Jyri Pakarinen* and David T. Yeh†
*Department of Signal Processing
and Acoustics
Helsinki University of Technology
P.O. Box 3000
FI-02015 TKK Finland
jyri.pakarinen@tkk.fi
†Center for Computer Research
in Music and Acoustics
Department of Music
Stanford University
Stanford, California 94305-8180 USA
dtyeh@ccrma.stanford.edu

A Review of Digital Techniques for Modeling Vacuum-Tube Guitar Amplifiers

Although semiconductor technologies have displaced vacuum-tube devices in nearly all fields of electronics, vacuum tubes are still widely used in professional guitar amplifiers. A major reason for this is that electric-guitar amplifiers are typically overdriven, that is, operated in such a way that the output saturates. Vacuum tubes distort the signal in a different manner compared to solid-state electronics, and human listeners tend to prefer this. This might be because the distinctive tone of tube amplifiers was popularized in the 1950s and 1960s by early rock and roll bands, so musicians and listeners have become accustomed to tube distortion. Some studies on the perceptual aspects of vacuum-tube and solid-state distortion have been published (e.g., Hamm 1973; Bussey and Haigler 1981; Santo 1994).

Despite their acclaimed tone, vacuum-tube amplifiers have certain shortcomings: large size and weight, poor durability, high power consumption, high price, and often poor availability of spare parts. Thus, it is not surprising that many attempts have been made to emulate guitar tube amplifiers using smaller and cheaper solid-state analog circuits (e.g., Todokoro 1976; Sondermeyer 1984). The next step in the evolution of tube-amplifier emulation has been to simulate the amplifiers using computers and digital signal processors (DSP).

A primary advantage of digital emulation is that the same hardware can be used for modeling many different tube amplifiers and effects. When a new model is to be added, new parameter values or program code are simply uploaded to the device. Furthermore, amplifier models can be implemented as software plug-ins so that the musician can connect the guitar directly to the computer's sound card, record the input tracks, add effects and/or virtual instruments, and then compile the song as a CD or upload it to the Internet. This is especially useful for home studios and small ad hoc recording sessions, because it eliminates several tedious tasks of acoustic recording, such as setting up the amplifier and recording equipment, selecting a microphone position, finding a recording room, etc.

This article attempts to summarize real-time digital techniques for modeling guitar tube amplifiers. Although a brief overview was presented in Pakarinen (2008), to the authors' knowledge, there are no previous works that attempt a comprehensive survey of the topic. Because this topic is relatively new and commercially active, most of the reference material can be found in patents rather than academic publications. Judging from the large number of amateur musicians and home-studio owners, as well as the huge number of discussion threads on Internet forums, this topic is potentially interesting for a wide spectrum of readers. Thus, a conscious choice has been made to try to survey the modeling techniques at an abstracted level, without delving into the underlying mathematics or electric circuit analysis.

This review is organized into four sections. We first describe the sources of the nonlinearities in guitar amplifier circuits. Then, we review published methods for modeling the linear stages of guitar amplifiers. The heart of this survey is the review of methods for nonlinear modeling. Finally we

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mention various other guitar-amplifier related technologies and present conclusions.

Vacuum-Tube Amplifiers

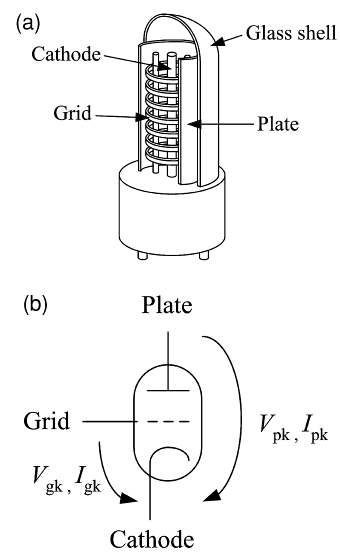
The purpose of this section is to present an overview of the operation of vacuum-tube amplifiers and to illustrate the complex nature of their important nonlinearities. An overview of vacuum tubes used in audio applications can be found in Barbour (1998), and a detailed tutorial on classic vacuum-tube circuits is provided in Langford-Smith (1954). The physical principles governing the operation of vacuum tubes are reviewed in Spangenberger (1948). Excellent Internet articles discussing the design of guitar tube amplifiers can be found online (e.g., at www.aikenamps.com and www.ax84.com).

A typical guitar tube amplifier consists of a preamplifier, a tone-control circuit (i.e., tone stack), a power amplifier, and a transformer that couples to the loudspeaker load. The preamplifier magnifies the relatively weak signal from the magnetic guitar pickups and provides buffering so that the pickup response is not altered by the amplifier circuitry. The preamplifier is usually realized with triode tubes. The tone stack provides a typical V-shaped equalization for compensating the pickup's resonance at mid-frequencies, and it gives the user additional tonal control. The power amplifier boosts the signal so that it is powerful enough to drive a loudspeaker. In the so-called all-tube guitar amplifiers, both the pre- and power-amplifier circuits use tubes instead of transistors in amplifying the signal. Typically, these amplification circuits contain one or more tube stages, namely, circuit blocks that consist of a tube connected to resistive and capacitive (RC) components.

Vacuum Tubes

Vacuum tubes, or thermionic valves, were invented in the early 1900s for amplifying low-level voltage signals. Structurally, they consist of two or more electrodes in a vacuum enclosed in a glass or metal shell. A two-terminal device is a diode, commonly used for signal rectification. Three-terminal devices are known as triodes and are primarily used in preamplifier circuits. Four- and five-terminal devices (tetrodes and pentodes, respectively) are used mainly for power amplification purposes to drive a loudspeaker, for example.

Figure 1. Physical construction (a) and electrical representation (b) of a triode tube. (Figure (a) is adapted from en.wikipedia.org/wiki/Vacuum_tube.)



The operation of vacuum tubes is analogous to water flow on a slope. First, the electrode termed the cathode is heated, and the process known as thermionic emission acts like a pump that forms a pool of electrons at the top of a hill. A second terminal called the plate (or anode) is at the bottom of a slope. Electrons will flow from the cathode to the plate depending upon the relative height of the plate, which is controlled by the voltage applied to it. Note that because a pump is at the cathode, electrodes can never flow backward from the plate to the cathode even though the plate may be raised uphill of the cathode. This describes the rectification behavior of a diode tube.

