A new inverse DEA method for merging banks

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This study suggests a novel application of Inverse Data Envelopment Analysis (InvDEA) in strategic decision making about mergers and acquisitions in banking. The conventional DEA assesses the efficiency of banks based on the information gathered about the quantities of inputs used to realize the observed level of outputs produced. The decision maker of a banking unit willing to merge/acquire another banking unit needs to decide about the inputs and/or outputs level if an efficiency target for the new banking unit is set. In this paper, a new InvDEA-based approach is developed to suggest the required level of the inputs and outputs for the merged bank to reach a predetermined efficiency target. This study illustrates the novelty of the proposed approach through the case of a bank considering merging with or acquiring one of its competitors to synergize and realize higher level of efficiency. A real data set of 42 banking units in Gulf Corporation Council countries is used to show the practicality of the proposed approach.

Keywords: data envelopment analysis (DEA); inverse DEA; bank; M&As; GCC.

1. Introduction

Data envelopment analysis (DEA), as reported by Charnes et al. (1978) and extended by Banker et al. (1984), is a recognized tool for the assessment of the performance of organizations. The DEA has gained a wide range of successful applications measuring comparative efficiency of multiple inputs and outputs of a homogeneous set of decision making units (DMUs), resulting in an abundant literature as reported by Gattoufi et al. (2004a); Emrouznejad et al. (2008) and Emrouznejad & De Witte (2010), and analysed by Gattoufi et al. (2004b). [For some of the recent applications using DEA, see (Behera et al., 2011; Sufian, 2011; Tsolas, 2011; Yeung & Azevedo, 2011)].

As more analysts apply the DEA methodology, new, genuine and interesting theoretical issues are discussed and addressed in the literature. However, some of those interesting theoretical advances remained without direct applications with real world. Among these recent developments, our interest in this paper is in the Inverse DEA (InvDEA), hereafter, a variety of conventional DEA that uses the inverse linear programming (LP). An inverse programming problem consists of inferring the values of the model’s parameters such as cost coefficient, right-hand side vector and the constraint matrix given the values of observable parameters, as described by Zhang & Liu (1996); Huang & Liu (1999) and Ahuja & Orlin (2001).
The basic idea in an InvDEA is to find the required level of inputs and outputs for a given DMU in order to reach a predetermined efficiency target (Wei et al., 2000). Unlike the conventional DEA, where the objective is to find the efficiency coefficient, the InvDEA assumes given the efficiency of a DMU defined as a preset target and determines the corresponding parameter that leads the DMU to realize the efficiency level. This paper extends the concept of an InvDEA for a case of merger and acquisition (M&A) in banking in order to find the required level of inputs and outputs of the merged bank for a given efficiency target. We applied the proposed method to the real data set banks in Gulf Cooperation Countries (GCC) where a large consolidation is ongoing due to the global financial crisis.

Despite the sound theoretical developments, initially suggested by Wei et al. (2000) and developed further by Pendharkar (2002) and Amin & Emrouznejad (2007), this paper is the first application of InvDEA in the case of merging using a real data set.

The rest of this paper is organized as follows: Section 2 highlights the literature review of M&As in banking. Section 3 presents the motivation of using the InvDEA method for merging banks. This is followed by presenting the general InvDEA models for merging banks in Section 4. Section 5 provides an application for the proposed InvDEA method in merging GCC banks. Concluding remarks, limitations and directions for future research are given in Section 6.

2. M&A in banking

The M&A in banking has been a popular research area for the last two decades in the Anglo-Saxon academia that was prolific in producing an abundant literature addressing different related issues and applied to a variety of economic sectors including the financial sector in general, and the banking sector in particular. There was a large debate about the possible existence of positive impact of M&A on the performance of the firms engaged in these types of consolidation. There are, in fact, substantial numbers of studies that have been published and trying to assess the argument of achieving positive gains through M&A in banking. The efficiencies, economies of scale and improved management are reported to be the main motivations as reported in Madura & Wiant (1994).

However, studies in this area have shown conflicting findings. Although some studies confirm the existence of such an impact, others, though they do not exclude it, document its limitation. Hence, there is a general agreement about the existence of positive impact of M&A on banks’ performance.

The general literature about M&A that contributed to this debate was initiated by the pioneering work of Healy et al. (1992) who documented that there is a clear positive link between abnormal stock gains at merger announcements, and the after-merger raises in operating cash flows, while Rhoades (1998) came to the conclusion that M&As in banking did not enhance performance.

The specific issue of impacts on economic–technical efficiency was studied by Rhoades (1998) and Zhu (1999), who analyzed nine cases of M&A in banking in USA. The results suggest that the motives behind the mergers along with consolidation process could influence the cost-efficiency effects. More recently, this was confirmed by Al-Sharkas et al. (2008) and Lin (2010), who investigated and confirmed the existence of positive impacts on both efficiencies (cost and profit) of bank consolidation on the American banking sector, concluding that M&As have enhanced the banks cost and profit efficiencies.

