Forest Land Conversion Over Time: Implications for the Faustmann Formula

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ABSTRACT. The Faustmann timber production model has been used to examine the allocation of land between alternative forest land use options. This article provides a clarification and extension of this previous work. We use the basic forest land use options model to allow for the changes in forestland use over time. First, we modify Faustmann’s original assumptions to allow for a one-off change in prices or an initial allocation constraint, and examine the forest land manager’s decision to switch to an alternative land use at some point in the future. Second, we look at the case where prices evolve over time and it is optimal for the forest land manager to switch between land uses on an equilibrium path. This second extension provides a new application of the Faustmann timber production model. We discuss the theoretical insights and the implications for forest conversion.

Key Words: Forest land-use options, land-use switching.

There is currently a great deal of interest among policy makers and academics about deforestation, and indeed about many other examples of switching land use. In order to analyze this phenomenon and to develop appropriate policies, we need to understand what factors are driving owners to switch land use and how these forces operate. In particular, it is important to understand whether switching land use is some kind of disequilibrium behavior (that is, part of an adjustment process from one equilibrium to another), or whether switching is consistent with equilibrium behavior. Therefore, one would like to understand under what circumstances switching land use emerges as a feature of behavior on the privately optimal path of some land owner.

Several recent studies have extended the Faustmann timber production model (Faustmann 1849) to examine the allocation of forestland between alternative options, such as forest preservation, natural forest management for timber and environmental benefits, plantation timber production, and conversion to agriculture (Burgess 2000, p. 104–125, Parks et al. 1998, Barbier and Burgess 1997, Mendelsohn 1994). In these models it is assumed that the forestland manager chooses the land-use option that yields the highest net present value at the outset and then maintains the land use forevermore. One concern raised about these analyses is that they are comparative static in nature and so switching land use is analyzed as a shift in the equilibrium, rather than as part of an equilibrium. The aim of this article is to extend the existing theoretical framework of forest land use to provide a dynamic model in which switching land use emerges as a phenomenon on the equilibrium/privately optimal path.

First, we restate the basic forest land-use options model. This is based on the Faustmann bare land value model which assumes that the forestland manager starts out with a tract of forestland that is clear of any commercially valuable vegetation (Faustmann 1849, Gaffney 1957, Samuelson 1976, Chang 1984, 1998). We explore the situation where the forestland manager has the opportunity to switch between forestland uses over time. As one would expect, simply allowing the possibility of switching does not change the forestland manager’s choice of optimal type of forestland use and rotation age. We provide a numerical illustration of this analysis based on empirical data drawn from recent case studies.

In our first case of switching between forestland uses, we modify Faustmann’s original assumptions to allow for a one-off change in timber prices or costs after the forestland use...
has been established or to take account of initial constraints on forestland use, such as concession agreements or property right requirements. Under these conditions, the forestland manager may initially allocate the bare land to one use, but this is not the preferred long-term use. Therefore, there is an incentive to switch from the initial suboptimal land use to the long-term optimal land use immediately or after some “holding” period. We show that the initial holding period is determined by analysis based on the value of the forestland under the initial suboptimal and the long-term optimal management regimes. This result differs from that derived by Faustmann (1849) but supports the approach taken by Chang (1998).

In our second case of switching between forestland uses, we assume that timber prices and costs change at different constant exponential rates. Under these conditions, relative timber prices and costs change over time, and the underlying problem is nonstationary. Therefore, it could be optimal for the forestland manager to switch between forest land uses on an equilibrium path. We show that in order to fully solve this problem we have to take two steps. First, we need to work out for each forestland use the length of the first rotation (or initial holding period) that maximizes its value function. Second, we need to choose the forestland use that has the greater maximum value at the start of the second rotation and thereafter. In order to determine the optimal choice of forestland uses and rotation lengths, the forestland manager needs to consider a combination of forest land-use options. This result differs significantly from the basic forest land-use options model and the conventional Faustmann formula and provides a “new” application of Faustmann’s timber production model.

The article is structured as follows. In the next section, we restate the forest land-use options model and allow the forest land-use manager to switch between alternative forest land uses over time. In the third section, we examine our first case of forest land-use switching due to a one-off change in a key parameter or to initial land-use constraints. In the fourth section, we present our second case of forest land-use switching due to timber prices and costs evolving at different rates. The last section discusses the theoretical insights derived from this analysis.

