Abstract: Most financial asset returns exhibit volatility persistence. This article investigates this phenomenon in the context of daily returns in lumber futures markets. The volatility of lumber futures is found to vary with different phases of the United States-Canada softwood lumber trade dispute. Daily price volatility was the highest in the post-Softwood Lumber Agreement period (2001–2005), followed by the Softwood Lumber Agreement period (1996–2000) and the period with trade disputes and temporary tariffs (1992–1995). An inverse relationship between inventory levels and volatility is found. Further, the inventory effect on volatility varies across the different periods of the trade dispute. An inverse relation between time to delivery and volatility is also found. Moreover, it is shown that although volatility persistence exists in the lumber futures market, the time gap between the arrival of news to the markets and the delivery time of futures contracts is the fundamental variable in explaining volatility persistence.

Keywords: futures markets, lumber, trade dispute, theory of storage, volatility, volatility persistence

Lumber plays an important role in new residential construction. Because of its reliability and safety, more than 90% of homes in the United States are built with lumber. However, lumber prices fluctuate over time, causing firms engaged in producing, processing, marketing, or using lumber and lumber products to face price risk. To help to manage this risk due to spot price volatility, the Chicago Mercantile Exchange launched lumber futures contracts in 1969. This was the first financial tool for price protection of forest products.

Volatility of financial asset returns persists. High-volatility periods are apt to be followed by high-volatility periods and similarly for low-volatility periods. This pattern has been observed for returns to publicly traded stocks (Engle 2004) and for returns to futures contracts, both those written on financial assets (Li and Engle 1998) and those written on physical commodities (Pindyck 2001, 2004, Ng and Pirrong 1994).

Explanations for volatility persistence have been proposed from microstructure models. The process of market price reaction to information flows is argued to result in such persistence. See, for example, Kyle (1985) and Andersen and Bollerslev (1997). However, there are other reasons to expect volatility persistence in the case of futures contracts written on commodities. Aggregate physical inventories play a shock-absorbing role in commodity markets, implying that when physical inventories are large, the size of commodity futures price changes is small (see Thurman 1988, Williams and Wright 1991, Karali and Thurman 2009). Further, aggregate inventories evolve slowly. Thus, commodity futures volatility is characterized by phases of varying length, depending on the speed of inventory changes for the particular commodity.

A second factor influencing volatility in commodity futures is the time gap between when news arrives to the market and when the contract calls for delivery. The major futures exchanges trade several contracts for a commodity, differing only by delivery date. Near-term information shocks should have greater impact on contracts for near-term delivery than on contracts for farther-out delivery because of the smaller elasticities of supply and demand for shorter runs. The implications for the dependence of volatility on time to delivery can only be studied by analyzing multiple contracts that are simultaneously traded.

In the case of lumber, price volatility can be affected by supply factors, such as stumpage costs, railroad strikes or railcar shortages, and demand factors such as residential construction (Rucker et al. 2005), interest rates, and prices of substitutes such as steel or aluminum. Further, uncertainty due to softwood lumber trade disputes between the United States and Canada is another source of volatile lumber prices (Zhang and Sun 2001, Rucker et al. 2005). Lumber futures markets and their forecasting efficacy of spot prices as well as a private cash forward market for lumber are studied in the literature to determine optimal hedging of the price risk in this market (Oliveira et al. 1977, Deneckere et al. 1986, Manfredo and Sanders 2008). In the current study, the focus is shifted in a new direction to test the two theoretical implications mentioned above, inventory and time-to-delivery effects, in the lumber futures market. Karali and Thurman (2009) studied the effect of housing starts announcement surprises relative to Money Market Survey forecasts on lumber futures prices. They found statistically and economically significant announcement, inventory, and time-to-delivery effects and showed that the price response to observed information flows depends on inventories and time to delivery. This article extends this earlier work by studying the response of lumber futures prices to unobserved information flows. It is hypothesized that, as in the case of observed information, the price response to unobserved information flows should also depend on inventories and time to delivery. Because the sign of the unobserved shock is unknown, one is naturally led to
use the absolute value of log price changes, a commonly used measure for volatility in the literature. Further, the current article compares daily lumber futures volatility across different periods of the United States-Canada softwood lumber dispute. In particular, daily lumber futures prices from the Chicago Mercantile Exchange are analyzed from 1992 to 2005, defining volatility as the absolute value of log price changes over a day. Studying time-to-delivery effects requires use of data on all contracts traded on a given day. This, in turn, requires a recognition of the correlation among price observations from the same day, which are subject to common shocks. A generalized least-squares procedure similar to the one in Karali and Thurman (2009) is used to take this contemporaneous correlation into account, resulting in efficient use of futures price data and consistent standard errors of the estimates.

Daily lumber futures volatility is found to vary in different phases of the United States-Canada trade dispute. Daily volatility is higher in both the Softwood Lumber Agreement (SLA) period (1996–2000) and the post-SLA period (2001–2005) compared with the period with trade disputes and temporary tariffs (1992–1995). In addition, volatility in the post-SLA period is higher than the volatility in the SLA period. An inverse relationship between inventory levels and lumber futures volatility is found, as predicted by the theory of storage. As inventory levels become smaller, lumber futures contracts become more volatile. The relationship is both statistically and economically significant. Moreover, the inventory effect on volatility varies across different periods of the trade dispute. An inverse relation between time to delivery and volatility is also found, as suggested by Samuelson (1965). The closer the contract trade date is to the delivery time, the higher is price volatility. This result can be interpreted in standard microeconomic terms. Lumber supply and demand curves become more inelastic as time to delivery nears. Thus, shocks originating from either the supply or demand side of the market have a larger price impact as time to delivery nears. Further, it is found that although volatility persistence is statistically significant in the marginal distribution of lumber futures returns, much of that persistence can be explained by the dependence on time to delivery.

A Three-Period Storage Model

The simple, finite-horizon storage model presented by Williams and Wright (1991) is used to derive optimal storage rules via a social planner’s perspective. These storage rules are then used in a simulation study to investigate the relationship between expected price changes and inventories, which plays an important role in the empirical analysis of futures prices presented here.

