Deaf Children’s Informal Knowledge of Multiplicative Reasoning

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Multiplicative reasoning is required in different contexts in mathematics: it is necessary to understand the concept of multipart units, involved in learning place value and measurement, and also to solve multiplication and division problems. Measures of hearing children’s multiplicative reasoning at school entry are reliable and specific predictors of their mathematics achievement in school. An analysis of deaf children’s informal multiplicative reasoning showed that deaf children under-perform in comparison to the hearing cohorts in their first two years of school. However, a brief training study, which significantly improved their success on these problems, suggested that this may be a performance, rather than a competence difference. Thus, it is possible and desirable to promote deaf children’s multiplicative reasoning when they start school so that they are provided with a more solid basis for learning mathematics.

The aim of this investigation was to analyze the development of multiplicative reasoning in young deaf children. In the first section of this paper, we define multiplicative reasoning and its importance for children’s mathematics learning. In the second section, we review studies with hearing children that investigate the schemas of action that constitute children’s informal knowledge of multiplicative reasoning used in solving multiplication and division problems before they are taught about these arithmetic operations in school. The third section provides an overview of research on deaf students’ performance in tasks that involve multiplicative reasoning. Two new empirical studies are then presented. The first compares the performance of deaf and hearing children on multiplicative reasoning problems at the beginning of primary school, before they have been taught about multiplication and division. This study established that deaf children under-perform, in comparison to hearing children, and therefore could benefit from instruction that supports their ability to use their informal mathematics knowledge to solve such problems. The second study reports a brief intervention that was relatively successful in promoting this ability.

Multiplicative Reasoning and its Importance for Mathematics Learning

Mathematics educators (e.g., Behr, Harel, Post & Lesh, 1994; Steffe, 1994; Vergnaud, 1983) include under the term “additive reasoning” those problems that are solved by addition and subtraction and under the term “multiplicative reasoning” those that are solved...
by multiplication and division. This way of thinking, focusing on the structures related to additive and multiplicative reasoning rather than on operations, has become dominant in mathematics education research in the last three decades or so. It reflects some assumptions about how children learn mathematics, two of which are made explicit here. First, it is assumed that in order to understand addition and subtraction properly, children must also understand the inverse relation between them; similarly, in order to understand multiplication and division, children must understand that they are the inverse of each other. Thus, a focus on specific and separate operations, which was more typical of mathematics education thinking in the past, is no longer justifiable. Second, it is assumed that, even though there are connections between additive and multiplicative reasoning, these two forms of reasoning are distinct enough to be considered as separate conceptual domains.

Multiplicative reasoning is important in many ways in mathematics learning, besides forming a foundation for learning about multiplication as an arithmetic operation. Two major roles are attributed to multiplicative reasoning in the development of children’s mathematical thinking.

First, it has been proposed that multiplicative reasoning is implicit in understanding place value: for example, 52 is interpreted as “five tens and two units.” Each “ten” is a multipart unit, and the expression “five tens” means 5 \times 10. Children do not need to know multiplication facts to understand this, but they do need to understand one-to-many correspondence: that is, they need to understand that each value in the tens place corresponds to 10 units (Carraher, 1985; Kamii, 1981; Nunes & Schliemann, 1990; Resnick, 1983; Steffe, 1994). Some researchers have referred to the idea that units can be made of more than one object as the concept of composite units (Behr et al., 1994; Kaput, 1985; Steffe, 1994) and have proposed that this conception builds a bridge between additive and multiplicative reasoning. Steffe and his colleagues (Olive & Steffe, 2002) have extended this idea to the measurement of length and suggested that such units can also establish a connection between natural and rational numbers. The idea of multiplication can remain entirely implicit in children’s knowledge developed in such situations: Steffe suggests that they are counting composite units, not multiplying the number of units by the value of the multipart units in an explicit fashion. This work focuses on the structure of number representations (i.e., the base-ten structure of our counting system or the use of repeated units in measurement) and thus makes a case for the omnipresent significance of multiplicative reasoning: any time we use the number system or measurement, we are using multiplicative reasoning, even if we do so implicitly.

A second, important role for multiplicative reasoning has been examined by other researchers (Confrey, 1994; Vergnaud, 1983, 1994) who maintain that it forms the basis for children’s understanding of other mathematical concepts. They argue that additive and multiplicative reasoning are fundamentally different: additive reasoning is used in one-variable problems, when quantities of the same kind are put together, separated or compared, whereas multiplicative reasoning involves two variables in a fixed ratio to each other. Even the simplest multiplicative reasoning problems involve two variables. For example, in the problem “Hannah bought 6 sweets; each sweet costs 5 pence; how much did she spend?,” there are two variables, number of sweets and price per sweet. Thus, it is argued that multiplicative reasoning forms the foundation for children’s understanding of proportional relations and linear functions (Kaput & West, 1994; Vergnaud, 1983) because these concepts involve two (or more) variables with a fixed rate connecting them.

The emphasis on the differences between multiplicative and additive reasoning does not mean that there are no connections between them: for example, multiplication is distributive in relation to addition, so we can multiply 25 by 9 by multiplying 20 by 9 and 5 by 9 and then adding the two products.

It is generally accepted (Carpenter, Hiebert, & Moser, 1981; Riley, Greeno, & Heller, 1983; Vergnaud, 1982) that children’s understanding of addition and subtraction starts from the actions of joining and separating sets. Young preschoolers show considerable success in solving simple addition and subtraction problems with manipulatives by modeling the problem situations through these actions. There is comparatively less research (reviewed below) on the origin of children’s schemas of multiplication. Researchers
pursuing the notion that multiplication plays a role in understanding numeration and measurement systems have focused on the counting of composite units, whereas those investigating the development of children’s understanding of relations between variables focus on the scheme of one-to-many correspondence. This paper investigates deaf children’s understanding of the latter aspect of multiplicative reasoning.

