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We present a new Dynamic Tonality MIDI sequencer, Hex, that aims to make sequencing music in and across a large variety of novel tunings as straightforward as sequencing in twelve-tone equal temperament. In order to enable the intuitive visualization and dynamic manipulation of tuning, it replaces the piano roll used in conventional MIDI sequencers with a two-dimensional lattice roll (see Figure 1). It is compatible with the Dynamic Tonality line of software—which currently consists of the microtonal synthesizers TransFormSynth, The Viking, and 2032—and, for static tunings, with any synthesizer that handles channel pitch bend.

In conventional piano roll sequencers, a piano keyboard is displayed on the left side of the window, and white and black note lanes extend horizontally to the right, into which a user can draw a sequence of notes. Similarly, in Hex, a button lattice is displayed in its own pane on the left side of the window, and horizontal lines are drawn from the center of each note to the right. These lines function as generalized note lanes, just like in piano roll sequencers, but with the added benefit that each note lane's height is always proportional to its pitch, even if the user changes the tuning. The presence

A MIDI Sequencer That Widens Access to the Compositional Possibilities of Novel Tunings

of the button lattice on the left side of the window illustrates exactly which buttons a performer would play in order to replicate the sequence when playing a physical button lattice instrument, such as the C-Thru AXiS-49 (www.c-thru-music.com) or the Thummer (www.thummer.com).

Dynamic Tonality is an audio synthesis and control framework that helps musicians explore novel tunings using a small number of intuitive parameters, and provides several new musical opportunities. First, it enables users to morph between many well-known tunings and demonstrates both their structure and their relation to a broad continuum of tunings. For example, a single parameter moves the tuning of tones in a repeating scale through a continuum containing a variety of notable tunings, such as seven-tone equal temperament (7-TET), 19-TET, various meantones, 12-TET, Pythagorean, 17-TET, 22-TET, and 5-TET (Milne, Sethares, and Plamondon 2007). Second, it affords a two-dimensional *isomorphic note layout*—a representation (see Figure 2) for visualizing, manipulating, and fingering pitch sets in a way that is consistent across key transpositions (Keislar 1987) and many diverse tunings (Milne, Sethares, and Plamondon 2007, 2008). Finally, it can temper the partials of individual tones to match the underlying scale's tuning, which allows sensory dissonance to be minimized in any tuning, and

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Figure 1. Hex uses a lattice roll in place of the traditional piano roll. This enables unfamiliar microtonal scales to be intuitively visualized, and their tuning to be

dynamically manipulated. In this example, three octaves of a ten-tone microtonal moment of symmetry (MOS) scale are indicated by the light-colored buttons and

lanes. The light buttons/lanes can be thought of as generalized “diatonic” tones, the dark buttons/lanes as generalized “chromatic” tones.

Figure 2. A two-dimensional isomorphic note layout.

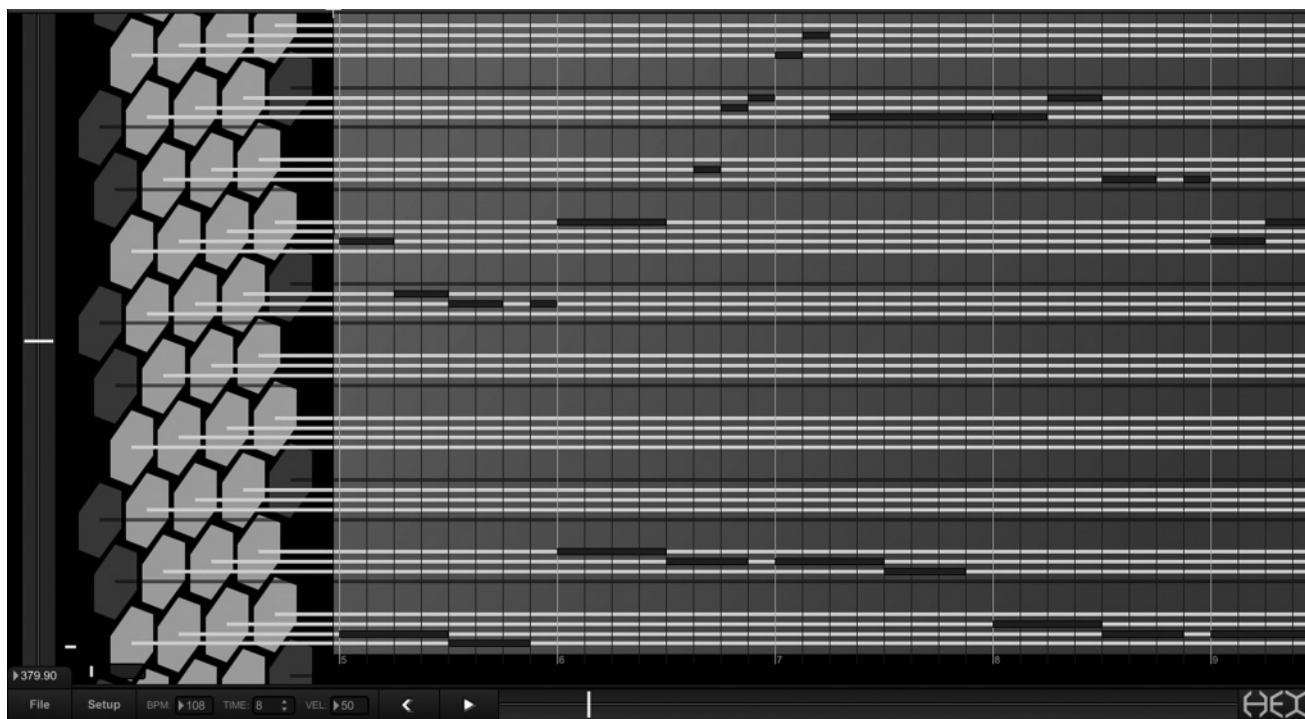


Figure 1.

