Errata: Convergence Analysis of Evolutionary Algorithms That Are Based on the Paradigm of Information Geometry

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Four lines below Equation (7), page 682, it must correctly read:

$$\mathbf{Q} = -\frac{1}{2}\mathbf{H}_f(\boldsymbol{x}) = -\frac{1}{2}\nabla\nabla^{\mathrm{T}} f(\boldsymbol{x}).$$

In Equation (27), page 685, the logarithms are missing, it must correctly read:

$$I_{ij}(\boldsymbol{\theta}) = \int \frac{\partial \ln p(\boldsymbol{x}|\boldsymbol{\theta})}{\partial \theta_i}\frac{\partial \ln p(\boldsymbol{x}|\boldsymbol{\theta})}{\partial \theta_j} p(\boldsymbol{x}|\boldsymbol{\theta})\,\mathrm{d}^N\boldsymbol{x} = -\int \frac{\partial^2 \ln p(\boldsymbol{x}|\boldsymbol{\theta})}{\partial\theta_i \partial\theta_j} p(\boldsymbol{x}|\boldsymbol{\theta})\,\mathrm{d}^N\boldsymbol{x} \tag{27}$$

On page 687, below Equation (36), the derivations leading to Equation (40) must be corrected[1] (the final result in Equation (41) does not change by this correction):

Treating the **C**-related part in (35) using $\frac{\partial C_{ab}}{\partial C_{cd}} = \frac{1}{2}\left(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}\right)$ (here, symmetry of **C** must be taken into account) yields

$$\begin{aligned} I_{(\alpha_1\alpha_2),(\beta_1\beta_2)} &= \frac{1}{2}\sum_{k,l,m,n} C^{-1}_{kl}\frac{\partial C_{lm}}{\partial C_{\alpha_1\alpha_2}} C^{-1}_{mn}\frac{\partial C_{nk}}{\partial C_{\beta_1\beta_2}} \\ &= \frac{1}{8}\sum_{k,l,m,n} C^{-1}_{kl}(\delta_{l\alpha_1}\delta_{m\alpha_2} + \delta_{l\alpha_2}\delta_{m\alpha_1})C^{-1}_{mn}(\delta_{n\beta_1}\delta_{k\beta_2} + \delta_{n\beta_2}\delta_{k\beta_1}). \end{aligned} \tag{37}$$

Thus, one gets for the **C**-related part of $\boldsymbol{\theta}$ (taking the symmetry of $\mathbf{C}^{-1}$ into account)

$$\mathbf{C}: \quad I_{(\alpha_1\alpha_2),(\beta_1\beta_2)} = \frac{1}{4}\left(C^{-1}_{\alpha_1\beta_1}C^{-1}_{\alpha_2\beta_2} + C^{-1}_{\alpha_1\beta_2}C^{-1}_{\alpha_2\beta_1}\right). \tag{38}$$

The non-numbered equation below Equation (40), page 687, must be adopted accordingly:

The correctness of

$$\mathbf{C}: \quad I^{-1}_{(\alpha_1\alpha_2),(\beta_1\beta_2)} = 2C_{\alpha_1\beta_2}C_{\beta_1\alpha_2} \tag{40}$$

[1]The author is grateful to Zhenhua Li for pointing out this mistake to be corrected here.

is proven directly by checking

$$\sum_{\beta_1,\beta_2} I^{-1}_{(\alpha_1\alpha_2),(\beta_1\beta_2)}\, I_{(\beta_1\beta_2),(\gamma_1\gamma_2)} = \frac{1}{2}\left(\delta_{\alpha_1\gamma_2}\delta_{\alpha_2\gamma_1} + \delta_{\alpha_1\gamma_1}\delta_{\alpha_2\gamma_2}\right).$$

On page 694, the derivation of Equation (83) must be corrected (again without consequences for the result in Equation (84)):

$$\begin{aligned}
\frac{\partial\sigma_f}{\partial C_{mn}} &= \frac{1}{2}\frac{1}{\sigma_f}\frac{\partial}{\partial C_{mn}}\left(\sum_{i,j,k,l}(a_i - 2\overline{x}_k Q_{ki})C_{ij}(a_j - 2\overline{x}_l Q_{lj}) + 2Q_{ij}C_{jk}Q_{kl}C_{li}\right)\\
&= \frac{1}{2}\frac{1}{\sigma_f}\sum_{i,j,k,l}\Bigg((a_i - 2\overline{x}_k Q_{ki})\frac{1}{2}(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})(a_j - 2\overline{x}_l Q_{lj})\\
&\qquad + Q_{ij}(\delta_{jm}\delta_{kn} + \delta_{jn}\delta_{km})Q_{kl}C_{li} + Q_{ij}C_{jk}Q_{kl}(\delta_{lm}\delta_{in} + \delta_{ln}\delta_{im})\Bigg)\\
&= \frac{1}{2}\frac{1}{\sigma_f}\sum_{k,l}\left((a_m - 2\overline{x}_k Q_{km})(a_n - 2\overline{x}_l Q_{ln}) + 4Q_{mk}C_{kl}Q_{ln}\right) \qquad (83)
\end{aligned}$$