Experiments on the Effects of Reynolds Number and Advance Ratio on the Unfolding of Disorganization in Low-Speed Underwater Propulsors With Vibrating Blades

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1 Introduction

The world is becoming acoustically noisier. For example, due to the rapid rise in shipping, the low-frequency ambient acoustic noise in the ocean is increasing at an average rate of 0.5 dB per year [1]. Such noise occurs at high Reynolds numbers whose hallmark is seeming disorganization that is randomness which is thought to be statistical and not deterministic.

Experiments in turbulent boundary layers in the momentum thickness Reynolds number range of 500–17,500 have shown that organized hairpin vortex structures populate the boundary layer when viewed in orthogonal cross-stream planes one of them being slanted at 45 deg to the flow direction [2–6]. Therefore, organizations can remain masked in seeming disorganization unless viewed in appropriate orientations. In a recent dynamical analysis of such turbulent boundary layers, the experimentally determined spatiotemporal distributions of several state variables have been reproduced with remarkable accuracy but only at transitional low Reynolds numbers so far [7]. Furthermore, it was possible to reproduce accurately the dermal denticles and riblets of sharks [8] as devices for control of lateral diffusion of vorticities (called µ-control). This work indicates that: (1) the spatiotemporal evolution of the flow at least at low transitional Reynolds numbers is deterministic and not random and (2) lateral control of vorticity diffusion organizes the boundary layer turbulence at least in the wall-layers.

Therefore, in the context of a propulsor before we march to higher Reynolds numbers, first we need to pinpoint exactly when and how disorganization in thrust emerges after laminar to turbulence transition has already taken place. To simplify, in a propulsor, we consider temporal growth rather than spatiotemporal growth of the flow. We also explore devices that control lateral (spatial) diffusion of vorticity in lifting surfaces in vogue in nature and engineering.

Assuming a propulsor to be a point source of acoustic radiation related to the time derivative (′) of fluctuating (′) pressure (p′), we write p′ ∝ (F′/A), where F of is the fluctuating component of thrust F, and A is the cross-sectional surface area. To reduce error, instead of time derivative, we map temporal history (F′(t)), where t is the time. In this work, we look for the emergence of disorganization in F at low Reynolds numbers and advance ratios and the properties of their control.

Nonlinear time series analyses have led to chaos predictions [9]. Such approaches are useful when the underlying determinism cannot be extracted from the noise, and the initial condition is unknown. However, since chaotic phenomena are basically deterministic although complex, if the system is oscillatory and if we have the option of starting from the pristine (unchaotic) initial condition, then it may be possible to uncover the route to thrust disorganization. This is the approach we take in this work on an underwater rotating propulsor.

To understand the measurements, one could fully solve the Navier–Stokes equations at low Reynolds numbers, or carry out turbulence modeling. Instead, we have taken the dynamical...
approach, because the propulsor is oscillatory. Furthermore, the nonlinearly coupled blades and waves tend to be self-regulating. These are key features of many fluid dynamic systems [7,10]. We hope that these features would offer preferential properties to the flow thereby simplifying our search for organization. However, in dynamical analysis considerable insight of the flow mechanism and acknowledged key parameter ratios and of the interactions between the main and control oscillators are needed. For these reasons in engineering, dynamical analysis is not routinely used. However, in view of ours and others recent successes in oscillatory propulsion [7,10–12], here, we use the dynamical approach. We had success even in low Reynolds number turbulent boundary layers, where the oscillatory nature of production is masked in seeming randomness [7]. Although the following fluid dynamic example is distant, we are encouraged to notice that dynamical analysis considerable insight of the flow mechanism and knowledge of key parameter ratios and of the interactions between the blade vibration in otherwise steady rotational blades and the steady wings of an aircraft.