No Commitment to Tree Type

In this section we briefly revisit the forest land-use options model that is already established in the literature (Burgess 2000, p.104–125, Parks et al. 1998, Barbier and Burgess 1997). Finally, if we consider agricultural crops as a type of very fast growing “tree crop” that is ready to be harvested within a year, then our framework can be used to examine the conversion of forestland to agricultural production, which is a major concern throughout both the tropical and temperate forest countries (FAO 1997, Barbier et al. 1994, p. 5–21).

The objective of the forestland manager is to maximize the present value of a tract of “bare” land (i.e., clear of any commercially valuable timber) by choosing the optimal forest land-use option and rotation period. For now we assume that once the bare land has been allocated to this forest land-use option, it will be maintained under this use forever. All the standard assumptions about perfect information, perfect markets, and constant key parameters apply. Let there be n possible types of forestland use and let \( N^i(T) \) be the net revenue function if the forest land manager allocates the land to type \( i \) forest land use:

\[
N^i(T^i) = (P^i - C^i)Q^i(T^i)e^{-rT} - S^i \quad i = 1, \ldots, n
\]

(1)

where \( P \) is the price of timber, \( C \) is the cost of harvesting, \( Q(T) \) is the production of timber at rotation age \( T \), \( S \) is the regeneration cost, and \( r \) is the rate of discount. In line with the recent studies of forest land-use options we assume continuous compounding, rather than Faustmann’s assumption of annual compounding.
Let \( V_i(T^i) \) represent the net present value of the bare land that is "dedicated forever to type \( i \) forestland use and maintains the same rotation period \( T^i \) forever. This is referred to as the bare land expectation value:

\[
V^i(T^i) = (P^i - C^i)Q^i(T^i)e^{-rT^i} - S^i + V^i(T^i)e^{-rT^i}
\]  

which implies:

\[
V^i(T^i) = \frac{(P^i - C^i)Q^i(T^i)e^{-rT^i} - S^i}{1 - e^{-rT^i}} = \frac{N^i(T^i)}{1 - e^{-rT^i}}
\]  

Let \( \hat{V}^i \) be the maximum present value of profits from forest land-use type \( i \), by choosing the optimal constant rotation period, \( \hat{T}^i \):

\[
\hat{V}^i = \text{MAX}_{T^i} V^i(T^i)
\]  

Let \( \hat{T}^i \) be the optimum constant rotation period for forest land-use type \( i \):

\[
\hat{T}^i = \text{arg max}_{T^i} V^i(T^i)
\]  

The first-order condition for the optimal rotation period is found by differentiating (5) with respect to \( T^i \) and then setting this equal to zero: This yields the necessary condition for an efficient rotation age:

\[
\hat{T}^i = \frac{(P^i - C^i)Q^i(\hat{T}^i)}{e^{-r\hat{T}^i} - (P^i - C^i)Q^i(\hat{T}^i) - S^i} - r\hat{T}^i
\]  

which can be rewritten as:

\[
(P^i - C^i)Q^i(\hat{T}^i) = e^{-r\hat{T}^i}(P^i - C^i)Q^i(\hat{T}^i) + re^{-r\hat{T}^i} V^i(\hat{T}^i)
\]  

This is the conventional Faustmann formula for bare land immediately prior to regeneration, and it states that the timber should be harvested at the age when the rate of change in the present value of the timber harvest with respect to time is equal to present value interest on the stock of timber and the land. This optimal rotation period maximizes the value of the bare land when allocated to forest land-use type \( i \).

Now let \( \hat{V} \) be the maximum present value of the bare land that is derived by choosing the optimal type of forest land-use and rotation period that will be implemented forever:

\[
\hat{V} = \text{MAX}_{i} \{ \hat{V}^i \equiv V^i(\hat{T}^i) \}
\]  

The basic decision rule states that the bare land should be allocated to its highest valued use at the outset and be maintained under this use forever. For example, when the value of the land under forest land-use type 1 is greater than the value of the land under all other types of forestland use, that is, \( \hat{V}^1 > \hat{V}^j \), the forestland manager will allocate the bare land to type 1 forestland use.

Numerical Illustration

We develop a numerical example for tropical hardwood timber production, such as mahogany, grown in a "natural" forest setting, and softwood timber production grown in a plantation setting, in order to illustrate the theoretical model. The data for the numerical analysis are based on a number of case studies in the tropics and should be considered as representative examples rather than specific case studies (see for example, Browder et al. 1996, Wunder 1996, Pearce and Warford 1993, Vincent 1990, Boscolo et al. 1997, Ross 1984). Full details of this analysis are presented in Appendix 1.

