Neutrality of Forestry Income Taxation and Inheritable Tax Exemptions for Timber Capital

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ABSTRACT. This article examines the neutrality of income taxation in the so-called Austrian sector of the economy. By focusing on the forestry sector, it is shown that certain exemption practices of yield taxation can be modified to restore the efficiency of the taxation. Earlier research on the production efficiency relating to different capital income tax systems in the Austrian sector shows that yield taxes based on realized income lead to inefficient allocation of resources between the Austrian and the ordinary sector of the economy when pretax distortions are present. The models used in the earlier tax literature do not, however, differentiate between taxing capital income and taxing the initial capital under an individual occupancy of an asset. In this article, by explicitly separating timber capital taxation from timber capital income taxation, a modified yield taxation scheme is presented, which gradually corrects an initially distorted allocation of resources between forests and other assets in the economy.

Additional Key Words: Bequest, capital income taxes, inheritable tax liabilities, production efficiency.

Over the years, interest in the efficiency and neutrality aspects of different income taxes in the Austrian sector has generated a fair number of research papers. Neutrality was first defined with respect to the effects within the Austrian sector itself (Bentick 1980, Klemperer 1982). Consequently, when looked at within a single sector of the economy, it can be argued that yield taxes (taxes on the realized income from an Austrian asset) are neutral. For example, a forestry yield tax based on timber sales income is neutral and does not lead to allocative distortions within the forestry sector alone. A tax is sector-wise neutral if it retains the efficient pretax allocation of assets within the sector.

However, Kovenock and Rothschild (1983), and Kovenock (1986) expanded the point of view to cover the entire economy. They showed how a sector-wise neutral tax system may be inefficient when viewed from the viewpoint of the complete tax system. Specifically, Kovenock (1986) used a rotation model, and later Ovaskainen (1992) used a two-period intertemporal model, to show that if a capital income tax is levied at a certain rate in the rest of the economy and a yield tax on realized income is levied at the same rate in the Austrian sector, the social return to marginal resources used in the Austrian sector is less than the social return to resources used in the rest of the economy. This implies nonneutrality of the taxation. Instead, it is taxes that account for the growth of the asset (such as accrual income tax or ad valorem property tax) applied in the Austrian sector that would be able to bring about an economy-level neutrality. In practice, however, growth-based taxes are ruled out by legislators primarily because of their administrative difficulties in acquiring the needed periodical land-site level inventory data.

Koskela and Ollikainen (1997) examined the optimal design of forest and capital income taxes in an economy.
with an ordinary and an Austrian sector. Their model allowed for future timber price uncertainty and a preset budget constraint for the government. They suggested that under this second-best condition, tax neutrality may not be a desirable goal. In terms of optimality of taxation, the present study applies the so-called first-best solution. Therefore, the government is allowed to choose the best tax regime without a budget constraint. In contrast, the so-called second-best optimal forest income taxation minimizes the excess burden from taxes at a given level of tax revenues (besides Koskela and Ollikainen 1997, see, e.g., Gamponia and Mendelsohn 1987).

The yield tax forms analyzed in the previous literature do not, however, separate timber income taxation from timber capital taxation in the sense that when timber is harvested or sold, forest yield taxes applied in the models do not distinguish between timber accrued during and prior to an individual occupancy of a forest asset. This distinction is crucial in determining whether the government is taxing timber capital or related income, and is normally addressed in the tax policies of governments applying yield taxation in forestry.

In this article, a model is introduced that incorporates features applied in forest taxation to distinguish timber capital and capital income. More specifically, an n-period intertemporal tax model with a bequest motive is presented including partially exempted timber capital and upper limits on the annual tax deductions.

The article shows that an inheritable timber capital exemption scheme can be designed so that forestry yield taxation becomes nondistortionary across the whole economy. Within the n-period model, the privately held forest assets converge to what is socially optimal. Contrary to the growth-based taxes, which also can be neutral, a modified yield tax system with the introduced features would not require annual or periodical land-site information on the market value of a forest property.

Administrative simplicity and neutral appearance are typical arguments often put forward by legislators to support yield tax regime in forestry, even if by now it is known that the neutrality argument may not hold within the broader system of capital income taxation. However, in many countries, forestry income taxation is still organized around the idea of notional, or so-called cadastral, income taxation, in which the true income is imputed based on certain land productivity characteristics (Grayson 1993). As forestry taxation regimes are revised in some countries, yield taxes based on the realized income may become a preferred option. In this system, timber sales revenues form the tax base. Yield taxes are in operation, for instance in Sweden and Finland (Nilsson et al. 1994, Tax Laws 1995). New Zealand has also set up a yield taxation system (OECD 1994). In the United States, several states have long been applying yield taxes in forestry (Davis and Johnson 1987). Furthermore, in some of the former socialist countries in Eastern and Central Europe, where the state-owned forest land holdings are being (re-) privatized (OECD 1994), there is a discussion on the best forest tax systems. It seems, as described by Amacher (1997), that implementation of forest taxes is an acute problem in many parts of the world today. Even if it may be socially preferable—for carbon sequestration, recreation, or other externalities—to allow larger forest assets than implied by a straightforward intersectoral neutrality, policymakers should be aware of how allocation of resources between sectors is affected by tax parameters aimed at dealing with capital exemptions of the forest assets.

