Internal and external simultaneous optimization of an irreversible thermoelectric generator for maximum power output

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Abstract

A generalized finite-time thermodynamic model of irreversible multi-element thermoelectric generator is established taking into account inner and external factors. The inner effects include Seebeck effect, Fourier effect, Joule effect and Thomson effect. The irreversibility is caused by the finite-rate heat transfer between the heat reservoirs and the device. The theoretical iterative functions of the hot and cold junction temperatures and energy equations are obtained. The model is applied to the analysis of a multi-element thermoelectric generator, which is made of typical thermoelectric materials. It is found that, for given other parameters, there is an optimal electrical current, an optimal length of thermoelectric elements and an optimal ratio of thermal conductance allocation corresponding to the maximum power output. Thus, internal and external simultaneous multivariable optimizations are performed for a maximum power output. The effects of several important parameters on the optimal variables are analyzed in detail. The comparison between the optimized power and the non-optimized power shows that the multivariable optimization is necessary and effective for various working conditions. The model and optimization conclusions obtained herein can be applied to not only the analysis and optimization but also the design of thermoelectric generators.

Keywords: power generation; thermoelectric; power output; performance optimization

1 INTRODUCTION

The recent worldwide energy consciousness has focused attention on unconventional means of producing electrical energy. The photovoltaic process and the thermoelectric process are the only processes that convert solar energy directly into electrical energy. Thermoelectric generators are extremely useful and environment-friendly devices for converting heat energy, via the Seebeck effect, directly into electrical energy [1–3]. These last years, considerable efforts have been developed to identify thermoelectric materials [4–6] with enhanced performances. In addition to the improvement of the thermoelectric material, the system analysis and optimization of thermoelectric generators are equally important in designing high-performance thermoelectric generators [7, 8].

Non-equilibrium thermodynamics [9–12] is used to analyze the performance of single-stage one- or multiple-element thermoelectric generators traditionally. Considering the inner structure of a thermoelectric generator, a significant increase in the power output from a module can be achieved by modifying the geometry of the thermoelectric elements [13–15]. Rowe [16, 17] investigated the efficiency of a single couple solar-powered thermoelectric generator and reviewed US applications of nuclear-powered thermoelectric generators in space. Chen et al. [18] investigated the influence of Thomson effect on the maximum power output and maximum efficiency of a thermoelectric generator. Yamashita [19] derived the resultant Seebeck coefficient of a thermoelectric element analytically from the temperature dependence of the intrinsic Seebeck coefficient by taking into account the Thomson effect. However, the external heat transfer irreversibility was not considered in the literatures hereinbefore.

Like other traditional heat engines, thermoelectric devices cannot be used independently. They should be connected with heat exchangers to absorb and dissipate heat [1–5]. Considering the external irreversibility of a thermoelectric generator, much work has shown that the heat transfer irreversibility between the device and its external reservoirs affect the thermodynamic processes strongly. Some authors have investigated the effect of the external irreversibility on the

In practice, a commercial thermoelectric generator is a multi-element device, which comprises many thermoelectric elements. Many researchers investigated the characteristics of multi-element thermoelectric generators with the irreversibility of external finite-rate heat transfer, along with inner effect inside the thermoelectric elements. Chen et al. [26–28] analyzed the effects of external thermal conductance and the number of elements on the power output and the efficiency of multi-element single- and two-stage generators. Yu and Zhao [29] presented a numerical model, and Niu et al. [30] constructed an experimental unit incorporating the commercially available thermoelectric modules of thermoelectric generator with the parallel-plate heat exchanger. Hsiao et al. [31] and Astrain et al. [32] studied the influence of heat exchangers’ thermal resistances on a thermoelectric generation system.

Reviewing the former literatures concerning optimization of thermoelectric generators, some features can be concluded as follows:

(1) Researches on the topic of the inner geometry optimization of a thermoelectric generator, which are based on non-equilibrium thermodynamics, are without considering the effects of external heat transfer irreversibility.