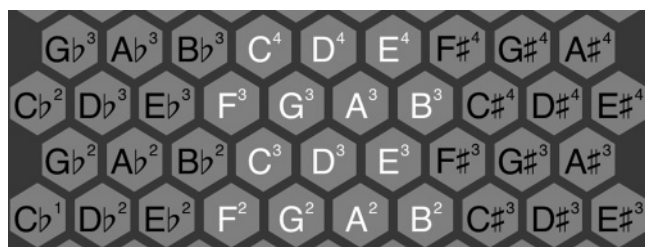


Figure 2.

introduces novel classes of timbre (Sethares 2004; Sethares et al. 2009).

We hope that this combination of features will facilitate the exploration of tuning as a creative tool in compositions and performances. For example, Andrew Milne’s *Magic Traveller* and *Hanson* demonstrate how Dynamic Tonality can be used to explore scales and tunings radically different from those used in conventional

Western music. (Hex project files are available online at www.dynamictonality.com.) William Sethares’ *C to Shining C* (www.cae.wisc.edu/~sethares/spectoolsCMJ.html) demonstrates how Dynamic Tonality can be used to create progressions of extravagant tuning bends that seem to function similarly to chord progressions in Western tonal harmony. *C to Shining C* actually gets its name from its use of a progression of C major chords tuned in different ways, rather than a progression of different chords. Dynamic Tonality can also dynamically adjust intonation for expressive purposes, as (for example) advanced string players commonly do (Sundberg, Friberg, and Frydén 1989). We hope the addition of Hex to the current line of software can enable Dynamic Tonality to reach a larger community, particularly those who prefer to compose music with a sequencer rather than by playing in real time, and those who do not have access to a specialized hardware controller.

Hex is a standalone application built in Cycling 74's Max/MSP, and runs on Windows and Macintosh systems. It can be downloaded as freeware from the Dynamic Tonality online resource at www.dynamictonality.com.

Tuning and Scale Theory

Hex's principal task is to represent a variety of microtonal tunings (specifically, two-dimensional tunings, which are defined subsequently) as effectively as piano roll sequencers represent the familiar 12-TET. In order to approach this, Hex utilizes isomorphic note layouts, a widely studied class of note layouts that are invariant over transpositions (Helmholtz 1954 [1877]; Bosanquet 1877; Wilson 1974; Keislar 1987) and different tunings (Milne, Sethares, and Plamondon 2008). In this article, we describe how Hex additionally introduces three novel extensions to the current theory of isomorphic note layouts. First, we describe how controlled transformations of isomorphic layouts can ensure that the pitch height of each note is proportional to its spatial height, and that notes an octave apart are always vertically aligned. Second, we introduce a class of isomorphic note layouts, *adjacent predominant steps layouts*, that generalize many of the useful properties of the Wicki (1896) accordion note layout over a wide variety of microtonal scales. Third, we show how note coloration can be used to indicate generalized diatonic and chromatic scales, called *moment of symmetry* (MOS) scales, which are formally defined subsequently. Before we describe how each of these extensions is achieved in Hex, we will first outline pertinent tuning and scale theory.

An *r-dimensional tuning* is one whose intervals can be generated by independent iterations (a linear combination) of at least r linearly independent intervals called *generators* (Milne, Sethares, and Plamondon 2008), which together constitute a *basis* of the tuning. *Linear independence* in this context simply means that the ratio of the generators is irrational when measured in a logarithmic unit like cents. In other words, starting from some origin, independently iterating the generators will never result in an exactly matching pitch. If the number

of iterations of each of the r generators is denoted by j, k, l, \dots , then the coordinates $(j, k, l, \dots) \in \mathbb{Z}^r$ uniquely specify every possible pitch—there is a one-to-one mapping between coordinates and pitch. For example, $(2, -2, 3)$ represents a uniquely sized interval created by combining the values of two of the first generator, negative two of the second generator, and three of the third generator (a positive number of iterations represents adding generator values in cents, or another logarithmic unit, while a negative number of iterations represents subtracting them).

An r -dimensional tuning may be specified by the sizes of its r generators, conventionally notated in cents. For example, 12-TET is a one-dimensional (1-D) tuning with a generator of 100 cents, because iterating that interval alone can generate every one of its tones. Indeed, all n -TETs are 1-D tunings and vice versa. However, 2-D and 3-D tunings are commonplace. For example, quarter-comma meantone, a common eighteenth-century tuning (Barbour 1951), is a 2-D tuning with generators of 1200 and approximately 696.6 cents: iterating those intervals can generate every one of its tones. Similarly, 5-limit just intonation is a 3-D tuning with generators of 1200 cents (a frequency ratio of 2/1), 702.0 cents (a ratio of 3/2), and 386.3 cents (a ratio of 5/4).

A 1-D tuning like 12-TET can also be generated by two or more non-linearly independent intervals, such as 1200 cents and 700 cents (which have a highest common factor of 100 cents). In such a case, these intervals do not constitute a basis of the tuning, but instead are a *spanning set*, meaning that the coordinates (j, k, l, \dots) always refer to a pitch, but any given pitch can be identified by infinitely many different values of the coordinates. In other words, there is a many-to-one mapping from coordinates to pitch. In this way, it is possible to treat any 1-D tuning as an instance of a *degenerate* higher-dimensional tuning; for example, 12-TET is a member of a degenerate 2-D tuning that occurs when one generator is 1200 cents and the other is 700 cents.

Two-dimensional tunings play an important role in historical Western music, as exemplified by tunings such as Pythagorean and the various meantones (Barbour 1951). However, they are equally important in modern microtonal music that uses tunings suitable for *temperaments*—mappings

Table 1. Generation of the Diatonic Scale Using a Period of 1200 Cents and a Generator of 700 Cents

Iteration	Interval size (cents)		Scale degree of the major mode	Example
	unreduced	period reduced		
0	0	0	4	F
1	700	700	1	C
2	1400	200	5	G
3	2100	900	2	D
4	2800	400	6	A
5	3500	1100	3	E
6	4200	600	7	B

Note how the starting point (iteration “0”) and iterations 1 through 4 form the pentatonic scale.

from higher-dimensional to lower-dimensional tunings—such as meantone, Mavila, porcupine, srutal, magic, Hanson, and so forth (Erllich 2006; Milne, Sethares, and Plamondon 2008). Each of these temperaments is a different mapping from 5-limit just intonation (which is 3-D) to a 2-D tuning. For reasons explained below, Dynamic Tonality and Hex focus specifically on 2-D and degenerate 2-D tunings.