Consolidation in GCC, a recent phenomenon recommended and encouraged by the public and regulating authorities being considered as appropriate to overcome the negative impacts of the global financial crisis and to hedge against its aftermaths, was not analysed scholarly and the authors were unable to identify relevant published studies.

In the case of banks in GCC countries, the positive impact of M&A on technical efficiency was analysed by Mostafa (2007); Ramanathan (2007) and Gattoufi et al. (2009) using financial ratios. Country
wise, Gattoufi & Al-Hatmi (2009) analysed the performance of Omani banks and came to the conclusion that there is room for efficiency improvement, through M&A, they suggest, considering the wide range in the size of banks operating in the Omani, and also due to the level of scale inefficiency of local banks. Moreover, the authors advised that to overcome the barriers to entry, local banks went through regional consolidation to improve their performances and gain market share.

3. Research motivation

It is important to mention that the general practice when banks decide to go through consolidation is to define an achievable target in terms of performance to be reached by synergizing with its acquirer/target. This target is usually defined based on the current performances of the two banks, acquirer and target. The performance is usually assessed using a variety of methods, and we consider here the method using the technical efficiency as an indicator of performance. The technical efficiency is a relative measure of the performance obtained by the DEA approach, and the efficiency coefficients are the optimal solutions of a set of linear programs, one for each bank included in the sample. An InvDEA model uses a feasible solution, not necessarily optimal, to determine the required changes on the parameter values of the corresponding DEA problem. In more technical words, ‘we have a given feasible solution which is not necessary an optimal solution, and we wish to adjust these parameter values, inputs, and outputs, as little as possible so that the feasible solution becomes the optimal one under the adjusted parameter values. Or, in a more general case, we wish that after adjusting the parameters as little as possible, the optimal solution should possess some required properties’ (Wei et al., 2000). In the context of InvDEA and M&A in banking, we set a target in terms of technical efficiency for the merged banks, and we determine the corresponding inputs and outputs. To illustrate how an InvDEA can be developed for merging banks, we consider the following hypothetic example. Table 1 shows six banks with two inputs and one output.

The input-oriented variable returns to scale (VRS) DEA model for bank A is as follows.

\[
\begin{align*}
\text{min } & \theta \\
\text{s.t. } & 20\lambda_A + 19\lambda_B + 60\lambda_C + 27\lambda_D + 58\lambda_E + 55\lambda_F - 20\theta \leq 0 \\
& 151\lambda_A + 131\lambda_B + 250\lambda_C + 168\lambda_D + 258\lambda_E + 255\lambda_F - 151\theta \leq 0, \\
& 100\lambda_A + 150\lambda_B + 120\lambda_C + 195\lambda_D + 95\lambda_E + 230\lambda_F \geq 100, \\
& \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F = 1, \\
& \lambda_j \geq 0, \quad j = A, \ldots, F.
\end{align*}
\]
The optimal value of the above model is $\theta^*_A = 0.95$. Also, the efficiency scores of the other banks are

$$\theta^*_B = 1, \quad \theta^*_C = 0.524, \quad \theta^*_D = 1, \quad \theta^*_E = 0.5078, \quad \theta^*_F = 1.$$

So, we have three efficient banks and three inefficient banks. Now, assume that the inefficient bank $C$ would like to take over the inefficient bank $E$. We denote the merged bank by $M$ and assume that in input-orientation it keeps the amount of output of both banks, that is $y_M = y_C + y_E = 215$, and looking to find the minimum amount of the first and second inputs of these banks in order to reach the desired given efficiency target. Suppose, bank $M$ keeps $\alpha_{1C}$ unit(s) of the first input of bank $C$ and $\alpha_{1E}$ from bank $E$. Similarly, we denote $\alpha_{2C} + \alpha_{2E}$ as the amount of the second output of the merged bank $M$. Therefore, the bank $M$ is a new DMU with the following information:

$$M = (\alpha_{1C} + \alpha_{1E}, \quad \alpha_{2C} + \alpha_{2E}, 215).$$

Therefore, the amount of reductions in the first and second inputs will be as follows:

$$(x_{1C} + x_{1E} - \alpha_{1C} - \alpha_{1E}, \quad x_{2C} + x_{2E} - \alpha_{2C} - \alpha_{2E}).$$

To find the optimal reduction, we propose the following input-oriented InvDEA model.

