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Optimization of natural frequencies of a plate structure by modifying boundary conditions

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Abstract: A combined approach based on finite element method and genetic algorithm (FEM-GA) is proposed for optimizing the natural frequencies of plate structures. This approach can identify the optimal boundary conditions so that the plate's natural frequencies can be adjusted simultaneously to their corresponding target values. In this approach, the natural frequencies of plates with arbitrary boundary conditions are calculated by FEM, while GA is employed for searching the optimal solutions of the multiple-objective optimization problem. The FEM is validated by comparing with previous results. The proposed approach is illustrated by numerical examples. The results demonstrate the effectiveness of this approach.

[GM]
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1. Introduction

Plate structures form an important class of structures in building, aerospace, and various engineering areas. Natural frequencies are fundamental properties affecting the dynamic behavior of a plate structure. Structural dynamic optimization by modifying the natural frequencies of a plate structure has been exhaustively researched by a variety of approaches. These approaches include adding external masses to the plate,[1] adjusting the size, thickness or shape of the plate,[2] optimizing the topology of the plate, and so on. Among these applications, the optimization of modifying the given natural frequencies to some particular desired values is important and worthy of study, which can have broad applications in engineering designs, such as adjusting and improving the acoustic performance of the panel sound absorbers[3] and music instruments.[4]

It is known that the natural frequencies of a plate are highly dependent on the plate's actual boundary conditions.[5] However, according to the authors' best knowledge, no method is available for optimizing simultaneously multiple natural frequencies of a plate by considering the effects of arbitrary boundary conditions. This kind of method however has many potential applications in structural optimization. It only requires the modification of boundary supports and does not need to make any change to the main body of the structure. This can be useful especially when the given constraints do not allow modifying the body and appearance of the structure, such as the plate's material, size, shape, mass and surface property.

The purpose of the present study is to develop an optimization approach for the control of natural frequencies of a plate structure by taking the plate's boundary conditions as design variables. Specifically, this approach aims to identify the optimal boundary conditions so as to simultaneously allow multiple natural frequencies to reach corresponding desired values. To this end, a finite element method and genetic algorithm (FEM-GA) combined optimization method is proposed, in which the FEM method can handle arbitrary complex elastic boundary conditions while the GA method can handle multiple-objective optimization problems.

2. Free vibration analysis model

2.1 FEM model

Consider a thin rectangular plate with arbitrary elastic boundary supports along the four edges, as shown in Fig. 1. The characteristic equation is given as

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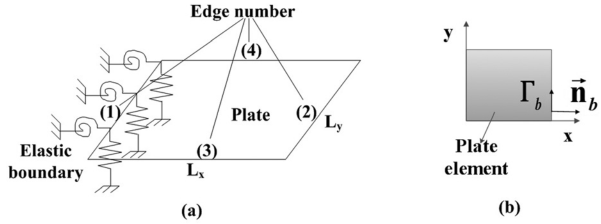

Fig. 1. Schematic illustration of a rectangular plate. (a) The plate with elastic boundary supports along the edges (for simplicity and clarity, only the supports along the left edge are shown). (b) The plate element.

$$(-\omega^2[M] + [K])\{\varphi\} = \{0\}, \tag{1}$$

where ω is the frequency of the free vibration, $\{\varphi\}$ is the amplitude of the nodal displacement, and $[M]$ and $[K]$ are the mass and stiffness matrices of the plate structure.