In the interest of clarity, we will briefly review some conventions and terminology relevant to 2-D tunings. One of the generators in a 2-D tuning is typically referred to as the *period*, and the other simply as the *generator*, even though both are technically generators. Ideally, the period should be chosen so that notes separated by an integer multiple of it are *functionally equivalent* in some sense. The inspiration and model for the concept of a functionally equivalent interval is the octave. In most music, the period is typically an octave. Other intervals, however, such as the tritave (Moreno 1995; Pierce 2001) and octave subdivisions (e.g., $1200/2 = 600$, $1200/3 = 400$, and $1200/4 = 300$ cents), have also been suggested (Erllich 2006). In this article, with no loss of generality, and where not stated otherwise, we will assume that the period is always an octave. By contrast, the other generator can be any interval.

To construct a *scale* using a 2-D tuning, the generator is iterated a chosen number of times, then reduced back into the span of a single period using period equivalence, and finally ordered by ascending pitch. The intervals between adjacent notes in the

scale are referred to as *steps*. For different purposes, the distance between any two notes in a scale can be expressed either as a *generic size*—an integer number of scale steps—or as a *specific size*—a real number in a logarithmic unit such as cents or semitones. A *2-D scale* is defined as any selection of notes taken from a 2-D tuning.

Moment of Symmetry Scales

In order to narrow the enormous range of different scales found within 2-D tunings and isolate the ones that are the most musically interesting and practical, Hex utilizes a family of scales known as MOS scales (Wilson 1975), or *well-formed* scales (Carey and Clampitt 1989). These are special cases of 2-D scales, each of whose generic intervals (e.g., steps) comes in no more than two specific sizes. In order to construct an MOS scale given a specific period and generator, the generator must be iterated precisely a number of times that yields a scale satisfying these requirements. The familiar (anhemitonic) pentatonic and diatonic scales are MOS scales with a period of 1200 cents and a generator of approximately 700 cents—the generator is iterated four times for the pentatonic scale, and two additional times for the diatonic scale. Numerous unfamiliar possibilities become available with non-standard tunings (Erllich 2006), however. Table 1 demonstrates the generation of the diatonic scale using this method.

Any MOS scale can be characterized by two *co-prime* (i.e., with no non-unity common divisors) positive integers indicating the number of large and small steps. Together, these integers are known as the *MOS signature*, and their sum reveals the total number of notes in the scale. For example, because the diatonic scale contains five large and two small steps, its MOS signature can be written as 5L, 2s—this is the notation used (among other places) on the Xenharmonic Wiki at xenharmonic.wikispaces.com—and it has $5 + 2 = 7$ notes.

MOS scales have many properties widely thought to be desirable, of which we note five. First, as we have already mentioned, every generic interval (i.e., characterized by the number of steps it spans) in an MOS scale comes in one of just two different specific sizes, which is known as *Myhill's property* (Clough and Myerson 1986). Second, the two step sizes are distributed with *maximal evenness*; for example, in the diatonic scale, there is no way of distributing the two different step interval sizes more evenly than the circular pattern of M2-M2-m2-M2-M2-M2-m2. Third, they have *uniqueness* within each period, meaning that every scale degree is surrounded by a different set of specific intervals. This means each scale degree has the potential to serve a unique musical role within the scale, which may be a prerequisite for tonal functionality (Balzano 1982). Fourth, within a large portion of their valid tuning range (we will discuss this concept in the following section) they are *proper* (Rothenberg 1978)—also known as *coherent* (Balzano 1980)—which means that there is a monotonic relationship between generic and specific interval sizes. For example, in the diatonic scale with a generator less than 700 cents, all thirds are larger in cents than all seconds, all fourths are larger in cents than all thirds, all fifths are larger in cents than all fourths, and so on. Finally, they have *transpositional simplicity* (Balzano 1982), which means that transposition of the scale by one generator will produce a new scale that shares all but one pitch class with the untransposed scale. For example, if the diatonic scale is transposed by a perfect fifth (its generator), the resulting scale contains just one different pitch class—a sharpened or flattened version of one of the original scale's

pitch classes. This facilitates modulation because closely related keys are always available.

In combination, these properties indicate that MOS scales may be a good compromise between the simplicity of equal step scales, and the complexity of wholly irregular scales (Carey 2007). Furthermore, some MOS scales with relatively few tones can be tuned so that they contain many consonant intervals and chords (Erlich 2006), making them potentially rich resources for both melody and harmony in microtonal compositions.

Valid Tuning Ranges and Embeddings of MOS Scales

Varying the size of the generator in an MOS scale causes the sizes of the small steps and the large steps to co-vary such that when one is made smaller, the other becomes larger, and vice versa. Because of this relationship, each MOS scale, as characterized by its number of small and large steps, has a *valid tuning range*: a range of generator values over which its number of small and large steps is preserved. The boundaries of an MOS scale's valid tuning range occur where (a) the size of the small steps reduces to zero, and (b) the size of the small steps increases to the size of the large steps. In the diatonic scale (5L, 2s), for example, the two boundaries occur at 5-TET, which has a generator of 720 cents, and 7-TET, which has a generator of 685.7 cents.

Interestingly, the size of the small steps can always be legitimately increased even beyond the size of the large steps, in which case the numbers of large and small steps simply swap; for example, 5L, 2s becomes 2L, 5s. This reflects the fact that every MOS scale has an *inverse* in which the numbers of large and small steps in the MOS signature are reversed. The legitimacy of any MOS scale's inverse is guaranteed because, irrespective of order, its defining integers remain co-prime.

A final important property of MOS scales is that each one can be viewed as embedded in another MOS scale. For example, the pentatonic scale (2L, 3s) can be viewed as embedded in the diatonic scale (5L, 2s). In this case, the pentatonic scale's small steps (whole tones) correspond to the diatonic scale's large steps,

Figure 3. A conventional MIDI sequencer user interface, as used in Cakewalk's SONAR.