(2) Researches on the topic of the external heat transfer optimization of a thermoelectric generator, which are based on finite-time thermodynamics, are without considering inner geometry of the thermoelectric generator.

(3) Most of the researches are without considering the temperature dependence of thermoelectric properties, i.e. the Thomson effect of thermoelectric materials.

To sum up, there is a lack of a complete model, which can be applied to the internal and external simultaneous performance optimization of a multi-element thermoelectric generator, and can be applied to analyze the effect of important parameters on the optimal variables and the optimal performance, respectively.

This paper aims to establish a generalized finite-time thermodynamic model of an irreversible multi-element thermoelectric generator with internal and external irreversibilities by combining finite-time thermodynamics with non-equilibrium thermodynamics. The temperature dependence of thermoelectric properties especially the Thomson effect, the contact thermal resistance and the external heat transfer irreversibility are all taken into account in the model. Rowe [1] has argued that when the energy input of a thermoelectric generator is the waste or low-grade heat type, power rather than efficiency is the primary criterion. Thus, in this study, internal and external simultaneous performance optimizations are performed for maximum power output. The effects of several important parameters on the optimal variables, i.e. electrical current, optimal length of thermoelectric elements and optimal ratio of thermal conductance allocation and the maximum power output, are analyzed in detail, respectively. The results obtained herein can offer principles for the power optimization of practical thermoelectric generators.

2 A GENERALIZED FINITE-TIME THERMODYNAMIC MODEL

Thermoelectric generator works through the absorption and liberation of heat at the connection interface between compositionally distinct electrical conductors (thermoelectric elements) with a flowing current (Peltier effect). The electric current is generated by a voltage difference that is created within each conducting leg when subjected to a temperature gradient along the length of the leg (Seebeck and Thomson effects). The schematic diagram of a multielement thermoelectric generator with external heat transfer is shown in Figure 1. A thermoelectric generator consists of P-type and N-type semiconductor legs. The length and cross-sectional area of a thermoelectric semiconductor leg are L and A. The number of thermoelectric elements is N. The junctions of the thermoelectric elements are fixed at a thermal conducting and electrical insulating ceramic plate.

Figure 2 shows the generalized finite-time thermodynamic model of an irreversible thermoelectric generator established in this paper. The model can be divided into a thermal network, i.e. the heat flow system, and an electrical network, i.e. the electrical current system. The thermal rates are plotted by hollow arrows and the electrical currents are plotted by solid arrows.

The hot and cold junction temperatures are $T_H$ and $T_L$. The heat source and heat sink temperatures are $T_H$ and $T_L$. The heat flow rate absorbed from the heat source to the thermoelectric generator is $Q_H$. The heat flow rate dissipated from the
The generalised finite-time thermodynamic model of an irreversible thermoelectric generator.

is the thermoelectric generator to the heat sink is \( Q_h \). The heat flow rate through the hot and cold junctions of the thermoelectric elements are \( Q_h \) and \( Q_c \). The heat transfers between the heat reservoirs and the thermoelectric generator are finite-rate irreversible heat transfer obeying Newtonian heat transfer law \( Q \propto \Delta T \).

The electrical resistance of the P- and N-type semiconductor legs are \( R_p \) and \( R_n \). The contact electrical resistance is \( R_{cl} \). The output electrical voltage and current are \( U \) and \( I \), respectively. The load resistance is \( R_L \).

For a well-designed thermal insulation packaging module, the heat leakage from the surround of the elements and module can be neglected, so the heat transfer of the whole device can be treated as one dimensional. According to the non-equilibrium thermodynamics, the inner effects of the thermoelectric elements include Seebeck effect, Fourier effect, Joule effect and Thomson effect. The increment rate of internal energy of the infinitesimal is zero at steady state. One can obtain the energy conservation equation of P-type semiconductor leg as follows [1, 2]:

\[
Q_{\text{Kin}} - Q_{\text{Kout}} + Q_I + Q_{\mu} = 0
\]  

