

# Geometry and Meaning

**Dominic Widdows**

(MAYA Design)

Stanford, CA: CSLI Publications (CSLI lecture notes, number 172), 2004, xxii+319 pp; hardbound, ISBN 1-57586-447-9, \$70.00; paperbound, ISBN 1-57586-448-7, \$25.00, £17.50

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*Let no man enter here who is ignorant of geometry.*

—Plato

*Geometry and Meaning* is an interesting book about a relationship between geometry and logic defined on certain types of abstract spaces and how that intimate relationship might be exploited when applied in computational linguistics. It is also about an approach to information retrieval, because the analysis of natural language, especially negation, is applied to problems in IR, and indeed illustrated throughout the book by simple examples using search engines. It is refreshing to see IR issues tackled from a different point of view than the standard vector space (Salton, 1968). It is an enjoyable read, as intended by the author, and succeeds as a sort of tourist guide to the subject in hand.

The early part of the book concentrates on the introduction of a number of elementary concepts from mathematics: graph theory, linear algebra (especially vector spaces), lattice theory, and logic. These concepts are well motivated and illustrated with good examples, mostly of a classificatory or taxonomic kind. One of the major goals of the book is to argue that non-classical logic, in the form of a quantum logic, is a candidate for analyzing language and its underlying logic, with a promise that such an approach could lead to improved search engines. The argument for this is aided by copious references to early philosophers, scientists, and mathematicians, creating the impression that when Aristotle, Descartes, Boole, and Grassmann were laying the foundations for taxonomy, analytical geometry, logic, and vector spaces, they had a more flexible and broader view of these subjects than is current. This is especially true of logic. Thus the historical approach taken to introducing quantum logic (chapter 7) is to show that this particular kind of logic and its interpretation in vector space were inherent in some of the ideas of these earlier thinkers.

Widdows claims that Aristotle was never respected for his mathematics and that Grassmann's *Ausdehnungslehre* was largely ignored and left in obscurity. Whether Aristotle was never admired for his mathematics I am unable to judge, but certainly Alfred North Whitehead (1925) was not complimentary when he said:

*The popularity of Aristotelian Logic retarded the advance of physical science throughout the Middle Ages. If only the schoolmen had measured instead of classifying, how much they might have learnt! (page 41)*

On the other hand, Grassmann was recognized for his foundational work on analytical geometry; for example, Hermann Weyl (1949) comments:

*Today probably the best approach to analytic geometry is by means of the vector concept, following the procedure of Grassmann's Ausdehnungslehre. (page 68)*

Weyl of course contributed extensively to quantum mechanics and already then held the view that “classical logic does not fit in with quantum physics and is to be replaced by a kind of ‘quantum logic’ ” (page 263). More recently Suppes et al. (1989) refer to Grassmann’s ground-breaking work on geometrical structures, which are now called Grassmann Structures. I think the case for Grassmann’s obscurity is unproven.

Chapters 1–6 are really an elementary introduction for those without much experience in mathematics and logic to prepare them for the novel work presented on quantum logic and concept lattices in Chapters 7 and 8 (but see Courant and Robbins [1941] and Marciszewski [1981] for further introductory material). The elementary concepts are well illustrated with examples from search engines and with problems of ambiguity of words. The “killer application” for quantum logic in chapter 7 is the modeling of negation, and in chapter 8 the interpretation of taxonomic structures as non-Boolean lattices. In both cases the applications are convincing. The basis for the representation in quantum logic is the lattice of subspaces of a vector space endowed with a geometry which determines a logic (Birkhoff and von Neumann 1936). It is a pity that Widdows did not complete the story, which was started by Birkhoff and von Neumann, and introduce the probability measure on this lattice of subspaces. One of the striking results in quantum theory is that this can be done consistently and uniquely (Gleason 1957). Von Neumann was well aware of this although he could not prove it; in 1954, he said:

*In other words, probability corresponds precisely to introducing the angles geometrically. Furthermore, there is only one way to introduce it. The more so because in the quantum mechanical machinery the negation of a statement, so the negation of a statement which is represented by a linear set of vectors, corresponds to the orthogonal complement of this linear space. (von Neumann, 1954, reproduced in Rédei and Stölzner 2001, page 244)*

Had probability been introduced, and therefore the role of measurement, it would have helped explain the somewhat tantalizing expression on page 238: “It follows that every ‘experimental proposition’ in a quantum mechanical system corresponds to a subspace of the vector space in which the states of the system are represented mathematically.” It is curious that in the entire book, probability is mentioned only twice in passing. Returning to Whitehead and his beef about Aristotle, when he complains about Aristotle not “measuring,” one needs to take this seriously because the machinery of quantum mechanics and its observables only makes sense when one considers the observation of attributes with a probability of success or failure. Heisenberg’s uncertainty principle, one of the foundation stones of quantum mechanics, was about observables and their probabilistic interaction. A quantum logic without a theory of observation (or interaction) is somewhat empty. Readers might like to pursue this line of reasoning by consulting Beltrametti and Cassinelli (1981), one of the seminal works on quantum logic.