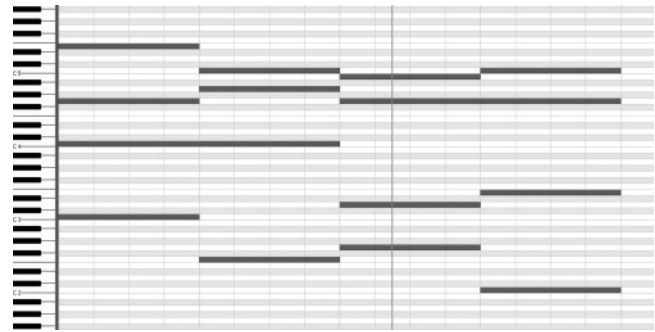
while the diatonic scale's small steps (semitones) are smaller than any steps in the pentatonic scale. The diatonic scale's small steps emerge because the generator is stacked for exactly two iterations more than for the pentatonic scale; each of these final iterations splits one of the pentatonic scale's large steps (an interval of three semitones) into one small step in the pentatonic scale and a remainder that becomes a small step in the diatonic scale. This splitting process occurs whenever an MOS scale is embedded in another MOS scale.

Another example is the embedding of the diatonic scale (5L, 2s) in the chromatic scale (either 5L, 7s, or 7L, 5s). It may seem puzzling to think of the chromatic scale as an MOS scale with two step sizes, because it is typically considered a regular 12-TET grid from which the notes of the diatonic scale are taken, and against which the asymmetries and irregularities of the diatonic scale are measured. The MOS perspective, however, emphasizes that 12-TET is only one case of the chromatic scale, yielded by a generator of 700 cents iterated eleven times. In this case, the "small" and "large" steps are the same size; however, when the generator reduces to less than 700 cents, the MOS signature becomes 7L, 5s, and when it increases to more than 700 cents, the MOS signature becomes 5L, 7s. Depending on which of these versions of the chromatic scale is used, it can even be viewed as embedded in either a 17-tone or a 19-tone MOS scale (Carey and Clampitt 1989).

In this way, any MOS scale and its embedding MOS scale (i.e., the larger scale that embeds the smaller one) can be thought of as generalizations of diatonic and chromatic scales, respectively. Furthermore, valid tuning ranges, coherence, and embeddings are intertwined properties (Milne et al. 2011), which demonstrate that MOS scales are deeply structured, not just in themselves, but also in relation to each other. They form a deeply structured and interconnected scale universe.

Spatial Mapping of 2-D Tunings in Hex

Given the advantages of 2-D tunings and MOS scales, an interface that could facilitate their exploration



would likely have considerable artistic and research potential. As we will discuss in the next subsection, the traditional piano keyboard is not particularly suitable for this task. Instead, we have found that a 2-D lattice affords much more intuitive visualization and manipulation of both generator tuning and MOS structure.

The Traditional Piano Roll

Conventional MIDI sequencer user interfaces, like the one shown in Figure 3, feature a piano roll in which horizontal position indicates notes' start and end times, and vertical position indicates pitch. Horizontal, light-colored note lanes indicate diatonic notes, and horizontal, dark-colored note lanes indicate non-diatonic (chromatic) notes. This layout is sufficient for 12-TET, but not for non-degenerate 2-D tunings or n -TETs where $n \neq 12$. The piano roll faces three problems when attempting to represent such tunings: (1) it has an insufficient number of keys to distinguish between notes that are enharmonically equivalent in 12-TET, (2) its pitch heights are not proportional to the sizes of non-12-TET intervals, and (3) it is impossible to map non-standard MOS scales to a piano roll in such a way that their spatial representation preserves any semblance of their structure. We will discuss each of these points in detail now.

In tonal music, it is common for enharmonically equivalent notes to appear in close succession; for example, a familiar chord progression such as C major–E major–A minor–A-flat seventh–G seventh–C major contains the enharmonically equivalent

notes G-sharp (the third of E major) and A-flat (the root of A-flat seventh). In any non-degenerate 2-D tuning—such as the historically important quarter-comma meantone tuning, which has a period of 1200 and generator of 696.6 cents (Barbour 1951)—enharmonically equivalent notes like G-sharp and A-flat have different tunings. Because the piano only has one black note between G and A, however, it cannot unambiguously differentiate between G-sharp and A-flat. Even as early as the 15th century, gamuts of 14 and 17 tones were presented in treatises (Dahlhaus 1990) and encountered the same problem with the traditional keyboard layout.

Even if one were happy to work with a gamut of twelve meantone pitch classes, the piano roll would generally represent their pitch heights inaccurately. On a physical piano keyboard, the horizontal position of each key is actually not proportional to its pitch. For example, the physical distance between the center lines of the keys for A and B-flat is substantially less than that between B and C, even though both intervals are exactly one semitone. In most sequencers, this problem is ameliorated by modifying the displayed width of the keys to ensure all semitones are truly of equal distance; the resulting layout is still only accurate in 12-TET, however. For example, in non-12-TET meantone tunings, each step (e.g., C–D) consists of two differently sized semitones (e.g., the augmented unison C–C-sharp, and the minor second C-sharp–D or, alternatively, the minor second C–D-flat, and the augmented unison D-flat–D). The different sizes of these semitones are not represented on the piano roll.

The piano roll is even more problematic when representing non-standard MOS scales, such as 3L, 7s, a scale that is useful because, with a generator of 380 cents, it contains ten major and minor triads close to just intonation. 3L, 7s contains ten notes per octave, but the piano roll has twelve notes—five black and seven white—per octave. One could map ten notes to a subset of these twelve, but there is no logical way to decide which two of the twelve keys should go unused, or how it is possible to reasonably represent a scale with three large and seven small steps on five black and seven white notes. Indeed, there is no way to map a non-standard MOS scale to a

piano roll such that its spatial representation reflects even a modicum of its structural characteristics.

