A Compatible Growth-Density Stand Model Derived from a Distance-Dependent Individual Tree Model

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ABSTRACT. A compatible growth-density stand model is developed for simulating forest stands with uniformly distributed trees (e.g., square spacing, equilateral triangle spacing, and 1:1.5 row to column ratio tree distribution patterns). The model is based on a distance-dependent individual-tree competition model. The independent variable in the individual-tree model is Hegyi's (1974) diameter-distance competition index with \( n = 8 \). The derived growth density stand model satisfies a necessary boundary condition (i.e., stand growth is zero either when stand density is zero or when stand density reaches a very high level). The stand density which maximizes stand growth rate can be calculated with this stand model. It is found that stands with an equilateral triangle tree distribution pattern have the greatest growth rate among the three uniform stands. Stand growth in measured forests is more than 10% lower than that predicted in uniform stands. Simulation comparisons suggest that the uneven distribution of trees in such stands can reduce stand growth. For. Sci. 43(3):443–446.

Additional Key Words: Plantation growth, tree spacing, competition, modeling, Pinus koraiensis.
vivors within a growth period was based on two logical considerations: (1) the model should be flexible enough to be able to fit different ranges of data sets for trees under various possible competition conditions; (2) the model should be able to meet the demands of two extreme boundary conditions, i.e., the minimum growth rate cannot be negative, and the maximum growth rate cannot be higher than the growth rate for open-grown trees. We used a three parameter exponential function:

$$g = g_{\text{max}} e^{bDCI}$$  \hspace{1cm} (1)

where $g = 3$ yr periodic increment of tree basal area (cm²); $g_{\text{max}}$, $b(<0)$, and $c$ are constants, in which $g_{\text{max}}$ is the upper asymptote and can be considered as the periodic growth for open grown trees (cm²). These three parameters were estimated with the least-square method. The diameter-distance competition index for each subject tree ($DCI$) is given by

$$DCI = \sum_{i=1}^{n} \frac{D_i}{D} \frac{1}{L_i}$$  \hspace{1cm} (2)

where $D = DBH$ of the subject tree (cm); $D_i = DBH$ for competitor tree $i$ (cm); $L_i = \text{distance between the subject tree and competitor tree } i$ (m); $n = \text{number of competitor trees}$ ($n = 8$ in this note for TOKOR). The higher the value of $DCI$, the more competition a subject tree suffers. The competition trees will be regrouped if more than one time step is applied because some trees die over time.

The tree mortality submodel was developed with the assumption that every tree has a mortality probability ($p_m$) that is related to the value of $DCI$. The value of $p_m$ is set to 1.0 if the $DCI$ value of a tree is higher than a threshold value, $DCI_0$. For the trees whose $DCI$ values are lower than $DCI_0$, the value of $p_m$ is:

$$p_m = aDCI^b, \quad DCI < DCI_0$$  \hspace{1cm} (3)

where $p_m$ = mortality probability for a tree ($0 \leq p_m \leq 1$); $a$ and $b$ are constants. $DCI_0$ is calculated when $p_m$ is set to 0. The survival probability ($p_s$) equals $1 - p_m$.

Using values for the parameters of the TOKOR model of $g_{\text{max}} = 95.91$; $b = -0.2704$; $c = 1.250$; $\alpha = 0.00428$; $\beta = 3.037$, and testing against independently measured data sets, the relative error of predicting stand basal area growth is less than ±5% for stands under relevant site conditions and between 18 and 24 yr of age (Shao 1985).

### A Compatible Growth Density Stand Model

The net increase of stand basal area was defined by Husch et al. (1982) as:

$$G_t = B_2 - B_1$$  \hspace{1cm} (4)

where $G_t = \text{periodic increment of stand basal area per unit area}$; $B_1 = \text{stand basal area per unit area at beginning of growth period}$; $B_2 = \text{stand basal area per unit area at end of growth period}$.

If the initial stand density is $N$, the density at end of growth period is $N_s$ ($N_s$ is also referred survivor number), and the number of dead trees is $N_m$, then $N = N_s + N_m$, then Equation (4) can be approximated as:

$$G_t = \bar{B}_2 N_s - \bar{B}_1 N_s = \bar{B}_2 N_s - \bar{B}_1 (N_s + N_m)$$  \hspace{1cm} (5)

where $\bar{B}_i = \text{average tree basal area for a stand at time } i$; $\bar{D}_k = \text{average } DBH \text{ for diameter class } k \text{ at time } i$; $g = \text{periodic increment of basal area growth}$; $n = \text{the number of diameter classes}$; $N_{s}$ and $N_{m}$ are the numbers of survivors and dead trees in diameter class $k$ from time 1 to time 2.