### The Model: N-Periods with a Bequest Motive

The model setup includes a forest owning consumer endowed with (e.g., by inheritance, purchase, or donation) two types of holdings: forest wealth plus financial assets. The consumer lives n periods and derives utility from consumption and the bequest, which is passed on to the next generation. To maximize lifetime utility, the individual allocates the assets between the periodic consumptions and the bequest. In the beginning of each period, he adjusts both the forest holding and the financial assets to support the optimal periodic consumption. The decision is constrained by the given growth function of the forest and the given interest rate on the financial markets. At the end of the nth period the consumer passes away and leaves a bequest to his heirs.

The main purpose of this paper is to study the optimality of forest taxation within the system of capital income taxation. The consumer in the model is thus not assumed to sell his labor, and is therefore not earning wage income. Furthermore, for simplicity and to keep the analysis more tractable, the model abstracts from timber price or other uncertainties (for a forest taxation model with price uncertainties, see, e.g., Koskela 1989, Koskela and Ollikainen 1998).

The consumer derives utility from the periodic consumptions \( C_t \), and the bequest \( B \). Consumer preferences are described by the following intertemporally additive utility function:

\[
U = \sum_{t=1}^{n} \frac{1}{(1 + \rho)^t} u(C_t) + \frac{1}{(1 + \rho)^n} u(B) \tag{1}
\]

where \( \rho \) is the consumer’s time preference factor. Thus, the utility function \( U \) describes the discounted utility from consumption and the bequest motive. There is no present time utility from the bequest. \( U \) is assumed to be well-behaved with all the regular differentiability and concavity properties.

Next, the size of the initial forest holding (measured by the size of the standing timber stock) is denoted by \( Q_0 \), and forest-holding size after the fellings at the beginning of each period by \( Q_t \). Correspondingly, the periodic financial

\[ \text{Amacher (1997)} \text{ contains an analysis of the stylized features used in forest tax literature.} \]

\[ \text{A yield tax is here understood as a realized income tax paid at a given percent of the net value of sold timber (c.f. Klempner 1976). In some countries, such as Finland and Sweden, this tax is referred to as the timber sales income taxation. In some states in the United States, yield taxes are based on publicly registered average stumpage rates (Wisconsin Dept. of Natural Resources 1996).} \]

\[ \text{The principles used in applying yield taxes as well as other forest taxes in the United States are discussed in detail in Boyd (1986). USDA (1995) demonstrates in detail the forest taxes in the federal income taxation.} \]
asset sizes are \( W_0 \) and \( W_i \); these assets are also adjusted in the beginning of each period. \( \pi \) and \( \pi^w \) are the general inflation rate and the nominal change rate of the timber price, respectively. The general and the timber prices are indexed to unity for the first period. Furthermore, let \( F(Q) \) be a concave growth function yielding the volume growth of the forest assets. Then, the constraints of the above utility function in terms of the periodic consumptions and the final bequest can be written as:

\[
C_i = Q_i + W_0 - Q_i + W_i \\
(1 + \pi)^{i-1} C_i = (1 + \pi^w)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i] \\
+ (1 + \pi) W_{i-1} - W_i \quad \text{for } i = 2, ..., n \\
(1 + \pi)^{n} B = (1 + \pi^w)^{n} [Q_n + F(Q_n)] \\
+ (1 + \pi) W_n
\]

(2)

For example, for \( i = 2 \), the budget constraint is:

\[
(1 + \pi) C_2 = (1 + \pi^w)(Q_1 + F(Q_1) - Q_2) + (1 + \pi) W_1 - W_2.
\]

In (2) the periodic fellings are given by the term \( [Q_{i-1} + F(Q_{i-1}) - Q_i] \), while the size of the bequest forest is given by \( [Q_n + F(Q_n)] \). The \( Q \)'s only assume nonnegative values, but the \( W \)'s can also be negative, in which case the landowner would be a borrower in a particular period. The price-changes in the forest sector are assumed to be constant over the periods. Also, the nominal interest rate \( r \) is expected to remain constant, and the individual is assumed to be able to borrow and lend as he likes at a fixed market rate of interest (i.e., perfect capital markets prevail).

The above model bears a resemblance to the lifecycle models used in public economics literature (see, e.g., Yaari 1964, Atkinson and Stiglitz 1980). What is essential is that the model accounts for individual "history" and "future" with respect to ownership of the incoming and outgoing wealth, consisting of financial and forest assets. In the following tax analysis, it is useful to account for the endowment and bequest (each consisting of two elements), when studying the efficiency properties of the modified yield tax systems with inheritable tax exemptions on the forest capital. More specifically, we want the model to be able to distinguish between tax incidence on the incoming forest wealth and the part of the timber stock that accrues during the lifetime of an individual forest ownership.

Conditions (2) define separate flows-of-funds equations for the periodic consumptions and the bequest. An intertemporal budget constraint can be derived from (2) by iterative substitution (starting by first solving for \( W_i \) from the first equation and then using it in the second):

\[
\sum_{i=1}^{n} \left( \frac{1 + \pi}{1 + r} \right)^{i-1} C_i + \left( \frac{1 + \pi}{1 + r} \right)^{n} B = \\
W_0 + \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + r} \right)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i] \\
+ \left( \frac{1 + \pi^w}{1 + r} \right)^{n} [Q_n + F(Q_n)]
\]

(3)

According to (3), the forest owner's consumption pattern must be such that the present value of consumption plus the total bequest (left-hand side of the equation) is equal to his endowment of financial assets (\( W_0 \)) plus the present value of the periodic fellings and the final forest holding.