As can be seen in Table 1, in the Base Case scenario, the net present value from allocating the land to selective timber harvesting is $1,530/ha, with a constant optimal rotation length of 36 yr. The net present value from allocating the land to plantation timber production is $4,138/ha, also with a constant optimal rotation length of 36 yr. The returns to plantation timber production are clearly greater than the returns to natural forest management, and the forestland will be allocated to plantation timber production at the outset and maintained under this land-use option forever. The fact that the optimal rotation length of both selective harvesting and plantation timber production is 36 yr is purely a coincidence and a result of the key parameter values chosen for the analysis. In particular, the slower growth but higher value of the timber in the natural forest stand offsets the faster growth but lower value of the plantation timber.

Forest Land-Use Options Model with Switching

We now adapt the forest land-use options model developed previously to allow for the possibility of switching between different types of forestland use over time. We again assume that the forestland manager starts with a bare tract of land, but is not committed to maintaining the land in one type of forestland use over time. Therefore, there is nothing to stop the forestland manager from establishing one type of forestland use and then switching to another type of forestland use immediately or after an initial "holding" period. Note that in order to avoid confusion with the long-term rotation period, we have renamed the first rotation period the initial holding period.

Let \( v^i(t^i) \) represent the present value of the bare land if the forester establishes forest land-use type \( i \) for the initial holding period, harvests the timber after \( t^i \) years and switches to the optimal land use thereafter. It is important to notice that in this definition, \( t^i \) is just the length of the initial holding rotation. Then:

\[
v^i(t^i) = N^i(t^i) + \hat{V}e^{-r\hat{T}^i} = (P^i - C^i)Q^i(t^i)e^{-r\hat{T}^i} - S^i + \hat{V}e^{-r\hat{T}^i}
\]
Table 1. Comparative returns to plantation and natural forest.

<table>
<thead>
<tr>
<th>Case</th>
<th>Optimal rotation length</th>
<th>Maximum value of bare land</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>36 yr for plantation forest</td>
<td>$4,138/ha</td>
</tr>
<tr>
<td>—plantation forest only</td>
<td>36 yr for natural forest</td>
<td>$1,530/ha</td>
</tr>
<tr>
<td>—natural forest only</td>
<td>32 yr for natural forest, then plantation forest</td>
<td>$2,009/ha for initial rotation</td>
</tr>
<tr>
<td>Case 1: Established in initial Period</td>
<td>36 yr for plantation forest thereafter</td>
<td>$5,076/ha for plantation forest</td>
</tr>
<tr>
<td>—initial rotation of natural forest, then switch to plantation forest</td>
<td>36 yr for plantation forest then 44 yr for natural forest</td>
<td>then natural forest</td>
</tr>
<tr>
<td>Case 2: Increasing prices</td>
<td>36 yr for plantation forest then 40 yr for natural forest</td>
<td>$2,498/ha for natural forest</td>
</tr>
<tr>
<td>—plantation forest only (annual rate of price increase 0.001)</td>
<td>36 yr for plantation forest then 37 yr for natural forest</td>
<td>$2,616/ha for natural forest</td>
</tr>
<tr>
<td>—natural forest only (annual rate of price increase 0.02)</td>
<td>36 yr for plantation forest then 36 yr for natural forest</td>
<td>then plantation forest</td>
</tr>
<tr>
<td>—initial rotation of natural forest, then switch to plantation forest</td>
<td>36 yr for plantation forest then 36 yr for natural forest</td>
<td>then 40 yr for natural forest</td>
</tr>
<tr>
<td>Case 3: Increasing prices</td>
<td>36 yr for plantation forest then 47 yr for natural forest</td>
<td>$8,702/ha for natural forest</td>
</tr>
<tr>
<td>—plantation forest only (annual rate of price increase 0.01)</td>
<td>36 yr for plantation forest then 37 yr for natural forest</td>
<td>$5,855/ha for natural forest</td>
</tr>
<tr>
<td>—natural forest only (annual rate of price increase 0.03)</td>
<td>36 yr for plantation forest then 35 yr for natural forest</td>
<td>then plantation forest</td>
</tr>
<tr>
<td>—initial rotation of natural forest, then switch to plantation forest</td>
<td>36 yr for plantation forest then 36 yr for natural forest</td>
<td>then 40 yr for natural forest</td>
</tr>
<tr>
<td>Case 4: Increasing prices</td>
<td>36 yr for plantation forest then 44 yr for natural forest</td>
<td>$5,076/ha for plantation forest</td>
</tr>
<tr>
<td>—plantation forest only (annual rate of price increase 0.001)</td>
<td>36 yr for plantation forest then 47 yr for natural forest</td>
<td>$5,076/ha for plantation forest</td>
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</tr>
<tr>
<td>—initial rotation of natural forest, then switch to plantation forest</td>
<td>36 yr for plantation forest then 36 yr for natural forest</td>
<td>then 40 yr for natural forest</td>
</tr>
</tbody>
</table>