(1)

where \( Q_{\text{Kin}} \), \( Q_{\text{Kout}} \), \( Q_I \) and \( Q_{\mu} \) are the Fourier heat input (heat input to the hot junction of the element by heat conduction), Fourier heat output (heat output from the cold junction of the element by heat conduction), generated Joule heat (generated heat when electrical current flows through the conductor) and generated Thomson heat (generated heat because of Thomson effect). Equation (1) can be written as

\[
\frac{d}{dx} \left( T_p + dT_p \right) k_p A_p - \frac{dT_p}{dx} k_p A_p + \left( \epsilon_f T_p \right)^2 A_p L_p \frac{dx}{\sigma_p L_p} + \mu_p I dT_p = 0
\]  

(2)

where \( k_p \), \( \sigma_p \), \( \mu_p \), \( A_p \), \( L_p \), \( T_p \), \( \epsilon_f \) are the thermal conductivity, electrical conductivity, Thomson coefficient, cross-sectional area, length, temperature and electrical current density of the P-type semiconductor leg. Reforming Eq. (2) and making the same analysis on the N-type semiconductor leg, one can obtain the thermal conduction differential equations (3) and (4), with boundary conditions (5) and (6) as follows [11]:

\[
\frac{d}{dx} \left( k_p \frac{dT_p}{dx} \right) + \epsilon_f \mu_p \frac{dT_p}{dx} + \left( \epsilon_f T_p \right)^2 = 0
\]  

(3)

\[
\frac{d}{dx} \left( k_n \frac{dT_n}{dx} \right) - \epsilon_n \mu_n \frac{dT_n}{dx} + \left( \epsilon_n T_n \right)^2 = 0
\]  

(4)

\[
T_p(0) = T_n(0) = T_i
\]  

(5)

\[
T_p(L_p) = T_n(L_n) = T_h
\]  

(6)

where \( k_n \), \( \sigma_n \), \( \mu_n \), \( A_n \), \( L_n \), \( T_n \), \( \epsilon_f \) are the thermal conductivity, electrical conductivity, Thomson coefficient, cross-sectional area, length, temperature and electrical current density of the N-type semiconductor leg.

Taking into account the effect of temperature dependence of thermoelectric properties, \( k_p \), \( \sigma_p \) and \( \mu_p \) are functions of \( T_p \), while \( k_n \), \( \sigma_n \) and \( \mu_n \) are functions of \( T_n \). However, such a differential equation cannot be solved analytically. When the functions \( k(T) \), \( \sigma(T) \) and \( \mu(T) \) can be expressed by a quadratic, replacing \( k \), \( \sigma \) and \( \mu \) with the mean values \( \bar{k} \), \( \bar{\sigma} \) and \( \bar{\mu} \) cause little error and give approximation of Eqs. (3) and (4) as follows:

\[
\bar{k}_p A_p \frac{dT_p^2}{dx^2} + \bar{\mu}_p I \frac{dT_p}{dx} + \frac{I^2}{\bar{\sigma}_p A_p} = 0
\]  

(7)

\[
\bar{k}_n A_n \frac{dT_n^2}{dx^2} - \bar{\mu}_n I \frac{dT_n}{dx} + \frac{I^2}{\bar{\sigma}_n A_n} = 0
\]  

(8)

where

\[
\bar{k}_p = k_p |_{T=T_h+T_c}/2, \bar{\sigma}_p = \sigma_p |_{T=T_h+T_c}/2, \bar{\mu}_p = \mu_p |_{T=T_h+T_c}/2
\]

\[
\bar{k}_n = k_n |_{T=T_h+T_c}/2, \bar{\sigma}_n = \sigma_n |_{T=T_h+T_c}/2, \bar{\mu}_n = \mu_n |_{T=T_h+T_c}/2
\]

for P-type semiconductor leg and

\[
\bar{k}_n = k_n |_{T=T_h+T_c}/2, \bar{\sigma}_n = \sigma_n |_{T=T_h+T_c}/2, \bar{\mu}_n = \mu_n |_{T=T_h+T_c}/2
\]

for N-type semiconductor. Alteration methods along with the physical parameters fitted formulas are adopted to determine the junction temperatures \( T_h \) and \( T_c \). It is assumed that the heat generated in the semiconductor legs are absorbed and dissipated through the hot and cold junctions of the thermoelectric elements. The total heat flow rates through the hot and cold junctions are

\[
Q_h = N \left( \alpha_{gh} \alpha_{nh} T_h I + k_{p_1-s_1} A_p \frac{dT_p}{dx} |_{x=s_1} + k_{n_1-s_1} A_n \frac{dT_n}{dx} |_{x=s_1} \right)
\]  