In this work, the boundary supports are supposed to have no mass, as treated in the literature.[6,7] The mass matrix $\{M\}$ of the whole plate structure only contains the mass matrix of the plate. The stiffness matrix $\{K\}$ of the whole plate structure is decomposed into plate and boundary supports, and can be expressed as $\{K\} = \{K_p\} + \{K_b\}$, where $\{K_p\}$ and $\{K_b\}$ are the stiffness matrices for the plate and boundary supports, respectively. The arbitrary elastic supports, are modeled as a combination of translational and rotational springs,[8,9] with k_{tb} and k_{rb} being the translation stiffness and rotational stiffness per unit length, respectively. $\{K_b\}_e$ is the element stiffness matrix of $\{K_b\}$, which can be expressed by

$$\{K_b\}_e = \int\left(k_{tb}\{N_w\}^{\mathrm{T}}\{N_w\} + k_{rb}\left\{\frac{\partial N_w}{\partial \vec{\mathrm{n}}_b}\right\}^{\mathrm{T}}\left\{\frac{\partial N_w}{\partial \vec{\mathrm{n}}_b}\right\}\right)d\Gamma_b, \tag{2}$$

where $\{N_w\}$ is the shape function vector for the plate element and $\vec{\mathrm{n}}_b$ is the normal unit vector of the element boundary contour Γ_b (see Fig. 1). It is noted that the elastic parameters (k_{tb} and k_{rb}) along the contour can arbitrarily be varied to reproduce simply supported ($k_{tb} = \infty$ and $k_{rb} = 0$), clamped ($k_{tb} = \infty$ and $k_{rb} = \infty$), free ($k_{tb} = 0$ and $k_{rb} = 0$), and guided edges ($k_{tb} = 0$ and $k_{rb} = \infty$), or, more important, any intermediate situation (i.e., general boundary condition). Moreover, these parameters can spatially vary along each edge to represent arbitrary non-uniform elastic restraint.

In the FEM model, the four-node rectangular Kirchoff plate element is used and the stiffness and mass matrices of the plate structure are constructed by using the standard finite element assembly procedure.[10]

2.2 Model validation

The first six natural frequencies of plates with different boundary conditions and different aspect ratios are calculated by the proposed FEM model and given in Table 1. In the calculations the infinite large value is represented by a very large number, 1×10^{10}, the element number is 256 (16×16), and the boundary parameters are given in the dimensionless forms $\bar{k}_{tb}$ and $\bar{k}_{rb}$.[6–8] The subscript n of $\bar{k}_{tbn}$ and $\bar{k}_{rbn}$ represents the edge number of the plate (see Fig. 1). ω_n is the nth order natural frequency (in radians/s) of the plate. L_x, h, and D are the plate's length, thickness and flexural rigidity, respectively. The existing published results by the previous researchers[6,7] are also given in Table 1 for comparison. The results given in Ref. 6 are obtained using the Rayleigh-Ritz method based on the products of beam functions, while the results in Ref. 7 are obtained based on Fourier series expansions. The C, F, and S in Table 1 represent the clamped, free and simply supported boundary conditions, respectively. Good agreement can be seen between the calculation results and those of the previous researchers.

3. Optimization methodology

The objective is to optimize the boundary conditions of a plate structure within the prescribed design domain so as to make multiple natural frequencies of the plate to be of their corresponding target values simultaneously. The main challenges in achieving this objective can be seen as follows: (1) although only the boundary conditions are set as the design variables, there can be multiple boundary parameters for a practical plate structure and each boundary parameter can take on an infinite number of different values, which makes it very difficult to apply an exhaustive search to the task of boundary condition optimization. (2) This is a multi-objective optimization problem since more than one natural frequency is required to be optimized at the same time. Every

Table 1. Results of natural frequencies.