Optimal forest holding levels (optimal levels of \( Q_i \)) determine the optimal periodic timber supplies. In turn, the optimal forest holding levels are implicitly defined by the following first-order conditions derived by maximizing (1) subject to (3) in terms of \( Q_i \) (\( i = 1, \ldots, n \)):

\[
F'(Q_i) + \pi^w + \pi^w F'(Q_i) = r
\]

(4)

where \( F'(Q_i) \) is the derivative of the growth function at a certain level of the timber stock (defined by the period \( i \)), and can be understood as the periodic percentage growth of the timber stock. From (4) the optimum cutting policy can be read: timber supply in any period is determined by the level of the timber stock at which the sum of the percentage growth of the physical stock, the percentage change of the timber price, and their interaction term equals the market rate of interest. The interaction term represents the incremental value of the percentage growth of the standing timber. Intertemporally the left-hand side of (4) is the marginal cost of cutting more timber in period \( i \), while the right-hand side—the interest rate—is the marginal revenue of cutting more in period \( i \).

The optimal cutting rule (4) also implies that within the model the harvesting decision of the individual is independent of the consumption decisions, and thus the Fisherian separation theorem holds. It is also noteworthy that by setting \( \pi^w = 0 \) (i.e., assuming constant timber prices, Equation (4) is reduced to the standard first-order condition describing the optimal use of an Austrian-sector asset).

**Derivation of the Nonneutrality Property of the Yield Taxation when Forest Owners Have a Bequest Motive**

In this section the above model is used to demonstrate nonneutrality properties of a regular, unmodified yield tax. The economy-wide approach to the question is adopted. Accordingly, a forestry income tax is neutral only if it is such in a system where capital income taxes exist also for other, nonforestry incomes. In this, the results based on the introduced intertemporal model with the bequest motive do conform to those derived earlier with rotation models (Kovenock 1986, Gamponia and Mendelsohn 1987), as well as with two-period intertemporal models with future

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5 An alternative approach to model the bequest motive plus the incoming and outgoing wealth would be the application of the overlapping-generations model (OLG). In forestry, the overlapping-generations model has been applied by Löfgren (1990) and Hultkrantz (1992), for example. In the present case, social utility is not derived from the utility functions of the consumer-forest owners, but can be thought of as exogeneously given.
price certainty (Ovaskainen 1992) and uncertainty (Koskela and Ollikainen 1997).

Yield tax based on realized timber sales earnings is a proportional tax levied on the (net) revenues from timber sales within a certain period (usually a year). Let $t_f$ be the proportional yield tax rate, and $r$ the tax rate on financial asset earnings (both tax rates are assumed to take values between 0 and 1). Then, to describe yield tax in forestry, the constraints (2) of the utility maximization problem in terms of the periodic consumptions and the final bequest can be rewritten as:

$$C_i = (1-t_f)(Q_0 - Q_i) + W_0 - W_i$$

$$(1 + r)^{i-1} C_i =$$

$$(1 - t_f)(1 + r^n)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i]$$

$$+ (1 + r^*) W_{i-1} - W_i$$

for $i: 2, ..., n$  

$$(5)$$

$$(1 + r^*) B = (1 - t_f)(1 + r^n)^n [Q_n + F(Q_n)]$$

$$+ (1 + r^*) W_n$$

(6)

The multiperiod budget constraint (3) of the utility maximizing consumer can be rewritten as:

$$\sum_{i=1}^{n} \left( \frac{1 + \pi}{1 + r^*} \right)^{i-1} C_i + \left( \frac{1 + \pi}{1 + r^*} \right)^n B = W_0$$

$$+ (1-t_f)\sum_{i=1}^{n} \left( \frac{1 + \pi}{1 + r^*} \right)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i]$$

$$+ (1-t_f)\left( \frac{1 + \pi}{1 + r^*} \right)^n [Q_n + F(Q_n)]$$

$$\sum_{i=1}^{n} \left( \frac{1 + \pi}{1 + r^*} \right)^{i-1} C_i + \left( \frac{1 + \pi}{1 + r^*} \right)^n B = W_0$$

$$(6)$$

In (5) and (6), $r^*$ equals $r(1 - t)$, i.e., the tax-adjusted interest rate. The values of the periodic harvests are taxed with the yield tax rate $t_f$. Also, the bequeathed timber stock at the end of the nth period is depreciated to account for the tax, and to obtain the value of the inherited forest to the heirs prior to possible inheritance taxes. The consumer is viewing the value of the bequest from the heirs' viewpoint, and is setting the time for valuation at his own death.

From (1) and (6), the following first-order condition for the utility maximization in terms of $Q_i$ can be derived to describe the optimal level of the forest holding when the yield tax is applied:

$$F'(Q_i) + \pi^w + \pi^n F'(Q_i) = r^*$$

$$\sum_{i=1}^{n} \left( \frac{1 + \pi}{1 + r^*} \right)^{i-1} C_i + \left( \frac{1 + \pi}{1 + r^*} \right)^n B = W_0$$

According to the rule given by (7), timber supply is determined so that the incremental value of the remaining timber stock equals the tax-adjusted interest rate. Since $r^*$ is smaller than $r$ (when $t$ is larger than 0), and since $F(Q)$ is concave, the optimal timber stock implied by (7) is larger than that implied by (4). The result implies that a regular timber yield tax system is unable to restore an optimal allocation of capital resources between forestry and the rest of the economy under pre-existing distortions. It is clear that this holds regardless of the size of the yield tax rate, since $t_f$ does not appear in (7). Rather, the taxation system leads to over-investments in forestry in the sense that the privately optimal timber stock levels are socially too large.