In the simple-individual tree model, the growth rate of a tree is the function of $DCI$ for that tree (see Equation 1). Same sized trees can have different growth rates if they have different $DCI$ values due to differences in their neighboring trees. For this study, the mean $DCI$ for trees with $DBH = \bar{D}_k$ as well as the relations between tree distance and stand density are derived for the three uniform tree distribution patterns (Figure 1):

1. **For square distribution patterns with each tree in the stand on the corners of a square grid**, the expected value of $DCI$ for an average tree is:

$$E(DCI_{\bar{D}_k}) = \frac{1}{\bar{D}_k} \left( \sum_{i=1}^{4} \frac{D_i}{L} + \frac{8}{\sqrt{2L}} \sum_{i=5}^{8} \frac{D_i}{\sqrt{2L}} \right)$$  \hspace{1cm} (6)

where $E(DCI_{\bar{D}_k}) = \text{diameter-distance competition index for an average tree in diameter class } k$; $\bar{D}_k = \text{minimum distance between neighbor trees (m)}$; $N = \text{stand density (n ha}^{-1})$; and $E(\cdot) = \text{expected value function}$.

The distance between neighbor trees is calculated as the function of stand density:

$$L = \frac{100}{\sqrt{N}}$$  \hspace{1cm} (7)

2. **For equilateral triangle patterns**, similar to the square pattern with trees on the points of equilateral triangles tessellating the stand, the equivalent equations are:
For 1:1.5 row-to-column ratio patterns, the appropriate equations are:

\[
E(DCI_{\beta}) = E \left( \sum_{i=1}^{8} \frac{D_i}{D_k} \frac{1}{L} + \frac{8}{\sqrt{3}} \frac{D_i}{D_k} \frac{1}{L} \right)
= \frac{D}{\overline{D}_k} \frac{1}{L} \left( 6 + \frac{2}{\sqrt{3}} \right) = 7.1547 \frac{D}{\overline{D}_k} \frac{1}{L} \tag{8}
\]

\[
L = 100 \sqrt{\frac{1}{N}} = 107.5 / \sqrt{N} \tag{9}
\]

3. For 1:1.5 row-to-column ratio patterns, the appropriate equations are:

\[
E(DCI_{\beta}) = E \left( \sum_{i=1}^{8} \frac{D_i}{D_k} \frac{1}{L} + \frac{8}{\sqrt{2.5}} \frac{D_i}{D_k} \frac{1}{2L} \right)
= \frac{D}{\overline{D}_k} \frac{1}{L} \left( 2 + \frac{4}{\sqrt{2.5}} + 1 \right) = 5.5298 \frac{D}{\overline{D}_k} \frac{1}{L} \tag{10}
\]

\[
L = 100 / \sqrt{1.5N} \approx 81.6 / \sqrt{N} \tag{11}
\]

Recall that \( p_m + p_s = 1, p_s = 1 - p_m \) and \( N_k = (1 - p_m)N \), then, according to Equation (5), the periodic growth of trees in diameter class \( k \) for the square tree distribution pattern is:

\[
G_{kn} = g_{max} b^{DCI_{\beta}} \left( 1 - \alpha(DCI_{\beta}) \right) N_k \pi \frac{D_k^2}{4}
= g_{max} b^{6.8284 / 100} \left( 1 - \alpha \left( \frac{6.8284 / 100}{N} \right) \right) \tag{12}
\]

\[
N_k = \alpha \left( \frac{6.8284 / 100}{N} \right) N_k \pi \frac{D_k^2}{4}
= g_{max} b^{6.8284 / 100} \left( 1 - \frac{6.8284 / 100}{N} \right) N_k \pi \frac{D_k^2}{4}
\]

where

\[
k_1 = b \left( \frac{6.8284}{100} \right) \tag{13}
\]

\[
k_2 = \alpha \left( \frac{6.8284}{100} \right) \tag{14}
\]

\[
k_3 = \alpha \left( \frac{6.8284}{100} \right) \pi \frac{D_k^2}{4} \tag{15}
\]

and \( N_k \) is the number of trees in diameter class \( k \).

For the other two uniform distribution patterns, the growth-density stand models are the same in structure as Equation (12), but parameter values of \( k_1, k_2 \) and \( k_3 \) are different.