Boundary condition	Aspect ratios Lx/Ly	Method	$\Omega_n = \omega_n (L_x)^2 \sqrt{\rho_p h/D}$ 1	2	3	4	5	6	Maximum percentage difference
C-C-C-C	1	Present	35.84	73.00	73.00	106.88	130.88	131.56	1.22%
		Li (Ref. 6)	35.99	73.40	73.40	108.20	131.60	132.20	
	1.5	Present	60.52	93.11	148.16	148.44	177.32	225.09	1.27%
		Li (Ref. 6)	60.76	93.84	148.80	149.70	179.60	226.80	
C-S-S-F	1	Present	16.64	30.70	51.09	63.38	66.69	99.27	1.90%
		Li (Ref. 6)	16.87	31.14	51.64	64.03	67.64	101.20	
	1.5	Present	18.24	49.89	53.07	87.47	107.12	125.27	1.61%
		Li (Ref. 6)	18.54	50.43	53.72	88.78	108.20	126.10	
$\bar{k}_{tb1} = \infty, \bar{k}_{tr1} = 10$ $\bar{k}_{tb2} = \infty, \bar{k}_{tr2} = 10$ $\bar{k}_{tb3} = \infty, \bar{k}_{tr3} = 10$ $\bar{k}_{tb4} = \infty, \bar{k}_{tr4} = 10$	1	Present	28.44	60.03	60.03	90.12	110.82	111.04	0.76%
		Li (Ref. 6)	28.50	60.22	60.22	90.81	111.20	111.40	
$\bar{k}_{tb1} = 100, \bar{k}_{tr1} = 1000$	1	Present	17.47	25.27	25.27	33.86	46.24	46.80	0.24%
$\bar{k}_{tb2} = 100, \bar{k}_{tr2} = 1000$		Li *et al.* (Ref. 7)	17.51	25.30	25.30	33.90	46.29	46.86	
$\bar{k}_{tb3} = 100, \bar{k}_{tr3} = 1000$	2	Present	23.19	29.33	48.73	50.11	59.61	85.27	0.97%
$\bar{k}_{tb4} = 100, \bar{k}_{tr4} = 1000$		Li *et al.* (Ref. 7)	23.22	29.35	48.77	50.24	60.02	86.10	
$\bar{k}_{tb1} = \infty, \bar{k}_{tr1} = \infty$ $\bar{k}_{tb2} = 10, \bar{k}_{tr2} = 100$ $\bar{k}_{tb3} = 0, \bar{k}_{tr2} = 0$ $\bar{k}_{tb4} = 10, \bar{k}_{tr4} = 100$	1	Present	7.94	12.55	30.02	34.38	37.05	60.07	1.65%
		Li *et al.* (Ref. 7)	7.90	12.60	30.22	34.54	37.57	61.08	

slight change of the boundary conditions leads to a new value of each concerned natural frequency.

Therefore, global search algorithms that can search intelligently for the optimal solution within the search space are needed. For this reason, a genetic algorithm (GA) method is employed and combined with the proposed FEM method.

3.1 Genetic algorithm

Genetic algorithms (GAs) are heuristic search techniques inspired by natural evolution. Compared to other stochastic methods, a practical benefit of GAs is that it is extremely easy to parallelize the algorithm since the calculations (individuals) of each iteration (generation) are independent of one another.[11] The procedure of genetic algorithm (GA) generally consists of four bio-inspired operators, which are initialization, crossover, selection and mutation. In the whole process, the fitness function which states the purpose of the problem is another important concept. The fitness, calculated by the fitness function, indicates the quality of individuals. This procedure is inspired by the processes of natural selection and genetics and creates "environments" where the best fit individuals tend to pass on their data (genes) to the next generation. The quality (or performance) of the population can therefore increase over time.

3.2 Fitness functions

The objective is to optimize boundary parameters of the plate structure for achieving desired natural frequencies. It is a multi-objective optimization problem since the target natural frequencies can simultaneously include multiple modes. The objective functions can be given as follows: minimize $g_n = |(f_n - f_n^{\text{target}})/f_n^{\text{target}}|$, where f_n and f_n^{target} are the nth order natural frequency and target natural frequency, respectively. The simplest method for this multi-objective optimization problem is to combine these multiple objective functions into one weighted objective function, as

$$\text{Minimize } G = \sum (w_n g_n), \tag{3}$$

where w_n is the weighting coefficient of nth order natural frequency which expresses the relative importance of the nth objective. Users can set the weighting coefficients according to their actual needs. The fitness function can thus be given as $F = 1/G$. It is obvious that when F is minimized, the user has obtained the optimal results.