Analytic Solution

The model has three periods. Because the solution is derived by backward induction, the third and last period is denoted by $T$, the second period is denoted by $T - 1$, and the first period is denoted by $T - 2$. A social planner tries to maximize the discounted expected social surplus by choosing optimal storage level in each period. Thus, the social planner’s optimal choice for storage in any period $t$ is determined by the first-order condition of the objective function,

$$V_i(A_i) = \max_{S_i} \left( \int_0^{A_i - S_i} P(q_i) \, dq - cS_i + (1 + r)^{-1} E_t \left[ V_{t+1}(y_{t+1} + S_t - S_{t+1}) \right] \right),$$

(1)

subject to $S_i \geq 0$. The variable $A_i$ is the total availability in period $t$ and is equal to $y_t + S_{t-1}$, $y_t$ is the total production in period $t$ and is equal to $\bar{y} + q_t$, where $q_t$ is a uniformly distributed random disturbance with mean zero and SD $\sigma$, $S_t$ is the storage level in period $t$, $P(q_t)$ is the inverse consumption demand defined as $y_t = \alpha + \beta q_t$, and $c$ and $r$ are the assumed constant cost of storage and interest rate per period. The optimal choice in period $t$ then satisfies

$$P_t + c = (1 + r)^{-1} E_t [P_{t+1}], \quad S_t > 0.$$  

P_t + c = (1 + r)^{-1} E_t [P_{t+1}], \quad S_t > 0. \quad (2)$$

The carry-out from the last period has no value because the world ends beyond that time. Therefore, carry-out in the last period is zero, and everything available in the last period is consumed; that is, $S_T = 0$. Optimal storage rules for the second period $T - 1$ and for the first period $T - 2$ are found as

$$S^*_T = \max \left\{ \frac{(1 + r)A_{t-1} - \bar{y}}{2 + r} + \frac{1 + r}{2 + r} \frac{c}{\beta} + \frac{r}{2 + r} \frac{\alpha}{\beta}, 0 \right\},$$

(3)

$$S^*_T = \max \left\{ \frac{(1 + r)A_{t-1} - \bar{y}}{2 + r} + \frac{1 + r}{2 + r} \frac{c}{\beta} + \frac{r}{2 + r} \frac{\alpha}{\beta}, 0 \right\}. \quad (4)$$
As seen from Equation 4, the optimal level of storage in the first period, \( T - 2 \), depends on current availability \( A_{T-3} \) and model parameters. Availability in period \( T - 2 \) equals the sum of supply in that period and carry-in from period \( T - 3 \). Formally,

\[
A_{T-2} = y_{T-2} + S_{T-3} = \bar{y} + v_{T-2} + S_{T-3}. \tag{5}
\]

The level of carry-in into the first period will affect optimal storage in this period, which, in turn, will affect optimal storage in the second period. Therefore, one should expect varying storage and price paths depending on initial carry-in levels.

**Simulation Results**

With a starting value of carry-in into the first period, \( S_{T-3} \), one can simulate the three-period model by drawing from the probability distribution of the random supply shock. The explicit storage rules derived in the previous section show how many units will be stored in each period, given availability in those periods. The amount stored in each period then will determine consumption and price in those periods. In this way, one obtains a path for each endogenous variable. When this process is repeated many times with the same initial values but with different draws of the random shock, one obtains a conditional frequency distribution for each variable in each period. For instance, for price, \( P_{i,t} \) is obtained with \( t = T - 2, T - 1, T \) and \( i = 1, 2, \ldots, n \), where \( n \) is the number of iterations.

This process is repeated for several values of initial carry-in, \( S_{T-3} \). For each value of \( S_{T-3} \),

\[
\left| \frac{1}{n} \sum_{j=1}^{n} (P_{T-2,i} - P_{T-3}) \right| \tag{6}
\]

is computed, where

\[
P_{T-3} = \left( \frac{1}{1 + r} \right) \left( \frac{1}{n} \sum_{j=1}^{n} P_{T-2,i} \right) - c, \tag{7}
\]

to represent \( E_{T-3}[P_{T-2} - P_{T-3}] \). Model parameters are chosen as in Williams and Wright (1991). The inverse consumption demand equation has an intercept of \( \alpha = 600 \) and slope of \( \beta = -5 \); that is, \( P_2 = 600 - 5q_2 \). Marginal physical storage cost, \( c \), is $2 per period. The interest rate, \( r \), is zero. The mean production level, \( \bar{y} \), is 100 units. The random disturbance \( v_2 \) lies between \(-15 \) and \(+15 \), that is, \( \sqrt{3}\sigma = 15 \), where \( \sigma \) is the SD of the random disturbance. The number of iterations, \( n \), is 100,000.

Figure 1a shows the relationship between the expected absolute price change from the initial period \( T - 3 \) to the first period \( T - 2 \) and the level of initial carry-in. As inventory levels become larger, the expected magnitude of price movement becomes smaller. Figure 1b shows how the variance of price movement declines with inventories. Thus, the absolute price changes seem to capture the variance pattern pretty well. This simulation result demonstrates one important conclusion: the assertion of the theory of storage that \( E_i[P_{t+1}] \) is decreasing in \( S_i \) also holds for price movements, \( E_i[P_{t+1} - P_t] \). This result is the main motivation behind the current study’s empirical analysis.

This relationship should also hold in commodity futures markets, which reveal the market’s expectation of future spot price changes. Further, the price response of a futures contract to a shock should be smaller when its delivery time is farther away due to greater elasticity of supply and demand curves over longer runs. These hypotheses can be tested in the following linear model of volatility:

\[
\ln F_t - \ln F_{t-1} = \alpha + f(S_t) + g(TTD_t) + \epsilon_t, \tag{8}
\]

where \( \ln F_t \) is the natural logarithm of the price of a futures contract \( 2 \) on day \( t \), \( S_t \) is the physical inventory level on day \( t \), \( f(S_t) \) is a function of inventories, \( TTD_t \) is time to delivery, the number of days remaining to contract expiration on day \( t \), and \( g(TTD_t) \) is a function of time to delivery. As an implication of the theory of storage and larger elasticities over longer runs, one should expect to see

\[
\partial(\ln F_t - \ln F_{t-1})/\partial S_t = \partial f(S_t)/\partial S_t < 0, \tag{9}
\]

\[
\partial(\ln F_t - \ln F_{t-1})/\partial TTD_t = \partial g(TTD_t)/\partial TTD_t < 0. \tag{9}
\]

**Measuring Inventory and Time-to-Delivery Effects on Volatility**

Lumber futures contracts are traded weekdays at the Chicago Mercantile Exchange between 9:00 am and 1:05 pm (US Central Time) in an open outcry trading pit. Daily settlement prices are used for analysis during the period when inventory data are available, resulting in a sample period of July 14, 1992–Nov. 15, 2005 with a total of 77 contracts. Lumber contracts expire every 2 months, and the delivery months are January, March, May, July, September, and November. The last trading day for any contract is the last business day before the 16th calendar day of the contract’s delivery month. On the Chicago Mercantile Exchange, a new contract is listed on the day after the front month expires. At any point in time, a total of seven contracts are listed, each with a different delivery date up to 14 months into the future. However, in the empirical analysis, the data set purchased from the Commodity Research Bureau is trimmed to include 170 observations (the number of trading days of the shortest-lived contract) for all contracts, resulting in at most five contracts on a given day [3].