Each of Widdows’ chapters ends with a delightful and useful “Wider Reading” section encouraging the reader to explore further afield; overall, these sections total 15 pages, or about 5% of the book. Below in the appendix to this review, I add a few references that complement some of Widdows’.

In his foreword to the book, Pentti Kanerva suggests that the substance of the book is “the exploration of mathematics that would be appropriate for describing concepts and meaning.” For this reviewer, this is only part of the story; the book goes beyond exploration and applies the relevant mathematics to computational problems in linguistics and information retrieval. It may be the first steps along the way to recasting some old problems in terms of some new mathematics.

## Appendix: Wider Reading

Possibly the most outstanding reference to numerical taxonomy is Sneath and Sokal (1973). For a more up-to-date and appropriate reference to information retrieval, I would propose Belew (2000). The use and application of non-classical logic in IR is well covered by Crestani et al (1998). An extremely relevant set of papers on quantum logic, in that they deal with taxonomy and various conditionals — for example, the Stalnaker conditional — are the papers by Hardegree, especially Hardegree (1976). His 1982 paper on natural kinds covers similar territory to Widdows’ discussion on “extent” and “intent” and its mathematical duality. An excellent source for papers on developments in quantum logic is Beltrametti and van Fraassen (1981). A critical view of the quantum logic enterprise is Gibbins (1987). To plug the gap on measurement in quantum mechanics one can do no better than Wheeler and Zurek (1983).

Although Widdows does an excellent job of introducing most of the elementary concepts needed in his book, there is room for some more guidance on where to go next. An excellent introduction to the broad field of mathematics, each chapter written by one of the masters of the field, is Newman (1988). More specifically, good introductions to the foundations of vector spaces can be found in Halmos (1958), and Isham (1989). Finally, it may be worth pointing out that the late Jon Barwise, who also worked at CSLI, introduced quantum logic as part of his work on information flow (Barwise and Seligman 1997).

## References

- Barwise, J. and J. Seligman (1997). *Information Flow: The Logic of Distributed Systems*. Cambridge University Press.
- Belew, R. 2000. *Finding Out About: A Cognitive Perspective on Search Engine Technology and the WWW*. Cambridge University Press.
- Beltrametti, E. G. and G. Cassinelli. 1981. *The Logic of Quantum Mechanics*. Addison-Wesley Publishing Company, London.
- Beltrametti, E. G. and B. C. van Fraassen, Eds. 1981. *Current Issues in Quantum Logic*. Plenum Press, New York.
- Birkhoff, G. and J. von Neumann. 1936. The logic of quantum mechanics. *Annals of Mathematics*, 37:823–843.
- Courant, R. and H. Robbins. 1941. *What is Mathematics? An Elementary Approach to Ideas and Methods*. Oxford University Press.
- Crestani, Fabio, Mounia Lalmas, and C. J. van Rijsbergen, editors. 1998. *Information Retrieval: Uncertainty and Logics: Advanced Models for the Representation and Retrieval of Information*. Kluwer, Boston.
- Gibbins, P. 1987. *Particles and Paradoxes: The Limits of Quantum Logic*. Cambridge University Press.
- Gleason, A. M. 1957. Measures on the closed subspaces of a Hilbert space. *Journal of Mathematics and Mechanics*, 6:885–893.
- Halmos, P. R. 1958. *Finite-Dimensional Vector Spaces*. D. van Nostrand Company, Inc.
- Hardegree, G. M. 1976. The conditional in quantum logic. In Patrick Suppes, editor, *Logic and Probability in Quantum Mechanics*, D. Reidel Publishing Company, pages 55–72.
- Hardegree, G. M. 1982. An approach to the logic of natural kinds. *Pacific Philosophical Quarterly*, 63:122–132.
- Isham, C. J. 1989. *Lectures on Groups and Vector Spaces for Physicists*. World Scientific, Singapore.
- Marciszewski, W., editor. 1981. *Dictionary of Logic — As Applied in the Study of Language*. Martinus Nijhoff Publishers.

- Newman, J. R., ed. 1988. *The World of Mathematics: A Small Library of the Literature of Mathematics from A'h-mosé the Scribe to Albert Einstein*. Tempus Books, Redmond.
- Rédei, M. and M. Stöltzner, eds. 2001. *John von Neumann and the Foundations of Quantum Physics*. Vienna Circle Institute Yearbook, Kluwer Academic Publishers.
- Salton, Gerard. 1968. *Automatic Information Organization and Retrieval*. McGraw-Hill.
- Sneath, P. H. A. and R. R. Sokal. 1973. *Numerical Taxonomy*. W. H. Freeman and Company, San Francisco.
- Suppes, Patrick., David H. Krantz, R. Duncan Luce, and Amos Tversky. 1989. *Foundations of Measurement, Vol. II: Geometrical, Threshold, and Probability Representations*. Academic Press, San Diego.
- Weyl, Hermann. 1949. *Philosophy of Mathematics and Natural Science*. Princeton University Press.
- Wheeler, J. A. and W. H. Zurek, editors. 1983. *Quantum Theory and Measurement*. Princeton University Press.
- Whitehead, Alfred North. 1926. *Science and the Modern World*. Cambridge University Press.

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