

## **Boosting**

### **Foundations and Algorithms**

## **Adaptive Computation and Machine Learning**

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# **Boosting**

## **Foundations and Algorithms**

**Robert E. Schapire**  
**Yoav Freund**

**The MIT Press**  
**Cambridge, Massachusetts**  
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*To our families*

On the cover: A randomized depiction of the potential function  $\Phi_t(s)$  used in the boost-by-majority algorithm, as given in equation (13.30). Each pixel, identified with an integer pair  $(t, s)$ , was randomly colored blue with probability  $\Phi_t(s)$ , and was otherwise colored yellow (with colors inverted where lettering appears). The round  $t$  runs horizontally from  $T = 1225$  at the far left down to 0 at the far right, and position  $s$  runs vertically from  $-225$  at the top to 35 at the bottom. An edge of  $\gamma = 0.06$  was used. [Cover design by Molly Seamans and the authors.]

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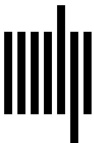
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