3.3 FEM–GA integrated optimization approach

The flowchart of the optimization approach is given in Fig. 2, in which the whole procedure is given on the left side while the core of the FEM-GA combined algorithm is given on the right side. The procedure can be described as follows: (1) input the known variables of the plate, such as the plate's Young's modulus, density, Poisson's ratio, length, width and thickness. (2) Input the constraints of design variables, i.e., the variable ranges of boundary parameters. (3) Input the order numbers of the concerned natural frequencies. (4) Calculate the achievable ranges for each of the concerned natural frequencies by FEM based on the information inputted in the first 3 steps. (5) Set target values for the concerned natural frequencies, as well as their weighting coefficients. The target values must be set within the achievable ranges, because it is not possible to obtain the natural frequency values beyond these ranges. (6) Set the termination criteria for the optimization algorithm. Users can set the criteria according to their actual needs. For example, users can set the maximum tolerable error for each concerned natural frequency and the maximum number of generations in GA method. The algorithm will terminate if the error of each concerned natural frequency falls within the tolerable range or the number of generations reaches the maximum number. (7) Run the optimization algorithm to search the optimal results.

4. Illustrative examples

The proposed FEM-GA combined approach is applied to natural frequency optimization of three plate structures in terms of the boundary conditions. Unless stated otherwise, the infinite large value of boundary condition is represented by a very large number, 1×10^{10}, and the element number used in the FEM is 256 (which are the same as used in Sec. 2). In the GA method, the initial population and maximum number of generations are set to be 100 and 1000, respectively. The rates of crossover and mutation are set to be 0.8 and 0.08, respectively. Users can set freely the target values for the concerned natural frequencies within the achievable ranges. In the following illustrative examples, we just arbitrarily select some numbers (in tens or in hundreds) within the achievable ranges as the target values. The termination criteria for the algorithm are setting as follows: (1) If the optimization target of $|(f_n - f_n^{\text{target}})/f_n^{\text{target}}| \leq 1\%$ is reached for all the concerned natural frequencies, the FEM-GA algorithm terminates immediately and outputs the finding results. (2) If the optimization target of $|(f_n - f_n^{\text{target}})/f_n^{\text{target}}| \leq 1\%$ cannot be reached, the FEM-GA algorithm terminates when the predefined maximum number of generations is reached and outputs the results.

As a general suggestion,[12] if the optimization target cannot be reached in one run of the genetic algorithm, users are suggested to start over the algorithm and run the algorithm multiple times independently to ensure finding the best results.

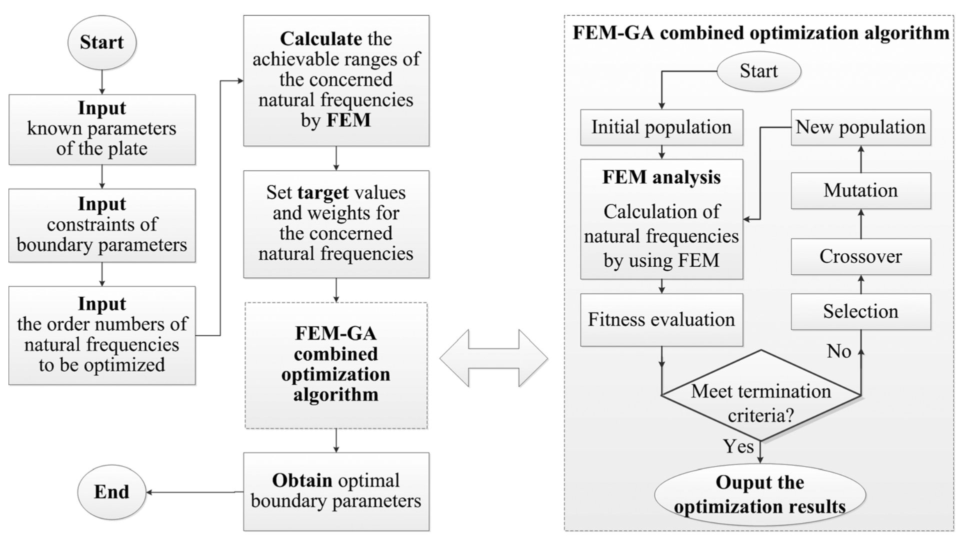

Fig. 2. Flowchart of FEM–GA integrated optimization.