Lumber contracts are based on the delivery of 110,000 board feet of two-by-fours of random length varying from 8 to 20 ft. The primary deliverable softwood species is Western spruce–pine–fir, but other Western species, hemlock–fir, Engelmann spruce, and lodgepole pine, may also be delivered. The deliverable grades are no. 1 and no. 2 of the structural light framing category. Wood should be kilndried to a moisture level of 19%. The lumber must be produced in the Canadian provinces of Alberta or British Columbia or in the states of California, Idaho, Montana, Nevada, Oregon, Washington, or Wyoming. The lumber must be grouped together according to length, wrapped by paper, and loaded on one 73-foot flatcar. If a futures contract buyer wants to take delivery, he or she is charged for
the lowest published freight rate for 73-foot flatcars from Prince George, British Columbia, to the specified destination. However, most of the lumber futures contracts are offset instead of ending with delivery.

Conditioning market’s response to unobserved information flows on inventories requires an empirical aggregate inventory measure. The US Census Bureau publishes Monthly Wholesale Trade reports in the first half of the month after the reporting period. These reports include merchant wholesalers, industrial distributors, exporters, and importers and exclude nonmerchant wholesalers such as manufacturer sales branches and offices. End-of-month inventory estimates represent stocks of merchandise owned by merchant wholesalers measured by a non-LIFO (last-in, first-out) method. Although the location of the merchandise does not matter, goods held outside the United States are excluded, as well as goods held on consignment and items not held for sale such as fixtures, equipment, and supplies.

This study uses the Lumber & Other Construction Materials inventory series (NAICS 4233) from Monthly Wholesale Trade reports from 1992 to 2005 [4]. The NAICS 4233 industry group includes lumber, plywood, millwork, and wood panel merchant wholesalers; brick, stone, and related construction material merchant wholesalers; roofing, siding, and insulation material merchant wholesalers; and other construction material merchant wholesalers [5]. The inventory series, released in current dollars, are converted into constant 1982 dollar values using the Lumber Producer Price Index published by the Bureau of Labor Statistics. The resulting monthly series are then interpolated by a cubic spline method to obtain daily inventories [6]. The daily inventory data are shown in Figure 2.

The US–Canada softwood lumber trade dispute has been the largest and longest lasting dispute between the two countries. The dispute is sometimes also referred to as the “softwood lumber war.” Canada, being the largest exporter of softwood lumber to the United States, provides more than 90% of US total imports (Yin and Baek 2004). The main

![Figure 1. Price movements at different levels of inventories.](https://academic.oup.com/forestscience/article-lookup/doi/10.5725/JAFSCD.57.5.379)
issue underlying the dispute has been whether the Canadian lumber is subsidized and, if so, whether the US lumber industry is adversely affected. There are numerous studies in the literature that have analyzed the importance of the trade dispute for both countries and the price, quantity, and welfare effects of the trade restrictions (Boyd and Krutilla 1987, Chen et al. 1988, Wear and Lee 1993, Myneni et al. 1994, Zhang 2001, 2007). See Yin and Baek (2004) for a review of published studies on the dispute. Zhang (2007) categorizes different episodes of the US–Canada softwood lumber dispute as Lumber I (1982–1983), Lumber II (1984–1986), Free Trade Agreement (1987–1991), Lumber III (1991–1994), the SLA (1996), and Lumber IV (2001–2006). Details on these different trade regimens can be found in Zhang and Sun (2001), Yin and Baek (2004), and Zhang (2007). Accordingly, indicator (dummy) variables are created to account for these different phases of the trade dispute during the sample period.

To combine simultaneously traded contracts, the full empirical version of Equation 8 is written as [7]

$$\% \Delta F_{it} = [100 \times (\ln F_{it} - \ln F_{i,t-1})]$$

$$= \alpha + \beta S_i + \gamma_1 \text{TTD}_t + \gamma_2 \text{TTD}^2_t + \delta_2 P_{2t} + \delta_3 P_{3t}$$

$$+ \theta_2 P_{2t} S_i + \theta_3 P_{3t} S_i + \epsilon_{it},$$

where $$\% \Delta F_{it}$$ is the approximate percentage change in the volatility of daily return on lumber futures, $$P_{2t}$$ and $$P_{3t}$$ are dummy variables for the periods 1996–2000 and 2001–2005, $$k_i$$ is the number of contracts traded on day $$t$$, between 1 and 5, and $$T = 3,365$$, the number of trading days in the sample. The total number of observations is $$\sum_{t=1}^{T} k_i = 13,090$$. Descriptive statistics of the variables are presented in Table 1.

If one just estimates Equation 10 via ordinary least squares, the large correlation among the price observations on a given day would have been ignored. To account for this contemporaneous correlation, a variant of the generalized least-squares method in Karali and Thurman (2009) is applied. Here, the covariance matrix of disturbances is defined in such a way that it assumes both covariance stationarity over time and identical covariances between contracts that have the same discrepancy in the delivery month. That is, the covariance between the first and second nearby contracts is assumed to be the same as that between the second and third nearby contracts as there is 2-month delivery discrepancy between these contracts [8].

Results from variations of Equation 10 are presented in columns a through c of Table 2 [9]. Column a shows the estimated parameters and their SEs and $$t$$ values from a model that excludes the time period dummy variables. As seen in the table, the inventory coefficient is negative. This

Figure 2. Lumber inventories (quantity index in billions of dollars).

Table 1. Summary statistics of daily variables.

<table>
<thead>
<tr>
<th>N = 13,090</th>
<th>%ΔF_{it}</th>
<th>%ΔF_{it}</th>
<th>Inventories</th>
<th>TTD</th>
<th>P1 (92–95)</th>
<th>P2 (96–00)</th>
<th>P3 (01–05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>−0.0105</td>
<td>1.2433</td>
<td>4.6154</td>
<td>0.8450</td>
<td>0.2503</td>
<td>0.3901</td>
<td>0.3596</td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
<td>0.9739</td>
<td>4.3835</td>
<td>0.8450</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Minimum</td>
<td>−7.8560</td>
<td>0</td>
<td>3.1054</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Maximum</td>
<td>14.1945</td>
<td>14.1945</td>
<td>7.5337</td>
<td>1.6900</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SD</td>
<td>1.6097</td>
<td>1.0225</td>
<td>0.9541</td>
<td>0.4908</td>
<td>0.4332</td>
<td>0.4878</td>
<td>0.4799</td>
</tr>
</tbody>
</table>

%ΔF_{it} = 100 × (ln F_{it} − ln F_{i,t−1}) and %ΔF_{it} = 100 × (ln F_{it} − ln F_{i,t−1}), i = 1, 2, ... , k_i, t = 1, 2, ... , T, where $$k_i$$ is the number of contracts traded on day $$t$$, $$T = 3,365$$, the number of days in the sample, and ln $$F_{it}$$ is the natural logarithm of futures price on day $$t$$, during which a total of $$k_i$$ contracts were traded. Inventories are measured in billions of 1982 dollars. TTD is measured as the number of trading days (i.e., not counting weekends and holidays) remaining to contract maturity divided by 100. P1, P2, and P3 are dummy variables for the periods 1992–1995, 1996–2000, and 2001–2005, respectively.
Table 2. Determinants of lumber price volatility.