Let \( \hat{v}^i \) be the maximum present value of the bare land if the forestland manager starts by establishing forest land-use type \( i \), and chooses the optimal length of the initial holding period:

\[
\hat{v}^i = \text{MAX}_i \hat{v}(t^i) \tag{11}
\]

Now \( \hat{t}^i \) let be the optimum initial holding period for forest land-use type \( i \):

\[
\hat{t}^i = \text{arg max}_i \hat{v}(t^i) \tag{12}
\]

Let \( \hat{v} \) be the maximum present value of the bare land that is derived by choosing the optimal forest land-use types and rotation periods, where the possibility of switching exists. Given this, then \( \hat{v} \) will be determined by choosing the optimal type of forestland use for the initial holding period:

\[
\hat{v} = \text{MAX}_i \hat{v}^i \tag{13}
\]

In order to illustrate this general approach, let us assume that the first forest land-use type, \( i = 1 \), provides the highest valued land use for the initial holding period. Then:

\[
\hat{v} = N^1(\hat{t}^1) + \hat{v} e^{-r\hat{t}^1} \Rightarrow \hat{v} = \frac{N^1(\hat{t}^1)}{1 - e^{-r\hat{t}^1}} = V^1(\hat{t}^1) \tag{14}
\]

Rewriting the left-hand side of (14) in full:

\[
\hat{v} = (P^1 - C^1)Q^1(\hat{t}^1)e^{-r\hat{t}^1} - S^1 + \hat{v} e^{-r\hat{t}^1} \tag{15}
\]

we can derive the first-order condition for \( \hat{t}^1 \) by differentiating (15) with respect to \( \hat{t}^1 \) and setting this equal to zero:

\[
(P^1 - C^1)Q^1(\hat{t}^1)e^{-r\hat{t}^1} = re^{-r\hat{t}^1}(P^1 - C^1)Q^1(\hat{t}^1) + re^{-r\hat{t}^1} \hat{v} \tag{16}
\]

By comparing (7) and (16), we can see that the initial holding period, \( \hat{t}^1 \), will be the same as the optimal rotation length for all consecutive harvests, \( \hat{T}^1 \), that is \( \hat{t}^1 = \hat{T}^1 \). This in turn implies that the maximum present value of the bare land where the possibility of switching exists, \( \hat{v} \), is the same as the maximum present value of the bare land where there is no opportunity for switching, \( V^1 \), and the optimal forest land-use type and rotation period will be implemented forever, that is \( \hat{v} = V^1 \).

Numerical Illustration

In our Base Case scenario, the returns to allocating the bare land to plantation timber production are greater than the returns to natural forest management, and the bare land will be allocated to plantation timber production at the outset and maintained under this land use forever (see Table 1). Therefore, even when the forestland manager has the opportunity to switch to an alternative forest land use in the future, the choice of optimal forest land use and rotation period in the initial holding period will be identical to the choice of optimal forest land-use type and rotation period in all the following timber harvests. The optimal choice of forest land-use type can be determined by the basic forest land-use options model and the choice of constant rotation age can be determined by the conventional Faustmann bare land formula. This result is to be expected given our underlying assumptions of perfect information, perfect markets and constant key parameters. Although this result in itself does not provide a case for switching, we have now established an appropriate framework to enable the analysis of forest land-use switching over time in the following sections.
Case 1: Initial Allocation Constraint or a One-Off Change in Key Parameter

In this section, we establish our first case of switching between alternative forest land-use options over time. Once again, we assume that the forestland manager starts with a bare tract of land and is not committed to any one type of forestland use forever. However, we now modify Faustmann’s original assumptions to allow for a one-off change in costs and timber prices after the forest land use has been established or to take account of initial forest land-use constraints, such as concession agreements or property right requirements. Under these conditions, the forestland manager may allocate the bare land to one use at the outset, but this is not the preferred long-term land use. Therefore, there is an incentive to switch from the initial suboptimal land use to the long-term optimal land use immediately or after some “holding” period.