**Yield Tax System with Capital Exemption**

**Why a Capital Exemption?**

The above analysis showed how the system of capital income tax combined with the standard yield tax leads to nonneutrality from the viewpoint of the entire economy. Despite this fact, yield taxes are currently a fairly common forestry tax regime applied in several countries. There is little doubt that it is the administrative feasibility and the seemingly neutral appearance of the yield tax system that have contributed to its success in many countries. The tax schemes that could be used to effectively "neutralize" taxation—ad valorem or accrual income taxation (growth taxation)—are impractical since they rely on periodic land-site level inventory data.

In reality, yield taxation is not applied in its standard form, but certain extensions, or special features, have been added to the system mainly to avoid double taxation or taxation of the timber capital. The conditions under which modified yield taxes can lead to efficient allocation of resources are studied in this section. It will be shown that tax exemptions granted on timber capital can be designed so that the model leads to the neutrality property without implied requirements on forest holding inventory data. The analysis of the different exemption systems is allowed by the structure of the introduced tax model distinguishing between the incoming and outcoming forest (and other) wealth.

If, in 1 year, an individual purchases a site with timber stock, and decides to sell the entire timber stock in the next year, then under a simple yield tax system—and assuming that tax capitalization will not correct the situation—she will be taxed on the entire income based on the value of the initial stock plus 1 year's growth of the forest. This does not seem fair, because clearly it is not only the forestry income based on the growth of the trees that will be taxed, but also the initial timber capital. The yield tax is not to be imposed on the capital itself, but rather on the capital income. In the Austrian sector this means that the tax be imposed on the value of growth realized during an individual occupancy of the capital. To avoid taxing timber

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6 Inheritance taxes are not analyzed here and are not included in the model.

7 The impacts of tax exemptions for timber capital were analyzed before by Klemperer (1982) in the case of property taxation. He found that partial timber capital exemption could bring greater neutrality of property taxation with respect to different land uses.
capital, yield taxation is practiced in some countries with timber capital or so-called substance exemptions (Finland, Sweden), with reduced tax rates on harvests in excess of the sustainable capacity of the land site (Austria, Germany), or by allowing a generally lowered tax base for capital gains income (e.g., the United States prior to 1987). The timber capital exemptions are specific tax allowances, or deductions, which are based on the value of the initial forest (e.g., at the time of purchase of the timberland) under an individual ownership. A percentage of the value of the initial forest is deductible when the tax base is determined for sold timber. Usually when applied, the exemption can be used cumulatively over several years.

Besides the attempt to target the capital income only, the described allowances can be justified because an individual occupancy of a forest asset may be substantially shorter than the lifecycle of the forest stand. However, over a very long period, it can be argued that only the forest growth will be taxed even under a simple yield taxation without any exemptions on the timber capital. This will be demonstrated shortly. Therefore, the longer the time horizon of forestry operations, the more justified yield taxation without exemptions becomes. Since institutional or corporate owners are held to outlive individuals, this perhaps explains why legislators consider it fair to let nonindustrial private forest owners have higher tax allowance rates on an initial forest than institutional forest owners. (After 1986, in U.S. federal taxation, both ordinary income and long-term gains are taxed at the same rates for corporate and noncorporate taxpayers; USDA 1995.)

To make the above point clearer and to illustrate the allowance system, consider the total amount of yield taxes collected from an individual timberland over a period of time, say $N$ years. Let the tax exemption percentage of the value of the timber capital be denoted by $\alpha (0 < \alpha < 1)$ and for notational simplicity the present value of yearly timber cuttings by $x_i$. Then the total amount of yield taxes, $T$, collected by the government over $N$ years from that particular forest site becomes:

$$T = \max \left[ 0, t^f \left( \sum_{i=1}^{N} x_i - \alpha Q_0 \right) \right] \quad (8)$$

where $t^f$ is the yield taxation rate and $Q_0$ is the value of the initial timber capital. The structure of (8) rules out negative taxes which would arise if $\alpha Q_0 > \sum^N x_i$, that is, if the value of the tax-free timber stock exceeds the value of the sold timber in $N$ years. In Equation (8), for simplicity, exemption is assumed to be applied against taxes only once. Assuming, again for simplicity, constant unitary timber prices (unit price $= 1$ for the entire period) and using the relationship between timber growth and fellings through the initial and final forest stands,

$$Q_0 + \sum^N F(Q_i) \equiv \sum^N x_i + Q_N$$

(the value of the initial timber capital plus the value of the accumulated cuttings plus the value of the final timber capital), expression (8) can be rewritten as follows:

$$T = \max \left[ 0, t^f \left( \sum_{i=1}^{N} F(Q_i) + (1-\alpha)Q_0 - Q_N \right) \right] \quad (9)$$

From (9) it can be seen that if the forest ownership begins with a nonforested land, i.e., the value of the initial timber stock, $Q_0$, is zero, then a forest owner, who finally at period $N$ sells all of his timber capital ($Q_N = 0$), is only taxed on the accrued forest growth $\sum^N F(Q_i)$. Alternatively, if the forest ownership begins with $Q_0 > 0$, and ends with $Q_N = 0$, then it is only when $\alpha = 1$ that the forest owner will pay taxes exclusively on the growth. Whenever $\alpha < 1$ and $Q_0 > 0$, part of the initial forest wealth ($1 - \alpha$), will be taxed. Expression (9) then also helps to see that in the long run, as $N$ approaches infinity (assuming monotonically increasing growth function and $Q_N = 0$), the yield-tax base approaches the total growth of the forest, as the relative share of $(1-\alpha)Q_0$ declines in the tax base. This last observation gives some justification to the argument that in the long run, only the forest growth will be taxed under yield taxation, even without any exemption for the timber capital ($\alpha = 0$).