One-to-many Correspondence and Multiplicative Reasoning

Piaget (1952) was a pioneer in the analysis of children’s understanding of correspondence. He investigated whether children understand multiplicative relations by observing their ability to make inferences about equivalence in situations where one-to-many correspondences were established between two quantities. For example, he asked children to place one yellow flower into each vase in a set, then put the flowers aside and asked the children to place one red flower into each vase in the same set. He then asked the children how many flowers would be in each vase if all the red and yellow flowers were now placed in the vases. Children who understood one-to-one correspondence had no difficulty in realizing that there would be two flowers in each vase. The crucial test of their understanding of multiplicative relations was obtained by Piaget’s next question: after putting all the flowers away and leaving only the vases on the table, he asked the children to take from a box the exact number of straws that would be required if the children wanted to place one flower in each straw. Children who understood one-to-many correspondence had no difficulty in solving this problem: all they had to do was to take two straws for each vase and establish the equivalence between the number of flowers and the number of straws. The children’s performance in this study was indeed impressive: many 5 and 6 year olds were able to make this inference, although presumably they had not been taught about multiplication in preschool at such young ages. Piaget also observed that the understanding of one-to-many correspondences should have no difficulty in solving this problem: all they had to do was to take two straws for each vase and establish the equivalence between the number of flowers and the number of straws. The children’s performance in this study was indeed impressive: many 5 and 6 year olds were able to make this inference, although presumably they had not been taught about multiplication in preschool at such young ages. Piaget also observed that the understanding of one-to-many correspondences was not an all-or-nothing type of development: many children could deal with situations where the inference involved a 2:1 ratio but their performance would break down when the ratios were 3:1 or more.

These results were replicated by Frydman and Bryant (1988, 1994) in another situation where children were asked to create correspondences between two variables. They used sharing in their study because young children seem to have much experience with correspondence when sharing. In a sharing situation, children typically use a one-for-you one-for-me procedure, setting the shared elements (e.g., sweets) into correspondence with the recipients (e.g., dolls). Like Piaget, Frydman and Bryant observed that children in the age range of 5–7 years became progressively more competent in dealing with one-to-many correspondences and equivalences. In their task, the children did not have to judge the equivalence of sets but were asked to construct equivalent sets when the units in each set were of a different value. For example, if the first recipient only liked his sweets in double units and the second recipient liked his sweets in single units, the children were able to adapt their correspondence procedure to construct equivalent sets and gave a double to the first recipient as they gave two singles to the second. This flexible use of correspondence to construct equivalent sets was interpreted by Frydman and Bryant as an indication that the children’s use of the procedure was not merely a copy of previously observed and rehearsed actions: it reflected an understanding of how one-to-many correspondences can result in equivalent sets. They also replicated another result obtained by Piaget: some children who succeed with the 2:1 ratio found the 3:1 ratio difficult. So the development of multiplicative reasoning in these situations does not happen in an all-or-nothing fashion.

Kouba (1989) also presented young children, in first and second grades, with multiplication and division problems before they had received school instruction on the operations of multiplication and division. She hypothesized that a key distinction among multiplicative reasoning problems is how clearly the one-to-many mapping relationship is identified.

In order to test this hypothesis, she analyzed in great detail the children’s strategies, which she classified in terms of the types of actions used and the level of abstraction. The level of abstraction varied from direct representation (i.e., all the information was represented by the children with concrete materials)
through partial representation (i.e., numbers replaced concrete representations and the child counted in groups) up to the most abstract form of representation, where multiplication facts were used. Kouba did not find support for the hypothesis that the use of one-to-many strategies depended on how clearly the mapping relationship was identified in the problem. For the children in first and second grades, who had not received instruction on multiplication and division, the most important factor in predicting the children’s solutions was which quantity was unknown. For example, in a problem involving six cups, with five marshmallows in each cup, there is a total of 30 marshmallows. When the size of the groups is known (i.e., the number of marshmallows in each cup), the children use correspondence strategies: they either pair objects (or tallies to represent the objects) and count or add, creating one-to-many correspondences between the cups and the marshmallows. For example, if they need to find the total number of marshmallows, they can point five times to a cup (or its representation) and count to five, pause, and then count from 6 to 10 as they point to the second “cup,” until they reach the solution. Alternatively, they may add as they point to the “cup.” In contrast, when the number of elements in each group is not known, the children use dealing strategies: they share out one marshmallow (or its representation) to each cup, and then another, until they reach the end and then count the number in each cup. Here they may also use trial-and-error, as they may share more than one at a time and then might need to adjust the number per cup to get to the correct distribution. Although the actions look quite different, their aims are the same: to establish one-to-many correspondences between the marshmallows and the cups.

Kouba observed that 43% of the strategies used by the children, including first, second, and third graders, were appropriate. Among the first and second grade children, the overwhelming majority of the appropriate strategies were based on correspondences, either using direct representation or partial representation (i.e., tallies for one variable and counting or adding for the other); few used recall of multiplication facts. The recall of number facts was significantly higher after the children had received instruction, when they were in third grade.

The level of success observed by Kouba among children who had not yet received instruction is modest, compared to that observed in two subsequent studies, where the ratios were easier. Becker (1993) asked kindergarten children, aged 4–5 years, to solve problems in which the correspondences were 2:1 or 3:1. As reported by Piaget and by Frydman and Bryant, the children were more successful with 2:1 than 3:1 correspondences, and the level of success improved with age. The overall level of correct responses by the 5-year olds was 81%.

Carpenter, Ansell, Franke, Fennema, and Weisbeck (1993) also gave multiplicative reasoning problems to kindergarten children involving correspondences of 2:1, 3:1, and 4:1. They observed 71% correct responses to these problems.

The success rates leave no doubt that many young children start school with some understanding of one-to-many correspondence, which they can use in school to learn about multiplication and division problems. However, the connection between this informal knowledge, which rests on counting (and addition as a shortcut counting strategy), and learning about multiplicative reasoning in school still requires further empirical research. Some researchers (e.g., Booth, 1981) have suggested that child methods based on counting, adding on, and building up could be an obstacle rather than a basis for understanding multiplicative reasoning; their view is that the children who use these methods are actually not using multiplication when they solve problems. In order to test whether correspondence plays a role in children’s understanding of multiplicative situations, it is necessary to analyze how understanding correspondence affects children’s success in solving multiplication problems.

Park and Nunes (2001) tested whether children’s understanding of one-to-many correspondences forms the basis for children’s learning of multiplication in an intervention study. They reasoned that, if this is the case, children taught to solve multiplication problems by using correspondences between two variables would improve more in solving multiplicative reasoning problems than other children, taught about multiplication as repeated addition. Children taught about correspondences would construct a model of how to establish relations between variables, and this model
could be used flexibly in solving multiplicative reasoning problems. Children taught to solve repeated addition problems would be essentially practicing addition, without creating a model for the relationship between two variables.