$$\begin{align*}
\min & \quad \alpha_{1C} + \alpha_{1E} + \alpha_{2C} + \alpha_{2E} \\
\text{s.t.} & \quad 19 \lambda_B + 27 \lambda_D + 55 \lambda_F + (\alpha_{1C} + \alpha_{1E}) \lambda_M - (\alpha_{1C} + \alpha_{1E}) \tilde{\theta} \leq 0 \\
& \quad 131 \lambda_B + 168 \lambda_D + 255 \lambda_F + (\alpha_{2C} + \alpha_{2E}) \lambda_M - (\alpha_{2C} + \alpha_{2E}) \tilde{\theta} \leq 0, \\
& \quad 150 \lambda_B + 195 \lambda_D + 230 \lambda_F + (120 + 95) \lambda_M \geq (120 + 95), \\
& \quad \lambda_B + \lambda_D + \lambda_F + \lambda_M = 1, \\
& \quad 0 \leq \alpha_{1C} \leq 60, \quad 0 \leq \alpha_{1E} \leq 58, \\
& \quad 0 \leq \alpha_{2C} \leq 250, \quad 0 \leq \alpha_{2E} \leq 258, \\
& \quad \lambda_B \geq 0, \quad \lambda_D \geq 0, \quad \lambda_F \geq 0, \quad \lambda_M \geq 0,
\end{align*}$$

(1)

where it is been assumed that there is no priority in the reduction of the first and the second inputs. That is, the objective function minimizes $(\alpha_{1C}, \alpha_{1E})$ and $(\alpha_{2C}, \alpha_{2E})$ with no priority. Assume that $M$ aims to obtain a target efficiency of

$$\tilde{\theta} = 0.65 > \max\{\theta^*_C = 0.524, \quad \theta^*_E = 0.5078\}.$$

Note that the merged bank $M$ is inefficient and, therefore, we can take $\lambda^*_M = 0$. This simplifies the non-linear InvDEA model (1) to be linear, and has the following optimal solution:

$$\lambda^*_D = 0.4286, \quad \lambda^*_F = 0.5714, \quad \alpha^*_{1C} = 60, \quad \alpha^*_{1E} = 6.1538, \quad \alpha^*_{2C} = 250, \quad \alpha^*_{2E} = 84.9451.$$

Hence, the merged bank $M$ will reach target $\tilde{\theta} = 0.65$ if and only if it uses the following optimal amount of inputs:

$$(\alpha^*_{1C} + \alpha^*_{1E}, \alpha^*_{2C} + \alpha^*_{2E}) = (66.1538, 334.9451).$$
to produce the output level $y_C + y_E = 215$. Also, if the merged bank $M$ would like to reach the target $\tilde{\theta} = 0.85$, the input-oriented InvDEA model (1) gives the following optimal reductions in two inputs:

$$x_{1C} - \alpha_{1C}^* = 60 - 50.5882 = 9.4118,$$
$$x_{1E} - \alpha_{1E}^* = 58 - 0 = 58,$$
$$x_{2C} - \alpha_{2C}^* = 250 - 250 = 0,$$
$$x_{2E} - \alpha_{2E}^* = 258 - 6.1344 = 251.8656.$$

On the basis of the inverse notion, the above InvDEA model (1) has an interesting interpretation. We have set a target for efficiency of bank $M$, and the model seeks the minimum amount of inputs to reach that target. According to the duality in DEA, this is equivalent to say that we have a DEA weights vector $(\bar{u}, \bar{v}_1, \bar{v}_2)$, obtained from dual DEA model corresponding to the given efficiency target, for example, corresponding to 0.85 (target of the merged bank $M$) and looking for the minimum changes in two inputs of banks $C$ and $E$. In a general inverse optimization problem (Ahuja & Orlin, 2001), a feasible solution, not necessary optimal, is given and we are looking to perturb data as little as possible in a way that the given feasible solution be an optimal solution for the perturbed data. This concept is directly used in Amin & Emrouznejad (2007) by using $(\bar{\lambda}_1, \ldots, \bar{\lambda}_j, \ldots, \bar{\lambda}_n) = (0, \ldots, 1, \ldots, 0)$ as a feasible solution in the standard DEA model in order to check whether the $j$th DMU (for any $j = 1, \ldots, n$) is efficient or not.

As is mentioned in the input-oriented InvDEA model (1), we suppose that there is no priority in keeping (or reduction) different inputs of the merged banks. In the case of priority, the following input-oriented InvDEA model can be used.

$$\min \ w_1\alpha_{1C} + w_2\alpha_{1E} + w_3\alpha_{2C} + w_4\alpha_{2E}$$
$$\text{s.t.} \ 19\lambda_B + 27\lambda_D + 55\lambda_F + (\alpha_{1C} + \alpha_{1E})\lambda_M - (\alpha_{1C} + \alpha_{1E})\tilde{\theta} \leq 0,$$
$$131\lambda_B + 168\lambda_D + 255\lambda_F + (\alpha_{2C} + \alpha_{2E})\lambda_M - (\alpha_{2C} + \alpha_{2E})\tilde{\theta} \leq 0,$$
$$150\lambda_B + 195\lambda_D + 230\lambda_F + (120 + 95)\lambda_M \geq (120 + 95),$$
$$\lambda_B + \lambda_D + \lambda_F + \lambda_M = 1,$$
$$0 \leq \alpha_{1C} \leq 60, \quad 0 \leq \alpha_{1E} \leq 58,$$
$$0 \leq \alpha_{2C} \leq 250, \quad 0 \leq \alpha_{2E} \leq 258,$$
$$\lambda_B \geq 0, \quad \lambda_D \geq 0, \quad \lambda_F \geq 0, \quad \lambda_M \geq 0,$$