Next, the capital exemption property is built into the tax model from the previous section. Various tax schemes are discussed, and a system will be presented which gradually restores the tax-distorted allocation of forest and nonforest assets in the economy. In this tax system, a forest owner can use annual tax deductions without a cumulative ceiling; if, at the end of the individual’s occupancy, the cumulative value of these deductions falls short of the initial tax exemption, the forest owner is also entitled to pass the leftover exemption to his heirs, or if the cumulative value of the annual tax reductions exceeds the initial tax exemption, then the government collects the balance from the bequest.

**Tax Schemes with Capital Exemption**

Three different capital exemption schemes are studied next. In the first one, an individual forest owner’s capital exemption rights are accumulated over his lifetime to benefit his heirs. In this case the tax exemption does not affect the annual yield tax rate but will be used to value the bequest. In terms of (5) above, this means that the periodic consumption constraints remain the same, but the bequest constraint will be changed as follows:

$$(1 + \pi)^n B = (1 - t^f)(1 + \pi^n)^n \left[ Q_n + F(Q_n) \right] + t^f \alpha Q_0 + (1 + \pi^n)W_n$$

(10)
where, again, $t'$ is the yield tax rate, and $\alpha (0 < \alpha < 1)$ is the exemption rate on the value of the initial timber capital. The bequest constraint now has the term $t/\alpha Q_0$ (= value of the initial tax exemption) added to it. The forest owner pays standard yield taxes on sold timber over $n$ periods, but transfers the redemption rights to the tax allowances to his heirs.

The first-order conditions for the utility maximization problem derived under the above tax scheme will be identical to (7). Therefore, we conclude that the tax exemption system described above corresponds to (with respect to the consumers' behavior) the standard yield taxation, and thus cannot be used to neutralize the tax system and restore the intersectoral efficiency.

The second scheme is one in which, instead of using the exemption to value the bequest, the exemption is used gradually over several years. In this scheme, forest owners can cumulatively deduct from yearly timber sales the value of the initially exempt timber capital. In fact, the exemption system is traditionally practiced in this way in several countries today. In the United States, for example, capital outlays for reforestation at harvest time can be deducted either at harvest or gradually over time during the next rotation.

When the annual deductions are applied in practice there usually are, however, upper limits on the percentage of the value of yearly cuttings that can be sold tax free.9 Over the years, the landowner uses the annual tax reductions (since a percentage of the income can be deducted from the taxable income base, the overall effect is a reduction in the annual tax rate) until their cumulative value equals the value of the capital exemption based on the initial timber stock value. More formally, if the annual tax-free percentage is denoted by $\beta (0 < \beta < 1)$ and the value of annual timber sales is denoted by $x_i$, then the forest owner can use the annual tax breaks until $\beta \sum x_i = \alpha Q_0$, where $\alpha$ is the above described exempted percentage of the initial timber stock value $Q_0$. In case any unused exemption could not be passed on to the bequest, the constraint structure of the maximization problem changes in terms of the periodic consumptions, but remains as in (5) in terms of the bequest constraint. The periodic consumption constraints are the following:

$$C_i = (1 - t')(Q_0 - Q_i) + W_0 - W_i$$

$$(1 + \pi)^{-1} C_i = (1 - t')(1 + \pi^w)^{-1}$$

$$[Q_{i-1} + F(Q_{i-1}) - Q_i] + (1 + r^s) W_{i-1} - W_i$$

for $i: 2, \ldots, n$

(11)

where $t' = t'/(1 - \beta)$ is the reduced tax rate due to the periodic deductions (when $\beta > 0$ it follows that $t' < t'$; e.g., if $t' = 0.25$, and $\beta = 0.4$, then $t' = 0.15$). In the described yield tax scheme, only the effective periodic yield tax rate is different (smaller) as compared to the "pure" yield tax case. Therefore, this scheme corresponds to the standard yield taxation in regard to the first-order condition of the utility maximization problem—except a temporary, one-period change, at the time when the effective tax rate shifts from $t'$ to $t$ as the total tax exemption on $\alpha Q_0$ is used up. Thus, the scheme does not seem to be usable to restore the allocative distortion caused by the taxation.

The third capital exemption scheme combines the two cases described above. Now, forest owners are assumed to use the annual tax reliefs and also pass on a possible remaining part of the timber capital exemption to the next generation. As indicated before, this is how an exemption system normally works in practice. Furthermore, this tax regime is modified here so that it allows forest owners to exceed the initial exemption with the cumulatively used annual tax deductions. In other words, tax liabilities are also allowed. When the value of the bequest is determined the initial tax exemption and the annual deductions are balanced. It is shown that this type of a yield tax model can be used in such a way that an initially inefficient allocation between forest and other assets will be corrected.