The children in their study were 6-year olds, who had not yet been taught about multiplication and division in school. They were given a pretest and a posttest, in which they were asked to solve addition, subtraction, and multiplication problems. Between the pre- and posttest, they participated in a brief intervention, during which they solved multiplication problems with the support of the researcher. The children in the two groups solved the same number of problems, which had the same numerical description. However, the children in the repeated addition group solved problems that were one-variable problems about joining sets, whereas those in the one-to-many correspondence group solved problems that involved two variables in one-to-many correspondence. For example, the children in the repeated addition intervention were presented with this problem: “Tom ate 3 apples in the morning. In the afternoon he ate 3 bananas. In the evening he ate 3 oranges. How many fruits did he eat altogether?” There is only one quantity in this problem, number of fruits, and three types of fruits are joined into a single set. In contrast, the one-to-many correspondence problem of the same numerical description \((3 \times 3)\) was: “Tom went to a sweet shop. He bought 3 sweets. One sweet costs 3 pence. How much money did he spend?” This problem involves two quantities, number of sweets and price per sweet, which have a fixed ratio: 3 pence for each sweet. During the intervention, when the children were not able to solve a problem, the researcher helped them construct a concrete representation of the story and then asked the question again. Further support was provided, if necessary (for the details, see Park & Nunes, 2001).

It was expected that both groups would make significant progress from pre- to posttest but that the children in the correspondence intervention group would make more progress in the multiplication problems than those in the repeated addition group. In contrast, the children in the correspondence group were expected to make less progress in addition and subtraction problems than the repeated addition group. The first prediction was supported by a significant difference between the groups in the posttest as a function of type of intervention. The children who participated in the correspondence intervention made significantly more progress in multiplication problems than those who participated in the repeated addition intervention. However, contrary to our expectation, the two groups made similar amounts of progress in addition and subtraction problems: both groups performed significantly better at posttest than they had performed at pretest, and there was no difference between the groups. It is possible that all the children already had a good grasp of how to use joining and separating schemes to solve addition and subtraction problems; so the experience of modeling problems in order to solve them, which both groups had during the intervention, helped the correspondence group learn how to use modeling more efficiently in multiplication problems as well as in the addition and subtraction ones. It can be concluded from this study that the correspondence schema plays a key role in the development of children’s multiplicative reasoning.

Finally, there is evidence from a longitudinal study that children’s multiplicative reasoning at school entry is a significant factor in the progress that they make in learning mathematics in school. Nunes et al. (2007) carried out a longitudinal study, in which the children were tested on their understanding of four aspects of logical-mathematical reasoning at the start of school: one of these was multiplicative reasoning. Two types of multiplicative reasoning task were used: one assessed children’s performance in a composite units task and a second assessed the understanding of correspondence. About 14 months later, the children were given a state-designed and teacher-administered mathematics assessment, which is entirely independent of the researchers and an ecologically valid measure of how much they have learned in school. The children’s performance in the two multiplicative reasoning tasks at school entry was a significant predictor of their mathematics achievement, after controlling for (a) age at the time of the achievement test, (b) performance in a general measure of cognitive ability (British Abilities Scale, BAS; Elliott, Smith, & McCulloch, 1997) excluding their knowledge of numbers at school.
start, (c) performance on the Number Skills subtest of the BAS, and (d) performance on a working memory measure (Pickering & Gathercole, 2001). Nunes et al. (2007) did not report the analysis of longitudinal prediction based only on the items that assess multiplicative reasoning; so these results are reported here in greater detail. The total variance explained in the mathematics achievement by these predictors was 65% (standardized multiple $r^2$); age explained 1% (nonsignificant), the BAS general score (excluding Number Skills) explained 49%; the Number Skills subtest explained 12%, working memory explained 1% (nonsignificant), and the children’s understanding of multiplicative reasoning at school entry explained a further 4% ($p = .02$). This result strongly supports the idea that children’s understanding of multiplicative reasoning at school entry is an important factor in their mathematics achievement: after 14 months and after controlling for general cognitive factors as well as specifically for number skills at school entry, performance on multiplicative reasoning assessments still explained a significant amount of variance in the children’s mathematics achievement in school.

In summary, two types of multiplicative reasoning contribute to young children’s mathematics learning when they start school: their understanding of composite units, which helps them make sense of numeration and measurement systems and their understanding of correspondences, which forms the basis for learning about relations between variables in multiplicative reasoning problems. Thus, it is quite important to know how deaf children perform in these tasks as they start to learn mathematics in school. This paper focuses on their understanding of correspondences; their understanding of composite units is not analyzed here.

**Deaf Children’s Multiplicative Reasoning**

Much research has shown that deaf children underachieve in mathematics in comparison to their hearing age cohorts (e.g., Gregory, 1998; Moreno, 2000; Nunes, 2004; Traxler, 2000; Wood, Wood, & Howarth, 1983). Many studies have investigated deaf children’s problem solving performance in addition and subtraction problems, analyzing comprehension processes or knowledge of algorithms (e.g., Ansell & Pagliaro, 2006; Frostad & Ahlberg, 1999; Garrison, Long, & Dowaliby, 1997; Hitch, Arnold, & Philips, 1983; Hyde, Zevenbergen, & Power, 2003; Nunes & Moreno, 1997, 1998b; Secada, 1984; Serrano Pau, 1995), but there is a relative paucity of studies on deaf children’s multiplicative reasoning. Previous work on 8-year-old deaf children’s solutions to multiplication and division problems reports that hearing children perform significantly better than deaf children (Nunes, 2004) even in the simplest problem types.

The weak performance of young deaf students observed in these studies could result from lack of informal knowledge of multiplicative reasoning at school entry, or from teaching that is designed for hearing students and does not match deaf children’s information processing preferences, or from the more restricted problem solving experiences that are offered to deaf students in the classroom (see, e.g., Kelly, Lang, & Pagliaro, 2003). Of course, all three factors might operate together. Thus, it is important to know whether deaf children show poorer performance than hearing children before instruction, and if so, whether it is possible to intervene early on.

Two studies are reported here, which can contribute to a better understanding of deaf children’s knowledge of multiplicative reasoning: the first compared young deaf children’s performance on multiplicative reasoning problems with a hearing cohort and the second assessed the effects of an intervention designed to promote young deaf children’s multiplicative reasoning.

**Study 1**

The aim of this study was to analyze whether deaf children underperform in multiplicative reasoning problems for their level of nonverbal intelligence in comparison to hearing peers.