Let us assume that the present value of the bare land under forest land-use type 1 exceeds the value of the bare land under all other types of forestland uses, such that \( V^1 > V^j \), where \( j \neq 1 \). If the forestland manager establishes a forest land-use type \( j \) at the outset, then there is an incentive to switch to the more valuable forest land-use type 1 at the end of the initial holding period. The forestland manager would then wish to maintain the land under this higher valued type of forestland use for evermore. The maximum present value of the bare land if it is initially allocated to tree type \( j \) is given by:

\[
\hat{V}^j = (P^j - C^j)Q^j(t^j)e^{-\hat{r}t^j} - S^j + \hat{V}e^{-\hat{r}t^j} \tag{17}
\]

Differentiating (17) with respect to \( t^j \) and setting this equal to zero gives the first-order condition:

\[
(P^j - C^j)Q^j(t^j)e^{-\hat{r}t^j} = re^{-\hat{r}t^j} + (P^j - C^j)Q^j(t^j) + re^{-\hat{r}t^j} \hat{V} \tag{18}
\]

We can compare the optimal initial holding period given by (18) to the optimal constant rotation age given by the conventional Faustmann formula in (7). If the opportunity cost of holding onto the bare land in terms of its optimum value, \( \hat{V} \), is greater than the value of the bare land if it is allocated to forest land-use type \( j \) forever, \( \hat{V}^j \), then the forester will harvest the timber from the initial forest land-use type sooner than usual in order to get access to bare land for replanting with the higher valued type of forestland use from then on. Thus, as one would expect, when \( \hat{V} \) is greater than or equal to the value of the bare land if it is allocated to forest land-use type \( j \) forever, \( \hat{V}^j \), the optimal initial holding period, \( t^j \), will be less than or equal to the optimal constant rotation length, \( \hat{t}^j \), as determined by the conventional Faustmann formula. That is \( t^j \leq \hat{t}^j \), with a strict inequality if \( V^1 \neq \hat{V} \).

Therefore, the initial holding period for the suboptimal forestland use is not equal to either the “original” rotation period of the suboptimal forestland use or the rotation age of the optimal regime. Instead, the initial holding period is determined by analysis based on the value of the forestland under the initial suboptimal regime and the long-term optimal management regime. If the forestland manager does not, or is unable to, modify the decision-making framework, for example due to a lack of flexibility or a lack of foresight, then the choice of “if and when” to switch may be inefficient.

This result can be compared and contrasted to that derived by Faustmann (1849) in “Section I: Intermittent Yield Management, Part B: Land Currently Carrying a Stand.” In this section of his paper, Faustmann clearly defines the problem that we have addressed, but there are three main differences between what Faustmann proposes and what we have proposed. First, as noted previously, he assumes annual compounding rather than continuous compounding. Second, he assumes all costs and timber prices are constant, whereas we allow them to vary after the initial forestland use has been established. Third, he assumes that the forestland manager possesses a tract of land that currently contains a suboptimal stand of timber, which is converted to the optimal timber harvesting regime at the end of the initial holding period.

In contrast to our model, the length of the initial holding period is the same as the length of all future rotations (minus the current age of the timber), once the land is converted to the optimal timber harvesting regime. That is, Faustmann assumes that the suboptimal timber stand will be harvested after \( u - n \) years, where \( u \) is the constant rotation age of the optimal stand and \( n \) is the current age of the suboptimal stand. Therefore, our result differs from that derived by Faustmann (1849). However, our result does support that derived by Chang (1998) in his generalized Faustmann model, which allows for changes in current and future stumpage prices, regeneration costs, and the rate of interest.

Numerical Illustration

If the bare land is allocated at the outset to the lower valued natural forest management regime, then forestland managers will want to switch at the end of the initial holding period to the higher valued plantation timber production. The optimal initial holding period for natural forest management is derived using (18), and the new value of the forest stand is then determined. In Table 1, this scenario is called Case 1, and the optimal time to switch from natural forest management to plantation forestry occurs at 32 yr. As the theory predicted and as one would expect, this is shorter than the previous optimal rotation length of natural forest management of 36 yr, as in the Base Case. Once the forestland use has switched to plantation timber production, the optimal constant rotation length is 36 yr, which maximizes the returns to plantation timber production thereafter. The new maximum present value of the forest stand with switching is $2,009/ha, which is greater than the value of the stand allocated forever to natural forest management but less than the value of the stand if it had been allocated to plantation timber production from the outset.