<table>
<thead>
<tr>
<th>Volatility Models</th>
<th>Volatility Models with Persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>2.195</td>
</tr>
<tr>
<td></td>
<td>[0.071]</td>
</tr>
<tr>
<td>( \beta )</td>
<td>(30.806)</td>
</tr>
<tr>
<td></td>
<td>-0.094</td>
</tr>
<tr>
<td></td>
<td>[0.015]</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>(-6.448)</td>
</tr>
<tr>
<td></td>
<td>[0.038]</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>(-21.536)</td>
</tr>
<tr>
<td></td>
<td>[0.021]</td>
</tr>
<tr>
<td></td>
<td>(9.024)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.193</td>
</tr>
<tr>
<td></td>
<td>[0.021]</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td>0.108</td>
</tr>
<tr>
<td></td>
<td>[0.064]</td>
</tr>
<tr>
<td>( \delta_3 )</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>[0.123]</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>0.763</td>
</tr>
<tr>
<td></td>
<td>[0.110]</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>(6.922)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0693</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>0.0690</td>
</tr>
</tbody>
</table>

The full model is \( \% \Delta F_{it} = 100 \times (\ln F_{t-1} - \ln F_{t-1}) = \alpha + \beta S_t + \gamma_1 TTD_{it} + \gamma_2 TTD_{it}^2 + \psi_1 \Delta F_{t-1} + \psi_2 \Delta TTD_{t-1} + \psi_3 \Delta TTD_{it}^2 + \psi_4 \Delta F_{t-1} + \psi_5 \Delta TTD_{t-1} + \psi_6 \Delta TTD_{it}^2 + e_{it} \%
\( \% \Delta F_{it} \) is the approximate percentage change in the volatility of daily return on lumber futures. In \( F_{t-1} \) is the natural logarithm of futures price on day \( t \), during which a total of \( k_t \) contracts were traded, \( S_t \) is the lumber inventory level on day \( t \), and \( TTD_{it} \) is time to delivery, the number of days remaining to contract expiration on day \( t \). \( P_j \) and \( P_{j+1} \) are dummy variables for the periods 1996–2000 and 2001–2005, respectively. \( P_j \) is the dummy variable for the period 1992–1995, is used as the base period. SEs and \( t \) values of estimates are given in the brackets and parentheses, respectively. \( R^2 \) is the coefficient of determination, and Adj. \( R^2 \) is the adjusted \( R^2 \).

is the sign the theory of storage predicts for inventories. The smaller the inventories are, the higher the volatility is and vice versa. When inventories are large enough, demand and supply shocks cause larger fluctuations in price [10]. The coefficient on the linear TTD term is negative, and the one on the quadratic term is positive. The marginal time-to-delivery effect on volatility, computed as \( \partial \% \Delta F_{it} / \partial TTD_{it} = \gamma_1 + 2 \gamma_2 TTD_{it} \), stays negative over the range of the TTD variable, indicating that as futures contract approaches delivery its volatility rises [11]. The \( R^2 \) and adjusted \( R^2 \) of the regression are 6.96 and 6.90%.

Column b of Table 2 reports the results from the model with time period dummy variables. The period 1992–1995 is chosen as the base category for the regression. As before, the inventory effect is negative, and its magnitude is larger when dummy variables are included. Time-to-delivery effect is also negative throughout the sample period. The coefficient estimate for the time period 1996–2000 (\( \delta_3 \)) is negative and statistically significant, indicating lower volatility in this period compared with that in the base category. The coefficient estimate for the time period 2001–2005 (\( \delta_3 \)) is positive but statistically insignificant at the 5% level. The overall fit of the model improves compared with that of the previous model as both \( R^2 \) and adjusted \( R^2 \) increase to 11.54 and 11.50%.

Finally, column c of Table 2 presents the results from the full model. Interaction variables between inventories and time period dummy variables are added to the model in column b [12]. As seen in the table, the model fit improves even further with \( R^2 \) and adjusted \( R^2 \), increasing to 12.08 and 12.03%. Time-to-delivery effect shows a pattern similar to those obtained in the previous two models with a negative linear term and a positive quadratic term. Both estimates are highly statistically significant. The marginal time-to-delivery effect on volatility over the range of the TTD variable using column c estimates is shown in Figure 3a. As the figure shows, holding everything else constant, the time-to-delivery effect is negative and increases from 0.2 to 0.8 in magnitude over the life of a contract. Figure 3b shows predicted volatilities in different time period categories over the range of the TTD variable. As seen in Figure 3b, when evaluated at the mean value of inventories, volatility of a futures contract increases from 0.3 to 1.2 percentage points from the first to the last trading day in the 1992–1995 period. Likewise, volatility of a futures contract over its life increases from 0.6 to 1.5 and from 1.1 to 1.9 percentage points in the 1996–2000 and 2001–2005 periods, respectively. This result indicates that as a contract...
approaches delivery its volatility rises. When a contract is far from its delivery date its volatility is lower. This confirms the so-called Samuelson hypothesis (Samuelson 1965).

The inventory coefficient estimate of $-0.89$ ($\beta$) is again negative, and its magnitude is larger compared with those in columns a and b. This finding represents the inventory effect on volatility during the base period of 1992–1995. The inventory effects for the other two time periods should be computed by taking into account the coefficient estimates of the interaction terms. The inventory effect during the period of 1996–2000 ($\beta + \theta_2$) is $-0.20$ and during the
period of 2001–2005 ($\beta + \theta_3$) is $-0.13$. Thus, the effect of inventories on daily lumber futures volatility is higher in the base period, during which trade was governed by temporary tariffs between the United States and Canada, and lower in the SLA period, during which a tariff-rate quota system was applied. The inventory effect in the post-SLA period is lower compared with that in both the base period and the SLA period. Figure 3c shows predicted volatilities computed using the results in column c of Table 2 for these three time periods over the range of the inventory variable in the data. As seen in the Figure 3c, all three volatilities are decreasing as inventory increases, confirming the theory of storage. Further, the volatility in the 2001–2005 period is always higher than the volatility in the 1996–2000 period. The predicted volatility in the 1992–1995 period, on the other hand, is higher than the volatilities in the other two periods up to an inventory level of approximately $4.2$ billion, after which it becomes lower than the other two.