The problems presented to the children were of the type described by Vergnaud (1983) as isomorphism of measures, in which two variables are in a fixed ratio correspondence to each other.

**Method**

**Participants.** Deaf children ($N = 28$; mean age 6 years 5 months; standard deviation [SD] = 0.7 years) in grades 1 and 2 and hearing children ($N = 78$; mean
age 6 years 2 months; SD = 0.3 years) in grade 1 participated in this study. The deaf children were recruited from seven schools for the deaf or mainstream schools with units for deaf and hard of hearing; total communication was used in all schools. None had a documented disability beyond hearing loss. Twelve deaf children had cochlear implants (their level of loss is not described as it is not considered reliable information), three had moderate loss, and the remaining 13 had severe to profound loss. The hearing children were recruited from three schools with a varied intake in terms of their socio-economic background.

Measures. The children were given the Matrices subtest of the BAS (Elliott et al., 1997) as a measure of nonverbal cognitive ability and 12 word problems requiring multiplicative reasoning. Because this was part of a larger study, the children were also given number representation and additive reasoning problems, which are not described here.

A measure of nonverbal intelligence was chosen as the control measure for the comparison with hearing children for four reasons.

First, if the hearing children were found to perform significantly better than the deaf children in multiplicative reasoning tasks without controlling for general cognitive ability, this finding would be of lesser interest if the differences between the two groups were simply a consequence of differences in cognitive ability. Thus, we thought it necessary to control for the children's cognitive ability when carrying out the comparison between the groups in the multiplicative reasoning task.

Second, measures of nonverbal intelligence are significant predictors of both hearing and deaf children's mathematics achievement. Among hearing children, Elliott et al. (1997) found a correlation of .43 between the Matrices subtest of the BAS (which is part of the Nonverbal Reasoning Ability Scale and was used in this study) at age 5 and children's later performance in Early Number Concepts at age 7; the correlation between the overall Nonverbal Ability Scale and Early Number Concepts was .54. For deaf children aged 8 and 9 years, Moreno (2000) found a correlation of .38 (Spearman's rho) between the performance scale of the Weschler Intelligence Scale for Children-III and the nfer-Nelson Age-Graded Mathematics Tests; the nonverbal intelligence scale still explained 15% of variance in deaf children's performance in the mathematics achievement test, after controlling for differences in the children's ages. If we found differences in the multiplicative reasoning between deaf and hearing children and these differences were simply a result of discrepancies in cognitive ability, controlling for nonverbal reasoning should render the differences between deaf and hearing children nonsignificant.

Third, the correlation of .50 observed between the BAS verbal scale with early number concepts for hearing children is not significantly different from that with the nonverbal scale. There is therefore no case for attempting to use a verbal measure instead of the nonverbal one.

Fourth, little modification is required in the administration of the nonverbal subtest of the BAS, Matrices, to deaf children; this leaves us more confident in its results than if we were to use a measure of verbal ability.

All problems were presented with the support of drawings on a computer screen. Previous work showed that deaf children perform better in number representation problems when all the items are presented simultaneously than in succession (Zarfaty, Nunes, & Bryant, 2004) and also in addition and subtraction word problems when all drawings are presented simultaneously (Nunes, Bryant, & Pretzlik, 2006). In the number representation task, Zarfaty et al. (2004) found that deaf children performed better than hearing children when all the items were displayed simultaneously. In order not to disadvantage the deaf children in the comparison with hearing children and to obtain a better picture of their performance, in half of the problems all the drawings were displayed at the same time; in the other half, the drawings were presented in sequence. For example, one problem was: there are two houses in this street; in each house live three dogs; how many dogs live in this street? Figure 1 shows (here in black and white but the original was in color) the display as it was presented to the children in the simultaneous presentation condition. In the successive presentation, the three dogs would appear on the screen and then disappear under the house. The
movement of the dogs was activated by the researcher through the click of the mouse so that it could be coordinated with the explanations provided to the children. The movements were sufficiently slow for the children to see exactly what happened. In all the problems, the children were told the number of groups and the number of elements in each group; the children were asked to provide the total number of objects.

Problems were presented in the children’s preferred mode of communication in school. All schools used total communication but some children preferred oral and others preferred signed language in the classroom. The researchers were experienced signers and had passed the test for British Sign Language Stage 2, which is the qualification required of teachers of the deaf for primary school children.

Results

Preliminary analyses were carried out to test whether the hearing and deaf children differed significantly in age and BAS-Matrices scores. The SD for the children’s age was significantly different because deaf children’s school entry is sometimes delayed in order to give them more time to develop their language skills. The difference between the two samples was not significant but showed a trend (t = 1.92; df = 29.61; p = .06) and so it was decided that further comparisons should include age as a covariate.

The sample of deaf children in this study is too small to allow for analyses of how demographic variables affect the results; the deaf children will be treated in the subsequent analyses as a group. We think that there is good reason to do so. Their levels of hearing loss varied from moderate to profound. Wollman (1965) and Nunes and Moreno (1998a) found that the correlation between hearing loss and mathematics achievement in samples with comparable levels of loss was not significant; Wood et al. (1983) found a significant correlation but this was low. Nunes and Moreno (1998a) and Tymms, Brien, Merrell, Collins, and Jones (2003) found that the language spoken at home did not significantly affect deaf children’s mathematics performance. Nunes and Moreno (1998a) also report no effect of cochlear implants on the children’s mathematics achievement. So it seems appropriate to disregard these differences for our purposes and analyze the performance of the deaf children as a group. We recognize that they are a diverse group, and so are the hearing children, who were recruited from schools that serve a varied clientele in terms of socio-economic background. These variations will affect the statistical variance, which is considered when the group differences are analyzed.

The hearing children performed significantly better than the deaf children in the BAS, even when the comparison between their scores was carried out without controlling for age. Controlling for age, the adjusted mean raw scores were 6.36 for the deaf and 9.22 for the hearing children. An analysis of covariance showed that age was a significant predictor of BAS scores (F_{1,104} = 4.72; p = .03) and that the two groups differed significantly (F_{1,104} = 12.04; p = .001). Even though the hearing children were a bit younger, they scored significantly higher than the deaf children on this task. This result underscores the need to control for cognitive ability when comparing the deaf and hearing children.
hearing children’s performance in multiplicative reasoning.

Because our interest was to test whether the deaf children under-perform on these tasks for their level of intelligence, the statistical comparison on the multiplicative reasoning problems between the two groups controls for BAS results. Table 1 presents the adjusted means for the two groups in the two conditions of presentation, simultaneous, and successive.