Below, we describe the propulsor and explain how the blade vibration is simulated. We also describe the propulsor blade surface and drag calculation. We give the measurements and models of time-averaged and temporal thrust. The relationship of the time-averaged thrust to the amplitude of blade pitch vibration angle \( \theta \) is examined and modeled. The relationship between the time-averaged and temporal thrust is explored. Temporal thrust is also modeled. We explore the deterministic nature of the unfolding disorganization in temporal thrust. We draw an analogy between the blade vibration in otherwise steady rotational blades and the unstable propulsion mechanisms of the pectoral fins of penguins that undergo simultaneous orthogonal roll and pitch oscillations, where lift forces are enhanced due to the formation of leading edge vortices \([11,22]\). Finally, we show the common empirical relationship of the ratio of the boundary-layer control fence spacing to the fin/wing chord in unsteady whale flippers and in the steady wings of an aircraft.

2 Description of the Propulsor

This is a large-diameter, low-speed propulsor (Fig. 1). The propulsor has six untwisted blades of small aspect ratio (span/chord = 3) \([11,22]\). The pitch angle is variable. The propulsor is hung from a six-component load cell on a carriage in a tow tank. Measurements of forces, power, current, blade position, rotational frequency, and tow speed are recorded. Current is normalized with a time-mean value. The propulsor blades were smooth, or roughened using a sand-grain roughness \((60 \text{ grit})\), and a boundary layer fence (Fig. 1) was also used to make the flow chordwise in each otherwise smooth-walled blade. The fence is larger than the boundary layer thickness, but the roughness height is similar to the boundary layer thickness near the leading edges of the fins. The bare body drag versus speed was measured \((D = 8.243 U^{1.88})\), where \(D/N\) is the drag, and \(U\) (m/s) is the tow speed; the exponent is close to 2.0 (the theoretical value), and the bare body drag was removed from thrust measurements.

The blades have a National Advisory Committee for Aeronautics 0012-34 nominal section with a rounded leading edge and a rounded trailing edge (multiple layers of polyurethane coatings)

In Fig. 2 of Ref. [19], the sound pressure levels versus frequency due to commercial ships and frigates are compared with those of fish and sea state 3. Figure 45 of Ref. [23] shows schematically the three unsteady force levels versus frequency in a typical propulsor behind a cylinder. They are: (1) turbulence ingestion from any upstream body which is a broad band source; (2) blade tonals which show up as discrete spikes in the spectrum due to wake cutting if there is any upstream stator; and (3) vibration of trailing edges which shows up as a narrow band at higher frequencies. In the present laboratory experimental work, there is a small hub, but there is no upstream body, and there is no upstream stator either. The speeds are low, and we do not expect any trailing edge vibration. Hence, the present propulsor experiences no external perturbations apart from mechanical vibration of carriage.

In Fig. 1 Six-bladed propulsor with boundary layer fence in each blade, where \( c \) is the chord, and \( \lambda \) is the spacing from the hub
were applied to thicken and protect the trailing edge). The planform is an abstract penguin wing [11,22]. The blades are made of urethane of Shore hardness A20–A30, which is not as hard as metal. The chord, inner radius, outer radius, and planform area are 0.075 m, 0.0638 m, 0.22255 m, and 0.0104 m²/fin, respectively. The blades are hinged at a distance of ε/3 from the leading edge [11,22] to allow efficient coupling between spinning (which has the same axis as rolling oscillation of pectoral fins of penguins) and pitch oscillations.

The thrust coefficient was calculated using the planform area of the blades. With a total of six blades, the total planform area is 0.0624 m². A typical oscillating time trace of thrust is shown in Fig. 2. As the roll motor turns, the pitch motor applies a correction to maintain the amplitude of the pitch oscillation selected (see Ref. [14] on how roll and pitch motors can be made mechanically independent). This simulates blade vibration coupled to hub rotation.

3 Time-Averaged Thrust Measurements and Modeling

The variations in the measurements of the thrust coefficient with the effective pitch angle (EPA) when the fin surfaces are smooth, sand-grain roughened, or contains boundary layer fences (Fig. 1) are shown in Figs. 3(a)–3(c). They are overlaid in Fig. 3(d). The laminar-to-turbulence transition locations on the fins are influenced by sand-grain roughening and fencing, and they could affect the location of boundary layer separation if the boundary layer does separate. These variations are accounted for by deducting 5 deg from the effective pitch angle in the sand-grain roughened data and deducting 10 deg from the fence data. The angles 5 deg and 10 deg are determined by trial for best fit. The overlaid data provided in Fig. 3(e) show better collapse than in Fig. 3(d), meaning that all variables are being correctly scaled between different blade surface flows.