In the modified yield taxation, forest owners use the reduced annual or periodic tax rate $t'$ instead of the higher $t$ on the periodic timber sales proceeds even beyond the limit set by $\alpha Q_0$, in fact, for their entire lifetime. Then, to value the bequest, the cumulative value of the periodic deductions, $\beta \sum x_i (x_i$ are the periodic values of timber sales) is compared to the value of the initially tax-exempt timber stock, $\alpha Q_0$. If $\alpha Q_0 > \beta \sum x_i$, the government will reimburse the forest owner's bequest $t'(\alpha Q_0 - \beta \sum x_i)$ worth of unclaimed tax allowances. If, however, $\alpha Q_0 < \beta \sum x_i$, then the forest owner's heirs will be required to make the government $t'(\beta \sum x_i - \alpha Q_0)$ worth of outstanding tax liabilities.

Under the above-described yield taxation system, both the periodic consumption constraints and the bequest constraint will be changed as compared to the constraint system (5). The forest owner will maximize (1) subject to:

$$C_i = (1 - t')(Q_0 - Q_i) + W_0 - W_i$$

$$(1 + \pi)^{-1} C_i = (1 - t')(1 + \pi^w)^{-1}$$

$$[Q_{i-1} + F(Q_{i-1}) - Q_i]$$

$$+ (1 + r^s) W_{i-1} - W_i$$

for $i: 2, \ldots, n$

$$(1 + \pi)^n B = (1 - t')(1 + \pi^w)^n [Q_o + F(Q_o)] + (1 + r^s) W_n$$

$$+ t' \left\{ \alpha Q_0 - \beta \sum_{i=1}^{n} (1 + \pi^w)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i] \right\}$$

(12)

where $t' = t'(1 - \beta)$, i.e., the reduced annual income tax rate. The bottom line in (12),

9 For example, in Sweden a maximum of 50% of the annual timber sales can be sold tax-free, and in Finland the corresponding percentage is 40 (Nilsson et al. 1994, Tax Laws 1995).
\[ t_f \left[ \alpha Q_0 - \beta \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + \rho^*} \right)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i] \right] \]

is the final tax adjustment term including the initial tax exemption \( Q_0 \) and the annual tax deductions \( \beta \Sigma x_i \). Based on (12) the multiperiod constraint can be written as follows:

\[
W_0 + \left( 1 - \tau_f \right) \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + \rho^*} \right)^{i-1} \left[ Q_{i-1} + F(Q_{i-1}) - Q_i \right] + \left( 1 - \tau_f \right) \left( \frac{1 + \pi^w}{1 + \rho^*} \right)^{n} \left[ Q_n + F(Q_n) \right] + t_f \frac{1}{(1 + \rho^*)^n} \left[ \alpha Q_0 - \beta \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + \rho^*} \right)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i] \right]
\]

(13)

Using (1) and (13), it can be shown that the first-order conditions (FOC) for the utility maximization in terms of \( Q_i \) are (see Appendix):

\[
F'(Q_i) + \pi^w + \pi^w F'(Q_i) = r^* \left[ 1 + \beta \frac{t_f}{1 - t_f} \left( 1 - \frac{1}{(1 + \rho^*)^{i-1}} \right) \right]^{-1}
\]

(14)

Under the described taxation, the forest owners adjust their timber stocks, \( Q_i \) (\( i = 1, \ldots, n \)), so that the above condition is met. Two important points need to be made concerning (14). First, this condition is a dynamic one. When \( i = 1 \), the opportunity cost of leaving one unit of the forest assets to grow is the smallest. This opportunity cost increases until \( i = n \), at which point the term in the brace on the right-hand side (rhs) of (14) collapses to 1. Therefore, the FOC converges to

\[
F'(Q_i) + \pi^w + \pi^w F'(Q_i) = r^* \left[ 1 + \beta \frac{t_f}{1 - t_f} \right].
\]

The land owners under this tax system will gradually decrease the size of their forest assets to one implied by the FOC at the limit \( i = n \), since the growth function \( F(Q) \) was assumed to be concave. Second, the rhs in (14) is always larger than \( r^* \), which was the opportunity cost of not selling an additional unit of timber in (7), the FOC in the case of the standard yield tax system. The modified yield tax system with inheritable capital exemptions and tax liabilities gradually reduces the forest assets away from the level implied by standard yield tax system.

When the tax-adjusted interest rate is decomposed as \( r^* = r - rt \), the FOC in (14) can be expressed at the limit \( i = n \) as follows:

\[
F'(Q_i) + \pi^w + \pi^w F'(Q_i) = (r - rt) + (r - rt) \beta \frac{t_f}{1 - t_f}.
\]

(15)

Condition (15) is useful as it can easily be related to condition (4)—the general, undistorted optimality condition without the intervention of an income tax system in forestry. The modified yield tax per se is inefficient in allocating the economy-wide resources between forest and other assets whenever pre-tax distortions are present. However, (15) suggests that by setting \( \beta \), the annual tax deduction rate for timber sales income, and \( t_f \), the yield tax rate applied to forestry, in such a way that

\[
rt = (r - rt) \beta \frac{t_f}{1 - t_f}.
\]

the optimality condition reduces to (4) implying neutrality or allocative efficiency at the limit \( i = n \). The condition that guarantees this is given by the following expression:

\[
\beta = \left[ \frac{1 - t_f}{t_f} \right] \left[ \frac{t}{1 - t} \right]
\]

(16)

or alternatively expressed in terms of \( t_f \):

\[
t_f = \frac{t}{\beta(1 - t) + t}
\]

(17)

When (16) and (17) hold, the general FOC for the modified yield tax scheme can be expressed as follows (Appendix):

\[
F'(Q_i) + \pi^w + \pi^w F'(Q_i) = r \left[ 1 + \frac{t_f}{1 - t} \left[ 1 - \frac{1}{(1 + \rho^*)^{i-1}} \right] \right]^{-1}
\]

(18)

At \( i = n \) the rhs of (18) reduces to \( r \).