The triode, illustrated in Figure 1, introduces a third terminal called the grid between the two terminals. With the plate downhill of the cathode, the grid is like a raised barrier in the slope that limits the flow of electrons from the cathode to the plate. If this barrier controlled by the grid is high enough, it stops the electron flow completely. This water-flow analogy motivates the British term referring to vacuum tubes as "valves."

Nonlinear Amplification

The plate-to-cathode current is a nonlinear function of both the grid-to-cathode and plate-to-cathode voltages: $I_{pk} = f(V_{gk}, V_{pk})$. Note that a change in voltage on the grid causes a change in current flow between the cathode and plate. Amplification occurs when the change in current is converted to a change in voltage by a large-valued load resistor. Although amplification is nominally linear around a central operating voltage known as the *bias*, at extreme signal levels, the amplified output will saturate. When the grid-to-cathode voltage V_{gk} is very small, current flow cuts off sharply. Very large V_{gk} causes the plate voltage to approach that of the cathode again, limiting the current and resulting in a nonlinearly saturating characteristic. To find the full nonlinear transfer characteristic from input to output requires the solution of a nonlinear system of implicit equations, because in a typical amplifier circuit, V_{pk} depends on I_{pk} and vice versa.

In guitar-amplifier circuits, the operating point (bias), defined in terms of current through the tube device, is often set by a resistor connecting the cathode terminal to ground. The resistor introduces feedback into the circuit, and its value influences the shape of the input-output curve and determines the offset about which the signal varies. Amplifier designs often include an AC bypass capacitor to recover gain in the passband lost to the feedback, but this introduces memory effects into the nonlinear characteristic.

Dynamic Operation

Capacitive elements exist throughout the tube circuit, preventing it from being accurately modeled as a static waveshaper (a memory-less nonlinearity). If large transients are present in the input signal—as is often the case with the electric guitar—the grid-to-cathode voltage could become positive, and current I_{gk} will flow from the grid to the cathode, eventually causing the device to cut off, introducing an undesirable phenomenon called *blocking distortion* (Aiken 2006). Also, because a grid capacitor is often used to block the direct-current (DC) component of the input signal, the grid current I_{gk} charges the capacitor and dynamically varies the bias point of the tube, leading to dynamically varying transient distortion characteristics.

The cathode bypass capacitor retains memory of the tube bias and responds slowly to rapid changes in signal amplitude, causing signal history–dependent changes in distortion characteristics. Furthermore, there exist parasitic capacitances in the tube itself owing to the close proximity of its electrodes. The dominant effect, *Miller capacitance*, is a low-pass filter resulting from the amplified capacitance between plate and grid; this is discussed more thoroughly in Aiken (1999a).

Amplifier Power Stage

The power amplifier can use either a single-ended or push–pull topology. In the single-ended topology, the signal is amplified in a single vacuum tube. This tube conducts plate-to-cathode current during the whole signal cycle (Class A biasing). Parallel tube stages can also be added if more output power is required.

The push–pull topology, perhaps more commonly used, consists of two identical sets of output tubes driven in opposite phases. The output of one set is inverted and combined with the other through transformer coupling. When a push-pull power amplifier is operated in Class A biasing, both tubes are actively amplifying during the entire signal cycle. Alternatively, Class AB biasing can be used, where one tube handles the signal for positive signal excursions while the other tube is in a low current quiescent state, and vice versa for negative excursions. Leaving the quiescent tube in a low-power state gives Class AB operation higher power efficiency, but it may also introduce crossover distortion as the tubes transition between quiescent and amplifying states. Also, because Class AB amplifiers draw current from the power supply proportional to the signal amplitude, large input-voltage bursts can cause a momentary decrease in the supply voltage. This effect, called *sagging*, introduces further dynamic range compression (Aiken 1999b).

The power amplifier is coupled to a guitar loudspeaker through an output transformer, which introduces additional distortion and hysteresis (i.e., an increasing signal is distorted differently than a decreasing signal). Furthermore, the loudspeaker itself can also contribute significant nonlinear behavior both acoustically and electrically.

In conclusion, the complicated interdependencies and dynamic nonlinearities in vacuum-tube amplifiers make their accurate physical modeling extremely demanding. As a result, approximate models simulating only some of the most noticeable phenomena have been developed by the amplifier-modeling community.

Modeling of Linear Filters in Amplifiers

To better understand nonlinear distortion modeling later in this article, we will first consider the simulation of the linear part of the amplifier, namely, the tone stack. The characteristics of linear filtering greatly influence the tonal quality of electric-guitar amplifiers. Often, switches will be provided to allow a guitarist to choose between different component values in a circuit to vary its frequency response. Certain frequency responses are associated with particular genres or styles of music and are often associated with specific guitar-amplifier models.

The unique quality of the tone stack of the electric-guitar amplifier is significant enough to warrant several attempts in the patent literature to invent methods to make a digital tone-stack model. The tone-stack configurations in guitar amplifiers are all very similar. Amplifiers are mainly differentiated by the component values of the circuit and the mapping from the controls to these values. The tone stack typically has up to three knobs controlling the gains of three bands, loosely called bass, middle, and treble. The middle band is a notch in the frequency response.

Digital Filtering

A system that introduces no new frequencies to the signal is linear and can be characterized completely by its impulse response. The impulse response describes how the system reacts to a unit impulse. The frequency representation of this impulse response is known as the frequency response and describes the gain or attenuation applied to the input signal at various frequencies. Once the impulse response is known, e.g., on the computer in digital form, convolution with this impulse response will recreate the effect of this filter.

There are two general methodologies of modeling linear systems in guitar circuits. The *black-box* system identification approach views the system as an abstract linear system and determines coefficients replicating the system. A *white-box* approach derives a discretized frequency response transfer function for the system based upon knowledge of its linear, constant-coefficient differential equations. Because the linear systems in guitar amplification are often parametrically controlled (e.g., by potentiometers in tone or volume controls), the modeling approach must be parametric.

Black-Box Approach

In the black-box approach, the linear system is excited with a test signal that covers all frequencies of interest. This signal is usually a frequency sweep of a low-amplitude sinusoidal input or broadband white noise. A set of measurements is obtained for various settings of the parameters, which may be multivariate as for the low, mid, and high tone knobs of the guitar tone stack. Various techniques are well known for extracting a frequency response from these measurements (Foster 1986; Abel and Berners 2006).

