OCTOBER 09 2009

A Bayesian approach to modal decomposition in ocean acoustics

Zoi-Heleni Michalopoulou

J. Acoust. Soc. Am. 126, EL147–EL152 (2009)
https://doi.org/10.1121/1.3244037

A Bayesian approach to modal decomposition in ocean acoustics

Zoi-Heleni Michalopoulou
Department of Mathematical Sciences, New Jersey Institute of Technology, Newark, New Jersey 07102
michalop@njit.edu

Abstract: A Bayesian approach is developed for modal decomposition from time-frequency representations of broadband acoustic signals propagating in underwater media. The goal is to obtain accurate estimates and posterior probability distributions of modal frequencies arriving at a specific time and their corresponding amplitudes, which can be employed for geoacoustic inversion. The proposed approach, optimized via Gibbs sampling, provides uncertainty information on modal characteristics via the posterior distributions, typically unavailable from traditional methods.

PACS numbers: 43.60.Hj, 43.30.Pc, 43.60.Jn [JC]
Date Received: July 23, 2009 **Date Accepted:** September 9, 2009

1. Introduction

The evolution of the frequency content of an acoustic signal with time often acts as a fingerprint of the propagation medium, particularly with broadband signals with frequencies of a few hundreds of hertz propagating long distances in underwater environments.[1] Intra- and inter-modal dispersions reveal significant information on properties of the waveguide. Dispersion is captured in arrival time differences between distinct frequencies within the same mode or across a number of propagating modes. Estimates of such time differences and also modal amplitudes can be employed in conjunction with optimization for source localization and environmental parameter estimation;[2] dispersion estimation for inversion in underwater acoustics has been typically pursued with simple short time Fourier transforms (STFTs) and wavelet analysis.[3–5]

In Ref. 6, it was demonstrated that, although arrival time-frequency estimation is limited in terms of resolution when Fourier transforms are used, more reliable time-frequency pairs can be extracted via the stationary phase approximation. It was shown that the spectrum of one mode in the received acoustic signal calculated with a STFT is a squared, shifted, and scaled sinc function centered at the modal frequency arriving at that particular time. Accurately estimating the center of the sinc pulse with a fitting process provides a modal frequency-time pair that can be used for successful geoacoustic inversion with limited uncertainty.[6] Estimation of the scaling of the squared sinc provides information on the modal amplitude, linked to attenuation in the propagation medium. Extending this approach, we can formulate a superposition of similar functions to represent multiple modes arriving simultaneously. In Ref. 7, we followed a similar concept, empirically fitting a sum of Gaussian pulses to a STFT frame, forming the observation equation of a particle filter.

In this work we improve on the modal frequency and amplitude estimation of Zorych and Michalopoulou[7] by employing smoothed Wigner-Ville distributions (WVDs) and an approximation similar to the one proposed in Ref. 6. Working in a Bayesian framework for dispersion estimation,[8] we calculate posterior probability distributions of the unknown modal frequencies and amplitudes and optimize the process with a Gibbs sampler in a manner that is analogous to the time-delay estimation approach of Michalopoulou and Picarelli.[9] This paper is organized as follows: Sec. 2 briefly presents the sinc approximation for modal Wigner-Ville time-frequency representations. Section 3 discusses the statistical model employed in this work for spectral observations and describes the Bayesian modal decomposition approach using the introduced models and a Gibbs sampler. Section 4 presents results of frequency and amplitude estimation of distinct modes. A summary and conclusions follow in Sec. 5.

2. Approximating the acoustic signal in time-frequency

The WVD is selected for calculating the time-frequency content of acoustic signals because it does not have the resolution limitations of Fourier transforms and can accurately represent dispersion at any pair $(\omega, t) = (2\pi f, t)$.[10] Such a time-frequency representation of an analytic signal $s_{a,n}$ is obtained as[10] $WVD(\omega,t) = 1/2\pi \int_{-\infty}^{\infty} s_{a,n}(t-\tau/2)^* s_{a,n}(t+\tau/2)\exp(-i\tau\omega)d\tau$, where * indicates conjugate. In our application, employing a normal mode approach to model sound propagation in the ocean, we receive acoustic signal $p(r,z,z_0,t)$, where

$$p(r,z,z_0,t) = \sum_n s_{a,n}(r,z,z_0,t) = \frac{1}{2\pi}\sum_n \int_{-\infty}^{\infty} \mu(\omega) G_n(r,z,z_0,\omega)\exp\left(i\left(\omega t - k_n r - \frac{\pi}{4}\right)\right) d\omega. \quad (1)$$

Quantity r is the distance between source and receiver, z and z_0 are the source and receiver depths, respectively, k_n is the modal wavenumber, μ is the source spectrum, $\omega = 2\pi f$ is frequency, and $G_n(r,z,z_0,\omega) = (\beta/\sqrt{k_n r})\Psi_n(z)\Psi_n(z_0)$, where Ψ_n are orthogonal, depth-dependent functions that are appropriately normalized and β is a constant (the waveguide is assumed to have a constant density).