(9)

\[
Q_c = N \left( \alpha_{pc} \alpha_{nc} T_c I + k_{p_1-c_1} A_p \frac{dT_p}{dx} |_{x=c_1} + k_{n_1-c_1} A_n \frac{dT_n}{dx} |_{x=c_1} \right)
\]  

(10)

where \( \alpha_p \) and \( \alpha_n \) are the Seebeck coefficient of the P- and N-type semiconductor leg, and the subscript h and c represent the hot and cold side.
It is assumed that the external thermal conductance of the hot and cold side are $K_H$ and $K_c$, respectively. Then one has

$$Q_H = K_H(T_H - T_c)$$  \hspace{1cm} (11)  \\
$$Q_L = K_c(T_c - T_L)$$  \hspace{1cm} (12)

According to the heat flow balance, one can obtain the control equations of the system as follows:

$$Q_H = Q_h$$ \hspace{1cm} (13)  \\
$$Q_L = Q_c$$ \hspace{1cm} (14)

The temperature distribution of the P- and N-type semiconductor leg can be solved by Eqs. (7) and (8) as follows:

$$T_p(x) = T_c - F_p x + \frac{T_H - T_c + F_p \mu_p}{(e^{-\alpha_p x} - 1)}$$ \hspace{1cm} (15)  \\
$$T_n(x) = T_c + F_n x - \frac{T_H - T_c - F_n \mu_n}{(e^{-\alpha_n x} - 1)}$$ \hspace{1cm} (16)

where $\alpha_p = \mu_p I/(k_p A_p)$, $F_p = I/(\sigma_p \mu_p A)$, $\alpha_n = \mu_n I/(k_n A_n)$ and $F_n = I/(\sigma_n \mu_n A)$.

For commercially available thermoelectric modules, the physical property and geometry of the P- and N-type semiconductor legs can be approximated as: $\alpha_p = \sigma_p$, $k_p = k_h$, $\mu_p = -\mu_h$, $\mu_p = -\mu_n$, $A_p = A_n = A$ and $L_p = L_n = L$. Based on the above assumption, Eqs. (9) and (10) can be approximated as follows:

$$Q_h = N[\alpha_n I T_H + K(T_h - T_c) - 0.5 I^2 R - 0.5 \mu I(T_h - T_c)]$$ \hspace{1cm} (17)  \\
$$Q_c = N[\alpha_c I T_c + K(T_c - T_h) + 0.5 I^2 R + 0.5 \mu I(T_h - T_c)]$$ \hspace{1cm} (18)

where $\alpha_n = \alpha_n - \alpha_h$ and $\alpha_c = \alpha_c - \alpha_c$ are the Seebeck coefficient of the thermoelectric elements at hot and cold side. $K$, $K$ and $R$ are the total Thomson coefficient, thermal conductance and electrical resistance of a thermoelectric element. $\alpha T_H$, $I R^2$, $I R$ and $\mu I \Delta T$ are the rates of Peltier heat, Fourier heat, Joule heat and Thomson heat, respectively. $K$ and $R$ is given by $K = 2K/k_h$ and $R = 2L/(\sigma A)$.