Once the impulse response is found, it can be used directly as a finite impulse response (FIR) filter to simulate the measured system. Because the original systems are typically low-order infinite impulse response (IIR) systems, it is computationally advantageous to identify IIR filters corresponding to the measured response. The digital filter system identification process optimizes either the error in impulse response (time-domain identification) or frequency response (frequency-domain identification) over the set of digital filter coefficients, given a desired filter order. Preferably, optimizing over the impulse response captures phase information and is a simpler, more robust formulation.

Because the parameterized filter coefficients are usually implemented as lookup tables, the patents covering linear modeling of amplifier components generally concern methods to reduce table size and storage costs in a practical implementation. The Fender tone-stack patent (Curtis, Chapman, and Adams 2001) covers an active filter topology that replicates the range of frequency responses of a tone stack. Assuming this filter structure, system identification comprises obtaining coefficients for various knob settings by manual tuning to match the resulting frequency responses. The mapping from parameters to coefficients is compressed for implementation by sparse sampling (a suggested five points per knob) and 3D linear interpolation of the coefficients.

The Gustafsson et al. (2004) patent also describes multidimensional linear interpolation for the compression of mapping from parameters to filter coefficients. This approach improves upon the accuracy of classical linear interpolation and reduces the number of entries needed in the table by warping each parameter dimension using nonlinear mapping functions prior to interpolated table lookup. The patent also describes the decomposition of the resulting filter into a linear combination of Kautz basis filters, a particular form of second-order digital filter, for stability in implementation. This is a special case of the general technique in digital signal processing to ensure numerically stable filter implementations by decomposition into second-order sections. More information concerning Kautz filters in audio applications can be found in Paatero and Karjalainen (2003).

A *gray-box* approach incorporating some insight into the structure of the circuit, described in a patent application by Gallien and Robertson (2007), divides the tone stack into a parallel bank of two first-order filters, one high-pass and one low-pass, which are weighted and added. The filters are cleverly devised approximate equivalent circuits comprising resistors and capacitors that allow for implementation of the parameter mapping. The equivalent circuits are simulated and compared to a simulation of the full circuit to derive component values for the equivalent circuits and the filter weights so that the resulting response matches that of the actual circuit. The circuits, which are defined using capacitors and resistors, are taken into the discrete time domain by the bilinear transform for digital implementation.

In summary, black-box approaches decide on a particular filter structure, and then they decide on coefficients for that structure to match the response of the target system. Ad hoc mappings from parameter space to coefficient space parameterize the filter.

White-Box Approach

Yeh and Smith (2006) propose an analytical approach to the full tone-stack circuit and suggest that the resulting parameter update equations are not prohibitively complicated. This approach derives the full third-order transfer function with no approximations for the filter by symbolic circuit analysis. Because the coefficients are described as algebraic functions of the parameters, this method is fully parametric. Yeh, Abel, and Smith (2007) applied this approach to filters based upon operational amplifiers. The tone stack for the Boss DS-1 distortion pedal was implemented by interpreting the analog filter as a weighted sum of high-pass and low-pass functions and implementing the analogous structure digitally.

Nonlinear Modeling

Nonlinear signal processing is at the heart of tube-amplifier modeling. Here, we review static waveshaping with memoryless nonlinearities, which is a fundamental technique in digital-distortion implementations, and several categories of methods to reintroduce memory into the nonlinearity: ad hoc nonlinear filters based upon the circuit signal path, analytical approaches, and nonlinear filters derived from solving circuit equations using numerical methods.

Static Waveshaping

The most straightforward method for obtaining signal distortion with digital devices is to apply an

Figure 2. Construction of the digital effects device described in Araya and Suyama (1996). The distortion block consists of three identical nonlinearities and suitable scaling coefficients. The amount of distortion can be varied by changing the scaling coefficients.

Figure 3. Solid line: input–output plot of the nonlinear function of Equation 1 used in Araya and Suyama (1996); dotted line: the input-output plot of the symmetric nonlinearity in Equation 2, used in Doidic et al. (1998). Dash-dotted line: the asymmetric nonlinearity in Equation 3, also used in Doidic et al. (1998). The allowed operation range is denoted with dashed lines.

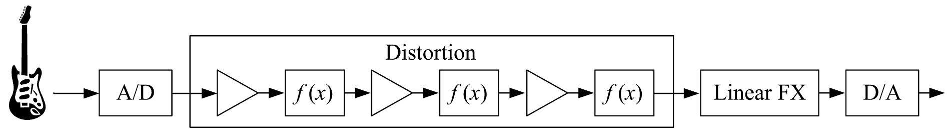


Figure 2

instantaneous nonlinear mapping from the input variable to the output variable. This type of timbre alteration is called *waveshaping* (Arfib 1979; Le Brun 1979). If the mapping does not change in time, this method is called static waveshaping. An early Yamaha patent (Araya and Suyama 1996) describes a digital guitar effects device using this technique. This is illustrated in Figure 2.

In Figure 2, the signal from the instrument is first fed to the distortion block through an analog-to-digital (A/D) converter (including an analog amplifier for setting a suitable input level). The distortion effect is obtained by feeding the signal into a nonlinear function through a scaling coefficient. The nonlinear function used in Araya and Suyama (1996) is of the form

$$y = \frac{3x}{2}\left(1 - \frac{x^2}{3}\right) \tag{1}$$

where x is the input (bounded between $[-1, 1]$) and y is the output signal. The nonlinear curve produced by Equation 1 is illustrated in Figure 3 with a solid line. Because the curve is fairly linear in the operation range of the device, the scaling and nonlinearity is applied three times in cascade (i.e., sequentially) for obtaining more distortion. After leaving the distortion block in Figure 2, the signal is fed to a collection of linear effects (e.g., chorus or reverberation) and finally to a digital-to-analog (D/A) converter. Araya and Suyama also suggest adding a digital equalizer between the A/D converter and the distortion.