Since we have multiple modes in our received acoustic field, the Wigner-Ville representation will include in the integrand a signal that is formed by the superposition of several modal contributions $s_{a,n}: s_a = p(r,z,z_0,t) = \Sigma_n s_{a,n}$. The time-frequency distribution will then have numerous cross-factors that will make its interpretation very difficult.[10] To avoid this complication, we use a smoothed, pseudo-Wigner-Ville Distribution (SPWVD), applying windows of finite length to multimodal signals s_a, smoothing away cross-terms. Assuming a window of length $2\Delta\tau$ and employing the stationary phase approximation,[11,6] the SPWVD of mode n in our signal becomes

$$\mathrm{SPWVD}_n(\omega,t) \approx \alpha_n \frac{\sin(\omega - \omega_n)\Delta\tau}{\omega - \omega_n}, \quad (2)$$

where α_n contains all multiplicative constants. Optimal window lengths $2\Delta\tau$ depend on the actual modal dispersion and separation (they, here, are empirically selected); specific window types (Kaiser, Hamming, and Blackman, for example) can be incorporated in the SPWVD.

The SPWVD of an acoustic signal can then be written as a superposition of weighted and shifted sinc functions, with a_n containing all multiplicative constants:

$$\mathrm{SPWVD}(\omega,t) \approx \sum_n \alpha_n \,\mathrm{sinc}((\omega - \omega_n)\Delta\tau). \quad (3)$$

3. The statistical model

The conventional assumption is that the time domain acoustic signal is distorted by additive, complex, zero-mean Gaussian noise for each channel (real and imaginary). Frequency representations, involving squared signal components, are typically then related to χ^2 distributions. The integral engaged in the calculation of WVDs (and SPWVDs) implies a summation of several χ^2 random variables. Consequently, following Ref. 7, a Gaussian model is used to describe random fluctuations in our observations. Thus, the misfit between our calculated SPWVDs and our parametric model is Gaussian distributed with zero mean and a variance of σ^2.

Given the sinc expression, the M elements of data vectors $\mathbf{W}$ can be written as

$$W_i = \sum_{n=1}^{N} \alpha_n \,\mathrm{sinc}((\Omega_i - \omega_n)\Delta\tau) + U_i, \quad (4)$$

where $\mathbf{U} = (U_1, U_2, \ldots, U_M)$ is a white Gaussian noise vector with $\mathbf{0}$ mean and variance σ^2, N is the number of considered modes, and M is the number of frequencies Ω_i at which we sample the SPWVD.

Once $\mathbf{W}$ is observed, Eq. (5) provides the likelihood function for unknown modal amplitudes, frequencies, and variance σ^2 $[\mathbf{\Omega}=(\Omega_1,\ldots,\Omega_M)]$:

$$l(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2|\mathbf{W}) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^M \exp\left(-\frac{1}{2\sigma^2}\left\|\mathbf{W}-\sum_{n=1}^{N}\alpha_n\,\mathrm{sinc}((\mathbf{\Omega}-\omega_n)\Delta\tau)\right\|^2\right). \tag{5}$$

Working in a Bayesian framework,[12,13] after establishing a statistical model for the time-frequency representations [Eq. (5)], we formulate the posterior probability distribution of the unknown parameters α_n,ω_n, $n=1,\ldots,N$, and σ^2. Bayes theorem allows us to express the posterior distribution as $p(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2|\mathbf{W})=Kl(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2|\mathbf{W})\times p(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2)$, where $p(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2)$ is the joint prior distribution on the unknown parameters and K is a normalizing constant such that $p(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2|\mathbf{W})$ integrates to 1.