Condition (16) implies that when the yield tax rate equals the general capital income tax rate in the economy, the neutrality property of the yield taxation is obtained by setting \( \beta = 1 \). This means that there should be no annual or periodic limits put to the tax-free timber sales that the landowners are using to "redeem" the granted tax exemption. On the other hand, condition (17) implies that when \( \beta < 1 \), the yield tax rate should be set higher than the general capital income tax rate to obtain the optimality property. For example, if \( \beta \) assumes the value 0.67, and the capital income tax rate is 0.25, then the approximate neutral yield tax rate would be 0.33. In fact, the yield tax rate could be set high enough so as to imply suboptimal levels of forest assets at the limit.

Within the model framework, it is likely that intergenerational shifts in the implied optimal level of forest occur. However, the optimal level is always smaller than under the standard yield tax. This is seen from (18) by noticing that \( r^* = r(1 - t) \), the adjusted interest rate, equals

\[
r(1 + \frac{t}{1 - t})^{-1}.
\]
Table 1. Effects of yield taxation on the allocative efficiency of forest assets.

<table>
<thead>
<tr>
<th>Mode of taxation</th>
<th>Value of bequest</th>
<th>First-order condition</th>
<th>Forest size compared to social optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard yield taxation</td>
<td>((1 - t)(1 + \pi^w)^n [Q_n + F(Q_n)] + (1 + r^w)W_n)</td>
<td>(F'(Q_n) + \pi^w + \pi^w F'(Q_n) = r^*)</td>
<td>Larger</td>
</tr>
<tr>
<td>With capital exemption</td>
<td>((1 - t)(1 + \pi^w)^n [Q_n + F(Q_n)] + t/fQ_0 + (1 + r^w)W_n)</td>
<td>(F'(Q_n) + \pi^w + \pi^w F'(Q_n) = r^*)</td>
<td>Larger</td>
</tr>
<tr>
<td>Exemption used for bequest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noninheritable exemption</td>
<td>((1 - t)(1 + \pi^w)^n [Q_n + F(Q_n)] + (1 + r^w)W_n)</td>
<td>(F'(Q_n) + \pi^w + \pi^w F'(Q_n) = r^*)</td>
<td>Larger</td>
</tr>
<tr>
<td>Inheritable exemption and tax liabilities</td>
<td>((1 - t)(1 + \pi^w)^n [Q_n + F(Q_n)] + (1 + r^w)W_n)</td>
<td>(F'(Q_n) + \pi^w + \pi^w F'(Q_n) = r^*)</td>
<td>Larger</td>
</tr>
<tr>
<td></td>
<td>(\alpha Q_0 - \beta \sum_{i=1}^{n} (1 + \pi^w)^{i-1} [Q_{i-1} + F(Q_{i-1}) - Q_i])</td>
<td>(r \left{1 + \frac{t}{1-t} \left[1 - \frac{1}{(1 + r^w)^{n-1}}\right]\right}^{-1})</td>
<td>i &lt; n: converging from above</td>
</tr>
</tbody>
</table>

Table 1 summarizes the key results concerning the neutrality properties of different tax schemes analyzed in this article.

**Discussion**

A detailed analysis of the practical plausibility of the described modified tax system is beyond the scope of this article. However, some aspects of how realistic this kind of a tax regime would be in practice are briefly discussed. This is all the more important because the theoretical modifications in the tax model presented were based on certain applications of yield taxation used in practice.

Theoretically, based on the model framework used throughout the preceding analysis, the government’s interest with respect to the different tax regimes lies in the economy-wide question of optimal allocation of assets. In practice, however, the government is also interested in how well the different tax regimes can guarantee a steady and secured inflow of tax incomes. Since the modified tax regime allowed forest owners to accrue tax debts or liabilities, it seems that the scheme would need some sort of a built-in collateral system. Most naturally, the collateral could be in the form of the timber stock. Then, some regulations would be needed to prevent forest owners (these can be forest-owning dynasties) from abusing or downsizing the collateral. However, these regulations could easily interfere with standard forestry practices. Also theoretically, incorporating such regulations into the model could alter the optimization set-up, and lead to deviations from the presented results.

In some tax systems, like the U.S. or Canadian, taxes paid to one level of government, such as federal or state (provincial), may allow a complete offset or a partial deduction against taxes owed to another level of government, such as state or municipality. The presented model does not account for these offsets, and, therefore, the model may very well overstate forest owners’ responses in these cases.

For future work, to cope with the limitations of the model applied here, some approaches seem possible. The model could be generalized to include the case in which the landowner is also selling his labor. It would also be interesting to see how amenity or nontimber values integrated into the model would affect the efficiency of various tax regimes. Finally, one could try to build a model to study the links between different exemption systems and the time paths of revenues collected by the government. This could be based on a second-best situation where the government faces revenue constraints, and the exemption schemes could be combined with the use of an inheritance tax parameter.

