Elements of Formal Semantics: An Introduction to the Mathematical Theory of Meaning in Natural Language

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Yoad Winter’s Elements of Formal Semantics is a lucid, well-organized, rather concise, and opinionated introductory textbook of formal natural language semantics. “Formal semantics” here refers to the study of sentence meaning and its tight connection to sentence structure, described with precision using mathematical methods. Elements introduces the central questions, concerns, and techniques of formal semantics, motivated by discussion of successively more complex phenomena in English. The end result presented is a particular flavor of categorial or type-logical grammar.

Elements is organized as a textbook and I think this certainly will be its primary use. The book claims to “not presuppose any prior knowledge in logic or theoretical linguistics” (p. 7). It would be appropriate for a first course in semantics at the graduate or upper undergraduate level. In addition to linguistics programs, the book would be an excellent text for courses on natural language meaning and related topics in computer science, artificial intelligence, or philosophy—and in fact, the required level of fluency with mathematical notation and symbolic manipulation may make it more suited to such students, at least at the undergraduate level. This is not a text introducing computational semantics and there is no discussion of practical implementation details. In addition to its primary use as a textbook, this coverage of key topics makes the book a useful primer to the central concerns and results of formal semantics for researchers in a range of closely related disciplines.

The 200-odd-page body of the text is organized into five core chapters, sandwiched between brief introduction and conclusion chapters. After the introductory chapter, Chapter 2 introduces us to the study of sentence meaning using judgments of entailment between sentences as the main source of evidence. In a sleight of hand, Winter introduces the “truth-conditionality criterion” (p. 20)—the central hypothesis that links these intuitive judgments of entailment to abstract models—without introducing or using the notion of a “truth-condition.” The idea of compositionality is introduced, motivated by the study of a toy model of simple copular sentences, negation, and conjunction, with discussion of structural ambiguities.

Chapter 3 introduces much of the common technical concepts and notation of formal semantics, including types, (higher-order) functions and sets, Currying, and λ-notation. This chapter is particularly well organized and thought through, introducing these technical notions step by step, with examples and motivation from linguistic phenomena where possible. This chapter is one of the clearest...
presentations of these tools that I have seen, with the caveat that it presupposes a certain fluency with the underlying mathematical notation and concepts. It concludes with an application of these tools to reflexive pronouns and intersective and non-intersective attributive adjectives, as well as propositional vs. predicate negation and conjunction.

Chapter 4 is concerned with the interpretation of quantifiers in natural language. Generalized quantifier theory is discussed, first with descriptions of quantifiers as relations between sets, followed by discussion of quantifiers as sets of predicates. This is followed by clear and rather standard exposition of monotonicity, negative polarity item licensing, the denotation of quantificational determiners, and conservativity.

Despite my saying that Chapters 3 and 4 are very clear—or maybe because of this—I want to reiterate again that some hand-holding will be required for students without backgrounds in, for example, mathematics or computer science. Simple familiarity with the basic concepts and notation of sets and functions is in fact insufficient; the reader must be comfortable moving quickly between these characterizations and translating between them. Take for example the discussion of quantificational determiner meanings, where in the span of just three pages (p. 116ff), the text switches between (a) a Curried, higher-order function (a function from predicates to functions from predicates to truth values), (b) a binary function (a function mapping two sets of individuals to truth values), and (c) a relation (a set of pairs of sets of individuals), choosing each formulation where convenient. Typographic conventions do not help the reader either. Sometimes the variables $A$ and $B$ are functions (as in [4.47a]) and sometimes they are sets (as in [4.47b] right below!). I find these issues unfortunate because it would not have taken an inordinate effort to make these (otherwise great) chapters friendlier towards readers who are less mathematically inclined.