The Button Lattice

Because of these problems, the piano roll is not a suitable interface for Hex. Instead, Hex's 2-D tunings lend themselves well to being displayed and manipulated by a representation of a *button lattice*: an array of buttons laid out in a regular lattice. A *lattice* is just the mathematical term for a set of points with a repeating structure in r -dimensional space (this geometrical definition of *lattice* should not be confused with the unrelated algebraic definition, in which it represents a type of partially ordered set). Because Hex focuses specifically on 2-D tunings, we restrict our analysis to 2-D button lattice layouts, whereby each tone and corresponding button can be identified by two integer coordinates (j, k) . Furthermore, we restrict Hex to mappings that demonstrate an *isomorphism* between the tuning and the layout, for reasons we will describe subsequently.

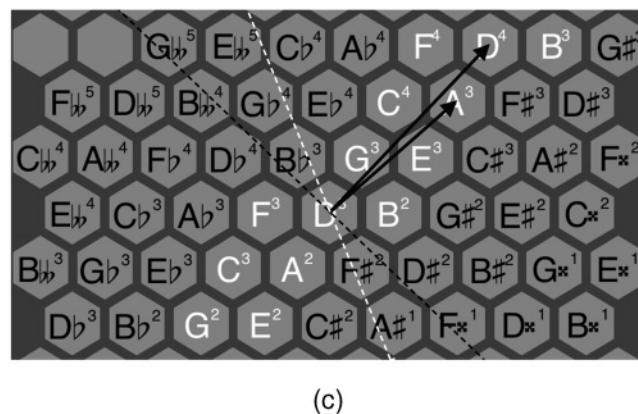
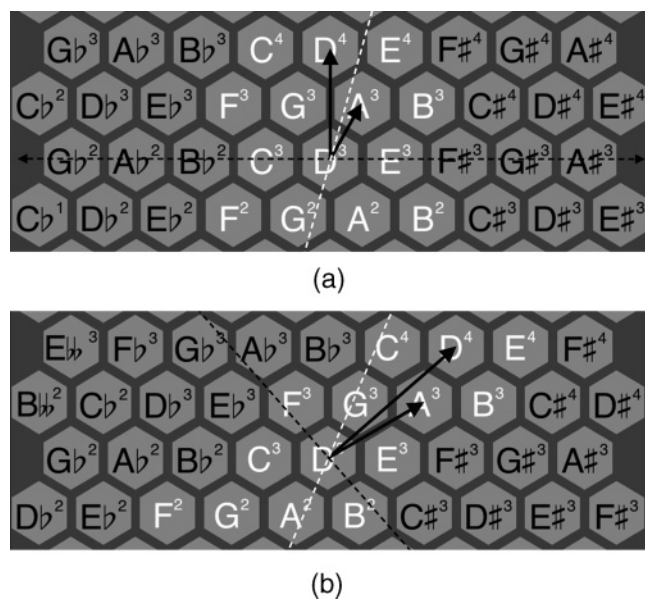
Any *isomorphic* (linear and invertible) mapping from an r -dimensional tuning to an r -dimensional button lattice yields an *isomorphic note layout* in which each interval, chord, or scale has the same spatial shape (i.e., fingering) over different transpositions (Keislar 1987), and, when categorized according to reasonable criteria, a broad continuum of tunings (Milne, Sethares, and Plamondon 2007, 2008). In order to construct an isomorphic mapping, the basis of the tuning—for our purpose, the period and generator—must be mapped to a spatial basis of the button lattice. Doing so ensures that every pitch is uniquely mapped to a button, and that every button plays a unique pitch; in other words, there is a one-to-one mapping between pitches and buttons.

One of the most historically successful isomorphic layouts—at least in the case of the twelve-fold chromatic scale—is the Wicki-Hayden hexagonal button accordion layout, shown in Figure 4a. The two black arrows indicate the spatial mappings of the period (an octave) and generator (a perfect fifth). Other possible isomorphic layouts, such as those shown in Figures 4b and 4c, map the period and

Figure 4. Three different layouts for the diatonic scale. Figure 4a is the Wicki mapping, and (b) and (c) are novel mappings. The spatial

basis vectors, to which the tuning basis (here the octave and perfect fifth) are mapped, are indicated by black arrows. Each diagram also shows the

pitch axis (dashed white line) and the generator span axis (dashed black line), both discussed in correspondingly titled sections.



generator to different spatial bases of the button lattice, and thus provide different spatial layouts for a given MOS scale.

Also shown in Figure 4 are two important axes that naturally emerge from any isomorphic layout: the *pitch axis* and the *generator span axis*. The former orders buttons by pitch height, and the latter by number of generator iterations, as described in more detail in subsequent sections. Under normal circumstances, these two axes are not perpendicular (see Figure 4), but by applying a controlled rotation and *shear*—a transformation in which points are shifted parallel to an axis by a distance proportional to their perpendicular distance from that axis—to the lattice, Hex ensures that the pitch axis is always vertical and the generator span axis is always horizontal (see Figure 5, which shows a transformed version of the Wicki layout in Figure 4a). In the following sections, we describe in detail these two axes and the implications of their orientation.

The Pitch Axis and Isotones

We have already demonstrated that, in an isomorphic 2-D mapping, the period and generator are mapped to the spatial basis of a layout. Con-

sequently, a pitch axis emerges whereby the pitch difference between any two notes is proportional to their spatial distance on this axis (Milne, Sethares, and Plamondon 2008). For example, two buttons a whole tone apart on the pitch axis are twice the distance as two buttons a semitone apart on the pitch axis, assuming a 12-TET tuning. This phenomenon has far-reaching consequences for the design of Hex.

An easy way to understand how the pitch axis works is to rotate Figure 4a, 4b, or 4c so that the pitch axis—the dashed white line—is vertical, then to place a horizontal ruler passing through the center of some reference button. If the ruler is slid vertically upwards, but kept horizontal, the centers of the buttons will always be encountered in ascending pitch order, irrespective of the numbers of periods and generators that produce them. Another notable example is provided by the Bosanquet layout (see Helmholtz 1954 [1877] for an illustration), which has a perfectly horizontal pitch axis—vertically aligned keys have the same pitch—when tuned to 12-TET.