Imposing no *a priori* correlation structure between unknowns, the joint prior distribution can be written as $p(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2)=p(\alpha_1)\cdots p(\alpha_N)p(\omega_1)\cdots p(\omega_N)p(\sigma^2)$, where the factors are the prior distributions for the individual variables. We select uniform prior distributions within chosen intervals (positive for the amplitudes and between 400π and 1200π for ω_n, because of the frequency content of our synthetic signals). The prior distribution for σ^2 is $1/\sigma^2$, which is the conventional non-informative prior for variance.[14]

To make the implementation of the Bayesian processor practical in the presence of multiple modes, we build a Gibbs sampler[15,16] for the estimation of $p(\alpha_1,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2|\mathbf{W})$. Gibbs sampling is a Markov chain Monte Carlo approach that estimates the full joint posterior distribution of all unknown parameters given a set of observations, a statistical model describing uncertainty in the data, and prior distributions on the unknowns. This posterior distribution can be maximized in a straightforward manner for the calculation of maximum *a posteriori* (MAP) estimates.

Gibbs sampling is an iterative approach that starts with a set of randomly selected initial values for the unknown parameters. Sequentially, samples are drawn from the posterior distributions of each parameter conditional on all other parameters. For our purposes, we need to identify the $2N+1$ conditional marginal posterior distributions for α_n, ω_n, $n=1,\ldots,N$, and σ^2.

Focusing on parameter α_1 as an example, its posterior distribution conditional on all other parameters is found to be a Gaussian distribution with variance σ^2 and mean $(\Sigma_{i=1}^{M}W_i\,\mathrm{sinc}((\Omega_i-\omega_1)\Delta\tau)-\Sigma_{j=2}^{N}\alpha_j\,\mathrm{sinc}((\Omega_i-\omega_1)\Delta\tau)\mathrm{sinc}((\Omega_i-\omega_j)\Delta\tau))$,[9] from which we can expeditiously draw samples. Conditional distributions for $\alpha_2,\ldots,\alpha_N$ are similarly obtained. The conditional distribution of σ^2 is simply expressible in a closed form as well and is identified as an inverse χ^2 distribution.[14,9] The distributions for ω_n cannot be expressed in a closed form and are evaluated on a grid.[9]

Gibbs sampling starts with a set of randomly chosen initial conditions for all unknown parameters $(\alpha_1,\alpha_2,\ldots,\alpha_N,\omega_1,\ldots,\omega_N,\sigma^2)$. The process as implemented here first draws a sample from the conditional distribution of α_1 given the initial values for all other parameters. Subsequently, a sample is drawn from the marginal conditional posterior of α_2 given initial values for the other parameters and the updated value for α_1, obtained during the previous step. The process is repeated for remaining α_n and ω_n and σ^2 as well. For a large number of iterations, the obtained sample sequence converges to the true joint posterior distribution. We can use this sequence to calculate marginal distributions and moments.

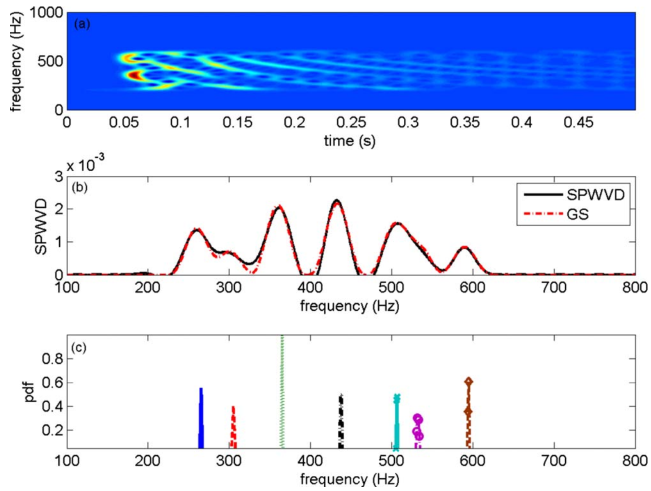

Fig. 1. (Color online) (a) The SPWVD of the acoustic signal. (b) A "slice" from the SPWVD representation of (a) (solid line); the best fit calculated via the Gibbs sampler is superimposed (dot-dashed line). (c) The marginal posterior probability distributions for $f_1, \ldots, f_7$.

4. Gibbs sampling results in modal decomposition

To first provide an example for the SPWVD of an acoustic signal, we have generated a synthetic acoustic reception in a shallow water environment similar to that for the Gulf of Mexico experiment.[17] The simulated source had a uniform, flat spectrum with frequency content between 200 and 600 Hz; the sampling rate was 2000 Hz, suitable for preventing aliasing in the WVD calculations for an analytic signal. The signal was calculated at a distance of 20 km from the source. Receiver and source depths and the water column sound speed profile are provided in Ref. 17; the seafloor sediment was a thin layer of sand over limestone. Figure 1(a) shows the SPWVD of the received signal, which was calculated with Kraken[18] and Fourier synthesis. A "slice" of the distribution at a selected time t is demonstrated in Fig. 1(b) (black, solid line).