More nonlinear functions are suggested in Doidic et al. (1998), including a symmetric function of the form

$$f(x) = (|2x| - x^2)\,\mathrm{sign}(x) \tag{2}$$

where $\mathrm{sign}(x) = 1$ if $x > 0$, and $\mathrm{sign}(x) = -1$ otherwise. Alternatively, a hard-clipping function or a piecewise-defined asymmetric static nonlinearity of

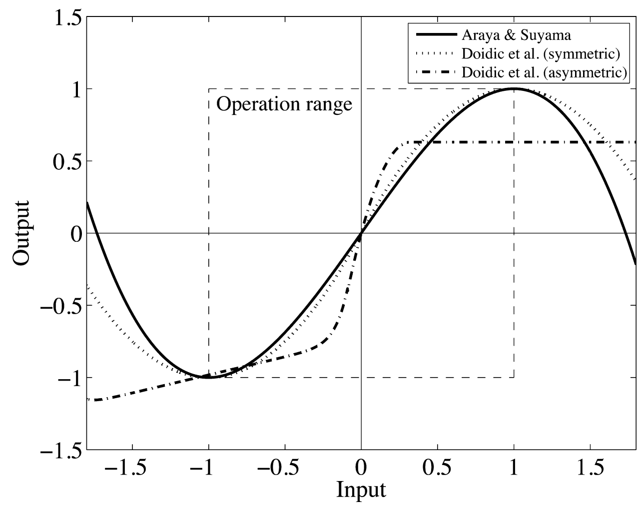


Figure 3

the form

$$\begin{aligned} f(x) &= -\frac{3}{4}\left\{1 - [1 - (|x| - 0.032847)]^{12} \right.\\ &\quad \left. + \frac{1}{3}(|x| - 0.032847)\right\} + 0.01, \\ &\quad \text{for} -1 \le x < -0.08905 \\ f(x) &= -6.153x^2 + 3.9375x, \\ &\quad \text{for } -0.08905 \le x < 0.320018, \quad \text{and} \\ f(x) &= 0.630035, \text{ for } 0.320018 \le x \le 1 \end{aligned} \tag{3}$$

can be used. Figure 3 illustrates the input-output curve defined by Equation 2 using a dotted line and the curve defined by Equation 3 using a dash-dotted line. It must be noted that the original patent (Doidic et al. 1998) has some typographical errors in the equation of the asymmetric nonlinearity, and thus it does not produce the input–output relationship illustrated in Figure 3.

As displayed in Figure 3, all the input–output curves are fairly linear for small-amplitude signals, that is, signal values near the origin. This obviously means that the smaller the signal is, the less it is distorted. A patent by Toyama (1996) uses a signal-dependent scaling procedure with a nonlinear function to also distort small-amplitude signals. This technique can add harmonic content to various signals regardless of their amplitude levels, although it does not resemble the behavior of vacuum-tube distortion. A further Yamaha patent (Shibutani 1996) describes a computationally simple method for creating piecewise-linear distortion functions by branching the signal via various scaling coefficients and adding the output. Graphically, this means that each of the scaling coefficients determines a slope for a linear segment in the input–output plot.

Another simple digital distortion circuit, "mantissa fuzz," is described in Massie (1996). This exotic algorithm uses a simple bitshifing operation in distorting the input signal. Although the mantissa-fuzz technique is computationally extremely efficient, it seems virtually impossible to match the distortion curve to a desired nonlinearity.

Möller, Gromowski, and Zölzer (2002) describe a technique to measure static, nonlinear transfer curves from all stages of a guitar amplifier. Their goal is to mimic the nonlinearities and filters in the signal path of the amplifier, approximating the nonlinearities as static, the filters as linear, and neglecting loading between stages. Santagata, Sarti, and Tubaro (2007) introduce a model of the triode preamplifier with an added hard-clipping feature. This model uses an iterative technique for evaluating the nonlinear tube equations, but it does not incorporate the capacitive effects of the triode stage; therefore, it can be considered as computing the implicitly defined waveshaping curve "on the fly," based on parameters measured from an actual tube.

Lookup-Table Nonlinearity

Preceding the patent by Araya and Suyama (1996), there had already been some studies on how to obtain digital distortion effects. Kramer (1991) introduced a simple method for obtaining arbitrary nonlinear distortion in real time using a lookup table. This means that instead of applying a nonlinear algebraic function, such as the one in Equation 1, the system reads the input–output relation from a pre-stored table, for example, a digitized version of Figure 3. The advantage of this technique is that it is easier to obtain a desired type of input-output relation, because the designer can freely draw the input–output curve for the lookup table.

On the other hand, a high-resolution lookup table would consume an excessive amount of memory, so low-resolution lookup tables and interpolation algorithms must be used. Also, run-time modification of the nonlinearity becomes difficult. Digidesign implemented this type of lookup-table waveshaping in their early software synthesizer Turbosynth in 1989.

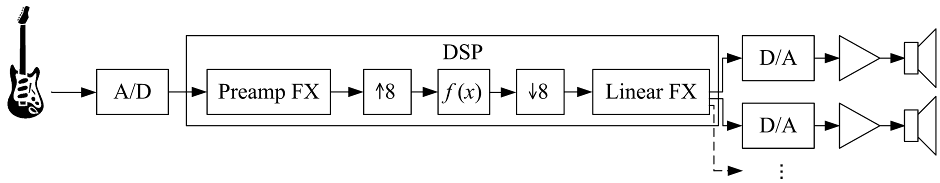
In an early study by Sullivan (1990), a simple nonlinear function or a lookup table is used in distorting the output of a synthesized guitar string. In fact, the nonlinear function in Equation 1 can be seen as a scaled version of the one suggested in Sullivan (1990). Sullivan's article also introduces a system for simulating the acoustic feedback between synthesized guitar strings, amplifier, and a loudspeaker.

Oversampling

Nonlinear signal processing blocks are known to expand the bandwidth of the incoming signal, which in a DSP system can cause aliasing if the bandwidth of the output exceeds the Nyquist frequency (i.e., half the sampling rate). An amplifier model can distort harmonic signals such as a guitar tone and produce many new harmonics in the output that, through aliasing into the audio range, are no longer harmonically related to the original tone. The resulting noisy, "dissonant" sound owing to aliasing is characteristic of low-cost digital implementations of strong distortions and is typically mitigated through running the distortion algorithm at an oversampled rate, which is computationally expensive.

In the late 1990s, the Line 6 Company patented a digital guitar amplifier, i.e., an amplifier and effects emulator combined with a loudspeaker (Doidic et al. 1998). This device used a sampling rate of 31.2 kHz for most of the signal processing, but it included

Figure 4. Tube-amplifier modeling scheme, as suggested in the Line 6 TubeTone patent (Doidic et al. 1998). The nonlinearity is evaluated at a higher sampling rate to avoid aliasing. Multichannel output can be used, for example, in conjunction with stereo effects.



an eight-times oversampling circuit for evaluating a static nonlinearity at 249.6 kHz, thus attenuating the aliased distortion components. This straightforward technique, named TubeTone Modeling, was used in several commercially successful Line 6 digital guitar-amplifier emulators.