I think of Chapter 5, entitled “Long-Distance Meaning Relationships,” as logically two chapters and will discuss them in turn. The first half introduces the process of function abstraction through hypothetical reasoning in the tradition here called the Lambek-Van Benthem Calculus, motivated by the discussion of non-subject quantifiers and relativization. A brief detour introduces the idea of introducing a hypothetical premise $\varphi$ into a proof of conclusion $\psi$ and then “discharging” $\varphi$ later, resulting in the conclusion $\varphi \rightarrow \psi$. By analogy, in the computation of many sentences with quantifiers and relativization, we introduce a hypothetical argument (variable) $u$ to yield the $u$-dependent meaning $z$, and then later “discharge” the hypothesis of $u$ to yield the function $\lambda u . z$, abstracting over the value of $u$. This presentation is friendly and successful. There is no mention of the relation of this process of variable introduction and abstraction to syntactic movement as discussed in much contemporary work on the syntax/semantics interface, save for one brief mention later (p. 175).

The second half of Chapter 5 uses questions raised in the first half regarding the relationship between semantics and surface word order as motivation to introduce the framework known as Abstract Categorial Grammar (ACG) or also Lambda Grammar. Under this approach, lexical items (signs) have perceptual denotations that can be (higher-order) functions over strings, not just strings themselves. For example, the transitive verb praise has the perceptual representation $\lambda x . \lambda y . y \cdot praised \cdot x$, where $\cdot$ is concatenation; this encodes that the first argument of praise will follow the verb in surface word order and its second argument will precede the verb. This treatment of the syntax of word order will strike readers with any previous training to the intricacies and
regularities of natural language syntax as terribly naive. But to be fair, these limitations
are readily acknowledged by Winter himself:

...when we talk about ‘Abstract Categorial Grammar,’ we use the term ‘grammar’ in the
sense of a formal grammatical framework, with no pretensions to the wide empirical
coverage that we normally expect from grammatical descriptions of natural
languages... at present there is no commonly accepted way of embedding ACG within
linguistic theory. (p. 167)

The chapter ends with discussion and treatment of scope ambiguities.

Chapter 6 is an introduction to intensional semantics, motivated by classic problems
of sense versus reference. After a brief and approachable survey of intensional contexts,
a domain of possible worlds (or indices) is added to our models and individual concepts
are introduced, deriving the desired result that Lewis Carroll is C. L. Dodgson and John
believes Lewis Carroll wrote Alice does not entail John believes C. L. Dodgson wrote Alice.
This is followed by a more general discussion of intensionalization using the ACG type
system introduced in Chapter 5. The chapter concludes with de re/de dicto ambiguities
and their analysis as scope ambiguities.

Finally, Chapter 7 suggests directions for further study, devoting a paragraph each
to a range of additional topics in formal semantics. This is followed by a brief appendix
to Chapter 3 with formal definitions for the basic typing and interpretation system.

Each of the core chapters ends with pointers to recommended further reading,
at both introductory and advanced levels, which include key works in linguistics,
computer science, and philosophy of language. This is followed by a set of exercises,
immediately followed by selected solutions. Some exercises emphasize mechanical
practice in engaging with technical notions, some are more conceptual, and some intro-
duce new linguistic phenomena to consider. All are clear and of appropriate difficulty.
Some are downright clever, like problems 5 and 6 of Chapter 2 (p. 38ff). The inclusion of
selected solutions—just final results, not intermediate steps or detailed justification—
will be appreciated by conscientious students and those using the text for self-study.

In reading the book as a potential text for students, a few gaps stand out. In
Chapter 2, where entailment is given a central role in the empirical investigation of
meaning, there is no discussion of presupposition. In fact, the definition of “entailment”
(p. 16) as any indefeasible consequence will include both presuppositions and entail-
ments proper. Also missing is discussion of motivation and diagnostics for constitu-
cy. Readers with little or no linguistics background would have to simply accept
the bracketings and trees given in the text, without reliable tools to work out
novel sentences on their own. There is also no discussion of languages other than
English. Of course, the methods presented here are broadly applicable to other lan-
guages, but in my experience it is worth making this point. Still, at the end of the day,
I generally think the choice of concepts and topics covered is good, as is their depth.

The physical book (I reviewed the paperback) is relatively compact, with nice, large
text. The prose feels mature and well edited, with just a small handful of typos.

Overall, Elements is a generally clear, precise, and friendly introduction to formal
natural language semantics for students with mathematical maturity. I recommend it.

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ceived his Ph.D. in Linguistics from MIT in 2014. His research interests lie in the syntax/semantics
interface and linguistic theory, with a particular interest in long-distance dependencies including
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