The Gibbs sampler developed here was applied to the slice of the SPWVD (obtained by selecting a specific time t) of Fig. 1; the MAP estimate calculated with the method proposed in this work is superimposed in Fig. 1(b) (dot-dashed line) on the actual SPWVD representation at the selected time, indicating an excellent fit between the data and the parametric model involving sinc pulses. Figure 1(c) shows the marginal posterior probability distributions for seven modal frequencies.

The same technique was applied to a SPWVD of a signal with a low signal-to-noise ratio (SNR); Figure 2(a) demonstrates the MAP estimate of the SPWVD superimposed on the noisy SPWVD. The match is still excellent despite the noise level, demonstrating the potential of the proposed approach in parametrizing received acoustic time series in a manner that will enable geoacoustic inversion even for a low SNR. Figure 2(b) shows marginal posterior distributions for modal frequencies $f_1, \ldots, f_7$ as extracted from the noisy time-frequency representation. Figure 2(c) shows the joint posterior distribution for α_1 and α_2; amplitude information can be extracted in this manner to be subsequently employed in attenuation estimation.

Figure 3 illustrates real data results from application of the method to data from the Gulf of Mexico experiment (the frequency content was between 100 and 600 Hz). Figure 3(a) shows the SPWVD slice and the best fit. Data and parametric fit match very closely. Figure 3(b) presents the modal frequency distributions, indicating that the first "peak" in the selected SPWVD slice corresponds to two modes that are close to each other. Figure 3(c) shows the joint posterior distribution for α_3 and α_5. Although probability is mostly concentrated around 0.55

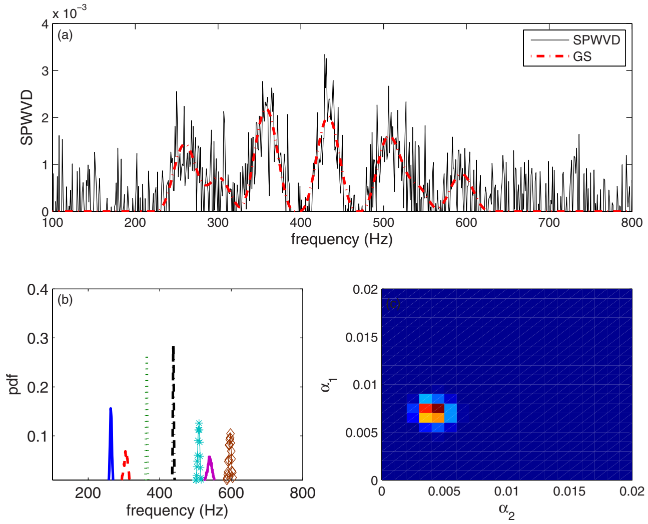

Fig. 2. (Color online) (a) A "slice" of a noisy SPWVD representation (solid line); the best fit calculated via the Gibbs sampler is superimposed (dot-dashed line). (b) The marginal posterior probability distributions for $f_1, \ldots, f_7$. (c) The joint marginal distribution for α_1 and α_2.

and 0.09 for the two amplitudes, secondary distribution peaks appear at 0.68 and 0.24, demonstrating uncertainty that would not have been captured with conventional amplitude and frequency extraction methods.

Results are conditional on the considered number of modes present in the data. An analysis is performed including the number of modal arrivals as an unknown in addition to modal amplitudes, frequencies, and noise variance. The presented results correspond to the number of arrivals for which we calculate the highest posterior probability.

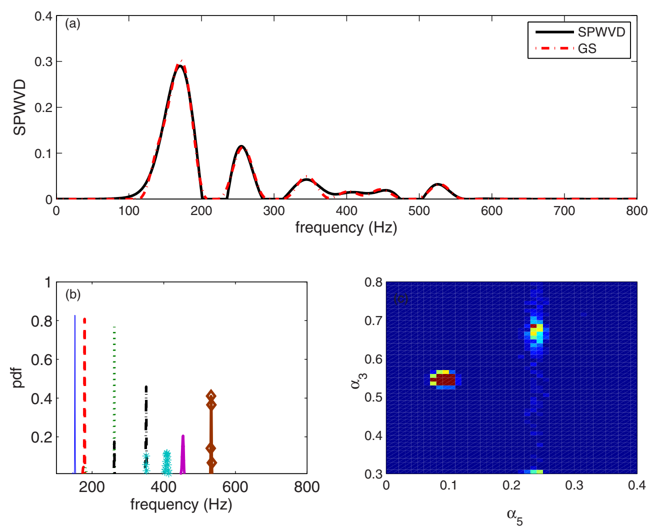

Fig. 3. (Color online) (a) A "slice" from the SPWVD representation of an acoustic signal from the Gulf of Mexico experiment (solid line); the best fit calculated via the Gibbs sampler is superimposed (dot-dashed). (b) The marginal posterior probability distributions for $f_1, \ldots, f_7$. (c) The joint marginal distribution for α_3 and α_5.