Figure 4 illustrates the system described in Doidic et al. (1998). Here, the digital signal is first fed to a collection of preamplifier effects—that is, effects that are typically located between the guitar and amplifier, such as a noise gate, compressor, or a wah-wah. Next, eight-times oversampling with linear interpolation is applied to the signal, and it is fed to a nonlinearity. After the nonlinearity, the signal is lowpass-filtered using an antialiasing FIR filter, and it is downsampled back to the sampling rate of 31.2 kHz.

Figure 5 visualizes what happens to the waveform and spectrum of a sinusoidal input signal when distorted by the nonlinear Equations 2 and 3. The top row illustrates the waveform (left) and spectrum (right) of a 1.2-kHz sinusoidal signal with an amplitude of 0.8. The middle row shows the signal after the symmetric distortion defined by Equation 2. As expected, the symmetric distortion creates a "tail" of odd harmonics in the output signal spectrum. For frequencies above the Nyquist limit (a sampling frequency of 44.1 kHz was used here), the harmonics fold back to the audio band, resulting in frequency components that are not in any simple harmonic relation with the input tone. The bottom row shows the input signal after the heavy-clipping asymmetric distortion defined by Equation 3. As can be seen in the lower right graph, the asymmetric distortion creates even and odd harmonic components. The upper components are again aliased back to the audio band, resulting in an inharmonic spectrum.

In Doidic et al. (1998), the output signal from the distortion is fed to a collection of linear effects, such as tremolo, chorus, or delay. If headphones or line output are used, a simple low-pass filter can also be applied for simulating the effect of the loudspeaker cabinet. Finally, the signal drives a loudspeaker (or several loudspeakers, if for example stereo effects are used) after a D/A conversion and amplification.

Customized Waveshaping

An interesting method for obtaining a highly customized type of distortion has been introduced in Fernández-Cid and Quirós (2001). This technique, illustrated on the left of Figure 6, decomposes the input signal into frequency bands using a filterbank, and it then applies a different static nonlinearity for each band separately. Thus, only narrow-band signals are inserted to the nonlinear waveshapers, and the perceptually disturbing intermodulation distortion is minimized. The authors call this technique *multiband waveshaping*. The delay imposed on the direct signal in Figure 6 equals the delay caused by the filterbank, so that the signal phase is correctly preserved after the final summation.

Fernández-Cid and Quirós (2001) suggest using Chebychev polynomials as the nonlinearities. These polynomials are a special type of function allowing the designer to individually set the amplitude of each harmonic distortion component, provided that the input signal is purely sinusoidal with unity amplitude. Furthermore, using this type of polynomial approximation, aliasing can be avoided for sinusoidal input signals, because the designer can simply choose not to synthesize the highest harmonics. The right part of Figure 6 illustrates the construction of a single Chebychev-based waveshaper used in Fernández-Cid and Quirós, where the overall signal level is set between [–1, 1] prior to the evaluation of the nonlinearity. Dynamic nonlinearities can be imitated by using two different polynomials ($f_A(x)$ and $f_B(x)$ in the right part of

Figure 5. Signal waveforms (left pane) and the corresponding frequency spectra (right pane) for sinusoidal input-output signals. Top row: a sinusoidal input signal with a frequency of 1.2 kHz; middle row: the input signal after the symmetric distortion defined by Equation 2; bottom row: the input signal after the heavy-clipping asymmetric distortion defined by Equation 3.

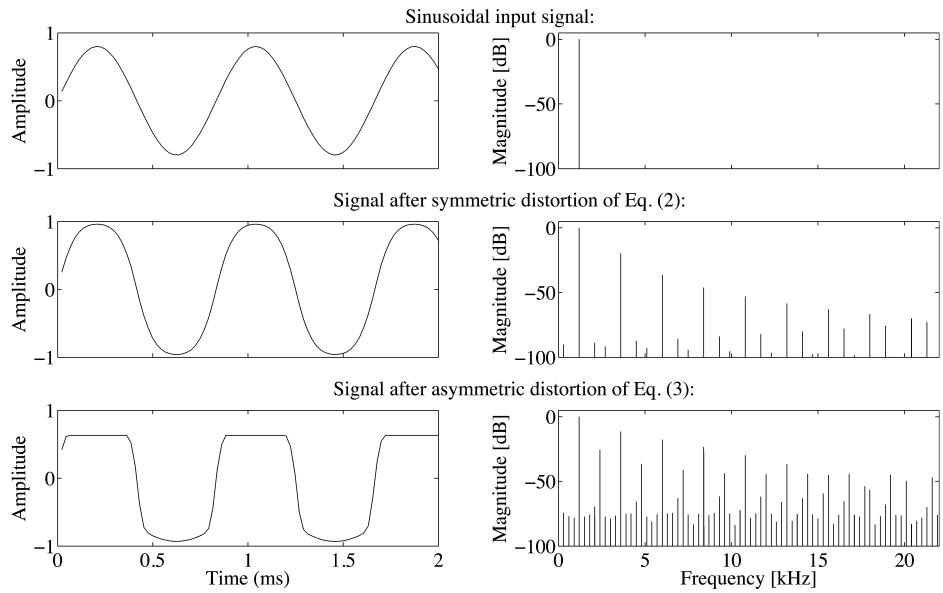


Figure 6) in parallel, and varying their mix ratio according to the signal level of the corresponding band. Finally, the original dynamics of the signal are restored by multiplying the polynomial output with the signal level, as shown in the right part of Figure 6. The authors claim that the waveshapers perform well, even though their input is not a sinusoid but rather a narrowband signal.

Patents by Jackson (2003) and Amels (2003) present trigonometric functions for creating static waveshapers where the distortion component levels can be set by the designer. Schimmel and Misurec (2007) implemented and analyzed static nonlinearities using piecewise-linear approximations of the nonlinear input-output curves. These three methods use oversampling to suppress aliasing. Also, a polynomial approximation of a static nonlinearity without aliasing suppression is presented in Schimmel (2003).