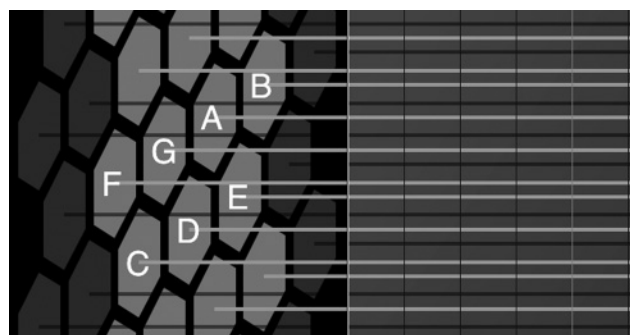
In cases where the period and generator are linearly independent (i.e., they have no common divisors), there will never be two buttons with exactly the same pitch, strictly speaking. However, by considering infinitely distant regions of the plane, buttons can be found as close as desired in pitch to any

Figure 5. The diatonic scale arranged in an APS (Wicki) layout that has been rotated and sheared to give a vertical pitch axis and a horizontal generator

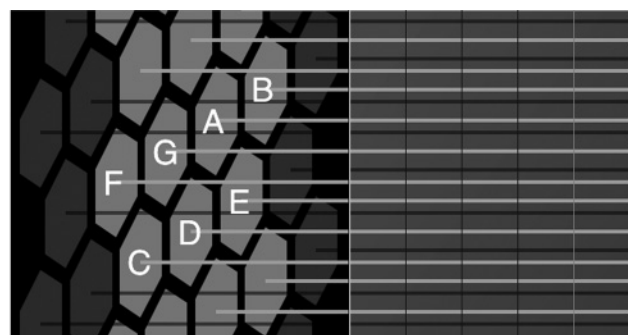
span axis. In (a), the tuning is Pythagorean (the generator is 702.0 cents); in (b), the tuning is quarter-comma meantone (the generator is

696.6 cents). The height of the center of each button corresponds to its pitch, so horizontal lines drawn from them can be used as note lanes. Looking at the

light colored note lanes, observe how the Pythagorean tuning has wider major seconds and narrower minor seconds than the meantone tuning.



(a)



(b)

other; so, for practical purposes, we can talk about two buttons having equal pitch. In cases where the period and generator are not linearly independent—in other words, they are a spanning set—then buttons can be found with exactly the same pitch, assuming the plane is sufficiently large. Thus, we can define an *isotone*: an axis that passes through the center of each button with equal or infinitely near-equal pitch. The pitch axis and the isotones are perpendicular, by definition.

The angle of the isotones and the perpendicular angle of the pitch axis depend on the layout used and the tunings of the period and generator, but it is straightforward to calculate these angles. Let the sizes of the period and generator, in a logarithmic frequency unit such as semitones or cents, be denoted α and β , respectively. If we imagine any regular 2-D lattice as laid over a reference Cartesian coordinate system, we can specify its spatial layout simply by giving the x and y coordinates of vectors representing the period and generator, respectively. Symbolically, we can express this with the two equations $\psi = (\psi_x, \psi_y)$ and $\omega = (\omega_x, \omega_y)$, with the former vector being the spatial coordinates of the period, and the latter vector the coordinates of the generator. From here, it is a matter of elementary geometry to show that the angle of the isotone θ , for any 2-D tuning and any layout, is given by

$$\theta = \arctan((\omega_y - \psi_y\beta/\alpha)/(\omega_x - \psi_x\beta/\alpha))$$

(from Milne, Sethares, and Plamondon 2008). Given this relationship, it follows directly that for any

2-D tuning and layout, varying the generator's size—while keeping the period constant—causes all isotones to rotate smoothly about the button acting as the origin for both period and generator. This fact is central to the design of Hex, as we will discuss later.

Figure 4 shows the pitch axes (with a dashed white line) for three different layouts, all with the same tuning: a period of 1200 cents and a generator of 700 cents, which corresponds to 12-TET.

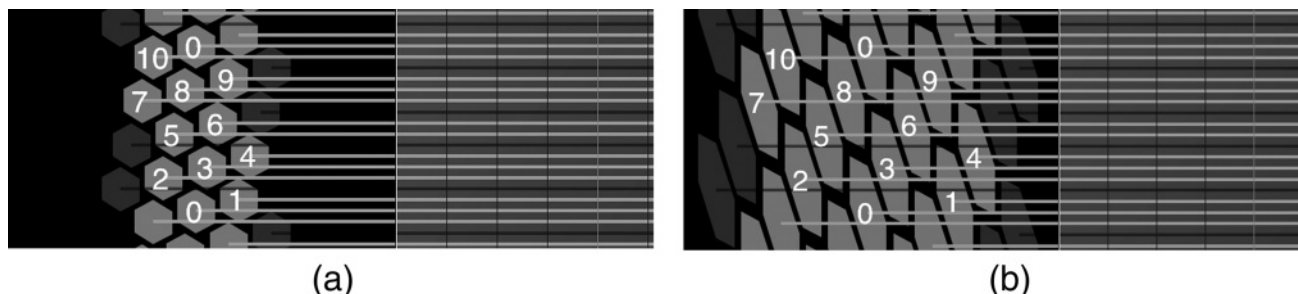
Generator Span Axis

Another axis that emerges from any isomorphic 2-D button lattice layout is the generator span axis, which orders the entire plane of buttons by the number of generator iterations between their notes, irrespective of the period in which they occur. In fact, this axis is simply perpendicular to that of the period, which, by definition, is specified in the layout. This idea can be illustrated by rotating Figure 4a, 4b, or 4c so that the generator span axis—the black dotted line—is horizontal (it already is in Figure 4a), and placing a vertical ruler passing through the centers of some reference buttons separated by octaves. If the ruler is kept vertical and is slid horizontally to the right, then the centers of buttons will be encountered in ascending order of the number of generator iterations needed to produce their pitches, irrespective of the number of periods between them. As before, the Bosanquet layout provides an alternative example because

Figure 6. Two different layouts of the 4L, 7s MOS scale (numbered by scale degree, starting at 0) with a generator of 320 cents (it is, therefore, an 11-tone subset of 15-TET). In

(a), an APS layout is used; in (b), the Wicki layout is used. The pattern of notes in the APS layout is more compact, and the scale step pattern is easier to read, when compared to

the Wicki layout. Note that the dark buttons are part of the fifteen-tone embedding scale, not of the 4L, 7s MOS scale itself.



its generator span axis is perfectly vertical (in contrast to the horizontal orientation of the Wicki layout).

