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# Visual Cortex and Deep Networks

## Learning Invariant Representations

By: Tomaso A. Poggio, Fabio Anselmi

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**Visual Cortex and Deep Networks**  
**Learning Invariant Representations**

**Tomaso A. Poggio**  
**Fabio Anselmi**

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