization of dynamic systems based on the H-infinity norm concept of state-space models in the frequency domain. A minimax formulation is developed to search for the configuration with the minimum H-norm of dynamic compliance with the worst case of maximum load. The sensitivities of frequency response functions with respect to design parameters are estimated and applied for optimization. The results from the numerical examples of a square cantilever and the Messerschmidt-Bolkow-Blohm beam show that the optimal topology with static or dynamic compliance minimized is sensitively affected by load uncertainty.

**Uncertainty Quantification.** The seven papers in this category highlight the quantification and propagation of model-form and

parameter uncertainty and discrepancy prediction, often based on surrogate modeling methods such as generalized polynomial chaos, Dynamic Kriging (DKG), and radial basis function (RBF). For UQ, variabilities in both input parameters and initial states are considered. Applications include dynamic response of flexible multibody systems, stochastic speed-made-good maps, motion planning strategies for parallel manipulators, noise radiated by a plate, and MEMS switches.

In the work entitled "Multiple Dynamic Response Patterns of Flexible Multibody Systems With Random Uncertain Parameters," by Wang, Tian, and Hu, parameter uncertainty in flexible multibody systems, as a result of geometric variation and material inhomogeneity, is quantified with the polynomial chaos expansion method. Instead of only one surrogate model, multiple models are constructed with the clustered collocation points and combined based on the Dirichlet process mixture model, in order to improve the accuracy of polynomial chaos expansion. The new method is applied to two numerical examples, and the accuracy is comparable to the Monte Carlo sampling.

In "Discrepancy Prediction in Dynamical System Models Under Untested Input Histories," by Neal, Hu, Mahadevan, and Zumberge, the authors presented a discrepancy prediction methodology for dynamic system based on two surrogate modeling methods: observation surrogate and bias surrogate.

The paper "Generalized Polynomial Chaos With Optimized Quadrature Applied to a Turbulent Boundary Layer Forced Plate," by Wixom, Walters, Martinelli, and Williams uses generalized polynomial chaos expansion with stochastic collocation for modeling the uncertainty in the noise while significantly reducing the number of evaluations of the deterministic model compared to Monte Carlo sampling. The application of new quadrature rules to compute the generalized polynomial chaos expansion coefficients accurately reconstructs the output statistics by propagating the input uncertainty through a computational physics model while using far fewer quadrature nodes than with traditional methods.

In the paper entitled "Uncertainty Quantification Using Generalized Polynomial Chaos Expansion for Nonlinear Dynamical Systems With Mixed State and Parameter Uncertainties," by Bhusal and Subbarao, the generalized polynomial chaos collocation based on the mixed sparse grid is applied to propagate parameter uncertainty to state variables in nonlinear dynamical systems. The evolution of probability densities for state variables can be estimated and is demonstrated with examples of harmonic oscillators and two-degree-of-freedom airfoil oscillation. Compared to traditional Monte Carlo sampling, the described pseudospectral stochastic collocation framework can significantly reduce the number of samples and save simulation time.

The paper "Framework of Reliability-Based Stochastic Mobility Map for Next Generation NATO Reference Mobility Model," by Choi, Jayakumar, Funk, Gaul, and Wasfy, introduces a computational framework for propagation of the variability in

terrain and soil properties in ground vehicle simulation for the generation of stochastic off-road mobility maps. Based on the constructed dynamic Kriging surrogate models, the inverse reliability analysis is carried out using Monte Carlo simulations. The proposed procedure is demonstrated in the generation of stochastic speed-made-good maps for a prototype wheeled vehicle platform over a significantly large geographical area.

In "Radial Basis Functions Update of Digital Models on Actual Manufactured Shapes," by Biancolini and Cella, the authors consider the problem of including the actual manufactured geometry obtained from optical or contact based metrology in a CAE model, as opposed to merely the nominal design intent. Using a method for approximating a field of scattered points in space, based on radial basis functions, this automated approach minimizes expert user input and facilitates the inclusion of uncertainties and variable geometries in CAE-based analysis tools, as demonstrated in the problem of reconstruction of the wet surface of a manufactured wind tunnel model.

The paper entitled "Uncertainty Considerations for Nonlinear Dynamics of a Class of MEMS Switches Undergoing Tip Contact Bouncing," by Bognash and Asokanathan, treats the problem of predicting uncertain measures for MEMS switches based on their transient bouncing dynamic response. This study quantifies the second order statistics for the output measures of a complex microswitch model that includes flexibility and an asperity-based contact model with parameters of material and geometric properties, actuation voltage, and tip asperity contact parameters.

**Radu Serban**  
Department of Mechanical Engineering,  
University of Wisconsin–Madison,  
Madison, WI 53706  
e-mail: serban@wisc.edu

**Yan Wang**  
Woodruff School of Mechanical Engineering,  
Georgia Institute of Technology,  
Atlanta, GA 30332  
e-mail: yan.wang@me.gatech.edu

**Kyung K. Choi**  
Department of Mechanical Engineering,  
The University of Iowa,  
Iowa City, IA 52242  
e-mail: kyung-choi@uiowa.edu

**Paramsothy Jayakumar**  
US Army RDECOM TARDEC,  
Warren, MI 48397  
e-mail: paramsothy.jayakumar.civ@mail.mil