The importance of generator distance (i.e., distance measured on the generator span axis) is exemplified by the fact that, in familiar tunings with a perfect fifth generator, intervals with low generator distance, like the octave and the perfect fifth, are typically considered to be closely related. Furthermore, in Western music theory the circle of fifths is a common representation of pitch, chord, and key distance (Krumhansl 1990). Although the extent to which the correlation of generator distance and pitch and chord relatedness generalizes to unfamiliar tunings is unclear, it is true that MOS scales transposed one generator apart have maximally similar pitch class content due to the aforementioned property of transpositional simplicity—in other words, they would always have only one different pitch class. This means that closely related modulations are achieved simply by shifting the scale the smallest possible distance along the generator span axis. Additionally, the spatial pattern produced by the differing left–right offsets of the scale degrees provides a useful cue for navigating through the pitch space of a possibly unfamiliar MOS scale.

Adjacent Predominant Steps (APS) Layouts

When used for the diatonic scale (5L, 2s), the Wicki layout exhibits a desirable property that qualifies it as what we call an *adjacent predominant steps* (APS) layout. In order to explain this property, we will first describe the fingering of a diatonic major scale in the Wicki layout.

As shown in Figure 4a, for each octave in the major scale, there are two rows of white notes: one with three notes, and one with four. The major scale is played by starting at the left of the three-note row and proceeding rightwards by major seconds to the end of the row. The next note—a minor second above—is reached by what might be called a “carriage return” to the first note in the next row above. After that, movement proceeds along this new row until its end is reached and another carriage return makes the final minor second step up to the octave. Crucially, the most numerous scale step (a major second) corresponds to movement along a row, and the least numerous scale step (a minor second) corresponds to a carriage return. The same phenomenon occurs with the pentatonic scale: The most numerous scale steps (major seconds) still run along rows, whereas the least numerous scale steps (minor thirds) correspond to carriage returns. This pattern of motion is visually easy to comprehend and spatially concise. In the Wicki layout, however, it does not occur when using most other MOS scales—specifically, those that require at least three generator iterations to generate seconds, because this requirement means that two adjacent buttons cannot form the interval of a second.

Fortunately, for any given MOS signature, it is straightforward to construct an APS layout, which generalizes the Wicki layout’s pattern of stepwise motion when used with the diatonic major scale, and extends it to all MOS scales. In any APS layout, the most numerous (i.e., “predominant”) scale steps—whether they be large or small—always correspond to movement along rows, and the least numerous scale steps always correspond to “carriage returns.” Figure 6 compares the APS layout and the Wicki

layout of the 4L, 7s MOS scale (note that the Wicki layout is an APS layout for 5L, 2s, but not for the 4L, 7s MOS scale).

There is only one APS layout for each MOS scale, but there are several MOS scales for each APS layout. Indeed, the 120 different MOS scales with 19 or fewer notes use only 13 different APS layouts. In Hex, the APS layout may be automatically found and applied to any scale simply by clicking “APS layout” in the Setup dialog. On the Dynamic Tonality online resource (www.dynamictonality.com), we have also provided an additional application, Relayer, that implements APS layouts for the C-Thru AXiS-49, the Thummer, and even a standard computer QWERTY keyboard.

The Visualization and Manipulation of MOS Scales in Hex

In Hex’s Setup dialog—accessible via a button at the bottom left of the window (see Figure 1)—the user can enter a pair of integers in order to specify an MOS signature (the default is 5L, 2s). To ensure the validity of the specified signature, Hex checks whether the two integers are co-prime, and, if not, it reduces them. Visually, the specified scale is represented in the button lattice by columns of light-colored notes, and the additional notes of its embedding scale are represented by columns of dark-colored notes. The number of dark-note columns in the lattice always corresponds to the number of large steps in the specified scale. This relationship holds because each large step must be split—by a single black note—to create the embedding scale, as outlined previously in the diatonic/chromatic embedding example. Hex arranges the dark note columns evenly on the left and right of the specified MOS scale; if the number of dark note columns is odd, it places the extra one on the left.

The user can vary the size of the generator in real time by moving or automating a large slider directly to the left of the button lattice (see Figure 1). Whenever this happens, an appropriate rotation and shear is automatically performed on the button

lattice to keep the pitch axis vertical and the generator span axis horizontal. This is one of Hex’s most important features, as it allows the user to simultaneously visualize several important aspects of the tuning in real time.

Keeping the pitch axis vertical ensures that pitch height is always proportional to vertical height, which, in turn, keeps all isotones horizontal, since the pitch axis and isotones are perpendicular by definition, as noted previously. This means that Hex’s note lanes are always proportional to the pitch heights of the notes they represent. Keeping the generator span axis horizontal allows the user to visualize generator distance along the horizontal dimension of the button lattice. Because the button lattice is displayed in its own pane, this does not conflict with how the horizontal dimension of the lattice roll is used to represent time. Furthermore, because by definition the period axis is always perpendicular to the generator span axis, notes one or more periods apart are always vertically aligned in the button lattice. Although periods are only separated by two rows in the familiar diatonic (5L, 2s) scale (see Figure 5), they may be separated by more in other MOS scales, in which case this extra visual cue would likely become particularly helpful (e.g., Figure 6a).