Table 2. Known parameters and optimization results of Case 1.

Known plate parameters (Procedure 1)	Density (kg/m^3)	Young's modulus (GPa)		Poisson's ratio		Lx (m)	Ly (m)	h (mm)	
	7800	216		0.28		1	1	10	
Constraints of boundary conditions (Procedure 2)	Edge No.	1		2		3		4	
	Boundary parameter	$\bar{k}_{tb1}$	$\bar{k}_{rb1}$	$\bar{k}_{tb2}$	$\bar{k}_{rb2}$	$\bar{k}_{tb3}$	$\bar{k}_{rb3}$	$\bar{k}_{tb4}$	$\bar{k}_{rb4}$
	Variable Range	∞	∞	$0 \sim 1000$	$0 \sim 1000$	∞	∞	$0 \sim 1000$	$0 \sim 1000$
Natural frequencies to be optimized (Procedure 3)		f_1 and f_2							
Achievable range of each concerned natural frequency (Hz) (Procedure 4)		$f_1 \in [17.44, 82.97], f_2 \in [60.16, 152.94]$							
Optimization targets (Procedure 5)	Target values (Hz)	$f_1^{\text{target}} = 50, f_2^{\text{target}} = 100$							
	Weights	$w_1 = 1, w_2 = 1$							
Optimization results (Procedure 7)	Boundary condition	$\bar{k}_{tb1} = \infty, \bar{k}_{rb1} = \infty, \bar{k}_{tb2} = 642.63, \bar{k}_{rb2} = 5.46,$ $\bar{k}_{tb3} = \infty, \bar{k}_{rb3} = \infty, \bar{k}_{tb4} = 3.00, \bar{k}_{rb4} = 918.18.$							
	Natural frequencies (Hz)	$f_1 = 50.35, f_2 = 101.00$							
	Percentage Deviation	0.70% and 1.00% for n = 1 and 2, respectively.							

4.1 Case 1 (two natural frequencies are optimized simultaneously)

The plate's known parameters, design constraints, target values, and the final optimization results of this case are shown in Table 2. The procedures given in this table are the same as defined in Sec. 3.3. Procedure 6 is given in the beginning of Sec. 4 and is not repeated in the table. It can be seen from this table that, by using the proposed FEM-GA method, the optimal boundary parameters can be found, which can adjust the concerned natural frequencies, f_1 and f_2, to their corresponding target values (with both of their percentage deviations from the target values being not larger than 1%).

4.2 Case 2 (three natural frequencies are optimized simultaneously)

The plate's known parameters, design constraints, target values, and the final optimization results of this case are shown in Table 3. It can be seen from this table that the

Table 3. Known parameters and optimization results of Case 2.

Known plate parameters (Procedure 1)	Density (kg/m^3)	Young's modulus (GPa)		Poisson's ratio		Lx (m)	Ly (m)	h(mm)	
	2700	68.5		0.34		0.8	0.6	10	
Constraints of boundary conditions (Procedure 2)	Edge No.	1		2		3		4	
	Boundary parameter	$\bar{k}_{tb1}$	$\bar{k}_{rb1}$	$\bar{k}_{tb2}$	$\bar{k}_{rb2}$	$\bar{k}_{tb3}$	$\bar{k}_{rb3}$	$\bar{k}_{tb4}$	$\bar{k}_{rb4}$
	Variable Range	∞	0	$0 \sim \infty$	$0 \sim \infty$	∞	0	$0 \sim \infty$	$0 \sim \infty$
Natural frequencies to be optimized (Procedure 3)		f_1, f_2 and f_4							
Achievable range of each concerned natural frequency (Hz) (Procedure 4)		$f_1 \in [16.71, 145.57], f_2 \in [76.19, 268.88], f_4 \in [195.32, 472.44]$							
Optimization targets (Procedure 5)	Target values (Hz)	$f_1^{\text{target}} = 100, f_2^{\text{target}} = 200, f_4^{\text{target}} = 300$							
	Weights	$w_1 = 1, w_2 = 0.75, w_4 = 0.5$							
Optimization results (Procedure 7)	Boundary condition	$\bar{k}_{tb1} = \infty, \bar{k}_{rb1} = 0, \bar{k}_{tb2} = 1241.90, \bar{k}_{rb2} = 7145.30,$ $\bar{k}_{tb3} = \infty, \bar{k}_{rb3} = 0, \bar{k}_{tb4} = 162.81, \bar{k}_{rb4} = 1800.70.$							
	Natural frequencies (Hz)	$f_1 = 99.62, f_2 = 199.85, f_4 = 302.98$							
	Percentage Deviation	0.38%, 0.08% and 0.99% for n = 1, 2 and 4, respectively.							