If the temperature dependence of thermoelectric properties is not considered, the model changes into a temperature-independent model and then Eqs. (17) and (18) change into:

$$Q_h = N[\alpha_n I T_H + K(T_h - T_c) - 0.5 I^2 R]$$ \hspace{1cm} (19)  \\
$$Q_c = N[\alpha_c I T_c + K(T_c - T_h) + 0.5 I^2 R]$$ \hspace{1cm} (20)

The power output is given by

$$P = Q_h - Q_c = NI[\alpha_n T_h - \alpha_c T_c - IR - \mu(T_h - T_c)]$$ \hspace{1cm} (21)

The external heat transfer is considered in this paper. For given heat source and heat sink temperatures $T_H$ and $T_L$, the junction temperatures of thermoelectric element $T_h$ and $T_c$ are unknown, thus the thermoelectric properties $\alpha_n$, $\alpha_c$, $K$, $\mu$ and $\sigma$ are unknown. Alteration method is adopted here to determine the junction temperatures $T_h$ and $T_c$. The alteration formula can be solved by Eqs. (13) and (14) as follows:

$$T_h = N[\alpha_n I I + (-2N^2 R K - NRK)]I^2 + 2N\alpha_n K_i T_i$$  \hspace{1cm} (22)

where $\alpha_n$, $\alpha_c$, $K$, $R$ and $\sigma$ are functions of $T_h$ and $T_c$. For given initial value of $T_h$ and $T_c$ ($T_h = T_h$, $T_c = T_c$, for example), $\alpha_n$, $\alpha_c$, $K$, $R$ and $\sigma$ can be calculated by the fitting formula of the thermoelectric material. Then $T_h$ and $T_c$ can be calculated by Eqs. (24) and (25). Repeat the process until the required precision is obtained. The numerical calculations in this research proved that, when the heat reservoir temperature difference of the thermoelectric device is between 0 and 300 K, the iterative procedure converge for all parameter values ($error < 10^{-8}$) within 20 iterations.

The model built in this paper is a temperature-dependent model, so it is more helpful than a temperature-independent model. If the temperature dependence of thermoelectric properties is not considered, the model changes into a temperature-independent model and then Eqs. (24) and (25) change into:

$$T_h = 0.5N^2 R \alpha I^2 + (-N^2 R K - 0.5N K_i) I^2 + m N K_i T_i + m N K_i T_i$$  \hspace{1cm} (23)

$$T_c = 0.5N^2 R \alpha I^2 + (N^2 R K - 0.5N K_i) I^2 + m N K_i T_i + m N K_i T_i$$  \hspace{1cm} (24)
In this case, the junction temperatures $T_h$ and $T_c$ can be solved directly.

3 INTERNAL AND EXTERNAL SIMULTANEOUS OPTIMIZATION

3.1 Design variables and fixed parameters

The power output of a thermoelectric generator depends on the output electrical current $I$. For a given thermoelectric generator, there exists an optimal electrical current corresponding to the maximum power output, which has been proved previously [18, 21, 23, 25, 27, 28, 30]. Thus, the optimization of electrical current is a basic problem of the internal optimization of a thermoelectric generator.

Previous studies concerning the geometry of thermoelectric generator have argued that a significant increase in the power output from a thermoelectric generator can be achieved by optimizing the geometry of the thermoelectric elements [13–16]. The most effective approach of the optimization of internal geometry of a thermoelectric generator is the optimization of the length of the thermoelectric elements $L$.

More external thermal conductance means better heat exchange conditions but bulkier structures and higher cost of the device. It is an interesting and important problem that how to allocate fixed total external thermal conductance among hot and cold side of the thermoelectric generator for maximum power output. When the external heat transfer is considered, the allocation of thermal conductance of heat exchangers between the hot and cold side affects the performance of the device. To describe the allocation, a ratio of thermal conductance allocation is defined as $f = K_0/(K_H + K_L)$ [27–29].

For maximum power output, the design variables $I$, $L$ and $f$ should satisfy

$$\frac{\partial P}{\partial I} = \frac{\partial P}{\partial L} = \frac{\partial P}{\partial f} = 0$$

(28)

However, Eq. (28) cannot be solved analytically because of the iteration procedure. The following numerical calculations show that there exist optimum output electrical current $I_o$, optimum length of the thermoelectric elements $L_o$ and optimum ratio of thermal conductance $f_o$ corresponding to the maximum power output for given other parameters.