This case of forestland use switching illustrates the comparative static analysis of forest conversion due to a one-off change in timber prices or costs or due to initial allocation constraints. However, this case does not help to explain the situation where it is optimal for the forestland manager to...
Case 2: Costs and Timber Prices Evolving at Different Rates Over Time

Here we examine the situation where the forestland manager has an incentive to change from one type of forestland use to another at some point in the future because timber prices and costs change at different constant exponential rates (McConnell et al. 1983, Hardie et al. 1984, Newman et al. 1985). Under these conditions, the relative timber prices and costs of the various forestland uses also change over time, and it may be optimal for the forestland manager to switch between forestland use options on an equilibrium path. In this case the underlying problem specified is nonstationary, which gives rise to a dynamic land allocation decision. The existing literature has not fully addressed this problem.

Once again, we assume that the forestland manager starts out with a bare tract of land and that there are $n$ possible types of forestland use. We now introduce the possibility that for forest land-use type $i$, all timber prices and costs grow at a constant rate, $\pi_i$, but the rates of price and cost increases differ across types of forestland use. Under these conditions, it could be optimal to establish one type of forestland use initially and then switch to another type of forestland use when it becomes more valuable at a later date.

The price of tropical hardwood timber from naturally managed forests has been steadily increasing over time and is predicted to continue increasing in the future due to diminishing supplies. In contrast, the price of softwood timber from plantation forests is expected to remain fairly stable over time due to adequate supplies in temperate regions and competition from nonwood substitutes (Barbier et al. 1994). Differences in the rates of timber price increases across different forestland uses may also occur due to the establishment of timber certification or labeling schemes. Costs of production may be expected to rise over time due to the increasing scarcity of quality forestland, increasing input costs, and the removal of existing subsidies.

We assume that the forestland manager can establish whatever type of forestland use he wishes in the first rotation, or the initial holding period. The forestland manager can also establish whatever type of forestland use he wishes at the start of the second rotation. The type of forestland use established in the second rotation may be different to that established in the first rotation, but whatever type of forestland use is established in the second rotation must then be planted in all subsequent rotations. These assumptions imply that from the second rotation period onwards, there is no further change in forestland use, and a steady state is reached. This effectively reduces the model to two periods: before and after the second rotation.

We modify our model to allow prices and costs to increase at a constant exponential rate, $\pi_i$. Let $V^i(T^i)$ be the present value of the bare land at date zero if the forestland manager establishes type $i$ forestland use forever and uses the same rotation period $T^i$ forever. Then:

$$V^i(T^i) = \frac{(P_0^i - C_0^i)Q^i(T^i)e^{-(r - \pi_i)T^i} - S_0^i}{1 - e^{-(r - \pi_i)T^i}}$$

Let $\hat{V}^i$ be the maximum present value of the bare land allocated to forest land-use type $i$ by choosing the constant optimal rotation period that will be implemented forever:

$$\hat{V}^i = \max_T V^i(T^i)$$

Let $\hat{T}^i$ be the optimum constant rotation period for forest land-use type $i$:

$$\hat{T}^i = \arg \max_T V^i(T^i)$$

Suppose now that at date zero, the forestland manager establishes forest land-use type $j$ and harvests the timber after $t'$ years. As in the third section, the notation $\hat{t}$ used is such that denotes the length of the initial holding period, or first rotation, only. If at date $t'$ the forestland manager switches to forest land-use type $i$ forevmore, then the maximum present value of the bare land at date $t'$ will be $V^i e^{r\hat{t}'}$.

So, to determine which forestland use to switch to after the end of the initial holding period, the forestland manager will choose the value of $i$ which maximizes $\hat{V}^i e^{\pi_i t'}$. Notice that in general the type of forestland use that achieves this maximization will depend on the length of the initial holding period. Thus, the choice of type of forestland use after the initial holding period could depend crucially on how long this initial holding period lasts. Let

$$\hat{V}(t') = \max_i \hat{V}^i e^{\pi_i t'}$$

be the maximum present value of the bare land at date $t'$ if forest land-use type $i$ is established at the end of the initial holding period.