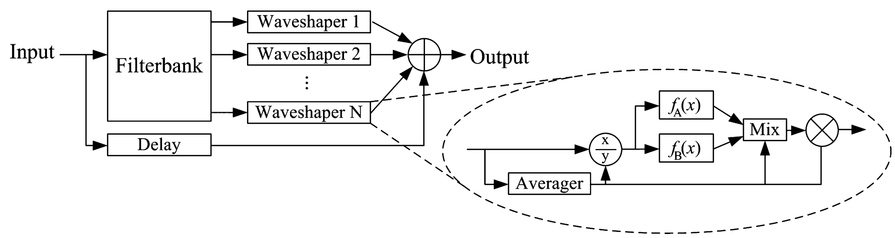
Ad Hoc Nonlinear Filters

Because the assumption that the nonlinearities are memory-less does not hold for describing the behavior of real tube amplifiers, researchers have proposed various dynamic waveshapers, namely, nonlinearities that change their shape according to the input signal or some system-state variables.

An early digital system for emulating a tube amplifier was outlined by Pritchard (1991). He suggested using two nonlinear distortion blocks with a digital equalization unit in between. Ideally, the first distortion block would have a high-pass filter with the cutoff frequency controlled by the input-signal polarity, and an asymmetric static nonlinearity for producing mainly even harmonics. The second distortion block would generate both even and odd harmonics and emulate the sagging effect of the power amplifier using a dynamic nonlinearity. Aliasing problems, however, are not addressed by Pritchard.

A more detailed description of a dynamic tube-amplifier model has been discussed in a Yamaha patent (Kuroki and Ito 1998). There, a single tube stage is again modeled using a lookup table, but the DC offset of the input is varied according to the input-signal envelope. The authors give the impression that this bias variation would be caused by grid capacitor charging owing to grid current, although a more realistic explanation would be the variation of the cathode voltage owing to a change in plate current. A tube preamplifier can be simulated by connecting several tube-stage models in cascade.

Figure 6. Construction of the multiband waveshaper distortion, described in Fernández-Cid and Quirós (2001). The overall structure is illustrated in the left half of the figure, and the signaling inside an individual waveshaper is depicted in the right half. In the right half, the output of the averager can be seen as a measure of the overall signal level.



Sign inversion is applied between tube stages for modeling the phase-inverting behavior of a real tube stage. Note that, owing to the dynamic nonlinearities (i.e., signal history-dependent DC offsets), the preamplifier stages cannot be combined as a single equivalent lookup table. A push–pull power amplifier can be simulated by connecting two tube-stage models in parallel and reversing the sign of the other branch. With suitable DC-offset values, crossover distortion can be emulated, if desired. The system proposed in Kuroki and Ito is illustrated in Figure 7.

Another dynamic model of a guitar preamplifier has been presented in Karjalainen et al. (2006). This model assumes that the plate load of the tube stage is constant and resistive, so that the tube nonlinearity simplifies to a mapping from the grid voltage V_{gk} to plate voltage V_p. This curve is measured from the tube by shorting the cathode to ground and varying the grid voltage. Grid current is also measured as a function of the grid voltage. These curves are combined in a single precomputed V_{gk}-to-V_p table. Bias variation is simulated using a feedback loop, as in Kuroki and Ito (1998). The filtering effect caused by the grid resistor and Miller capacitance is modeled with a low-pass filter at grid input, while a high-pass filter emulates the interstage DC-blocking filter. Three tube-stage models are used in series and connected to a loudspeaker model via an equalizer. A minimum-phase FIR filter is used as a loudspeaker model.

An interesting system-identification-based approach has been presented by Gustafsson et al. (2004), the founders of the Swedish company Softube AB (producers of Amp Room software). Here, the dynamic nonlinearity is simulated by feeding the signal through a nonlinear polynomial function and varying the polynomial coefficients according to the input signal. Figure 8 illustrates this. The signal-analysis block estimates the signal energy for the last few milliseconds, and it checks whether the input signal is increasing or decreasing. Next, the polynomial coefficients are interpolated from a set of pre-stored coefficient values according to the signal energy. The pre-stored coefficients are obtained from measured tube data using system-identification techniques (see, e.g., Nelles 2000). The hysteresis effect can be simulated by using a different set of polynomial coefficients for increasing and decreasing input signals. The authors suggest implementing the static nonlinearities with Chebychev polynomials to avoid aliasing, and also because the accuracy of the Chebychev polynomial approximation is highest near the signal extrema (i.e., around ± 1, near saturation).

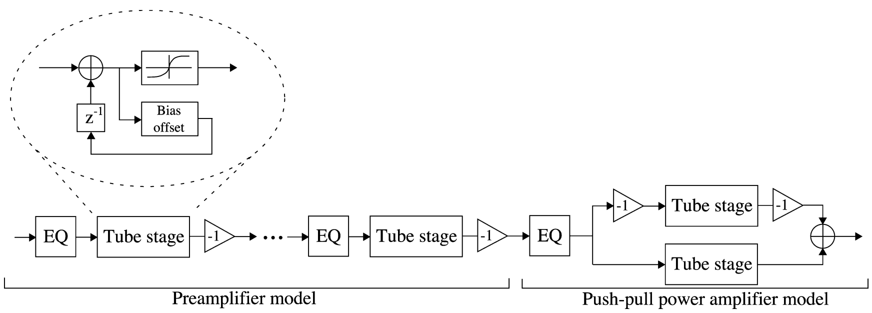
Analytical Methods

Several methods exist for analyzing a nonlinearity with memory. These are based upon Volterra series theory and can be used to implement nonlinear audio effects.

Volterra Series

The Volterra series expansion (Boyd 1985) is a representation of systems based upon a nonlinear expansion of linear systems theory. Analogous to convolution with the impulse response vector of a linear system, the Volterra series is a multidimensional convolution with nonlinear system-response matrices. Whereas in linear systems the impulse response fully characterizes the system and allows its output to be predicted given an input, Volterra systems are characterized by special functions, called kernels, that correspond to the multidimensional impulse response of the nonlinear terms. It can also

Figure 7. A dynamic tube amplifier model as described in a Yamaha patent (Kuroki and Ito 1998). The model of a single tube stage consists of a lookup table, added with a signal-dependent DC-offset. An entire preamplifier can be simulated by connecting tube-stage models in cascade with a phase inversion in between. A push–pull power amplifier is simulated by connecting the tube stage models in parallel and in opposite phase.



be regarded as a Taylor series expansion with the polynomial terms replaced by multidimensional convolution, accounting for the memory associated with different orders of nonlinearity.