where $w_1 + w_2 + w_3 + w_4 = 1$, and the weights can be suggested by experts. This means that the larger weight for an input implies the less priority for keeping it in the merged bank. Clearly, if $w_i = 0.25 (i = 1, 2, 3, 4)$, then the optimal solutions of models (1) and (2) will be the same. Now, consider the output-oriented IDEA model for merging banks $C$ and $E$. The standard VRS output-oriented model has the following optimal values for banks $C$ and $E$:

$$h_c^* = 1.899904, \quad h_E^* = 2.421053.$$

In output orientation, the merged bank $M$ keeps the amount of two inputs, $(x_{1M}, x_{2M}) = (x_{1C} + x_{1E}, x_{2C} + x_{2E}) = (118, 508)$, and a single output, $y_M = y_C + y_E = 215$, and tries to find the maximum amount of additional output, say $\beta$, in order to touch the predetermined target $\bar{h} \leq \min(h_c^*, h_E^*) = 1.899904$. For instance, assume that the merged bank $M$ would like to reach target $\bar{h} = 1.052631579$
(or equivalently to be 0.95 efficient). We propose the following output-oriented InvDEA model.

$$
\text{max } \beta \\
\text{s.t. } 19 \lambda_B + 27 \lambda_D + 55 \lambda_F + 118 \lambda_M \leq 118 \\
131 \lambda_B + 168 \lambda_D + 255 \lambda_F + 508 \lambda_M \leq 508, \\
150 \lambda_B + 195 \lambda_D + 230 \lambda_F + (215 + \beta) \lambda_M \geq (215 + \beta) \bar{h}, \\
\lambda_B + \lambda_D + \lambda_F + \lambda_M = 1, \\
\lambda_B \geq 0, \quad \lambda_D \geq 0, \quad \lambda_F \geq 0, \quad \lambda_M \geq 0.
$$

As the merged bank \( M \) is inefficient, we can simplify the above non-linear InvDEA model by taking \( \lambda_M^* = 0 \), and obtain the following optimal value:

$$
\beta^* = 3.499994.
$$

Note that this relaxation can be used, taking \( \lambda_M = 0 \), even if the merged bank is efficient. Therefore, \( M \) will touch the given target if it produces 3.499994 additional outputs. Despite the input-oriented InvDEA model (1) always being feasible, the output-oriented InvDEA model (3) may become infeasible. The reason for feasibility of the output-orientation InvDEA model (3) has an interesting interpretation. According to the output-oriented InvDEA model (3), the merged bank \( M \) keeps two inputs and one output of both banks \( C \) and \( B \). Therefore, the predetermined target, or \( \bar{h} \), for the efficiency of the merged bank \( M \) should be at least the efficiency of a virtual bank \((x_1C + x_1E, x_2C + x_2E, y_C + y_E) = (118, 508, 215)\); otherwise, the corresponding InvDEA model (3) becomes infeasible. In this case, if the mentioned virtual bank is on the frontier, this means that the merged bank \( M \) will be efficient as well. In the next section, we extend the idea for the input and output-oriented InvDEA models, and also address the feasibility of the output-oriented InvDEA model in the general case with multi-inputs and multi-outputs.

4. Merging using InvDEA: general case

There are three alternatives in practice in consolidation, either both banks remain and constitute a holding, or only one of them remains in the market or both of them disappear, and are replaced by a merging entity with a new name. Moreover, the consolidation can take place for more than two banking units or consolidation between different entities in different markets or from different sectors. For simplicity and without losing generality, we consider the last two alternatives, i.e. the case of consolidation when only one bank remains in the market as well as the case when both banks disappear, and are replaced by a merging entity. Also, the case of consolidation for more than two banks related to the two mentioned alternatives can be extended easily from the proposed InvDEA method in this section. Assume that banks \( k \) and \( l \) are consolidating their activities in the form of M&A. Let us denote the merged bank generated by the consolidation as \( M \).

The general input-oriented InvDEA model has the following form:

$$
\text{min } \sum_{i=1}^{m} (\alpha_{ik} + \alpha_{il}) \\
\text{s.t. } \sum_{j \in F} x_{ij} \lambda_j + (\alpha_{ik} + \alpha_{il}) \lambda_M - (\alpha_{ik} + \alpha_{il}) \bar{\theta} \leq 0, \quad i = 1, \ldots, m.
$$
\[
\sum_{j \in F} y_{ij} \lambda_j + (y_{rk} + y_{rl}) \lambda_M \geq (y_{rk} + y_{rl}), \quad r = 1, \ldots, s,
\]
\[
\sum_{j \in F} \lambda_j + \lambda_M = 1,
\]
(4)

where \( \bar{\theta} \) is a predetermined target for efficiency of the merged bank \( M \). Also, \( F \) denotes the set of existing banks in the evaluation process of the merged bank \( M \). Therefore, according to the mentioned consolidation alternatives, \( F \) can take the following forms:

(i) \( F = \{i : 1 \leq i \leq n, i \neq k, l\} \).