5. Conclusions

A new approach is proposed for decomposition of a multimodal signal into components revealing time-frequency and amplitude information for distinct modes. The approach is optimized with a Gibbs sampler that estimates posterior distributions of center frequencies and corresponding amplitudes arriving at a specific time *t*, and allows quantification of uncertainty in the determination of modal characteristics which can be subsequently employed in geoacoustic inversion interpretation. Our method was applied successfully to synthetic and real data collected during the Gulf of Mexico experiment.

Acknowledgments

This work was supported by the Office of Naval Research through Grant No. N000140510262. The author is grateful to Dr. Jack Ianniello for making the Gulf of Mexico data available.

References and links

[1]G. Okopal, P. J. Loughlin, and L. Cohen, "Dispersion-invariant features for classification," J. Acoust. Soc. Am. **123**, 832–841 (2008).
[2]S. D. Rajan, J. F. Lynch, and G. V. Frisk, "Perturbative inversion methods for obtaining bottom geoacoustic parameters in shallow water," J. Acoust. Soc. Am. **82**, 998–1017 (1987).
[3]C.-S. Chen, J. Miller, F. Boudreaux-Bartels, G. Potty, and C. Lazauski, "Time-frequency representations for wideband acoustic signals in shallow water," in *Proceedings of Oceans 2003* (2003), pp. 2903–2907.
[4]M. Taroudakis and G. Tzagkarakis, "On the use of the reassigned wavelet transform for mode identification," J. Comput. Acoust. **12**, 175–196 (2004).
[5]G. Potty, J. Miller, P. Dahl, and C. Lazauski, "Geoacoustic inversion results from the ASIAEX East China Sea Experiment," IEEE J. Ocean. Eng. **29**, 1000–1010 (2004).
[6]T. C. Yang, "Dispersion and ranging of transient signals in the Arctic Ocean," J. Acoust. Soc. Am. **76**, 262–273 (1984).
[7]I. Zorych and Z.-H. Michalopoulou, "Particle filtering for dispersion curve tracking in ocean acoustics," J. Acoust. Soc. Am. **124**, EL45–EL50 (2008).
[8]J. Candy and D. Chambers, "Internal wave signal processing: A model-based approach," IEEE J. Ocean. Eng. **21**, 37–52 (1996).
[9]Z.-H. Michalopoulou and M. Picarelli, "Gibbs sampling for time-delay and amplitude estimation in underwater acoustics," J. Acoust. Soc. Am. **117**, 799–808 (2005).
[10]L. Cohen, *Time Frequency Analysis: Theory and Applications* (Prentice-Hall, Englewood Cliffs, NJ, 1994).
[11]R. P. Porter, "Transmission and reception of transient signals in a SOFAR channel," J. Acoust. Soc. Am. **54**, 1081–1091 (1973).
[12]S. Dosso, "Quantifying uncertainty in matched field inversion. I. A fast Gibbs sampler approach," J. Acoust. Soc. Am. **111**, 129–142 (2002).
[13]D. J. Battle, P. Gerstoft, W. S. Hodgkiss, W. A. Kuperman, and P. L. Nielsen, "Bayesian model selection applied to self-noise geoacoustic inversion," J. Acoust. Soc. Am. **116**, 2043–2056 (2004).
[14]G. Box and G. Tiao, *Bayesian Inference in Statistical Analysis* (Wiley, Reading, MA, 1973).
[15]S. Geman and D. Geman, "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images," IEEE Trans. Pattern Anal. Mach. Intell. **PAMI-6**, 721–741 (1984).
[16]A. E. Gelfand and A. F. Smith, "Sampling-based approaches to calculating marginal densities," J. Am. Stat. Assoc. **85**, 398–409 (1990).
[17]Z. H. Michalopoulou and M. B. Porter, "Focalization in the Gulf of Mexico," in ICASSP-96, Atlanta, GA (1996), Vol. **6**, pp. 3086–3089.
[18]M. B. Porter, "The Kraken normal mode program," Naval Research Laboratory Mem. Report No. 6920, NRL, Washington, DC, 1991.