The objective function $P$ is three-variable function. The problem is how to determine $I$, $L$ and $f$, so that the objective function reaches the maximum. This problem can be solved by Matlab Optimization Toolbox.

The objective functions are set as follows:

$$\max P(I, L, f)$$

(29)

The lower bounds and upper bounds are

$$\text{LB} = [0 \ 0 \ 0]$$

(30)

$$\text{UB} = [\text{inf} \ \text{inf} \ \text{inf} \ 1]$$

(31)

The nonlinear inequalities are

$$\begin{bmatrix} Q_H \\ Q_L \end{bmatrix} > \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(32)

It should be noted that the optimization procedure must include the iteration procedure, i.e. the solving procedure of the hot and cold junction temperatures (see Eqs. (24) and (25)).

The physical properties of the commercially available material by Melcor [33] used for this calculation are shown as follows:

$$\alpha_p = (22224.0 + 930.6 T - 0.9905 T^2) \times 10^{-9} \text{ V K}^{-1}$$

(33)

$$\rho = (5112.0 + 163.4 T + 0.6279 T^2) \times 10^{-10} \text{ } \Omega \text{ m}$$

(34)

$$k = (62605.0 - 277.7 T + 0.4131 T^2) \times 10^{-4} \text{ W m}^{-1} \text{ K}^{-1}$$

(35)

where $\alpha_p$, $\rho$ and $k$ are the Seebeck coefficient, electrical resistivity and thermal conductivity. The Thomson coefficient is given by the second Kelvin relationship [1]

$$\mu = T \frac{d \alpha}{dT}$$

(36)

3.2 Effects of important parameters on the optimal variables and maximum power output

The cross-sectional area of the thermoelectric element $A$ represents the geometry feature of the thermoelectric elements; the number of thermoelectric element $N$ represents the power generation capacity of the thermoelectric generator; the heat reservoir temperature difference $\Delta T$ represents the working strength of the thermoelectric generator; the total external thermal conductance $K_T$ represents the external heat exchange condition; the coefficient of contact resistance $C_{ct}$ represents the effect of contact resistance. Thus, the parameters $A$, $N$, $\Delta T$, $K_T$ and $C_{ct}$ are the most important parameters of a multi-element thermoelectric generator with external heat transfer. The effects of the important parameters, i.e. $A$, $N$, $\Delta T$, $K_T$ and $C_{ct}$ on the optimal variables $L_o$, $L_o$ and $f_o$ and the maximum power output $P_{\text{max}}$ will be analyzed, respectively.

Figure 3a shows the optimal variables $L_o$, $L_o$ and $f_o$ versus cross-sectional area of the thermoelectric element $A$. In the optimization, fixed parameters are set as $N = 127$, $T_L = 300$ K, $K_T = 10$ W/K, $K_T = 10$ W/K and $C_{ct} = 1 \times 10^{-8}$ $\Omega$ m$^2$. It can be seen that when the cross-sectional area $A$ increase, the optimal electrical current $I_o$ and the optimal length of thermoelectric element $L_o$ increase linearly whereas the optimal ratio of thermal conductance allocation $f_o$ decrease. Figure 3b shows the maximum power output $P_{\text{max}}$ and the corresponding efficiency $\eta_p$ versus cross-sectional area of the thermoelectric element $A$. For comparison, the non-optimized power $P$ calculated at $I = 2A$, $L = 1$ mm and $f = 0.5$ is also presented in the figure. It can be seen that both the maximum power output $P_{\text{max}}$ and the corresponding efficiency $\eta_p$ increase slowly with the increase in cross-sectional area of the thermoelectric.
element. That is the power output can be improved by improving the cross-sectional area of the thermoelectric element. However, because of the increase of the volume of thermoelectric materials, it is not economical.