(ii) \( F = \{i : 1 \leq i \leq n, i \neq l\} \).

The first case shows a consolidation when both banks \( k \) and \( l \) disappear, and in the second form only bank \( k \) remains in the market.

In the real world, the most common consolidations happen between banking units to improve their respective performances and, in general, this naturally implies improving their technical efficiencies. Now, we show that the non-linear InvDEA model (4) can be simplified to a linear-programming form. Clearly, if \( \bar{\theta} < 1 \), which is the merged bank \( M \) is inefficient, then the corresponding \( \lambda_M \) will be zero in optimality (\( \lambda_M^* = 0 \)), and this simplifies the non-linear input-oriented InvDEA model (4) to the following linear form:

\[
\text{min } \sum_{i=1}^{m} (\alpha_{ik} + \alpha_{il})
\]
\[
\text{s.t. } \sum_{j \in F} x_{ij} \lambda_j - (\alpha_{ik} + \alpha_{il}) \bar{\theta} \leq 0, \quad i = 1, \ldots, m
\]
\[
\sum_{j \in F} y_{ij} \lambda_j \geq (y_{rk} + y_{rl}), \quad r = 1, \ldots, s,
\]
\[
\sum_{j \in F} \lambda_j = 1,
\]
\[
0 \leq \alpha_{ik} \leq x_{ik}, \quad i = 1, \ldots, m,
\]
\[
0 \leq \alpha_{il} \leq x_{il}, \quad i = 1, \ldots, m,
\]
\[
\lambda_j \geq 0, \quad j \in F.
\]
(5)

Also, if the merged bank \( M \) is efficient, or equivalently \( \bar{\theta} = 1 \), but it is still inside of the production possibility set (PPS), it can be presented in terms of the other efficient bank(s), and therefore in this case, we can still suppose that \( \lambda_M = 0 \) in optimality, and therefore the non-linear InvDEA model (3) will be simplified to the same LP model (5). Note that in this case, the nonlinear programming (NLP) model (3) has alternative optimal solutions and considering \( \lambda_M = 0 \) means ignoring only one optimal
solution where $\lambda^*_{M} = 1$. In this paper, we limit our development to the case of the consolidation where the merged bank $M$ is within the current PPS. Clearly, the merged bank $M$ will be inside of the current PPS, if and only if the virtual bank $(x_k + x_l, y_k + y_l)$ is within the PPS. This comes from the objective of the NLP input-oriented InvDEA model (4) as well as the objective of the relaxed input-oriented InvDEA model (5), where it tries to keep the minimum level of the inputs of banks $k$ and $l$ or equivalently the virtual bank. This assumption guarantees that the merged bank $M$ is inside the PPS when it is inefficient or on the frontier once it is efficient. Therefore, if we relax the NLP InvDEA model (4) to the LP InvDEA model (5), we will not lose generality. Now, we show that the input-oriented InvDEA model (5) is feasible.

**Theorem 1** Model (5) is feasible.

**Proof.** Consider the dual of model (5) as follows:

$$\begin{align*}
\max & \quad \sum_{r=1}^{s} (y_{rk} + y_{rl}) u_r - \sum_{i=1}^{m} (x_{ik} p_i + x_{il} q_i) \\
\text{s.t.} & \quad - \sum_{i=1}^{m} x_{ij} v_i + \sum_{r=1}^{s} y_{jr} u_r \leq 0, \quad j \in F \\
& \quad \tilde{\theta} v_i - p_i \leq 1, \quad i = 1, \ldots, m, \\
& \quad \tilde{\theta} v_i - q_i \leq 1, \quad i = 1, \ldots, m, \\
& \quad p_i \geq 0, \quad q_i \geq 0, \quad i = 1, \ldots, m, \\
& \quad u_r \geq 0, \quad v_i \geq 0, \quad r = 1, \ldots, s, \quad i = 1, \ldots, m.
\end{align*}$$

Note that the primal model (5) is bounded and the above dual formulation is feasible. This is achieved by taking all variables equal to zero. So, we conclude that the input-oriented InvDEA model (5) is feasible. \hfill \square

Now, consider the following output-oriented InvDEA model in the general form.

$$\begin{align*}
\max & \quad \sum_{r=1}^{s} \beta_r \\
\text{s.t.} & \quad \sum_{j \in F} x_{ij} \lambda_j + (x_{ik} + x_{il}) \lambda_M \leq (x_{ik} + x_{il}), \quad i = 1, \ldots, m \\
& \quad \sum_{j \in F} y_{jr} \lambda_j + (y_{rk} + y_{rl} + \beta_r) \lambda_M - (y_{rk} + y_{rl} + \beta_r) \bar{h} \geq 0, \quad r = 1, \ldots, s, \\
& \quad \sum_{j \in E} \lambda_j + \lambda_M = 1, \\
& \quad \beta_r \geq 0, \quad r = 1, \ldots, s, \\
& \quad \lambda_j \geq 0, \quad j \in F, \quad \lambda_M \geq 0.
\end{align*}$$
Similar to the above discussion, this model can also be simplified to the following LP by assuming the merged bank \( M \) is within the PPS.