There was a main effect of group membership: the hearing children performed significantly better than the deaf children in the multiplicative reasoning task ($F_{1,104} = 34.71; p < .001$). The difference between the simultaneous and successive conditions was not significant and the interaction between group and type of presentation was not significant but it showed a trend ($p = .06$); a further comparison between the means for the deaf children using a Bonferroni adjustment did not suggest that there is a difference for the deaf children across the two conditions of presentation. So there was no evidence that hearing and deaf children had different preferences for information processing in correspondence problems.

It is concluded that deaf children under-perform in comparison to hearing children on multiplicative reasoning tasks before they start to receive instruction on multiplication at school.

Discussion

There are two important findings of this study that we wish to highlight in this discussion. First, previous work by Zarfaty et al. (2004) comparing preschool deaf and hearing children’s number representation in a computer task showed that deaf children obtained significantly better results when the number of items to be represented later appeared simultaneously rather than successively on the computer screen. Nunes et al. (2006) also found that this variation affected deaf children’s performance in addition and subtraction story problems. This variation in mode of presentation was introduced in this study to investigate whether it is also relevant to deaf children’s performance in multiplicative reasoning problems. The difference in the children’s performance was not significant when the mode of presentation was varied. This could be due to the small sample of deaf children who participated in this study or to the fact that multiplicative reasoning situations may actually be too complex for simultaneous processing. In multiplicative reasoning problems, there are two variables and three values to be taken into account: the number of groups, the number of elements in each group, and the total number of elements. It is possible that encoding larger amounts of information is more difficult with simultaneous presentation. It is, of course, never possible to prove a null hypothesis, but when no effects are found with larger numbers of participants, it is possible to have a clearer interpretation for a negative result. Further research on this issue would be necessary to clarify this negative result.

Second, the deaf children’s performance was weaker than that of the hearing children even after controlling for differences in general cognitive ability. Thus, they seem to be under-performing for their level of cognitive ability. It is important to be cautious about the interpretation of this difference: it may be a performance rather than a competence difference. Hearing children may be more familiar with word problems in general and the assessment may underestimate the deaf children’s competence, in spite of our attempts to use visual materials to support the children’s understanding of the problem instructions and the recall of the information.

In order to check this interpretation, we decided to carry out a second study, which consisted of a pretest, an intervention, and two posttests, one immediate and one delayed. There is a long tradition of using interventions in assessments to help understand the nature of group differences in psychological research. Piaget’s clinical method was probably the first step toward this tradition. Piaget (1954; originally published in 1937) argued that when one is trying to understand a child’s

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<th>Mean (maximum 6)</th>
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cognitive processes and judgments, it is essential to engage with the child in a dialogue, where the interviewer poses to the child a different view in order to examine the child’s understanding of the question and certainty of the answer just given. Children who gave, for example, a nonconservation response to a conservation question would be asked what they thought of another child’s answer, when this other child had said the opposite. Some children might then realize that they misunderstood the question, whereas others might reaffirm their view with new arguments.

Bovet (1974) extended this idea in her studies of cognitive processes of unschooled children and adults. She argued that, in order for the investigator to avoid incorrect conclusions when working with groups that might not be exposed to the same cultural background, one must attempt to activate any cognitive structures that might not be evident. “Learning situations in the form of ‘exercises in operativity’ might serve the purpose of activating such mental structures” (Bovet, 1974, p. 314). She observed, for example, that some adults gave nonconservation responses to a conservation of weight task; however, all that was required for them to grasp the notion of conservation was to weigh the two pieces of clay in scales in front of them. They then accompanied their subsequent judgments by logical justifications and generalized their conservation responses to various changes in shape. This contrasts with observations of children, for whom demonstrations on the scale were not sufficient to elicit more advanced judgments (Smedslund, 1961). Kelly et al. (2003) noted that deaf children are exposed to more restricted problem solving experiences in school than hearing children. So, a study that includes an intervention could be valuable in interpreting the group differences observed in this study.

The use of interventions as part of dynamic assessments, as they are now referred to, is currently identified more with Vygotsky’s theory (Lidz, 1995). The characteristics of this form of assessment will be discussed in the introduction to Study 2.

Study 2

The aim of this study was to analyze whether deaf children can learn how to use correspondence reasoning to solve multiplicative problems after a brief intervention.

Intervention studies are of great importance in educational research. They can be used to test causal hypotheses: if A is hypothesized to be a cause of B, then changing A should lead to a change in B. Bradley and Bryant (1983) not only argued this case cogently but also provided the first unambiguous evidence for the causal role of phonological awareness in reading, through a combination of longitudinal and intervention methods.

Intervention studies are also important in helping interpret the nature of group differences. As discussed at the end of Study 1, Bovet (1974) used this approach in an attempt to interpret unschooled adults’ nonconservation responses. More recently, response to intervention has been used as a way of assessing whether children who show poor performance in school entry assessments are at risk for reading difficulties later or whether they may have not received sufficient exposure to activities that are predictors of and facilitate reading acquisition (Vellutino, Scanlon, Small, & Fanuele, 2006; see also Gresham, 2002).

Finally, intervention studies also test the relevance of psychological theories to education. There is a difference between proposing a psychological hypothesis and showing that it can be used in educational settings.

The intervention study described in this section will address the last two points: it will be used to clarify the nature of the difference between deaf and hearing children observed in Study 1 and to show the relevance of previous research in mathematics education to the education of deaf children.

According to Lidz (1995), the most salient characteristic of dynamic assessments is the use of a test-intervene-retest design, but the specific details may vary. There are different aims for and ways of using interventions in assessments. Some researchers (e.g., Feuerstein, 1979; Budoff, 1987) are interested in what they identify as learning potential. The tasks in these assessments are seen as content-free, in the sense that they are not subjects of school learning. In contrast, other researchers have sought to develop assessments that are more closely related to school learning (e.g., Campione & Brown, 1987; Kaminski & Good, 1996; Newman, Griffin, & Cole, 1989) because their aim was
to see how much children can do with the support of a more competent peer or adult and how responsive they are to brief interventions. The interventions are sometimes carried out in a one-to-one fashion and sometimes in groups. Sometimes they are highly scripted and sometimes more contingent on the student’s responses. Often the aim of the assessments is to obtain information about individuals (e.g., Lidz, 2002) or to reach educational decisions regarding whether children are at risk for learning in school (e.g., Good & Kaminski, 1996).