Figure 4a shows the optimal variables $I_o$, $L_o$ and $f_o$ versus the number of thermoelectric elements $N$. In the numerical optimization, fixed parameters are set as $A = 1 \text{ mm}^2$, $T_L = 300 \text{ K}$, $T_H = 450 \text{ K}$, $K_T = 10 \text{ W/K}$, $C_{ct} = 1 \times 10^{-8} \text{ W m}^2$. It can be seen that when the number of thermoelectric elements $N$ increase, the optimal length of thermoelectric element $L_o$ increases linearly whereas the optimal electrical current $I_o$ and the optimal ratio of thermal conductance allocation $f_o$ decrease slowly. The decrease of $f_o$ proportional to the number of couples can be reasoned as the increase of thermal conductance of the thermoelectric layer requires that the external conductance of cold side should increase to pump the additional heat being rejected by the thermoelectric layer. Figure 4b shows the maximum power output $P_{\text{max}}$ and the corresponding efficiency $\eta_p$ versus number of thermoelectric elements. For comparison, the non-optimized power $P$ calculated at $I = 1 \text{ A}$, $L = 0.5 \text{ mm}$, $f = 0.5$ is also presented in the figure. It can be seen that both the maximum power output $P_{\text{max}}$ and the corresponding efficiency $\eta_p$ increase slowly with the increase in the number of thermoelectric elements. That is the power output can be improved by improving the number of thermoelectric elements. It should be noted that the maximum power output $P_{\text{max}}$ is not in proportion to the number of thermoelectric elements because the fixed total external thermal conductance. To improve the power output effectively, the total external thermal conductance $K_T$ should be increased along with the number of thermoelectric elements $N$. The maximum power output occurs when the total thermal resistance of the heat exchangers must be equal to the thermal resistance of the thermoelectric layer.

Figure 5a shows the optimal variables $I_o$, $L_o$ and $f_o$ versus heat reservoir temperature difference $\Delta T$. In the numerical optimization, fixed parameters are set as $A = 1 \text{ mm}^2$, $N = 127$, $K_T = 10 \text{ W/K}$, $C_{ct} = 1 \times 10^{-8} \text{ W m}^2$. It can be seen that when the temperature difference $\Delta T$ increase, the optimal electrical current $I_o$ increases whereas the optimal length of thermoelectric element $L_o$ and the optimal ratio of thermal conductance allocation $f_o$ decrease. For comparison, the non-optimized power $P$ calculated at $I = 1 \text{ A}$, $L = 0.5 \text{ mm}$, $f = 0.5$ is also presented in the figure. Figure 5b shows the maximum power output $P_{\text{max}}$ and the corresponding efficiency $\eta_p$ versus heat reservoir temperature difference $\Delta T$. Because the thermal resistances of the thermoelectric elements and the external heat exchangers are constants, the junction temperature difference increases with the increase in heat reservoir temperature difference. It can be seen that the maximum power output increases linearly with the increase in temperature difference. That is the power output
can be improved by improving the temperature difference at all times with the optimal variables. The efficiency $\eta_P$ corresponding to the maximum power output increases to a maximum and then decrease due to the decline of figure of merit $Z = a^2 \sigma/k$ (can be calculated by Eqs. (33)–(35)) of the commercially available material at high temperature.

Figure 6a shows the optimal variables $I_o$, $L_o$ and $f_o$ versus the total external thermal conductance $K_T$. In the numerical optimization, fixed parameters are set as $N = 127$, $N = 127$, $T_L = 300 \text{ K}$, $T_H = 450 \text{ K}$ and $C_{ct} = 1 \times 10^{-8} \text{ \Omega m}^2$. It can be seen that when the total external thermal conductance $K_T$ increase, the optimal electrical current $I_o$ and the optimal ratio of thermal conductance allocation $f_o$ increase whereas the optimal length of thermoelectric element $L_o$ decreases. The number of couples and the cross-sectional area of the leg are fixed quantity, and the thermal conductance of the thermoelectric layer is proportional to the number of couples, cross-sectional area of the thermoelectric leg and inversely proportional to the length of the leg. As listed previously due to the thermal resistance matching requirement as the total external thermal conductance increases, the thermal conductance of the thermoelectric layer is increased by the decrease of leg length. Figure 6b shows the maximum power output $P_{max}$ and the corresponding efficiency $\eta_P$ versus total external thermal conductance $K_T$. For comparison, the non-optimized power $P$ calculated at $I = 1\text{ A}$, $L = 0.5 \text{ mm}$, $f = 0.5$ is also presented in the figure. The maximum power output increases more and more slowly with the increase in total external thermal conductance. That is there is a limit of maximum power output that cannot be overrun by improving the conditions of external heat exchange. When the total external thermal conductance tends to infinite, the maximum power output tends to a fixed value, i.e. the result of non-equilibrium thermodynamics in which the external heat transfer irreversibility is neglected. The efficiency $\eta_P$ corresponding to the maximum power output decreases with the increase in total external thermal conductance.