\[
\begin{align*}
\max \quad & \sum_{r=1}^{s} \beta_r \\
\text{s.t.} \quad & \sum_{j \in E} x_{ij} \lambda_j \leq (x_{ik} + x_{il}), \quad i = 1, \ldots, m \\
& \sum_{j \in E} y_{ij} \lambda_j - (y_{rk} + y_{rl} + \beta_r) \bar{h} \geq 0, \quad r = 1, \ldots, s \\
& \sum_{j \in E} \lambda_j = 1, \\
& \beta_r \geq 0, \quad r = 1, \ldots, s, \\
& \lambda_j \geq 0, \quad j \in E.
\end{align*}
\] (7)

The feasibility of the output-oriented InvDEA model (7) is shown in the following theorem.

**Theorem 2** Model (7) is feasible if and only if \( \bar{h} \leq h^* \), where \( h^* \) is the optimal value of the following model:

\[
\begin{align*}
\max \quad & h \\
\text{s.t.} \quad & \sum_{j \in F} x_{ij} \lambda_j + (x_{ik} + x_{il}) \lambda_{n+1} \leq (x_{ik} + x_{il}), \quad i = 1, \ldots, m \\
& \sum_{j \in F} y_{ij} \lambda_j + (y_{rk} + y_{rl}) \lambda_{n+1} \geq (y_{rk} + y_{rl}) h, \quad r = 1, \ldots, s, \\
& \sum_{j \in E} \lambda_j + \lambda_{n+1} = 1, \\
& \lambda_j \geq 0, \quad j \in F, \quad \lambda_{n+1} \geq 0.
\end{align*}
\]

**Proof.** According to the output-oriented InvDEA model (7), it is clear that the efficiency score of the merged bank \( M \) should be at least equal to the efficiency of a virtual bank, say \( n+1 \), with \( (x_{ik} + x_{il}) \) as the \( i \)th input \( (i = 1, \ldots, m) \) and \( (y_{rk} + y_{rl}) \) as the \( r \)th output \( (r = 1, \ldots, s) \). This arises from the assumption where bank \( M \) keeps the amount of inputs and outputs of both the banks \( k \) and \( l \). This completes the proof. \( \square \)

Note that we can extend the proposed InvDEA method in this paper to merge a series of banks, i.e., more than two banks, e.g. for three banks \( P, Q \) and \( R \), it is enough to solve the following input-oriented InvDEA model.

\[
\begin{align*}
\min \quad & \sum_{i=1}^{m} (\alpha_{ip} + \alpha_{iq} + \alpha_{ir}) \\
\text{s.t.} \quad & \sum_{j \in F} x_{ij} \lambda_j - (\alpha_{ip} + \alpha_{iq} + \alpha_{ir}), \quad \tilde{\theta} \leq 0, \quad i = 1, \ldots, m
\end{align*}
\]
\[ \sum_{j \in F} y_{ij} \lambda_j \geq (y_{irP} + y_{irQ} + y_{irR}), \quad r = 1, \ldots, s, \]
\[ \sum_{j \in F} \lambda_j = 1, \]
\[ 0 \leq \alpha_{ip} \leq x_{ir}, \quad i = 1, \ldots, m, \]
\[ 0 \leq \alpha_{iQ} \leq x_{iQ}, \quad i = 1, \ldots, m, \]
\[ 0 \leq \alpha_{iR} \leq x_{iR}, \quad i = 1, \ldots, m, \]
\[ \lambda_j \geq 0, \quad j \in F, \]

where \( F \) can be extended from the definition given in model (5). This extension can be defined for more than three banks as well as for output orientation easily. It is also clear that the proposed model in this paper is completely different from that illustrated by Bogetoft & Wang (2005). We use the concept of InvDEA, hence, we preset the level of efficiency first and then find the level of inputs and outputs that is feasible to reach the preset efficiency level. However, Bogetoft & Wang (2005) proposed to extend the PPS by adding a new merged bank, which is simply the sum of the current banks, and then calculate the efficiency score of the new merged bank using the conventional DEA in new PPS. Being completely two different ideas, the results are not really comparable, one focuses on the efficiency and find the data point while the other method focuses on the data and finds the efficiency score.

5. An application: merging GCC banks

In this section, the InvDEA approach explained so far is exemplified through a real-world data set, namely GCC commercial banks financial data obtained from BANKSCOPE database.\(^1\) The study was meant to analyse the efficiency of the GCC conventional commercial banking system over a period of 5 years, to assess and track the impact of its recent consolidations on the performance of the banking units, and to identify regional benchmarks for the sector. The study used the relative technical efficiency determined by adopting the DEA methodology as an indicator of performance.