In this study, the aims were to analyze the meaning of group differences, as in the study by Bovet (1974), and to gauge whether deaf children may be at risk for mathematics learning due to lack of informal knowledge of multiplicative reasoning. It was hypothesized that, if a brief intervention is effective in promoting deaf children’s performance in multiplicative reasoning tasks, the difference observed in the previous study could be safely interpreted as a performance difference. However, irrespective of the nature of the difference, if deaf children arrive in school without being able to use this informal knowledge, it would be desirable to know how to promote it. So it is important to assess the effectiveness of such an intervention for deaf children.

Brief interventions can be successful in promoting children’s problem solving, if they are designed to overcome the particular difficulties that children encounter in solving specific problems, and if they are designed on the basis of a developmental description of children’s progression in the task. Frydman and Bryant (1988), for example, observed that 4–year olds find it difficult to deal out equal shares of sweets when the recipients of these shares like their sweets in different sized units. If one recipient likes sweets that are single units and the other likes sweets that are double units, the children need to adapt the procedure one-for-me one-for-you to take into account the size of the units: they must deal twice to the recipient who likes single sweets and once to the one who likes double sweets. This difficulty was conquered by most 4–year olds (75%) if the single sweets were of different colors and the double sweets were double colored. In this case, when they dealt a double sweet, which was yellow and blue, to one recipient, they then gave one yellow and one blue sweet to the other recipient. After only two trials using color cues, all the children who succeeded with color cues were able to share fairly double and single sweets to these fussy recipients when all the sweets were of the same color. This intervention was carried out with hearing children and in a single session.

Nunes et al. (2008) recently reported a brief intervention to improve deaf children’s understanding of the inverse relation between addition and subtraction. Previous work by Bryant, Christie, and Rendu (1999) had shown that 4 and 5–year olds understand the inverse relation between addition and subtraction better if this uses identity as a cue than if it does not. If some bricks are added to a row of bricks and then the same bricks are removed (identity cue), the children realize that the number of bricks remains the same; they may not reach this conclusion if the same number of bricks is added and subtracted to a row but the bricks themselves are not the same. Nunes et al. (2008) designed an intervention to help deaf children understand the inverse relation between addition and subtraction. This intervention used identity and color cues at the start: not only the bricks added and subtracted were the same but they differed in color from those in the initial row. The children saw, for example, a row of eight yellow bricks being hidden under a cloth, with only the ends of the row visible; the researcher then added seven red bricks to one end and removed the same seven bricks from the row; the child was asked how many bricks were in the row afterwards; they then were allowed to count the bricks to see whether they were right. In order to make sure that the children did not simply answer by always saying the original number of bricks (eight in this example), sometimes one less or one more brick was subtracted from the row; the child could clearly see that there was one red brick still left on the row or that one yellow brick had been removed. These variations kept the children reasoning about the problems and offered them opportunities to reflect about the inverse relation between addition and subtraction under circumstances where they could manage it. After some problems with identity cues, they were presented with trials where the identity cues were no longer present. This brief intervention, carried out over two sessions, was
effective in helping deaf children improve their grasp of the inverse relation between addition and subtraction. They progressed both in problems with objects and in story problems, which had not been included in the intervention.

The intervention used in this study to promote deaf children’s informal multiplicative reasoning was based on one previously developed by Nunes et al. (2007) for hearing children at risk for difficulties in learning mathematics. The programme developed by Nunes et al. was based on a detailed analysis of the steps that children take in grasping how to use correspondences to solve multiplicative reasoning tasks. Their intervention was brief and was carried out with small groups of children by a researcher. The children, who had been identified as performing at the bottom 20% in their cohort on school entry assessments, were taught three concepts (including correspondences) in 12 sessions. The children made significantly more progress than a comparison group during this period and also showed higher levels of achievement in a standardized mathematics test given by the schools about 14 months later.

These studies illustrate that brief interventions can be effective if they are well designed. Previous research on children’s solutions to multiplicative tasks was used by Nunes et al. (2007) to design the instruction on one-to-many correspondences. It has been consistently found, for example, that the use of concrete representations facilitates children’s success in solving these problems. Kouba (1989) found that first graders can succeed in solving multiplicative reasoning problems when they use direct representations of both variables in the problem. So Nunes et al. (2007) started their intervention with the tasks found to be easier in previous studies, that is, tasks where all the materials required to fully represent the problems were provided. The children were prompted to represent the situations described in the problem using the materials and to check their counting, particularly if they had made counting errors.

Becker (1993) observed that full sets of materials could be replaced by invisible items. In his study, the children initially represented the situation fully and then the items were hidden, so that when the children answered the question, the items were no longer visible. The full representation helped the children to interpret the situation but prompted them to move in level of abstraction. So the second step in the Nunes et al. (2007) study involved tasks where the children first created a full representation of the situation and then the researcher hid some items before the children answered the question. For example, the children were presented with the problem: we are making sandwiches for the children who will come to a party; there are four plates and will put three sandwiches on a plate; how many sandwiches will we need? The child was encouraged to represent the situation using circles for the plates and bricks for the sandwiches; the researcher then removed the “plates” and asked the children to say how many sandwiches were needed. On a later problem, the researcher would remove the “sandwiches,” leaving only the “plates,” and ask the child to say how many sandwiches were needed.

In these two sets of tasks, the children were offered cut-out shapes that represent one of the variables (e.g., vans which are carrying chairs to a party); the other variable was represented by bricks. The bricks were used in different problems, so they sometimes represent chairs but at other times represent sandwiches, children, or balloons. In the final set of problems, the children were only provided with bricks. In order to solve the problems, they would have to construct groups of bricks and count them in two ways: the number of groups of bricks and the number of bricks in each group. If the children did not succeed under this condition, the experimenter provided materials for the child to represent the other variable.

The ratios used in the problems were initially easier (2:1 and 3:1) and became progressively harder. The quantity, which is unknown, was varied throughout the training sessions so that the children always needed to reflect upon their actions and could not establish correspondences and provide an answer mechanically. For example, sometimes the question was about the product (Three lorries were bringing tables to the school. Inside each lorry there are 4 tables. How many tables are they bringing to the school?) and sometimes the unknown quantity is one of the factors (There are 8 rabbits in the school. We need houses for the rabbits. The rabbits like to live
together, 2 rabbits in each house. How many houses do we need?