The overall effects of the time period dummy variables should also be computed, taking into consideration the interaction terms. When evaluated at the mean value of inventories, volatilities in the periods of 1996–2000 and 2001–2005 are 0.3 and 0.7 percentage points higher than in the base period. If the mean values of inventory and time-to-delivery variables are used, predicted volatilities in these three time periods are 0.6, 0.9, and 1.4 percentage points.

In terms of economic significance, a change in inventories from their minimum value of $3.1$ billion to their maximum value of $7.5$ billion causes a 3.9-percentage point decrease in daily volatility in the 1992–1995 period, whereas the same change in inventories causes 0.9- and a 0.6-percentage point decreases in volatility in the 1996–2000 and 2001–2005 periods, respectively. As to time-to-delivery effects, the estimates imply a 0.8-percentage point increase in volatility over the life of the contract. Compared with a typical day’s absolute log price change of 1.2 percentage points, these changes are considered to be economically significant.

**Volatility Persistence**

In empirical studies of financial assets, volatility persistence is a common concern. Most financial asset returns exhibit volatility persistence. Days with high volatility are followed by high volatility, and days with low volatility are followed by low volatility. To investigate this phenomenon in lumber futures contracts and to compare across other financial assets, Table 3 presents AR(1) regression results for volatility on selected financial assets. The specific regression equation is

$$\Delta P_t = a + b(\Delta P_{t-1}) + \epsilon_t,$$

where $\Delta P_t$ is the natural logarithm of the price of a financial asset on day $t$. As a sample of financial assets, the S&P 500 index, the Dow Jones Industrial Average index, the NASDAQ Composite index, the Canadian dollar–US dollar exchange rate, the Japanese Yen–US dollar exchange rate, and the returns to holding 3-month, 6-month, and 1-year treasury bills (secondary market) are chosen.

For lumber futures, the AR(1) model using data on all contracts in the sample is estimated (column d of Table 2). The coefficient of the lagged volatility is positive and statistically significant. However, the adjusted $R^2$ is only 2.76%. Given the importance of accounting for different time periods as shown in the previous section, an AR(1) model including time period dummy variables is estimated. These results are presented in column e of Table 2. As seen in the table, the model fit increases to 6.28%. The last column of Table 3 shows these results without reporting the dummy coefficient estimates.

Results given in Table 3 show that there is statistical evidence for volatility persistence in all the futures markets considered. For financial futures, the estimate of the lagged volatility coefficient lies in the range (0.20, 0.36). The largest movements in the lagged volatilities of these financial futures returns are 22.9, 25.6, 13.2, 1.9, 9.5, 16.8, 11.6, and 10.6 percentage points in the order presented in Table 3. Thus, the estimated AR(1) coefficients imply 5.0-, 7.2-, 4.8-, 0.5-, 1.9-, 5.4-, 3.0-, and 2.1-percentage point increases, respectively, in today’s volatility due to the largest movements in yesterday’s volatility in these markets.

Results given in the last column of Table 3 show that there is statistical evidence for volatility persistence in the

<table>
<thead>
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<th>Table 3. Volatility persistence in financial assets returns.</th>
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<tr>
<td><strong>S&amp;P 500</strong></td>
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<td>$a$</td>
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<td>[0.01]</td>
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Regression results from $\Delta P_t = [100 \times (\ln P_t - \ln P_{t-1})] = a + b(\Delta P_{t-1}) + \epsilon_t$, where $\Delta P_t$ is the approximate percentage change in the volatility of daily return on the financial asset on day $t$. Standard errors and $t$ values of estimates are given in the brackets and parentheses, respectively. Data periods for selected financial series are as follows: S&P 500, Jan. 3, 1950–June 12, 2006; Dow Jones, Jan. 2, 1930–June 12, 2006; NASDAQ, Feb. 5, 1971–June 12, 2006; Canadian dollar (CAD)–US dollar (USD) exchange rate, Jan. 4, 1971–June 12, 2006; Yen–USD exchange rate, Jan. 4, 1971–June 12, 2006; 3-month Treasury bill (T-bill), Jan. 4, 1954–June 9, 2006; 6-month T-bill, Dec. 9, 1958–June 9, 2006; 1-year T-bill, Feb. 1, 1962–Aug. 24, 2001; lumber futures, July 14, 1992–Nov. 15, 2005. Dummy variables $P_{t-1}$ and $P_{t-2}$ (not reported here) are also included in the regression equation for lumber. $R^2$ is the coefficient of determination, and Adj. $R^2$ is the adjusted $R^2$.
lumber futures market as well. The magnitude of the persistence coefficient estimate is similar to, albeit at the low end of, that seen in other markets. This result is expected as lumber futures contracts are based on a physical commodity, whereas financial futures are not. The estimates for lumber imply that a 1.1-percentage point increase in today’s volatility results from a 7.7-percentage point increase, the largest movement in the sample, in yesterday’s price movement.

To incorporate volatility persistence into the previous model with covariates, the lagged value of the dependent variable in Equation 10 is added to the right-hand side of 10. The full model is

$$\text{\%} \Delta F_t = [100 \times (\ln F_t - \ln F_{t-1})]$$

$$= \alpha + \beta S_t + \gamma_1 \text{TTD}_{1, t} + \gamma_2 \text{TTD}_{2, t} + \gamma \text{\%} \Delta F_{t-1}$$

$$+ \delta_1 P_{2t} + \delta_2 P_{3t} + \theta_2 P_{2t} S_t + \theta_3 P_{3t} S_t + \epsilon_i,$$

$$i = 1, 2, \ldots, k_t; \quad t = 1, 2, \ldots, T. \quad (12)$$

Results from variations of Equation 12 are presented in columns f through h of Table 2. As before, model fit improves when the time period dummy variables and interaction terms with inventories are included in the regression equation. The adjusted $R^2$ increases from 8.03 to 11.93% when the time period dummy variables are added to the regressors. It further increases to 12.36% when interaction terms are also included. Thus, the discussion of the results is based on the estimates presented in column h of Table 2 [13].