optimal boundary parameters can be found, which can adjust the concerned natural frequencies, f_1, f_2 and f_4, to their corresponding target values (with all of their percentage deviations from the target values being smaller than 1%).

4.3 Case 3 (four natural frequencies are optimized simultaneously)

The plate's known parameters, design constraints, target values, and the final optimization results of this case are shown in Table 4. For the first set of weighting coefficients ($w_1 = 1, w_2 = 1, w_3 = 0.2, w_4 = 0.2$), the optimal boundary parameters are found to be $\bar{k}_{tb1} = \infty$, $\bar{k}_{rb1} = 11.16$, $\bar{k}_{tb2} = \infty$, $\bar{k}_{rb2} = 393.40$, $\bar{k}_{tb3} = \infty$, $\bar{k}_{rb3} = 3.52$, $\bar{k}_{tb4} = 72.51$ and $\bar{k}_{rb4} = 12.29$, which can adjust f_1, f_2, f_3 and f_4 to 20.00, 40.00, 60.00, and 72.60 Hz, with their percentage deviations from the target values being 0.00%, 0.00%, 0.00%, and 10.22%, respectively. For the second set of weighting coefficients ($w_1 = 0.2, w_2 = 0.2, w_3 = 1, w_4 = 1$), the optimal boundary parameters are found to be $\bar{k}_{tb1} = \infty$, $\bar{k}_{rb1} = \infty$, $\bar{k}_{tb2} = \infty$, $\bar{k}_{rb2} = \infty$, $\bar{k}_{tb3} = \infty$, $\bar{k}_{rb3} = 0$, $\bar{k}_{tb4} = 69.22$, and $\bar{k}_{rb4} = 30.06$, which can adjust f_1, f_2, f_3, and f_4, to 20.00, 42.81, 60.00, and 78.20 Hz, with their percentage deviations from the target values being 0.00%, 6.56%, 0.00%, and 2.30%, respectively.

In this case, because the optimization target of $|(f_n - f_n^{\text{target}})/f_n^{\text{target}}| \leq 1\%$ cannot be reached, the FEM-GA algorithm is run ten times independently and gives the best results. In each time, the FEM-GA algorithm terminates when the number of generations is reached 1000. Actually, in this case the algorithm achieves similar optimal natural frequency results each time. Therefore, it is confident that the optimized results are hard to beat given the considered constraints. It is unlikely that the algorithm, given its broad search space, settles on identical locally optimal solutions each time.

5. Discussion

The examples given in Sec. 4 show the effectiveness of the proposed FEM-GA method. Plate natural frequencies are sensitive to the boundary conditions of the structure. For a given plate structure, the adjustable range of each natural frequency is not small

Table 4. Known parameters and optimization results of Case 3.