Volterra series have been used extensively to model nonlinear acoustic systems including loudspeakers. In particular, they can linearize low-order distortion circuits and loudspeakers in real time (e.g., Katayama and Serikawa 1997). Farina, Bellini, and Armelloni (2001) and Abel and Berners (2006) used a technique to identify parameters for a subclass of Volterra systems based upon a frequency-sweep excitation of the system. A similar technique is used in the Nebula effects sampler by Acustica Audio (www.acusticaudio.net), which allows the user to create soft-saturating models of several audio effects based on the system response. Hèlie (2006) applied a specific Volterra series expansion to create a real-time effect that includes the third-order nonlinearities of the Moog ladder filter. Schattschneider and Zölzer (1999) report an efficient implementation of a type of Volterra series and a system-identification technique to derive parameters for their model.

Although Volterra series are a theoretically valid black-box method for simulating various nonlinearities, real-time emulation of strongly saturating distortion poses a problem. This is because Volterra series involve a convolution of a dimension equal to the order of the nonlinearity for each nonlinear term in the model, making the number of coefficients and computational cost grow rapidly with increasing order of nonlinearity. Because guitar distortion often involves very strong, clipping-type nonlinearities, Volterra series are not the preferred technology for this application.

Dynamic Convolution

Kemp (2006) has patented a black-box method, dynamic convolution, for nonlinear system analysis and emulation. The basic idea of this technique is simple: several impulses with different amplitudes are inserted into the distorting system during the analysis, and the resulting impulse responses are recorded. System emulation is carried out using convolution, so that the amplitude of each input sample is detected and compared to the set of impulse amplitudes used in the analysis. Once the nearest measured impulse is found, the corresponding impulse response is used in evaluating the convolution. Because this procedure is applied for each input sample, the convolution coefficients change according to the input signal level during run-time.

Although a promising technique, dynamic convolution has some limitations. First, the amount of stored data can be prohibitively large if a high-amplitude resolution is used. Secondly, dynamic convolution can be used for modeling static nonlinearities, but it fails to model dynamic nonlinearities, namely, systems for which the shape of the nonlinearity changes due to the input signal (Berners and Abel 2004). Note that the nonlinear convolution introduced by Farina, Bellini, and Armelloni (2001) can be seen as the Volterra representation of the dynamic convolution method.

Circuit Simulation-Based Techniques

The preceding techniques have all treated the distortion device as a nonlinear black box, possibly with memory. Techniques based upon solving the

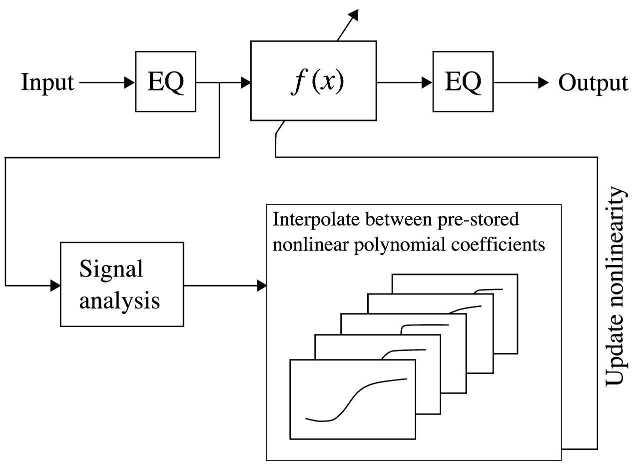


Figure 8. A dynamic amplifier stage model, described in Gustafsson et al. (2004). The nonlinear function f(x) *is varied each time sample according to the input signal characteristics. Chebychev polynomials are suggested for implementing the nonlinearities. A complete amplifier can be simulated by connecting several amplifier stage models in cascade.*

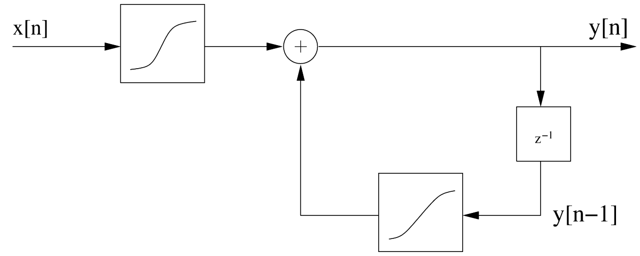


Figure 9. A single stage of the nonlinear digital Moog filter (Huovilainen 2004). The nonlinearity is embedded within the digital filter feedback loop. Equivalently, this is a nonlinearity with embedded memory, derived by discretizing the circuit equations.

ordinary differential equations (ODEs) that describe the behavior of the circuit have also been attempted.

Transient Modified Nodal Analysis

Integrated circuit design involves the engineering of analog and digital systems based upon highly nonlinear integrated circuit devices such as metal-oxide-semiconductor field-effect transistors (MOSFETs) and bipolar transistors. Verification of the designs depends critically on the accuracy of numerical circuit simulators, e.g., the Simulation Program with Integrated Circuit Emphasis (SPICE; Vladimirescu 1994). SPICE uses transient modified nodal analysis (MNA) with nonlinear components in audio circuit simulation. MNA solves the equations describing circuit behavior in matrix form, $GV = I$, where V is a vector containing the node voltages; I is a vector containing the current contributed by the nonlinear devices, capacitors, and sources; and G is the conductance matrix representing the linear current-to-voltage relation of each component in the circuit. MNA is particularly convenient, because the computer can easily derive the circuit equations given a circuit schematic.

The matrix G is typically sparse, because it encodes the connections between the components of the circuit, which are typically connected to just a few neighbors. MNA requires the solution of this equation, usually by LU decomposition. Although the complexity of a general matrix solve is $O(N^3)$, where N is the number of rows or columns of the square matrix G, it has been found empirically that for typical circuits a sparse LU solve is $O(N^{1.4})$, owing to the sparse nature of the matrix equations (White and Sangiovanni-Vincentelli 1987). As computational power increases and researchers model more complex circuits, MNA offers a simple way to construct circuit schematic-based audio effects.