Figure 3(e) shows a branching of the trends into modes of a lower and a higher thrust coefficient past the EPA of about 10 deg. The coefficient of thrust is weakly and strongly dependent on changes in the effective pitch angle, in the lower and upper modes, respectively. Note that the (lift-pitch) graph of a single steady (nonflapping) fin of similar dimensions as the propulsor blade also departs from linearity at 10 deg and maintains an approximately flat stall up to 50 deg [14]. Remarkably, the single fin measurements in the stall range of 25–30 deg show a slight dip in lift and that the lower mode measurements of thrust in the propulsor in Figs. 3 and 4 have even such a reduction in thrust. Hence, in the lower mode, the blades act like that they are in a steady flow and no leading edge vortex (LEV) forms enhancing lift force unlike that in the flapping case [14] and in the higher thrust mode.

We suggest that insight into the effects of blade vibration in propulsors may come from lift trends of isolated blades of such propulsors subjected to flapping motions. The two modes of time-averaged thrust (Fig. 3(e)) have been modeled (Fig. 4) assuming that the blades vibrate in pitch and that, in the mode of higher thrust coefficient, the blade motion due to spin and pitch oscillation couples but that it does not couple in the other mode of lower thrust coefficient (Fig. 2). This is justified because the mode correlates accurately with the amplitude of the blade pitch vibration angle; the coupled mode is produced when θ₀ is increased from 20 deg to 30 deg and 45 deg.

It is assumed that, when the pitch angle is large making the blade vibration large, an LEV forms on each blade as in flapping fins [10,11]. In the coupled mode modeling, we use the quasi-steady assumption found to be applicable in flapping fin propulsion where the spatiotemporal angles, rather than their rate of change, determine thrust and where stall is delayed up to large angles of attack [10,11,14,22]. The LEV is accounted for by the extrapolated steady lift curve beyond steady-state stall.

We make use of our bio-inspired single-fin lift and drag measurements [14] with and without flapping for modeling the coupled and uncoupled modes [11] (Fig. 4). It is assumed that when the spinning and pitch vibration effects in the blade boundary layer do not couple, the single-fin lift and drag behavior (versus angle of attack) in the (nonflapping) steady case can be used to resolve the thrust produced once the effect of hub rotation on the pitch angle is accounted for. Conversely, when the propulsor blade boundary layer does couple with the pitch oscillation, the hydrodynamic behavior can be modeled by the flapping-fin lift and drag characteristics. In both cases, viscous and pressure drags of the blades are included.

The thrust generated in the propulsor is given by

\[ C_{Tj} = K(C_{l,j} \cos x - C_{d,j} \sin z) \]

where \( C_{Tj} \) is the coefficient of thrust of the propulsor, whereas \( C_{l,j} \) and \( C_{d,j} \) are the lift and drag coefficients of the single fin, \( j = 0 \) or \( 1 \) (\( j = 0 \) when the blade boundary layer is not coupled to the pitch oscillation, and \( j = 1 \) when the fin boundary layer is coupled to the pitch oscillation), and \( z \) is the effective blade pitch angle. Here, \( K \) (\( < 1 \)) is a factor that accounts for the interference effects of the hub and the fins.

Define rotational velocity \( \nu_r = 2\pi R_{0}\nu_p/(N/60) \), where \( N \) is the blade revolutions per minute, effective pitch angle \( x = x_p - x_a \), \( x_p \) is the geometric pitch angle, and blade angle \( x_a = \tan^{-1}(v_r/U_{\infty}) \). The total thrust coefficient \( C_T = 2F_c/(\rho U_{\infty}^2 A_{\text{planform}}) \), where \( F_c \) is the measured force (baseline hub-propulsor drag is removed) in the tow direction; and \( U_{\infty} \) total = \( \sqrt{U_{\infty}^2 + v_r^2} \).