As before, a negative and statistically significant coefficient estimate for the linear TTD variable is obtained. The quadratic TTD term is positive and statistically significant. Figure 4a shows the marginal time-to-delivery effect ($\gamma_1 + 2\gamma_2 \text{TTD}_{1, t}$), which stays negative over the range of the TTD variable. Predicted volatilities over the range of the TTD variable in the three time periods are presented in Figure 4b. When evaluated at the mean value of inventories, volatility of a futures contract return increases from 0.4 to 1.2 percentage points in the 1992–1995 period, from 0.7 to 1.5 percentage points in the 1996–2000 period, and from 1.1 to 1.9 percentage points in the 2001–2005 period.

The coefficient estimate for the inventory variable ($\beta$) is again negative and statistically significant. As before, its magnitude is larger compared with the results without the time period dummy and/or interaction variables (columns f and g of Table 2). The inventory effect for the period of 1992–1995 is $-0.87$; that is, lumber futures volatility in this period decreases by 0.87 percentage point when inventory increases by 1 billion dollars. The estimated inventory effects for the periods of 1996–2000 and 2001–2005 are $-0.19$ and $-0.12$. As in the case without volatility persistence, the inventory effect is the highest in the base period with temporary tariffs, the second highest during the SLA period, and the lowest in the post-SLA period. Predicted volatilities in these three time periods over the range of the inventory variable, shown in Figure 4c, follow the same pattern as that in Figure 3c.

The overall impacts of the time period dummy variables are computed as $\delta_2 + \theta_2 S_t$ and $\delta_3 + \theta_3 S_t$ for the periods 1996–2000 and 2001–2005, respectively, where $S_t$ is the mean value of inventories. As before, the daily lumber futures volatility is 0.3 and 0.7 percentage points higher in the SLA and post-SLA periods than in the 1992–1995 period. When evaluated at the mean values of the inventory, time to delivery, and lagged volatility variables, predicted volatilities in these three time periods are 0.6, 0.9, and 1.3 percentage points.

A positive and statistically significant estimate for $\psi$ is obtained. Note that both the magnitude and significance of $\psi$ decline dramatically compared with the case when only intercept, lagged volatility, and time period dummy variables are included (column e of Table 2). The parameter estimate falls more than half, from 0.14 to 0.05, a 9 SD decrease in volatility persistence. These estimates imply that a 7.7-percentage point increase in yesterday’s price movement will cause only a 0.4-percentage point increase in today’s volatility, instead of the 1.1-percentage point increase reported before, when one corrects for correlation among contracts and accounts for inventories and time to delivery.

Another important result is seen by a comparison of columns c and h of Table 2. Including the lagged dependent variable in the volatility model does not have much effect on the parameter estimates for inventories and time to delivery. When evaluated at the sample range of inventories ($4.4$ billion), the estimates in column h of Table 2 imply a 3.8-percentage point decrease in volatility in 1992–1995 period. The same change in inventories causes volatility in the SLA period to decrease by 0.8 percentage point and in the post-SLA period by 0.5 percentage point. The time-to-delivery estimates imply a 0.8-percentage point increase in volatility over the life of the contract. These results are very similar to those found without the persistence variable.

**Conclusions**

Lumber is a major production cost in housing, and therefore it is important to understand price dynamics in this market. High volatility of lumber prices puts pressures on lumber producers to make better production and marketing decisions in the short run. These decisions would be improved if producers could anticipate future price changes correctly. Improved decisions, in turn, would result in a better allocation of scarce resources, such as lumber. Based on their future price expectations, producers can adjust their current production level. For instance, if a lumber producer anticipates a price decrease in future months, he might increase production and marketing now to avoid the lower price. On the contrary, if he expects the price to increase in the future, then he might cut back current production and marketing. In the lumber industry, the production and marketing decisions can also be adjusted accordingly when short-run price expectations change. For example, a lumber producer might delay processing stumpage for several weeks if
short-run prices are expected to increase. However, production levels cannot be adjusted instantly because converting standing timber into lumber takes about 1 year. Therefore, a producer needs to plan production up to a year ahead of time (Holtkamp 1984).

Futures markets provide information on future cash price expectations of traders on a daily basis and are found to be the primary center of price discovery for the underlying cash commodity (Yang and Leatham 1999). Thus, lumber futures prices are forecasts of future lumber cash prices,

**Figure 4.** Predicted volatility in lumber futures market with persistence.
adjusted for storage costs and anticipated supply and demand conditions. The results of this study then, can be used to better understand the behavior of lumber futures prices, which can be used to form future cash price expectations to make current production and marketing decisions.

Understanding volatility of lumber prices is also important from a hedging perspective. The variance of a hedging portfolio is a function of the variances of the returns on individual assets and the covariances among them. Thus, an increase in the variance of the return on lumber futures (i.e., volatility) would increase the variability of the return from the hedging portfolio, assuming the covariances did not change. This will also affect optimal hedge ratios. Deneckere et al. (1986), for example, using data for the period 1980–1985 found that lumber futures contracts were effective in reducing price risk, with an optimum hedge reducing the variance of a portfolio return by 50–90% compared with an unhedged position. So, market participants involved in trading lumber futures should understand the structure and the dynamics of the price volatility in this market.

After inventory, time-to-delivery, and persistence effects are taken into account, this study shows that the volatility of lumber futures prices was the highest in the 2001–2005 period, followed by the 1992–1995 and 1996–2000 periods. This result is consistent with the findings in Zhang and Sun (2001), whose sample period spans from May 1980 to August 2000. They reported that the period from September 1991 to March 1996 was the most volatile period for lumber prices, followed by the periods of April 1996–August 2000 and January 1987–August 1991 and stated that they expect to see volatile lumber prices in the future. The current study shows that the period after 2000 was, in fact, the most volatile period.

However, when the time periods representing various phases of the US-Canada trade dispute are interacted with lumber inventories, it is found that although the 2001–2005 period remains the most volatile, the 1996–2000 period becomes the second volatile period, and the 1992–1995 period the least volatile. This result shows that uncertainty about future trade terms contributes to the price volatility. However, and most importantly, even the SLA period created lumber price volatility rather than reducing it by eliminating uncertainty. The SLA period is characterized with a tariff-rate quota, creating a situation in which supply cannot turn by 50–90% compared with an unhedged position. So, market participants involved in trading lumber futures should understand the structure and the dynamics of the price volatility in this market.

In addition, the empirical findings confirm that lumber price volatility is inversely related to inventories. When inventories are low, futures contracts are relatively volatile. When inventories are large, their role in absorbing supply and demand shocks makes futures contracts less volatile. This observation empirically confirms a central prediction from the theory of storage. Further, it is found that the inventory effect on lumber futures volatility was the highest in the 1992–1995 period with temporary tariffs, followed by the SLA period (1996–2000). The post-SLA period (2001–2005) had the smallest inventory effect.