Custom, Simplified Ordinary Differential Equation Solvers

For commercial digital audio effects, the simplest acceptable implementation is desired, because companies boast of their capability to provide a multitude of real-time effects simultaneously. To this end, several researchers have developed effects based on simplifying the ODE model of the circuit and trading off accuracy for efficiency in the numerical ODE solvers.

Huovilainen reported nonlinear models of the Moog ladder filter (2004), as well as operational transconductance amplifier (OTA)-based all-pass filters (2005), by deriving a minimal ODE from the circuit equations and solving it using Forward-Euler numerical integration. The result is a nonlinear recursive filter structure with a nonlinearity embedded in the filter loop. Huovilainen's nonlinear Moog filter model is illustrated in Figure 9. A simplified version of this model has been presented in Välimäki and Huovilainen (2006).

Yeh et al. (2008) extended this approach to strongly clipping diode-based distortion circuits and found that for circuits in general, implicit ODE methods such as Backward Euler or Trapezoidal Rule are needed to avoid numerical instability at typical sampling rates. Implicit methods require the numerical solution of an implicit nonlinear equation by iterative fixed-point methods, a general subclass of which are the Newton–Raphson methods. Yeh and Smith (2008) also extended this approach to the triode preamplifier using a state-space approach with a memory-less nonlinearity (the vacuum-tube I_{pk} expression itself), demonstrating that implicit methods transform the ODEs for audio circuits into a recursive state-space structure with a multidimensional static nonlinearity embedded in the feedback loop. This approach accounts for both the implicit nonlinearity of the circuit and the memory introduced by bypass, coupling, and Miller capacitances in the circuit. It can be considered a brute-force, fixed-sampling-rate simulation of the circuit.

A recent patent by Gallo (2008), the founder of Gallo Engineering (producers of Studio Devil software), introduces a tube-stage emulation algorithm using a parametric nonlinear function. The bias variation is modeled by evaluating the cathode voltage ODE using a numerical solver, such as the fourth-order Runge–Kutta algorithm. The plate voltage variation is neglected here, as in Karjalainen et al. (2006).

Wave Digital Filters

Wave digital filters (WDFs; Fettweis 1986) are a special class of digital filters with parameters that directly map to physical quantities. Each of the basic electrical circuit elements has a simple WDF representation, and, through the use of "adaptors," the resulting filters connect to each other as real electric components do. Thus, the user can build the WDF circuit model by connecting elementary blocks (resistors, capacitors, etc.) to each other like a real amplifier builder. A real-time model of a WDF tube-amplifier stage has been presented in Karjalainen and Pakarinen (2006). Here, the tube is modeled using a two-dimensional lookup table for simulating the bias variation, while the effect of the grid current is neglected. Sound examples are available at www.acoustics.hut.fi/publications/papers/icassp-wdftube. Yeh and Smith (2008) demonstrated that the WDF can efficiently represent certain guitar circuits, such as the bright switch and the two-capacitor diode clipper.

Although WDFs are a computationally efficient, modular physical-modeling technique—and thus a promising method for flexible real-time audio circuit simulation—some barriers to widespread application of WDFs remain. Finding a general methodology in the WDF framework to model instantaneous feedback loops between different parts of the amplifier circuitry presents a significant challenge. Also, certain circuit topologies, such as bridges, do not easily map to connections of the adaptors commonly used for WDFs.

Other Models

A hybrid DSP/tube amplifier has been patented by Korg (Suruga, Suzuki, and Matsumoto 2002). Their system uses an upsampled nonlinear function in modeling the preamplifier, while the power amplifier is emulated using two push-pull triodes, connected to a solid-state power circuit via a transformer. A central processing unit (CPU) controls the biasing of the tubes and the filtering of the feedback from the output to the input. The power amplifier state can be switched between class A and class AB biasing by the CPU. Furthermore, the solid-state power circuit couples the output transformer to the loudspeaker so that the output power rating can be varied without altering the interaction between the tubes and the loudspeaker. Vox Amplification, a subsidiary of Korg, manufactures a hybrid DSP/tube amplifier modeling system called Valvetronix.

A recently introduced exotic sound effect (Pekonen 2008) uses a time-varying allpass filter in adding phase distortion to the input signal. Although various types of distortion could be emulated by suitably modulating the filter coefficients, the current usage of this effect does not allow convincing emulation of vacuum-tube distortion.

Summary and Discussion

Digital emulation of guitar tube amplifiers is a vibrant area of research with many existing commercial products. Linear parts of the amplifier, such as the tone stack, are modeled using digital filters, for which the parameters are found with system-identification methods or by using a priori knowledge of the underlying circuitry. In the simplest case, the distortion introduced by the tube stages is modeled using static waveshaping. Aliasing problems can be avoided using oversampling. More-sophisticated methods can be used for the simulation of dynamic nonlinearities. Most of these methods can be classified as being inspired by circuit signal paths, which try to model the signal path from the amplifier's input to the output. There are also some methods that attempt to simulate the operation of the underlying electric circuit, but these are often either greatly simplified or still too demanding computationally for real-time modeling of complex circuits. Alternatively, some analytical methods, such as Volterra series or dynamical convolution, have also been suggested. Owing to the complex dynamical nonlinearities of the tube-amplifier circuit, true physics-based models for accurate real-time simulation of the tube amplifier have yet to be discovered.

It must be noted that owing to the essentially nonlinear, complex nature of tube amplifiers, objective evaluation of their sound quality—and hence the sound quality of tube emulators—is extremely difficult. Thus, the best way to rate different emulation schemes is by listening. Marui and Martens (2002) have presented some studies discussing perceptual aspects of amplifier modeling. As a result of the subjectivity of human listeners, one should be careful not to underestimate certain amplifier-modeling schemes just because the method used is simple or physically inaccurate. Careful tuning of the emulation parameters can make a tremendous improvement in the resulting sound.

Existing emulation techniques are improving in both physical accuracy and sound quality. Owing to the easy distribution of digital media, software amplifier emulators are also constantly gaining new users. Although some tube-amplifier enthusiasts might feel that digital emulation is a threat to the tube-amplifier industry, the authors believe that it should rather be viewed as an homage. It can also be seen as a form of conservation, because the quantity and quality of available tube-amplifier components continues to dwindle. After all, the ultimate goal of amplifier emulation is to convincingly reproduce all the fine details and nuances of the vacuum-tube sound, and to make it widely available for use in artistic expression.

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