Fifth-order polynomials were fitted to the single-fin lift and drag versus angle of attack \( z_a \) measurements for both flapping and nonflapping cases. Their slopes are not much different, but the stall behavior is different. The modeled thrust is included in Fig. 4. Recall that, in the roughened and smooth-wall fenced blade cases, the pitch angle in the model is decreased by 5 deg and 10 deg, respectively, to account for the transition delay. A value of \( K = 0.3 \) is used.

Some justification for the choice of \( K = 0.3 \) can be found in our measurement of hydrodynamic efficiency in single fins and in a cylinder to which six fins are appended—three at each end [13]. The former has an efficiency of 0.60, while the latter has an efficiency of 0.42 at 0.4 m/s, which drops to about 0.15 at 0.1 m/s. In addition to blade–blade and blade-hub interference, there is a low-Reynolds-number effect; the efficiency is independent of speed above 0.46 m/s. The parameter \( K \) acts like hydrodynamic efficiency.

4 Relationship Between Time-Averaged and Temporal Thrust

In the unfolding of temporal thrust disorganization, Fig. 5(a) shows the first departure from a limit cycle (LCO), namely, the

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Fig. 2 Example of thrust oscillations in the spinning propulsor. The solid line denotes the long-time-averaged thrust of 0.62 N (the bare body drag has been removed). Count speed = 0.2 m/s; rotational speed = 15 rpm.
appearance of phase jitter—an effect of the coupling of the main oscillator with an external oscillator. Below the values of $J$ and $Re_c$, of Fig. 5(a), the amplitude of the external oscillator was likely insufficient for phase jitter of the coupling to occur, so the map was a pure limit cycle. As $J$ and $Re_c$ are increased (Fig. 5(c)), the disorganization takes hold. The thrust attractors of patterns in Figs. 5(a) and 5(c) are marked in Figs. 5(b) and 5(d).

Figures 6–8 show three collages of the thrust-current maps for amplitude of pitch angle vibration of 30 deg for smooth (Fig. 6), roughened (Fig. 7), and smooth-walled fenced blades (Fig. 8). Lines of constant $Re_c$ connect maps with increasing $J$. For example, at the bottom of Fig. 6, for $Re_c = 7470.1$, as $J$ increases to 1.08, 1.22, 1.41, 1.67, 2.04, and 2.62, the limit cycles shift bodily along the thrust axis, increasing the disorganization. The shift occurs when the current reaches a cycle-minimum value. The trend is similar at other values of $Re_c$ and surface condition (and at $\theta_s = 20$ deg, 45 deg).

4.1 Synthesis of How Disorganization in Thrust Unfolds. Measuring disorganization by the standard deviation of $C_T$ ($C_T$),
Fig. 9 shows the trend for all boundary conditions (BC) and all Reₐ. In Fig. 9, some of the trends have large amplification suggesting the presence of an underlying instability after laminar to turbulence transition has occurred. Figure 10 gives the individual trends. In Figs. 10(a)–10(c), the distributions of (\(J - C_T\)) are separated based on wall BCs—smooth (a), rough (b), and fenced (c). The unfolding of the disorganization occurs earlier at lower Reynolds numbers. In the present data range (\(J < 4\)), the \(C_T\) values are not high at the higher Re. This means that the unfolding process being observed is viscosity dominated, and the experimental data cover a regime where the viscous domination is giving way to inertia effects.

When the blades are fenced, the tipward flow is restricted, and the flow becomes chordwise (see schematic in Fig. 12(d)). The fencing attenuates disorganization (Fig. 10(c)). The rough wall BC both allows tipward flow and energizes the near-wall flow, the former being similar to the smooth BC case. Control of the boundary layer streamline curvature controls disorganization. From a dynamical consideration (see below), smooth and rough walls act as natural flows, where there is no control of lateral diffusion of vorticity. In contrast, the fence acts as a device for controlling the lateral diffusion of vorticity. Fencing is an external impetus of body scale that is countering the natural body force of curvature.

The trends in Figs. 10(a)–10(c) are synthesized in Fig. 10(d). Before the map becomes a blurred LCO (Fig. 5(a)), at lower Re and \(J\), a closed LCO without jitter must have occurred. For a blurred LCO (see below), \(C_T < 0.03\), approximately. The jump in
that occurs in some instances is followed by a reduction in $\overline{C_T}$. The jump in $\overline{C_T}$ is an indication of the onset of a change in the pattern. The later reduction in $\overline{C_T}$ is an indication of bistable behavior (hysteresis).