Having determined what happens at the end of the initial holding period, we can now let $\hat{v}^j(t')$ be the present value of the bare land at date zero if the forestland manager initially establishes forest land-use type $j$ and harvests the timber after $t'$ years:

$$\hat{v}^j(t') = (P_0^j - C_0^j)Q^j(t')e^{-(r - \pi_j)t'} - S_0^j + \hat{V}(t')e^{-\pi_j t'}$$

Let $\hat{v}$ be the maximum present value of the bare land that is established with forest land-use type $j$ for initial holding period by choosing the optimal initial holding period, $t'$:

$$\hat{v} = \max_T \hat{v}^j(T^j)$$

Let $\hat{t}'$ be the maximum present value of the bare land that is derived by choosing the optimal forest land-use type and rotation period for the initial holding period:

$$\hat{v} = \max_T \hat{v}^j(T^j)$$

Let $\hat{t}'$ be the optimum initial holding period if forest land-use type $j$ is chosen at the outset:

$$\hat{t}' = \arg \max_T \hat{v}^j(T^j)$$
We can derive the first-order condition for \( \hat{\tau}^j \) by differentiating (26) with respect to \( \hat{\tau}^j \):

\[
(P_0^j - C_0^j)Q^j e^{-(r-\pi_j)\hat{\tau}^j} = (r - \pi_j) e^{-(r-\pi_j)\hat{\tau}^j}(P_0^j - C_0^j)Q^j(\hat{\tau}^j) + r e^{-r\hat{\tau}^j} \hat{V}(\hat{\tau}^j)
\]

(27)

It is important to note that the function \( \hat{V}(\hat{\tau}^j) \) is not always differentiable if \( \hat{\tau}^j \) is a critical value at which the forester switches from one type of land use to another at the end of the initial holding period. In this analysis, we assume that \( \hat{\tau}^j \) is not such a critical value.

The result in (27) shows that the optimal length of rotation in the initial holding period occurs when the increase in value from current harvesting of forest land-use type \( j \) is just equal to the incremental cost of delaying the harvest. On the left-hand side, the benefits of delay consist of the gain in the present value of the net benefits from the additional timber volume growth from delaying the harvest one period. This is discounted at the effective rate of interest \( r - \pi_j \), which takes account of the constant rate of change in prices and costs of forest land-use type \( j \).

On the right-hand side, the opportunity cost of postponing the harvest consists of two components. First, the effective interest foregone from not being able to invest the present value proceeds from harvesting the timber. Once again, this is discounted at the effective rate of interest \( r - \pi_j \), which takes account of the constant rate of change in prices and costs of forest land-use type \( j \). Second, the interest lost from deferring the revenue from the present value of future rotations once the steady state is reached. Unlike the other components of (27), this is not discounted at the effective rate of interest \( r - \pi_j \), but just \( r \). However, it should also be noted that \( \hat{V}(\hat{\tau}^j) \) is the maximum present value of the bare land at date \( \hat{\tau} \) when type \( i \) forestland use is established forevermore, and from (22) we know that the value of \( \hat{V}(\hat{\tau}^j) \) is increasing at the constant exponential rate \( \pi_j \) over time.

The final step of this problem is to choose the type of land use to assign in the initial holding period. This is simply the value of \( \hat{\tau}^j \) that maximizes present value of the bare land for the initial holding period, \( \hat{\tau}^j \).

We have now developed a case for switching along an equilibrium path. This occurs when it is in the interest of the forestland manager to establish a type of forest use that is initially more valuable, but to switch to an alternative type of forestland use at the end of the initial holding period because its value is rising relatively more rapidly over time. In order to maximize the returns to the bare land, the forester needs to base her forestland use decisions on expected timber prices and costs and consider the returns to the bare land under a combination of forestland use choices. The forestland manager is unable to rely on the standard, unadjusted Faustmann timber production model. If the forestland manager does not adjust the decision-making procedure, for example due to high computation costs in terms of the time and effort required to implement the modified framework of analysis, then the forest land-use decision may be inefficient.

**Numerical Illustration**

We numerically illustrate this situation of switching by adapting our plantation and natural forest management examples, as shown by Case 2 in Table 1. Let us assume that the prices of both plantation timber and naturally managed timber now increase at a constant exponential rate over time, but the price of plantation timber increases at a lower rate of 0.01 and the price of naturally managed timber increases at a higher rate of 0.02. With these rates of price increases, the net present value of plantation timber production with no switching increases to $4,326/ha with an optimal rotation age of 36 yr, while the net present value of natural forest management with no switching increases to $4,383/ha with a much higher optimal rotation age of 43 yr. Therefore, if the forestland manager does not switch between the alternative management regimes over time, then the bare land would be allocated to natural forest management, which is now its highest valued use.

However, we can also examine the land-use decision when the forestland manager has the opportunity to switch at the end of the initial holding period. As shown by Case 2 in Table 1, it is not in the forestland manager’s interest to initially allocate the stand to natural forest and then switch to plantation forest, as the returns to this regime amount to only $3,719/ha. Under this pattern of land allocation, the optimal length of the initial holding period increases to 45 yr, as the forestland manager holds onto the land under its higher valued use longer before converting the stand to the lower valued plantation forest. In contrast, the forestland manager has an incentive to initially allocate the bare land to plantation timber production and then switch to natural forest management at the end of the initial holding period. This yields the highest returns of $5,076, assuming that the forestland manager harvests the plantation timber after 35 yr in order to switch to natural forest management sooner.