For the purpose of illustration, we use the data for 2006 only that are collected from BANKSCOPE, a public database providing financial reports about the banks around the world. The classification by country of the banks included in the study is provided in Table 2.

The intermediation approach, suggested by Berger & Humphrey (1997), is adopted for this study. Since the banking sector in GCC countries, as described in Hussain et al. (2002), is traditional in its form, the intermediation approach, claiming that banks are collecting funds and providing loans, is judged to be the most convenient for this study.

The two inputs considered for the analysis in this study are interest expenses \((X_1)\) and non-interest expenses \((X_2)\). Interest expenses include expenses for deposits and other borrowed funds while non-interest expenses represent the costs of converting deposits into loans, including service charges, commissions, expenses of general management affairs, salaries and other expenses. These inputs represent the costs of labour, administration, equipment and funds for operations, loans and for investment.

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\(^1\) BANKSCOPE is a comprehensive global database containing information on public and private banks. It includes the information on major banks around the world. For further details, see www.bankscope.bvdep.com.
Table 2  Number of commercial banks included in the sample per country

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bahrain</td>
<td>4</td>
</tr>
<tr>
<td>Kuwait</td>
<td>5</td>
</tr>
<tr>
<td>Oman</td>
<td>5</td>
</tr>
<tr>
<td>Qatar</td>
<td>5</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>9</td>
</tr>
<tr>
<td>United Arab Emirates</td>
<td>14</td>
</tr>
<tr>
<td>Total number of banks</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 3  Merging banks B002 and B003: input-orientation

<table>
<thead>
<tr>
<th>Target (θ̂)</th>
<th>α_{12}^*</th>
<th>α_{22}^*</th>
<th>α_{13}^*</th>
<th>α_{23}^*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>481.2388</td>
<td>319.9765</td>
<td>264.5743</td>
<td>138.6</td>
</tr>
<tr>
<td>0.75</td>
<td>347.9015</td>
<td>319.9764807</td>
<td>305.2</td>
<td>138.6</td>
</tr>
<tr>
<td>0.8</td>
<td>481.2388</td>
<td>319.9765</td>
<td>90.74</td>
<td>138.6</td>
</tr>
<tr>
<td>0.9</td>
<td>436.7745</td>
<td>319.9765</td>
<td>0</td>
<td>138.6</td>
</tr>
<tr>
<td>1</td>
<td>371.27</td>
<td>319.98</td>
<td>0</td>
<td>108.26</td>
</tr>
</tbody>
</table>

The two outputs considered for the analysis are interest income (Y₁) and non-interest income (Y₂). The interest income includes interest on loans and income from the government securities. The non-interest income includes service charges on loans and transactions, commissions and other operating income. These outputs represent bank revenues and the major profit generated by the banking service. Interest expenses can be seen as a proxy for deposits and interest income as a proxy for Loans. This makes the model in line with the intermediation approach traditionally using deposits, interest expenses, and non-interest expenses as inputs and loans, interest income and non-interest income as outputs (Yildirim, 2002; Avkiran, 2004; Kao & Liu, 2004).

In the following sections, we apply the InvDEA, input-oriented model (5) as well as the output-oriented InvDEA model (7), to come with some suggestion at the optimal policy for each target level of efficiency. First, consider the VRS efficiency scores of 42 banks for the year 2006, shown in Appendix 1.

According to the discussion in the general case, we can merge banks k and l if and only if the virtual bank (x_k + x_l, y_k + y_l) is within the PPS, regardless of any position of these banks. Assume that the bank B002 would like to consolidate with bank B003 to create bank M, that is, k = 2, and l = 3. The input-oriented InvDEA model (5) provides the following table by assuming different target levels of efficiency for the new banking unit M. Table 3 gives the minimum amount of inputs from each banks B002 and B003 that should be kept in order to reach the predetermined target as shown in the first column.

In the first row of Table 3, it is assumed that the merged bank M would like to reach the efficiency target θ̂ = 0.7. Using the input-oriented InvDEA model (5), we can determine the minimum amount of two inputs that should be included, or the maximum amount of inputs that should be dropped from both banks B002 and B003. The first row in Table 3 shows that this can be achieved if we keep the following amount of inputs.

\[(α_{12}^*, α_{22}^*) = (481.2388, 319.9765).\]
Table 4  Merging banks B002 and B003: output-orientation

<table>
<thead>
<tr>
<th>Target ($\bar{h}$)</th>
<th>$\beta_1^*$</th>
<th>$\beta_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.42857</td>
<td>0</td>
<td>214.1798</td>
</tr>
<tr>
<td>1.3</td>
<td>0</td>
<td>1299.769</td>
</tr>
<tr>
<td>1.25</td>
<td>36.4605</td>
<td>1437.601</td>
</tr>
<tr>
<td>1.1765</td>
<td>129.6556</td>
<td>1580.511</td>
</tr>
<tr>
<td>1.1111</td>
<td>222.8666</td>
<td>1723.568</td>
</tr>
<tr>
<td>1</td>
<td>409.24</td>
<td>2009.48</td>
</tr>
</tbody>
</table>