The emergence of blurred LCO in phase maps, that is departure from LCO, is the first indication that the oscillation is no longer locked-in, and the condition for phase equal to 0 deg does not return any more. In other words, there is excess energy in the system. If destabilizing forces such as inertial forces increase further, such departures would also increase. Eventually, the nature of coupling of oscillations in orthogonal axes would also change and new input–output patterns may emerge.

Fig. 6 Collage of transitional-thrust trajectories in smooth blades showing the effects of advance ratio and Reynolds number. $\theta_0$: 30 deg. The colored lines connecting the plots show that $J$ decreases (left to right and down) for fixed values of $Re$. The pair of small orthogonal lines at the left margin of the top left of plot indicate 0.05 of unit coefficient of thrust (ordinate) and normalized current (abscissa).
The blurring is also an indication of mismatch of all higher derivatives of the state variable at the end of the cycle of oscillation. New maps of acoustic radiation can then be expected. In flapping fin oscillations, for example, that is common in large swimming animals, initially, the blurring is systematic, because the system indicates lock-in after eight cycles of oscillation [11]. This discussion suggests a path how disorganization in the propulsor may unfold.

Recall that in flapping fin propulsion in the range of \[ \frac{3558}{C_20} Re_c \leq 70.895, \] bistable thrust fluctuations appear over \[ 8902 \leq \Re_c \leq 13,154 \] and \[ 31.345 \leq \Re_c \leq 35.686 \text{ [11,13].} \] For theoretical modeling, see Ref. [10]. They indicate that the vortices shed from the pressure, and the suction sides of the vibrating blades have unequal absolute values of circulation. The order of inequality is switching in these two ranges of bistability between the pressure and the suction sides of the blades. Therefore, the root-mean-square thrust fluctuation graphs are measures of vortex–vortex interactions in the blade wakes. In other words, the thrust patterns in Figs. 6–8 are the histories of interactions of the wake vortices and the process appears to be organized. The dynamical modeling below suggests that this organization is temporally deterministic.

### 4.2 Modeling of Temporal Thrust

Two models are given in Fig. 11. The first is systemic (Eq. (1); Figs. 11(a) and 11(b)), and the second (Eq. (2); Fig. 11(f)) is fluid-structural. Both models include effects that are internal and external to the main oscillator.

An oscillator of state \( x(t) \) under external periodic forcing is modeled as

\[
\frac{d^2 x}{dt^2} - (1 - x^2) \left( \frac{dx}{dt} + B \sin(kt) \right) + x = \tau_o - Bk \cos(kt) \quad (1)
\]

where the diffusion within the oscillator is a power relationship that is also a periodic function of the state and is given by \((1 - x^2)(dx/dt + B \sin(kt))\). The external forcing that is orthogonal to the main (roll) oscillator is also oscillatory and has some offset; it is given by \( \tau_o - Bk \cos(kt) \), where \( t \) is the time. In Figs. 11(a) and 11(b), which shows the results of Eq. (1), the parameters are: \( B = 0.15, \tau_o = -0.75, k = 1.8, \) and \( n = 2 \) in (a) and 8 in (b).

In Fig. 11, two sample measurements of thrust trajectories (Figs. 11(c) and 11(d)) for smooth fins at low values of \( J \) and \( Re_c \) are compared with the systemic model (Figs. 11(a) and 11(b); Eq. (1)). The model in Eq. (1) has two effects—a nonlinear diffusion internal to the thrust oscillator state \( x(t) \) (left-hand terms) and an external forcing (right-hand terms, where the state variable \( x(t) \) does not appear). The oscillator diffusion exponent \( n \) becomes more dominant with increasing \( J \) and \( Re_c \). The cosinusoidal external oscillator, which does not change with \( n \), is orthogonal to the
Fence, $\theta_0 = 30$, $J$ decreases left to right (and down).