The intervention designed by Nunes et al. (2007) was adapted for the deaf children in this study. Problems had been presented to the hearing children only orally: they were presented to the deaf children in their preferred mode of communication and with the support of pictures on a computer screen. The screen presentations were similar to the display in Figure 1. The hearing children in the study by Nunes et al. (2007) worked in small groups; the deaf children worked on a one-to-one basis with the researcher. Finally, the hearing children in the Nunes et al. study worked with three concepts, which were presented in short blocks of items across different sessions; multiplicative reasoning problems were presented in approximately four sessions. The deaf children worked only with multiplicative reasoning in this study, in two sessions. The focus on a single type of reasoning used in this study could facilitate learning.

Method

Participants. Deaf children (N = 27; mean age = 6 years 6 months; SD = 0.66 years) from seven special schools and mainstream schools with units for the deaf participated; two further deaf children were pretested but were not included in the study because their knowledge of counting was too limited. Eight children had cochlear implants (their level of loss is not recorded as it is not reliable information), 12 had severe to profound losses, the remaining 7 children had mild to moderate losses. None of the children had documented disabilities beyond hearing loss. Because of the differences in BAS scores observed in Study 1, the hearing children (N = 33; mean age = 5 years 7 months; SD = 0.31 years) were recruited from a younger cohort in order to obtain a sample matched in cognitive ability.

Design. The children were individually and randomly assigned either to the intervention or to the control group. Level of hearing loss was comparable across the two groups. All the children were assessed on three occasions: (1) pretest, (2) immediate posttest, on the first school day after the intervention had been completed, and (3) delayed posttest, about 2 weeks after the intervention. The intervention group received instruction on multiplicative reasoning. The control group received instruction on visual analysis aimed to improve their performance on tasks similar to those used in the BAS-Matrices subtest. This design allows both groups to have positive learning experiences and to maintain their interest during the study. Gains in the control task by the control group show that the group was motivated to learn, but they were learning a different task and so did not progress on the target task (in this case, multiplicative reasoning). Although there was a significant correlation between performance on the BAS-Matrices subtest and on the multiplicative reasoning problems in the previous study, this correlation was not sufficiently high (r = .4) to lead to an expectation of improvement on the multiplicative reasoning task by the children in the control group.

Assessments. The pretest and posttests consisted of 12 multiplicative reasoning problems; in six problems, the product was unknown, in three the number of groups was unknown, and in the remaining three the number of groups was unknown. Six addition and subtraction items were included to ensure that the children would not be simply repeating correspondence actions in an unreflective manner. If they did so, their performance in the addition and subtraction problems would decay from pretest to the posttests. The children were also given the BAS-Matrices task in order to assess whether the control group had a positive learning experience. This intervention is not analyzed here, except for a brief mention of the relevant results.

Intervention. Both interventions consisted of two teaching sessions, administered individually by a researcher who used oral and signed language with all the deaf children. In each session, the children solved five multiplication (product unknown) and five division problems; the type of problem was varied within the sessions so that, at the end of both sessions, the children had solved five problems where the size of the groups was unknown and five where the number of groups was unknown. The strategy that they were asked to use to solve these problems was always to
construct a representation of the situation, using manipulatives to represent the objects mentioned in the problems, and to set them in the appropriate form of one-to-many correspondence. For example, one multiplication problem was: 3 vans are bringing tables to the school; each van is carrying 4 tables; how many tables are they bringing to school? The children were given cut-out figures of vans and some cubes, and were encouraged to show what the problem had indicated. They received further directives, as required (e.g., if the child made the correspondence for one lorry and stopped, the experimenter suggested showing the correspondence for the other vans, too). For division problems, the same strategy was to be implemented but the question was not about the product: for example, a boy has 12 marbles; he wants to put the same number inside each of these 2 bags; how many marbles will go in each bag? The children had cut out circles to represent the bags and used the cubes to represent the marbles. The use of these simple manipulatives to represent different objects at different times during the sessions caused neither the hearing nor the deaf children any difficulty.

Results

At pretest, the BAS scores did not differ significantly between the hearing and deaf children ($p = .22$): the mean raw score for the deaf children was 6.63 (SD = 3.51) and for the hearing children was 5.73 (SD = 2.12). Thus, the hearing children were younger but had similar performance on the BAS-Matrices task.

There were no significant differences between the intervention and control groups at pretest in the multiplicative reasoning problems, but both hearing and deaf children in the intervention group performed slightly better than those in the control group, so it was decided to control for pretest multiplicative reasoning scores.

Both hearing and deaf children in the intervention groups made quite clear progress from pre- to posttests. The means for the groups on the pretest and both posttests are presented in Table 2.

To test the effects of the intervention, an analysis of covariance, controlling for pretest scores on the multiplicative reasoning task, was carried out. There were two between-participants factors, Intervention versus Control and Hearing versus Deaf, and one within-participants factor, Immediate versus Delayed post-test. The dependent measure was the number of problems solved correctly. This analysis showed a significant effect of intervention ($F_{1,53} = 24.50; p < .001$) and no significant overall difference between the hearing and deaf children. However, there was a significant interaction between hearing status, intervention group, and testing occasion. At the delayed posttest, the difference between the hearing and deaf children was significant ($B = 2.61; t = 2.19; p = .03$) and, for the deaf children, the difference between the intervention and control groups was no longer significant. The difference between the immediate and delayed posttest was not significant either for the hearing or for the deaf children.

The deaf children in the control group unexpectedly showed improvement at the delayed posttest, when their mean actually surpassed that for the hearing control group children.

The effect size of the intervention (Cohen’s $d$) for the deaf children at immediate posttest was 1.1 SD, which is large. At delayed posttest, a smaller effect size was observed: 0.30 SD.

There were no negative effects of this sustained practice of problem solving by means of correspondence on the addition and subtraction problems: in fact, the intervention group showed a small but significant progress across testing occasions, which presumably resulted from greater exposure to word problems during the intervention. The intervention

<table>
<thead>
<tr>
<th>Intervention group</th>
<th>Group</th>
<th>Pretest</th>
<th>Immediate posttest</th>
<th>Delayed posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
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<td>2.68</td>
<td>2.94</td>
</tr>
<tr>
<td></td>
<td>Deaf</td>
<td>1.29</td>
<td>1.92</td>
<td>3.71</td>
</tr>
<tr>
<td>Intervention</td>
<td>Hearing</td>
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<td>7.08</td>
<td>7.49</td>
</tr>
<tr>
<td></td>
<td>Deaf</td>
<td>1.92</td>
<td>5.96</td>
<td>4.88</td>
</tr>
</tbody>
</table>
group performed significantly better than the control group on both posttests ($F_{1,55} = 9.05; p = .004$). This suggests that the children used one-to-many correspondence after reflecting on the nature of the problems and did not simply repeat what they had been taught without any thinking.