The button lattice and lattice roll are illustrated in Figure 5, which shows two tunings of a light note diatonic scale embedded in a black note chromatic scale. Note how each button center’s height corresponds to its pitch, and its horizontal position in the button lattice corresponds to its location in the generator chain. To draw a note, the user simply clicks inside one of the note lanes in the lattice roll and drags rightwards. The left and right boundaries of the note indicates its start and end times, which may optionally be quantized during input via a toggle switch in the Setup dialog. Notes can be selected and deleted by clicking on them and pressing the Delete or Backspace key. When a sequence is complete, it may be saved as a Hex project (.hxp) file—a new ASCII file format—or exported as a MIDI file, making it easy for users to open, save, and share projects, as well as to use their sequences with any commercial digital audio workstation.

Conclusion

Hex is a MIDI sequencer with a graphical user interface that provides full control over the 2-D tunings that are central to Dynamic Tonality (as well as much historical music and contemporary microtonal music) while still allowing users to exercise their existing intuitions and experience with twelve-tone equal temperament and conventional piano roll sequencers. In addition to being the first sequencer specifically designed for microtonal MOS scales and isomorphic note layouts, it introduces three novel extensions. It organizes notes such that their spatial height is always proportional to their pitch height, and octave intervals are always vertically aligned in the button lattice. It features a novel class of note layouts, adjacent predominant steps layouts, that provide consistent, neat, and compact spatial representations of MOS scales. Lastly, it uses note coloration to visualize scale embeddings, essentially generalizing the notions of “diatonic” and “chromatic” across all MOS scales.

Ultimately, we hope that Hex will enable more musicians to think of tuning as a creative tool, rather than an unforgiving and fixed limitation. This is consistent with the overall goals of Dynamic Tonality, and we believe that Hex is an important and necessary addition to the Dynamic Tonality family of software.

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References

Balzano, G. J. 1980. “The Group-Theoretic Description of 12-fold and Microtonal Pitch Systems.” *Computer Music Journal* 4(4):66–84.

- Balzano, G. J. 1982. “The Pitch Set as a Level of Description for Studying Musical Perception.” In M. Clynes, ed. *Music, Mind, and Brain*. New York: Plenum Press, pp. 321–351.
- Barbour, J. M. 1951. *Tuning and Temperament: A Historical Survey*. East Lansing: Michigan State College Press.
- Bosanquet, R. H. M. 1877. *Elementary Treatise on Musical Intervals and Temperament*. London: Macmillan.
- Carey, N. 2007. “Coherence and Sameness in Well-Formed and Pairwise Well-Formed Scales.” *Journal of Mathematics and Music* 1(2):79–98.
- Carey, N., and D. Clampitt. 1989. “Aspects of Well-Formed Scales.” *Music Theory Spectrum* 11(2):187–206.
- Clough, J., and G. Myerson. 1986. “Musical Scales and the Generalized Circle of Fifths.” *The American Mathematical Monthly* 93(9):695–701.
- Dahlhaus, C. 1990. *Studies on the Origin of Harmonic Tonality*, trans. R. O. Gjerdingen. Oxford: Princeton University Press.
- Erlich, P. 2006. “A Middle Path Between Just Intonation and the Equal Temperaments, Part 1.” *Xenharmonikôn* 18:159–199.
- Helmholtz, H. L. F. 1954. *On the Sensations of Tone*, trans. A. J. Ellis. New York: Dover. (Originally published 1877.)
- Keislar, D. 1987. “History and Principles of Microtonal Keyboards.” *Computer Music Journal* 11(1):18–28. Published with corrections in 1988 as “History and Principles of Microtonal Keyboard Design,” Report STAN-M-45, Stanford University. Available online at ccrma.stanford.edu/STANM/stanms/stanm45. Accessed May 2011.
- Krumhansl, C. L. 1990. *Cognitive Foundations of Musical Pitch*. Oxford: Oxford University Press.
- Milne, A. J., et al. 2011. “Scratching the Scale Labyrinth.” In C. Agon et al., eds. *Mathematics and Computation in Music: Third International Conference, MCM 2011, Paris, France, June 2011 Lecture Notes in Computer Science*, vol. 6726. Berlin: Springer, pp. 180–195.
- Milne, A. J., W. A. Sethares, and J. Plamondon. 2007. “Isomorphic Controllers and Dynamic Tuning: Invariant Fingering Over a Tuning Continuum.” *Computer Music Journal* 31(4):15–32.
- Milne, A. J., W. A. Sethares, and J. Plamondon. 2008. “Tuning Continua and Keyboard Layouts.” *Journal of Mathematics and Music* 2(1):1–19.
- Moreno, E. I. 1995. *Embedding Equal Pitch Spaces and The Question of Expanded Chromas: An Experimental Approach*. PhD dissertation, Department of Music, Stanford University.

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- Pierce, J. R. 2001. "Consonance and Scales." In P. R. Cook, ed. *Music, Cognition, and Computerized Sound: An Introduction to Psychoacoustics*. Cambridge, Massachusetts: MIT Press, pp. 168–186.
- Rothenberg, D. 1978. "A Model for Pattern Perception with Musical Applications. Part I: Pitch Structures as Order-Preserving Maps." *Mathematical Systems Theory* 11:199–234.
- Sethares, W. A. 2004. *Tuning, Timbre, Spectrum, Scale*. London: Springer.
- Sethares, W. A., et al. 2009. "Spectral Tools for Dynamic Tonality and Audio Morphing." *Computer Music Journal* 33(2):71–84.
- Sundberg, J., A. Friberg, and L. Frydén. 1989. "Rules for Automated Performance of Ensemble Music." *Contemporary Music Review* 3(1):89–109.
- Wicki, K. 1896. *Tastatur für Musikinstrumente*. Swiss Patent 13329, issued 30 October. Available online at www.concertina.com/gaskins/wicki/Wicki-patent-13329-of-1896.pdf. Accessed May 2011.
- Wilson, E. 1974. "Keyboard Designs Related to the Scale Tree." *The Wilson Archives*. Available online at www.anaphoria.com/wilsonkeyboard.html. Accessed May 2011.
- Wilson, E. 1975. "Letter to Chalmers Pertaining to Moments Of Symmetry/Tanabe Cycle." *The Wilson Archives*. Available online at www.anaphoria.com/wilson.html. Accessed May 2011.