Fig. 8 Collage of transitional-thrust trajectories in fenced smooth-walled blades showing the effects of advance ratio and Reynolds number. $\theta_0$: 30 deg. The colored lines connecting the plots show $J$ decreasing, left to right and down, for fixed values of $Re_c$. The pair of orthogonal lines in top left plot indicate 0.05 of unit coefficient of thrust (ordinate) and normalized current (abscissa).

Fig. 9 Root-mean-square of thrust trace ($C_T$) versus $J$ for all data (all $Re_c$). $Re$ is $Re_c$. The $C_T$ values do not amplify much in the fenced case (dotted line, black fill) (see Fig. 10(c)).
main sinusoidal oscillator, which indicates its weaker strength but decisive role in control. The disorganization unfolds with increasing values of the power law exponent \( n \) of the diffusion (\( n \) from 2 in Fig. 11(a) to 8 in Fig. 11(b)). Hence, \( n \) is a measure of a change in patterns due to amplification of some yet unknown instability triggered by increasing values of \( J \) and \( \text{Re}_c \) (Figs. 6–8).

The trajectories for higher values of \( J \) and \( \text{Re}_c \) (Fig. 11(f)) were modeled using the damped oscillatory equation for a structure as follows:

\[
M\ddot{\phi} + c\dot{\phi} + k(\phi - \Omega_{\text{nom}}) = \tau_\phi
\]

(2)

where \( M \) is the moment of inertia, \( c \) is the damping, \( k \) is the stiffness, \( \Omega_{\text{nom}} \) is the attractor frequency (rad), \( \tau_\phi \) is the roll torque, \( \phi(t) \) is the roll rate, and \( \tau_\phi(\phi, U) \) comes from quasi-steady fin modeling [10,11], where the tow speed of 0.2 units is assumed to have 10\% noise. In (f) which shows the results of Eq. (2), \( M = 0.218 \), \( c = 0.1 \), \( \Omega_{\text{nom}} = 1/4\pi \), \( \phi \) is abscissa, and ordinate is thrust.

Figure 11(f) shows the fluid-structure model (Eq. (2)) compared with thrust measurements (Figs. 11(e) and 11(g)) at higher values of \( J \) and \( \text{Re}_c \), for fenced and roughened blades. The system is modeled as a damped oscillator. The cyclic bodily shift in thrust is reproduced. In the unfolding of disorganization, this model indicates the increasing roles: (1) the coupling of the fin boundary-layer with the oscillatory wake, (2) the transition location of the fin boundary layer, and (3) the oscillations in forward motion with increasing \( J \) and \( \text{Re}_c \). Recall that similarly, in bio-inspired flapping fin propulsion the shed wake vortices alter the incoming angle of attack, whereby the wake and the fin boundary layers become coupled [10–12].

For smooth blades in (c), \( J = 0.51 \), \( \text{Re}_c = 3735 \), effective angle of attack = 36 deg. For smooth blades in (d), \( J = 3.52 \), \( \text{Re}_c = 18,675 \), effective angle of attack = 25 deg. For fenced blades (e), \( J = 2.04 \), \( \text{Re}_c = 7470.1 \), and effective angle of attack = 30 deg. For roughened blades (g), \( J = 1.67 \), \( \text{Re}_c = 7470.1 \), and effective angle of attack = 32 deg.

Figures 6–8 show three effects on the unfolding of disorganization. In these three figures, the LCO patterns are “coat-hanger-like” when the blade is smooth but are less so when the blade is fenced or roughened. If two propulsor cases are compared, it is seen that their blade transition location also affects the unfolding of the disorganization, in addition to \( J \) and \( \text{Re}_c \).

5 Discussion

For a three-dimensional state variable \( w(t) \), the generalized spatiotemporal equation of motion is \( \frac{\partial w}{\partial t} = F + P \), where \( F \) is the body force, and \( P \) is the surface force. In vector notation, \( P = -\nabla p + \mu \nabla^2 w \), where \( \mu \) is absolute viscosity of the fluid, \( \nabla^2 \) is the Laplace operator \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \), and \( \mu \nabla^2 w \) is the viscous term.