Known plate parameters (Procedure 1)	Density (kg/m^3)		Young's modulus (GPa)		Poisson's ratio		Lx (m)	Ly (m)	h (mm)
	2500		65		0.25		2	1	10
Constraints of boundary conditions (Procedure 2)	Edge No.	1		2		3		4	
	Boundary parameter	$\bar{k}_{tb1}$	$\bar{k}_{rb1}$	$\bar{k}_{tb2}$	$\bar{k}_{rb2}$	$\bar{k}_{tb3}$	$\bar{k}_{rb3}$	$\bar{k}_{tb4}$	$\bar{k}_{rb4}$
	Variable Range	∞	$0\sim\infty$	∞	$0\sim\infty$	∞	$0\sim\infty$	$0\sim\infty$	$0\sim\infty$
Natural frequencies to be optimized (Procedure 3)	f_1, f_2, f_3 and f_4								
Achievable range of each concerned natural frequency (Hz) (Procedure 4)	$f_1 \in [9.96, 59.18], f_2 \in [28.57, 76.21], f_3 \in [45.62, 106.97], f_4 \in [58.49, 151.37]$								
Optimization targets (Procedure 5)	Target values (Hz)	$f_1^{\text{target}} = 20, f_2^{\text{target}} = 40, f_3^{\text{target}} = 60, f_4^{\text{target}} = 80$							
	Weights	$w_1 = 1, w_2 = 1, w_3 = 0.2, w_4 = 0.2$				$w_1 = 0.2, w_2 = 0.2, w_3 = 1, w_4 = 1$			
Optimization results (Procedure 7)	Boundary condition	$\bar{k}_{tb1} = \infty, \bar{k}_{rb1} = 11.16, \bar{k}_{tb2} = \infty, \bar{k}_{rb2} = 393.40, \bar{k}_{tb3} = \infty, \bar{k}_{rb3} = 3.52, \bar{k}_{tb4} = 72.51, \bar{k}_{rb4} = 12.29.$				$\bar{k}_{tb1} = \infty, \bar{k}_{rb1} = \infty, \bar{k}_{tb2} = \infty, \bar{k}_{rb2} = \infty, \bar{k}_{tb3} = \infty, \bar{k}_{rb3} = 0, \bar{k}_{tb4} = 69.22, \bar{k}_{rb4} = 30.06.$			
	Natural frequencies (Hz)	$f_1 = 20.00, f_2 = 40.00$ $f_3 = 60.00, f_4 = 72.60$				$f_1 = 20.00, f_2 = 42.81$ $f_3 = 60.00, f_4 = 78.20$			
	Percentage Deviation $\lvert(f_n - f_n^{\text{target}})/f_n^{\text{target}}\rvert$	0.00%, 0.00%, 0.00% and 10.22% for n = 1, 2, 3 and 4, respectively.				0.00%, 6.56%, 0.00% and 2.30% for n = 1, 2, 3 and 4, respectively.			

even only changing the plate's boundary conditions. By using the proposed method, users can set the constraint boundary conditions according to their practical needs, and identify the optimal boundary conditions within these constraints for their natural frequency targets. More than one natural frequency can be optimized at the same time.

The results also indicate that it cannot guarantee to achieve exactly the target natural frequency values in all cases (such as case 3 in Sec. 4), because some of the concerned natural frequencies can never reach the arbitrary target values at the same time within the given search space. However, even in these cases, the proposed method can find the optimal boundary conditions to ensure the weighted objective function is minimized (i.e., adjust the concerned natural frequencies to be very close to their corresponding target values), and in these cases the final optimal results depend on the weighting coefficients that are predefined by the user.

6. Conclusion

An FEM-GA optimization method is proposed for the control of natural frequencies of a plate, in which the FEM method is used for the free flexural vibration analysis of a plate with arbitrary elastic boundary conditions while the GA method is combined with the FEM method for searching the optimal boundary conditions so that the plate's natural frequencies can be adjusted simultaneously to their corresponding target values. Numerical examples are conducted which have demonstrated the efficiency and effectiveness of the proposed model. According to the authors' best knowledge, no optimization method is available in the literature for optimizing simultaneously multiple natural frequencies of the plate structure by considering the effects of arbitrary boundary conditions; therefore the proposed idea as well as the optimization model can be valuable for both academic and design practitioners.

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