Figure 1. Tree distribution patterns for 3 uniform stands (each symbol represents a tree location: symbol dot is a subject tree and dot within circle is 1 of 8 possible nearest competitor trees): a. square spacing, b. equilateral triangle spacing, and c. 1:1.5 row to column ratio spacing.
Table 1. Comparisons of 3 yr periodic increment for stands with a square-pattern spacing based on the growth-density stand model and on actual stands of the same densities (data from Shao 1985). SBA is Stand Basal Area.

<table>
<thead>
<tr>
<th>Stand density (n ha⁻¹)</th>
<th>Measured SBA growth (m² ha⁻¹)</th>
<th>&quot;Square&quot; SBA growth (m² ha⁻¹)</th>
<th>&quot;1:1.5 R&quot; SBA growth (m² ha⁻¹)</th>
<th>&quot;Triangle&quot; SBA growth (m² ha⁻¹)</th>
<th>Relative difference between measured and &quot;triangle&quot; (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,740</td>
<td>5.25</td>
<td>5.90</td>
<td>5.96</td>
<td>6.11</td>
<td>14.6</td>
</tr>
<tr>
<td>2,330</td>
<td>5.53</td>
<td>6.26</td>
<td>6.34</td>
<td>6.55</td>
<td>18.4</td>
</tr>
<tr>
<td>2,640</td>
<td>5.58</td>
<td>6.28</td>
<td>6.38</td>
<td>6.61</td>
<td>18.5</td>
</tr>
<tr>
<td>2,760</td>
<td>5.62</td>
<td>6.27</td>
<td>6.37</td>
<td>6.61</td>
<td>17.6</td>
</tr>
</tbody>
</table>

For equilateral triangle patterns:

\[ k_1 = b \left( \frac{D}{D_k} \right)^{107.5} \]

\[ k_2 = \alpha \left( \frac{D}{D_k} \right)^{107.5} \]

\[ k_3 = \alpha \left( \frac{D}{D_k} \right)^{107.5} \pi D_k^2 / 4 \]

For 1:1.5 row to column ratio patterns:

\[ k_1 = b \left( \frac{D}{D_k} \right)^{81.6} \]

\[ k_2 = \alpha \left( \frac{D}{D_k} \right)^{81.6} \]

\[ k_3 = \alpha \left( \frac{D}{D_k} \right)^{81.6} \pi D_k^2 / 4 \]

Then stand growth \( G_n \) is the summation of \( G_{kn} \) for all the diameter classes. The diameter classes for the forest stand used in this paper range from 6.0 to 17.0 cm, and the distribution pattern is shown to be {0%, 1%, 1%, 3%, 7%, 17%, 22%, 20%, 15%, 8%, 4%, 2%} when the diameter class interval is 1 cm.

Based on Equation (12), three curves can be created for the three types of uniformly spaced stands. These compatible growth-density stand models satisfy the boundary conditions that \( G = 0 \) when \( N = 0 \), and when \( N \) reaches to a large value (\( N = 7,000 \) trees/ha in this note, depending on tree distribution patterns). A forest stand with an equilateral-triangle spacing has the highest growth rate among the three patterns for densities higher than about 1000 trees/ha. Calculating the maximum growth of basal area and optimum stand density, 3 yr periodic basal area growth for equilateral triangle distributions is 6.61 m²/ha, which is higher than either of the other two spacing schemes (square = 6.28 m²/ha and 1:1.5 row-to-column ratio = 6.38 m²/ha). The optimum stand density for the equilateral triangle pattern is also higher than either of the other two patterns.

By comparing the uniformly spaced stand growth from Equation (12) with growth in actual stands (Table 1), one finds that, just as the differences in growth among the three uniform tree distribution patterns becomes lower when stand densities are lower, the difference between uniform stands and actual stands also becomes lower when stand density is lower. The growth expected in uniformly spaced stands from the compatible growth-density stand model can be about 16% to 18% higher than the observed stand growth. Recall that the individual-based model estimated stand basal area growth is within ± 5%. We believe the deviations of the growth-density stand model from actual independent observations may be due, in part, to deviations from an exact spacing pattern in the observed stands. In the actual stands, uneven tree spacing (observed in stem maps in Shao 1985) makes some trees subject to heavy competition stress and allows other trees to have surplus space. This feature is accounted for in the individual-based models by the exact computation of each tree’s actual position in the competition index. If this is the case, more exact spacing of trees could increase the forest net growth by more than 10% for the stands studied in this paper.

Literature Cited