An inverse relationship between price volatility and time to delivery, an empirical support for the Samuelson hypothesis, is also found. As futures contracts approach delivery, their price fluctuations become larger. When contracts are far away from their delivery date, they are less volatile. This result can be interpreted as an implication of lumber supply and demand curves becoming more inelastic as time to delivery approaches. Thus, supply and demand shocks have a larger price impact on near-term contracts than on those farther out.

As with other financial assets, there is strong statistical evidence of volatility persistence in the lumber futures market. However, much of the persistence in lumber futures can be explained by the dependence of volatility on inventories and time to delivery. Reported $R^2$ values imply that only a small fraction of lumber futures volatility has been explained, and there are some other unexplained factors that affect volatility. This phenomenon is not uncommon in the literature. Structural models derived from economic theory to explain price volatility are found to perform poorly in terms of model fit (Pindyck and Rotemberg 1990, Ai et al. 2006).

The effects of these observable and exogenous variables on the price volatility of lumber futures may be exploitable to improve hedging, options pricing formulas, and the setting of margin requirements. Market participants, including lumber mills, lumber wholesalers, home builders, and construction companies, who are concerned about managing price risk should take into account the volatility determinants presented in this study.

This study also has implications for forest management. Selling timber is a large component of income generated from a forestland. Even though the objectives for owning forestland might change over time, sooner or later most forest owners decide to harvest timber. The price the forest owners receive from their sale of standing timber depends on several factors, including the current state of the economy, market cycles, and lumber prices. The value of timber, in fact, changes frequently with changes in lumber prices. Any event that changes the demand for and supply of lumber, such as housing starts, tariffs, and other shifts in international trade conditions, causes fluctuation in lumber prices, which in turn results in fluctuations in prices received for standing timber. Therefore, a forest owner should keep a close eye on lumber price volatility to determine the best time to sell timber.

Endnotes

[1] The detailed computations can be found in the Appendix, which reproduces the derivations in Williams and Wright (1991).

[2] The natural logarithm of futures prices is used to eliminate the effects of inflation.

[3] Lumber futures prices are subject to limit moves. There is no price limit in the spot month for the expiring contract. However, there is a price limit on daily price changes of other contracts. According to this limit, the price of any lumber futures contract cannot change by more than $10 per 1,000 board feet above or below the previous day’s settlement price. If the contract closest to expiration settles at limit price 2 days in a row, the limit expands to $15 per 1,000 board feet for all contracts that are subject to daily limit. On the basis of the limit of $10 per 1,000 board feet and including all contracts, 10% of the price movements in the sample were limit moves. Because of this price change truncation, the estimated effects here might underestimate the true equilibrium adjustment. However, because they represent a small portion of the data, no adjustments are made for limit moves.
days. On this issue, see also Park (2000), who shows that price limits in grain futures markets do not directly affect price return volatilities, and volatility is more likely to be explained by factors relating to an imbalance of the supply and demand.


[5] Even though lumber futures contracts are based on a specific kind of lumber produced in the above-mentioned regions, volatility of futures prices should be affected by aggregate lumber inventories as they reflect the overall supply and demand conditions in the economy. Given that NAICS 4233 includes other construction materials as well as lumber, the inventory measure used in this study should be interpreted carefully. The estimated effects here might under- or overstate the exact response of volatility to changes in lumber inventories. However, less than 10% of lumber futures contracts result in physical delivery, and, therefore, homebuilders generally do not consider receiving the lumber they need for their construction from the futures contract. As a result, the difference between the inventory data used here and the lumber specified in futures contracts should not affect the results substantially. These readily available monthly national data provide a proxy for overall current inventory level and a signal for future production decisions.

[6] Linear and step function interpolation methods are also used, and few substantive changes in the results are found.

[7] Because Karali and Thurman (2009) found evidence of a nonlinear time-to-delivery effect on the lumber futures price response to housing starts announcement shocks, a quadratic function of time to delivery is chosen. Nonlinear time-to-delivery effects are also found in grain futures markets. See, for example, Streeter and Tomek (1992).


[9] To check the robustness of the results to the choice of the volatility measure, the models in Table 2 are also estimated with the squared returns, another common volatility measure used in the literature, and no change in the qualitative results is found.

[10] One might be concerned that slowly evolving inventories are proxying for an exogenous nonlinear trend. To investigate this possibility, a quadratic function of time trend is added to the model, and it is found to be statistically significant. However, inventory effects become even larger and stronger in this model. Thus, deviations of inventories from their trend also affect volatility inversely, indicating that the significant inventory effect in Table 2 is not simply an exogenous time trend.

[11] The relationship between trading volume and volatility has been an issue in the literature. To investigate whether the time-to-delivery effect is proxying for a volume effect, volume is added into the model for the sample period during which volume data were available (Jan. 1, 2001–Nov. 11, 2005). A positive and statistically significant volume effect is found. Holding everything else constant, a 100-unit increase in volume causes a 0.04-percentage point increase in volatility. The inventory effect does not change much with the addition of volume, whereas the time-to-delivery effect decreases in magnitude from −0.91 to −0.65 over the life of a contract. Even though time to delivery and volume are correlated, the time-to-delivery effect remains significant even with volume. Whereas the volume effect itself is statistically significant, it is decided to exclude it in the model because although time to delivery is exogenous, volume is endogenous: volatility and volume are jointly determined, and the goal of this study is to measure the total effect on volatility from a change in time to delivery.

[12] Models with interaction terms between the time to delivery and the time period dummy variables are also estimated. The coefficient estimates of these interaction terms are found not to be statistically significant and therefore are not reported here.

[13] Models with interaction terms between the time period dummy variables and the time to delivery as well as with the lagged volatility are also considered. The coefficient estimates of these interaction terms are found not to be statistically significant and therefore are not reported here.