According to the inputs of Bank002, we see that

$$x_{12} - \alpha_{12}^* = x_{22} - \alpha_{22}^* = 0.$$ 

This means that the merged bank $M$ will keep the entire amount of interest and non-interest expenses of bank B002. Also,

$$(\alpha_{13}^*, \alpha_{23}^*) = (264.5743, 138.6),$$

that is,

$$x_{13} - \alpha_{13}^* = 305.2 - 264.5743 = 40.6257,$$

$$x_{23} - \alpha_{23}^* = 0.$$ 

So, the merged bank $M$ will reach target $\bar{\theta} = 0.7$ if we cut the amount of 40.6257 from the interest expense of bank B003. Furthermore, the reference set of the merged bank $M$ is denoted in the last column of Table 3. The same interpretation is true for the other rows of the table. The second row shows that if the merged bank would like to reach target $\bar{\theta} = 0.75$, it only needs to cut the following amount of the interest expense from B002.

$$x_{12} - \alpha_{12}^* = 481.2388 - 347.9015 = 133.3373.$$ 

According to the second row, the merged bank $M$ should keep the other original input values. Now, consider the last row of Table 3 where the merged bank $M$ would like to be efficient, $\bar{\theta} = 1$. The optimal solution of the input-oriented InvDEA model gives the minimum amount of the interest and non-interest expenses of banks B002 and B003 that should be preserved by bank $M$. On the basis of the solution, we see that 481.24 $-$ 371.27 $=$ 109.97 and 319.98 $-$ 319.98 $=$ 0 are the amount of redundant interest, and non-interest expenses that should be ignored from bank $B$. Also, $\alpha_{13}^* = 0$ means that the merged bank $M$ will not use the available amount of interest expense of bank B003. Now, we use the output-oriented InvDEA model (7) for merging banks B002 and B003. First, note that the scores of banks B002 and B003 using the standard output-oriented VRS DEA model are as follows:

$$h_2^* = 1.4319, \quad h_3^* = 1.5601.$$ 

Table 4 provides part of the optimal solutions of the output-oriented InvDEA model (4) for different targets and improvement of the merged bank $M$.

For instance, the first row shows that the merged bank $M$ will reach the target of $\bar{h} = 1.42857$ (70% efficient) if it keeps the inputs and outputs of both banks B002 and B003, and be able to produce
214.1798 additional non-interest income. In this case, \( M \) will be presented in terms of efficient banks B001, B020, B031 and B039. Furthermore, according to the optimal solution of the output-oriented InvDEA model (7) shown in the last row of Table 4, the merged bank \( M \) will be efficient if it produces 409.24 and 2009.48 additional interest and non-interest income, respectively. This means that the merged bank \( M \) will be efficient if it has the following data, inputs and outputs.

\[
M = (x_{12} + x_{13}, x_{22} + x_{23}, y_{12} + y_{13} + \beta_1^*, y_{22} + y_{23} + \beta_2^*)
\]

\[
= (786.44, 458.58, 1863.9, 2859.4)
\]

6. Concluding remarks

Despite the wide applications of DEA models in banking, there is no single application of InvDEA models for merging banks, which was the aim of this study. It is shown that the merged bank \( M \) can reach a predetermined target, until being efficient, both in input and output orientations, by reducing some unused input(s) or by producing some additional output(s). Further, we have shown the applicability of the proposed model by investigating the InvDEA method for merging banks using a real data set of 42 GCC banks. The outcomes show that the merged bank \( M \) can reach any pre-defined target level, even efficient, if the corresponding input-oriented (or output-oriented) InvDEA model, proposed in this paper, is solved. Also we focused only on VRS, the proposed model can easily be extended to other returns to scale models. Further research is required to investigate the NLP InvDEA model in details, especially the potential use of this method for merging efficient banks.

REFERENCES


Appendix 1: GCC banks data and efficiency scores under the VRS assumption for the 2006

<table>
<thead>
<tr>
<th>Bank</th>
<th>Interest expenses</th>
<th>Non-interest expenses</th>
<th>Interest incomes</th>
<th>Non-interest incomes</th>
<th>Technical efficiency scores under VRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>B001</td>
<td>3956.796054</td>
<td>1894.4259</td>
<td>9001.0036</td>
<td>8701.496886</td>
<td>1</td>
</tr>
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<td>B002</td>
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<td>974.8543974</td>
<td>597.7262586</td>
<td>0.674</td>
</tr>
<tr>
<td>B003</td>
<td>305.2</td>
<td>138.6</td>
<td>479.8</td>
<td>252.2</td>
<td>0.64</td>
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<td>B004</td>
<td>4710.680232</td>
<td>3996.258941</td>
<td>12920.33718</td>
<td>6060.767712</td>
<td>0.8925</td>
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<td>B005</td>
<td>1.0179</td>
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<td>3.0537</td>
<td>0.377</td>
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<td>954.4368435</td>
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<td>1991.004009</td>
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<td>B009</td>
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