Finally, it is worth mentioning that the control group showed a significant improvement in their performance on the BAS-Matrices subtest. This is interpreted as showing that they continued to be motivated to participate in the study and had a positive learning experience.

**Conclusions and Discussion**

When hearing children start school, many already understand two types of multiplicative reasoning concepts: (1) the idea of composite unit and (2) how they can use correspondences between variables to solve multiplicative reasoning problems. Previous research has highlighted the importance of these concepts for mathematics achievement in school.

Theoretically, it is argued that the idea of composite units helps children understand numeration and measurement systems (Behr et al., 1994). Steffe and his colleagues (Steffe, 1994; Olive & Steffe, 2002) have shown that it is possible to improve children’s performance in numeration system and measurement problems by helping them construct this idea of composite units. Park and Nunes (2001) showed that helping children learn to use correspondence procedures to solve multiplication problems leads to significantly better problem-solving performance in multiplication problems than helping them implement repeated addition procedures.

Nunes et al. (2007) have further demonstrated the significance of both types of concept for children’s attainment in mathematics in school: a measure of their understanding of composite units and correspondence obtained at school entry is a significant and specific predictor of their mathematics attainment more than 1 year later, even after controlling for general cognitive ability, specific knowledge of numbers at school entry, and the children’s performance on a working memory task. Hearing children who have some grasp of multiplicative relations at school entry are much better placed to learn the concepts that they will be taught in school than those who do not have this understanding.

Deaf children under-perform in comparison to their hearing peers in multiplicative reasoning at the beginning of their primary school years, and many do not display this informal knowledge of correspondences. However, it appears that this is a performance discrepancy rather than a competence discrepancy: a brief intervention significantly improved their performance and brought it to the same level of hearing children’s performance when these were matched for cognitive ability. Although the hearing children were younger, they also profited significantly from this intervention, and their gains were more stable than those displayed by the deaf children. The fact that both hearing and deaf children profited from the intervention described in this study can be seen as good news for schools where deaf children are integrated in mainstream: activities that benefit deaf children also benefit hearing children. So teachers can confidently include such activities in the curriculum.

It is only possible to speculate about the reasons why the deaf children’s gains in the multiplicative reasoning tasks were less stable than those attained by the hearing children, although we cannot know the answer for certain. As suggested in the introduction, hearing children are exposed to a richer problem-solving environment in the mathematics classroom than deaf children (Kelly et al. 2003); so, the hearing children would continue to have opportunities to solve arithmetic problems and thus would continue to improve, whereas the opportunities for the deaf children to do so might be more restricted. A second factor may have been the short-term nature of the intervention: the children had only two teaching sessions, which lasted about 20–25 min each, and solved a total of 10 problems. It is actually remarkable that with such a brief intervention they could show such positive gains. It is possible that their gains would be more stable with a longer intervention. Finally, it should be remembered that interventions aimed to help children understand logical relations are difficult to develop and not always successful (Inhelder, Sinclair, & Bovet, 1974; Siegler & Stern, 1998).

This intervention used methods that had already been developed for children at risk for difficulties in
learning mathematics (Nunes et al., 2007). It was established in that study, and in another subsequent one (Bryant, Nunes, Evans, Bell, & Burman, 2008), that what the children have at their disposal to model the problem significantly influences their performance in multiplicative reasoning problems. For example, if the children only have materials to represent one variable in the problem, they find it more difficult to solve the multiplicative reasoning questions than if they have distinct manipulatives to represent each of the variables. In the teaching intervention in this study, the children were offered materials to represent both variables so that they could actually implement these correspondences with the materials. The manipulatives offered to them were limited and they had to use the same things to solve different problems: for example, they used cut-out circles to represent children in one problem and plates in another problem. This did not cause difficulty and may have helped them focus on the scheme of correspondence rather than on the actual objects, thereby facilitating abstraction.

Gravemeijer (1997) has pointed out that one of the difficulties of division, and perhaps even more so in situations where the division is not exact, is that children need to keep track of what the different numbers in a problem mean. For example, in the problem “there are 34 bottles to be arranged in boxes that take exactly 6 bottles each; how many boxes do we need to pack the bottles?”, the children could become easily confused about what is what if they have only one set of materials (e.g., small cubes) to work with. They would need to form groups of six cubes, with each group corresponding to one box, and then there would be two cubes left. How should they interpret this group of two: does it correspond to a box or not? If the children have manipulatives that help them represent both variables, the meaning of the numbers should be clear. The transition from representation with manipulatives to representation of numbers could make use of tables in order to help the children keep track of which variable is represented by which numbers: the children could represent in one column the number of boxes and in the second the number of bottles in each box. This approach was explored by Streefland (1985) and was tested more systematically by Kaput and West (1994) with hearing children. Kaput and West used a computer environment where the correspondences created by the children using icons (e.g., figures of animals and umbrellas) were presented next to a table that showed the number of animals in a column headed by an icon for the animals and the number of umbrellas in a column headed by an icon for the umbrellas. Their study shows positive results from a comparison between their intervention group and a class that was taught about multiplicative reasoning without deploying this coordination between iconic and numerical representation. It was also used by Nunes (2004) with deaf children who were older, as part of a larger intervention program, and was successful.

This study has two main implications for the education of deaf children. First, deaf children can profit from teaching that helps them develop their use of the logic of correspondences to solve multiplicative problems. This is possible from the time they start school: there is no need to wait until they start receiving instruction about multiplication and division in school. In view of the importance of multiplicative reasoning in different aspects of mathematics learning, including understanding place value and measurement, it would be of great value to offer deaf children these experiences from the time that they start school. Second, teaching deaf children about the use of the logic of correspondences to solve multiplication and division problems should be more extensive than what was provided in this study. This brief intervention showed positive results but there was a drop in performance after 2 weeks without further teaching. It is likely that further teaching, spread over more weeks, would result in a more stable acquisition of multiplicative reasoning by deaf children, providing them with a good basis for learning mathematics in school.

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