The existence of many niche actuators in low Reynolds numbers in aquatic swimming animals suggests a relationship with the ratio of the destabilizing and stabilizing forces, and that the relationship is like a staircase with the stairs being hysteretic [10,13].
Here, we are looking for an analogous behavior in the case of conventional propulsors.

In this work, $3735 < \text{Re}_c < 37,500$ covers the transition from the domination of viscous forces $\rho \nabla^2 W$ over the inertial forces $(\rho D w/\text{Dt})$ to the domination of $(\rho D w/\text{Dt})$ over $(\rho \nabla^2 W)$; here, $\rho$ is the fluid density. For $\text{Re}_c \geq 14,940$, the disorganization ($C_T$) does not show amplification as at lower values of $\text{Re}_c$. As inertial forces increase, structural vibration modes may act as external perturbations to thrust. Such effects are modeled as external oscillators in Eq. (1). Blade vibration is such an external oscillator.

The amplification of $C_T$ in the baseline case is large over $3735 \leq \text{Re}_c \leq 11,205$, but, if $F = 0$ (fenced case when streamlines are not curved), the amplification of $C_T$ is attenuated (Fig. 10(c)). The useful result is that, at least at very low Reynolds numbers, the growth of disorganization in thrust can be delayed by the reduction in streamline curvature.

Fig. 11 Dynamical modeling of temporal transitional-thrust compared with sample measurements. Systemic models (a) and (b) are compared with sample measurements of thrust trajectories (c) and (d) for smooth blades at low values of $J$ and $\text{Re}_c$. Fluid-structural model (f) is compared with thrust measurements (e) and (g) at higher values of $J$ and $\text{Re}_c$ for fenced and roughened blades, respectively.
Shark skins have numerous tiny, longitudinal ridges (also known as riblets) deeply buried in their turbulent boundary layers; these ridges are akin to wall-layer scale fences. When the ridges [19] are modeled [7,18] as devices for the control of the lateral diffusion of vorticity fluctuation (called \( \mu \)-control), the wanderings of the wall-layer streaks are remarkably stopped (Fig. B-4 in Ref. [18]).

Similarly, the propulsor fence, or the laterally arrayed protrusions of the whale flipper leading edge that produces streamwise vortices effectively acting as fences, can act as lateral diffusion control devices (boundary-layer scale \( \mu \)-control in Figs. 1 and 12). Such control is explicitly treated in the Stuart–Landau equation approach. The critical aspect in the \( \mu \)-control is the ratio \( L_c \) of the wavelength of the instability due to spanwise streamline curvature (fence spacing \( \lambda \)) to the mean flow wavelength (or the chord \( c \)). See schematic in Fig. 12(a). In the propulsor, the hub acts as a fence. The fence height is controlled by the mechanism of lateral instability. Note that quasi-steady assumptions apply to unsteady flapping propulsion [22]. The ratio \( \lambda/c \) is 2 in the propulsor (arbitrarily selected) and approximately 0.5–0.75 in the whale pictures in Fig. 12. It is 0.5 and 1.0 in the MiG-17 wings. All of these examples of wings have large aspect ratios (\( AR = \text{span}/\text{chord} \geq 3 \)). Hence, the control of lateral diffusion of vorticity fluctuation of body scale (curvature of streamlines) is the mechanism of the reduction in disorganization due to fencing. In an engineering context—steady or unsteady, if the aspect ratio is large, then, to accommodate several fences, the ratio \( L_c \) should be within 0.50–1.0.

To reduce the exposure of aquatic animals to ship radiated noise, vibration is sometimes isolated (as opposed to being introduced externally and minimally) to prevent its transmission from the propulsor to the hull (see Fig. 2 in Ref. [19]). But this remedy is not designed to organize the temporal propulsor thrust. This work, on the other hand, shows the disorganization in temporal thrust in a practical context as a deterministic process, albeit at low transitional Reynolds numbers, thus opening the door to rational control when transition cannot be delayed.

6 Conclusions

We have built a lowspeed underwater propulsor where the blades undergo synchronous pitching oscillations simulating blade vibration, while the hub is spinning. Three types of blades are used: smooth, or roughened, or smooth-walled and fenced. The advance ratios and Reynolds numbers are transitional: \( 0.51 \leq J \leq 4.89 \text{ and } 3.75 \times 10^3 \leq Re \leq 3.75 \times 10^4 \).