Figure 7a shows the optimal variables $I_o$, $L_o$ and $f_o$ versus the coefficient of contact resistance $C_{ct}$. In the numerical optimization, fixed parameters are set as $A = 1 \text{ mm}^2$, $N = 127$, $T_H = 450 \text{ K}$, $T_H = 450 \text{ K}$ and $K_T = 10 \text{ W/K}$. It can be seen that when the coefficient of contact resistance $C_{ct}$ increases, the optimal length of thermoelectric element $L_o$ and the optimal ratio of thermal conductance allocation $f_o$ increase whereas the optimal electrical current $I_o$ decreases. The changing features the optimal variables can be used as reference for analyzing the contact resistance. Figure 7b shows the maximum power output $P_{max}$ and the corresponding efficiency $\eta_P$ versus the coefficient of contact resistance $C_{ct}$. For comparison, the non-optimized
power $P$ calculated at $I = 1\, A$, $L = 0.5\, \text{mm}$, $f = 0.5$ is also presented in the figure. Both the maximum power output $P_{\text{max}}$ and the corresponding efficiency $\eta_p$ decrease with the increase in the coefficient of contact resistance $C_{\text{ct}}$. The decrease reflects the great influence of contact resistance on the performance of the device.

On the whole, there is a large difference among the optimized power and the non-optimized power. By optimization, the power is improved much. Therefore, the multivariable optimization is necessary and effective for various working conditions. Among the three design variables, parameters have significant effect on the optimal electrical current $I_o$ and optimal length of thermoelectric elements $L_o$ but little effect on the optimal ratio of thermal conductance allocation $f_o$. Among the several parameters, the heat reservoir temperature difference has biggest influence on the optimal ratio of thermal conductance allocation $f_o$.

4 CONCLUSIONS

Aiming at the limitations of single-variable optimization, a general finite-time thermodynamic model of irreversible thermoelectric generator is established, in which inner geometry dimension and external heat transfer are all taken into account. Applying the model to a typical thermoelectric generator, it is found that there are three optimal variables, i.e. an optimal electrical current, an optimal length of thermoelectric elements and an optimal ratio of thermal conductance allocation synchronously corresponding to the maximum power output. Thus, internal and external simultaneous optimizations are performed for maximum power output. The effects of several important parameters on the optimal variables are analyzed. It is found that different parameters have different effects on the design variables and the maximum power output. The multivariable optimization is necessary and effective for various working conditions. The changing features of the design variables obtained herein can be used as references for analyzing the effect of internal and external factors on the optimal performance of the thermoelectric generator.

This research is performed at a constraint on the total external thermal conductance. A constraint on the total amount of semiconductor material would be a further and interesting work. The area power density is another interesting optimization objective.

The general model and optimization method adopted herein may be applied to not only the analysis and optimization but also the design of practical thermoelectric generators. Following the present paper, further research work may analyze the performance of double- or multi-stage thermoelectric generators. Moreover, the performance optimization of thermoelectric refrigerators and heat pumps may be another interesting and important problem.

ACKNOWLEDGEMENTS

This paper is supported by the National Natural Science Foundation of P. R. China (Project No. 10905093) and the Natural Science Foundation of Naval University of Engineering (Grant No. HGDYDJJ10011). The authors wish to thank the reviewers for their careful, unbiased and constructive suggestions, which led to this revised manuscript.

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