**Literature Cited**


Yin, R., and J. Baek. 2004. The US–Canada softwood lumber
APPENDIX

Derivation of Optimal Storage Rules

The model has a finite horizon with three periods. The third and last period is period $T$, the second period is period $T - 1$, and the first period is period $T - 2$. The carry-out from the last period has no value because the world ends beyond that time. Therefore, carry-out in the last period is zero, and everything available in the last period is consumed; that is, $S_T = 0$. Inverse consumption demand is assumed to be linear in quantity consumed such that

$$P_t = \alpha + \beta q_t \quad \text{with} \quad \alpha > 0 \quad \beta < 0. \quad (A1)$$

Supply is perfectly inelastic with constant mean $\bar{y}$ and is subject to a random additive disturbance $v_t$. The supply equation is given by

$$y_t = \bar{y} + v_t, \quad (A2)$$

where $v_t$ is a uniformly distributed random disturbance with mean zero and SD $\sigma$. Thus, the probability density function of $v_t$, which is observable by everyone, is defined as

$$f(v_t) = \begin{cases} \frac{1}{2\sqrt{3}\sigma} & -\sqrt{3}\sigma \leq v_t \leq \sqrt{3}\sigma \\ 0 & \text{elsewhere}. \end{cases} \quad (A3)$$

In any given period, total availability in the market is the sum of production in that period and carry-in into that period; that is, $A_t = y_t + S_{t-1}$. Consumption in any period satisfies

$$q_t = y_t + S_{t-1} - S_t = A_t - S_t. \quad (A4)$$

Mean production level $\bar{y}$ and $v_t$ is observed by all market participants in the beginning of period $t$; thus, all decisions made in period $t$ are conditioned on the realization of $v_t$. Marginal physical storage cost is denoted by $c$ and is constant over time. The one-period interest rate is $r$.

To find optimal storage in period $T - 2$, the social planner uses backward induction. The planner first solves for the optimal storage rule in period $T - 1$ and then uses it to derive the optimal storage rule in period $T - 2$. The social planner’s optimal choice for storage in any period $t$ is determined by the first-order condition of the objective function,

$$V_t(A_t) = \max_s \left\{ \int_0^{A_t-s} P(q_s) dq - cS_t \right\}$$

$$+ (1 + r)^{-1} E_t \left[ V_{t+1}(y_{t+1} + S_t - S_{t+1}) \right], \quad (A5)$$

subject to $S_t \geq 0$. The first-order condition is then given by

$$\frac{\partial V_t}{\partial S_t} = -P_t - c + (1 + r)^{-1} E_t [P_{t+1}], \quad S_t > 0. \quad (A6)$$

Using this first-order condition, the planner’s optimal choice in period $T - 1$ satisfies

$$P_{T-1} + c = (1 + r)^{-1} E_{T-1} [P_T^*], \quad S_{T-1} > 0. \quad (A7)$$

Substituting the inverse demand Equation A1 into Equation A7 and then using supply Equation A2 in Equation A4 yields the optimality condition,

$$\alpha + \beta (A_{T-1} - S_{T-1}) + c = (1 + r)^{-1} E_{T-1} \left[ \alpha + \beta (\bar{y} + v_T + S_{T-1}) \right], \quad S_{T-1} > 0. \quad (A8)$$

The expectation on the right-hand side can be calculated using the probability density function of the random disturbance $v_T$, because $v_T$ is the only random variable in Equation A8. The optimal level of storage in period $T - 1$ is then found by solving the following equation for $S_{T-1}$:

$$\alpha + \beta (A_{T-1} - S_{T-1}) + c = \left( \frac{1}{1 + r} \right) \left( \frac{1}{2\sqrt{3}\sigma} \right) \int_{-\sqrt{3}\sigma}^{\sqrt{3}\sigma} \left( \alpha + \beta (\bar{y} + v_T + S_{T-1}) \right) dv, \quad (A9)$$

and the solution is given by

$$S_{T-1}^* = \max \left\{ \frac{(1 + r)A_{T-1} - \bar{y}}{2 + r} + \frac{1 + r}{2 + r} \left( \frac{c}{\beta} \right), \frac{r}{2 + r} \left( \frac{\alpha}{\beta} \right), 0 \right\}. \quad (A10)$$

If the interest rate is set equal to zero, as done by Williams and Wright (1991) for simplicity, the optimal storage rule for period $T - 1$ is the same as their Equation 3.8 on p. 59.

The first-order condition for optimal storage in period $T - 2$ is

$$P_{T-2} + c = (1 + r)^{-1} E_{T-2} [P_{T-1}], \quad S_{T-2} > 0. \quad (A11)$$

The solution for period $T - 2$ becomes complicated because of the possibility of storage in period $T - 1$ and a fundamental nonlinearity. As seen from Equation A10, optimal
storage in period $T - 1$ depends on total availability $A_{T-1}$, which, in turn, depends on the random disturbance to supply in that period, $v_{T-1}$. Depending on the random shock in period $T - 1$, one will either observe no storage or a positive amount of storage. This critical value of the random shock can be found by substituting total availability in period $T - 1$ into the optimal storage rule for that period and then by setting $S^*_{T-1}$ equal to zero. The critical value of $v_{T-1}$ is

$$v_{T-1} = -S_{T-2} - \left( \frac{c}{\beta} \right) - \left( \frac{r}{1 + r} \right) y - \left( \frac{r}{1 + r} \right) \left( \frac{\alpha}{\beta} \right).$$

(A12)

Whenever $v_{T-1}$ is less than the critical value, the planner will choose not to store in period $T - 1$; that is, $S_{T-1} = 0$. Whenever $v_{T-1}$ exceeds the critical value, the planner will choose to store a positive amount in period $T - 1$; that is, $S_{T-1} > 0$. After substituting the inverse demand function and consumption identity into Equation A11, the optimality condition becomes

$$\alpha + \beta (A_{T-2} - S_{T-2}) + c$$

$$= \left( \frac{1}{1 + r} \right) \left( \frac{1}{2\sqrt{3}\sigma} \right) \int_{-\sqrt{3}\sigma}^{\sqrt{3}\sigma} \left( \alpha + \beta (y) \right) \left( \alpha + \beta (y) \right)$$

$$+ v_{T-1} + S_{T-2}) dv + \int_{-\sqrt{3}\sigma}^{\sqrt{3}\sigma} \left( \alpha + \beta (\bar{y}) \right)$$

$$+ v_{T-1} + S_{T-2} - S^*_{T-1} ) dv \right).$$

The right-hand side of Equation A13 shows the weighted average of possible situations with and without storage in period $T - 1$. Substituting Equation A10 for $S^*_{T-1}$ in Equation A13 and integrating over $v_{T-1}$ results in a quadratic equation, the solution of which gives the optimal storage rule in period $T - 2$:

$$S^*_{T-2} = \max \left\{ \sqrt{3}\sigma \left( \frac{2r^2 + 7r + 7}{1 + r} \right) - \left( \frac{c}{\beta} \right) - \left( \frac{r \bar{y} + r \alpha / \beta}{1 + r} \right)$$

$$- \sqrt{12\sigma^2 \left( \frac{r^4 + 7r^3 + 19r^2 + 24r + 12}{(1 + r)^2} \right) - 4\sqrt{3}\sigma \left( 2 + r \right) \left( 2 + r \right) \left\{ \frac{2 + r}{1 + r} \right\} + v_{T-1} + S_{T-2} - S^*_{T-1} \right) \right\}. \quad (A14)$$