We find that large amplitude pitch vibrations act similar to flapping fin propulsion of penguins. We have examined the emergence of flow disorganization by examining the departure from LCO in a novel attempt at flow control of the oscillatory flow in this propulsor in order to gain some understanding of the general principles of unsteady flows.

Animals having a diverse array of propulsive actuators swim in the above range of Reynolds numbers. Theoretical oscillatory wake modeling and olivocerebellar control have similar self-regulating mechanisms in swimming animals [10–14]. For cruising, animals maintain LCO for optimal swimming. They maintain the Strouhal number of oscillation in a narrow range indicating resonant tuning [24,25]. Conversely, departure from LCO is costly and animals in their niche of low Reynolds numbers have evolved strategies for control of disorganization.

In this work, we have taken a similar approach to arrive at an internally consistent new method of analysis of temporal thrust measurements in a propulsor. Boundary layer fences are found to reduce the departure from LCO. The leading edge tubercles in whale fins and at least some aircraft wings of similar aspect ratios have fence spacing to chord ratios in the range 0.5–1.0. The suggestion is that in both nature and engineering, similar principles of control of disorganization at least at low Reynolds numbers are found to be beneficial when unsteadiness is present.

The leading edges of whale flippers have several tubercles that are similar to a boundary layer fence. The dermal denticles of shark skins have numerous tiny, longitudinal ridges (also known

**Fig. 12** The lateral diffusion control ratio \( L_c = \lambda/c \) (ratio controlling lateral diffusion of vorticity [7,18]). (a) Humpbacked whale flipper (photograph courtesy of Giles Breton). (b) Humpbacked whale in Atlantic; aspect ratio = 15 (photograph courtesy of Giles Breton). (c) MiG-17; aspect ratio = 3.7 (photograph courtesy of Balon Greyjoy). In both steady and unsteady wings, 0.50 ≤ \( \lambda/c \) ≤ 1.00. (d) Schematic of a streamline on a large aspect ratio wing and hydrofoil showing amplified boundary-layer instability of wavelengths \( \lambda \) and \( c \) of spanwise and chordwise flows in the unfenced smooth-wall case.
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Nomenclature

- \( c \) = damping
- \( c_f \) = fin chord
- \( C_{Dj} \) = drag coefficient of single fin
- \( C_{Lj} \) = lift coefficient of single fin
- \( C_T \) = total thrust coefficient
- \( C_T \) = coefficient of thrust of propulsor
- \( C_F \) = standard deviation of \( C_T \)
- \( D(N) \) = drag
- \( \text{deg} \) = degree
- \( f \) = frequency of oscillation
- \( F \) = body force
- \( F_G \) = thrust, measured force in the tow direction
- \( J \) = advance ratio
- \( k \) = stiffness
- \( K \) = factor accounting for interference effects of hub and blades
- \( L_n \) = ratio of fence spacing \( z \) to chord \( c \)
- \( M \) = moment of inertia
- \( n \) = oscillator diffusion exponent
- \( N \) = fin revolutions per minute
- \( P \) = surface force
- \( p' \) = fluctuating pressure
- \( r_{\text{root}} \) = radii at blade root
- \( r_{\text{tip}} \) = radii at blade tip
- \( R_{\text{avg}} \) = average fin radius
- \( R_e \) = Reynolds number based on chord length
- \( t \) = time
- \( U \) = tow speed
- \( U_{\infty} \) = tow velocity
- \( \alpha \) = effective fin pitch angle
- \( \gamma \) = angle of attack
- \( \theta_{\text{avg}} \) = amplitude of blade pitch vibration angle
- \( \lambda \) = fence spacing
- \( \mu \) = absolute viscosity
- \( \nu \) = rotational velocity
- \( \rho \) = fluid density
- \( \tau_\phi \) = roll torque
- \( \nu \) = kinematic viscosity of water
- \( \dot{\phi}(t) \) = roll rate
- \( \Omega_{\text{noon}} \) = attractor frequency (rad)

References