**Literature Cited**


Appendix

The Lagrange function based on maximizing (1) subject to (13) in the text is:

\[
L = \sum_{i=1}^{n} \left( \frac{1}{1 + r^*} \right)^{i-1} u(C_i) + \frac{1}{1 + r^*} u(B) + \lambda \left\{ \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + r^*} \right)^{i-1} C_i + \left( \frac{1 + \pi^w}{1 + r^*} \right)^{n} B - W_0 \right\} - \left(1 - \tau^f\right) \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + r^*} \right)^{i-1} \left[ Q_{i-1} + F(Q_{i-1}) - Q_{i} \right] - t^f \left[ \frac{1 + \pi^w}{1 + r^*} \right]^n \left[ Q_n + F(Q_n) \right] - \tau^f \frac{1}{1 + r^*} \beta \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + r^*} \right)^{i-1} \left[ Q_{i-1} + F(Q_{i-1}) - Q_{i} \right] \}
\]

(A1)

where \( \lambda \) is the Lagrange multiplier, and the other symbols are as defined in the text. To find the optimal cutting-policy rule, the first-order conditions of the function are derived with respect to \( Q_i \) which is the timber stock size after the period \( i \) fellings.

Using \( 1 - t^f = 1 - \tau^f - t^f \beta \), the expression for the present value of the bequest forest can be decomposed as follows:

\[
(1 - \tau^f) \left( \frac{1 + \pi^w}{1 + r^*} \right)^{n} \left[ Q_n + F(Q_n) \right] = (1 - \tau^f) \left[ \frac{1 + \pi^w}{1 + r^*} \right]^n \left[ Q_n + F(Q_n) \right] - \frac{1 - \tau^f}{1 + r^*} \beta \left[ (1 + \pi^w)^{n} Q_n + F(Q_n) \right]
\]

(A2)

Next, substituting (A2) into (A1), we can write the derivative of the Lagrange function with respect to \( Q_i \) as:

\[
\frac{\partial L}{\partial Q_i} = \frac{\partial}{\partial Q_i} \left( \sum_{i=1}^{n} \left( \frac{1 + \pi^w}{1 + r^*} \right)^{i-1} C_i + \left( \frac{1 + \pi^w}{1 + r^*} \right)^{n} B - W_0 \right) - \frac{1 - \tau^f}{1 + r^*} \beta \left[ (1 + \pi^w)^{i} Q_i + F(Q_i) \right]
\]

(A3)

Expanding by \( (1 + r^*)^i / (1 + \pi^w)^{-i} \), (A3) can be written as:

\[
\frac{\partial L}{\partial Q_i} = -\lambda \left( \frac{1 + \pi^w}{1 + r^*} \right)^{-i} \frac{\partial}{\partial Q_i} \left( (1 - \tau^f) - r^* Q_i + \pi^w Q_i + (1 + \pi^w) F(Q_i) \right) - \frac{t^f \beta}{(1 + r^*)^{n-i}} \left[ \pi^w Q_i + (1 + \pi^w) F(Q_i) \right]
\]

(A4)

Taking the derivative and setting it equal to 0 gives:

\[
(1 - \tau^f) \left[ -r^* Q_i + \pi^w Q_i + (1 + \pi^w) F(Q_i) \right] - \frac{t^f \beta}{(1 + r^*)^{n-i}} \left[ \pi^w Q_i + (1 + \pi^w) F(Q_i) \right] = 0
\]

(A5)

and using \( 1 - t^f = 1 - \tau^f - t^f \beta \) again:

\[
(1 - \tau^f)(-r^*) + (1 - t^f)[\pi^w + (1 + \pi^w) F(Q_i)] + t^f \beta [\pi^w + (1 + \pi^w) F(Q_i)] - \frac{t^f \beta}{(1 + r^*)^{n-i}} \left[ \pi^w + (1 + \pi^w) F(Q_i) \right] = 0
\]

(A6)

which is:

\[
\left[ \pi^w + (1 + \pi^w) F(Q_i) \right] + \frac{t^f \beta}{(1 - t^f)} \left[ \pi^w + (1 + \pi^w) F(Q_i) \right] - \frac{t^f \beta}{(1 - t^f)(1 + r^*)^{n-i}} \left[ \pi^w + (1 + \pi^w) F(Q_i) \right] = \frac{(1 - \tau^f) r^*}{(1 - t^f)}
\]

(A7)
Collecting terms on the lefthand side (lhs) and rewriting the rhs of (A7):

\[
\left[ \pi^w + (1 + \pi^w)F'(Q_t) \right] + \frac{t^f \beta}{(1 - t^f)} \left[ \pi^w + (1 + \pi^w)F'(Q_t) \right] \left[ 1 - \frac{1}{(1 + r*)^{n-i}} \right] = r^* (1 + \beta \frac{t^f}{1 - t^f}) \\
\text{(A8)}
\]

Further collecting terms to reduce the lhs:

\[
\left[ \pi^w + (1 + \pi^w)F'(Q_t) \right] \left[ 1 + \frac{t^f \beta}{(1 - t^f)} \left[ 1 - \frac{1}{(1 + r*)^{n-i}} \right] \right] = r^* (1 + \beta \frac{t^f}{1 - t^f}) \\
\text{(A9)}
\]

Now upon dividing by the \{ \} term, (A9) becomes the condition (14) in text.
By setting

\[
\beta = \left[ \frac{1 - t^f}{t^f} \right] \left[ \frac{t}{1 - t} \right],
\]

the rhs of (A9) reduces to \( r \), the standard interest rate, and \( t^f / 1 - t^f \) on the lhs becomes \( t / 1 - t \), which then confirms expression (18) in text.