By looking at the present value of the alternative forestland uses for different initial holding periods, we can determine the critical length of the initial holding period at which the forestland manager is indifferent between two types of forestland use after the initial holding period. For the parameter values used in this example, the critical value is 38 yr. That is, up to 38 yr, the present value of the bare land from allocating it to plantation timber production after the initial holding period is greater than from allocating it to natural forest management, whereas after 38 yr the reverse is true.

We can also show when it is not in the forestland manager’s interest to undertake switching. In Case 3 in Table 1, we assume that the rate of plantation price increase remains unchanged at 0.001. Through sensitivity analysis, we can show that when the rate of price increase for timber from natural forests is less than or equal to 0.01, then the forestland manager would allocate the stand to plantation forest at the outset and maintain it under this land use forevermore. We have already shown in Case 2 that when the rate of price increase for timber from naturally managed forest is 0.02, then the forestland manager would allocate the stand to plantation forest at the outset and then switch to natural forest management. If the rate of price increase for timber from natural
from the suboptimal forestland use holding period is based on the value of the timber harvest period. In line with Chang (1998), the length of the initial optimal forest land use at the end of an initial holding constraints. Here, the forestland manager has an incentive of the equilibrium due to either a one-off change in timber prices or costs or to initial allocation constraints. Here, the forestland manager has an incentive to convert from the suboptimal forestland use to the optimal forest land use at the end of an initial holding period. In line with Chang (1998), the length of the initial holding period is based on the value of the timber harvest from the suboptimal forestland use and the value of the bare land under the optimal forest land use. As one would expect, this results in a shortening of the initial holding period compared to the “original” rotation period of the suboptimal forestland use, so that the forestland manager can gain access sooner to clear ground on which to establish the more valuable type of forestland use. In contrast to Faustmann (1849), the length of the initial holding period is not the same as the length of all future rotations (minus the current age of the timber), once the land is converted to the optimal timber harvesting regime.

In our second case of forest land-use switching, we extend the existing theoretical framework to provide a dynamic model in which switching land use occurs on the privately optimal land-use path.

Conclusions and Policy Implications

This article has used the Faustmann bare land value model (Faustmann 1849) and the basic forest land-use options model (Parks et al. 1998, Barbier and Burgess 1997, Mendelsohn 1994, Burgess 2000, p. 104–125) to examine the decision to switch between alternative forestland uses over time. As we have previously discussed, this general problem was addressed by Faustmann in 1849. By relaxing some of the simplifying assumptions made by Faustmann (1849), this article provides a clarification of this previous work. In addition, we extend the existing theoretical framework of forestland use to provide a dynamic model in which switching land use occurs on the privately optimal land-use path.

Our analysis has confirmed that if it is optimal to allocate a bare tract of land to just one type of forestland use over time, then the optimal forestland use and rotation period can be derived using the basic forest land-use options model and the conventional Faustmann formula. Furthermore, as one would expect, simply providing the opportunity to switch does not change the optimal forest land-use and rotation decision.

Our first case of forest land-use switching is a standard comparative static analysis, whereby land-use switching occurs as a shift in the equilibrium due to either a one-off change in timber prices or costs or to initial allocation constraints. Here, the forestland manager has an incentive to convert from the suboptimal forestland use to the optimal forest land use at the end of an initial holding period. In line with Chang (1998), the length of the initial holding period is based on the value of the timber harvest from the suboptimal forestland use and the value of the bare land under the optimal forest land use. As one would expect, this results in a shortening of the initial holding period compared to the “original” rotation period of the suboptimal forestland use, so that the forestland manager can gain access sooner to clear ground on which to establish the more valuable type of forestland use. In contrast to Faustmann (1849), the length of the initial holding period is not the same as the length of all future rotations (minus the current age of the timber), once the land is converted to the optimal timber harvesting regime.

In our second case of forest land-use switching, we extend the existing theoretical framework to provide a dynamic model in which switching land use is analyzed as part of the equilibrium. Here, the forest land manager has an incentive to switch to an alternative forest land use at the end of the initial holding period because its value increases relatively rapidly over time. In order to determine the optimal land uses and rotation lengths we need to work out for each forest land use the length of the initial holding period that will maximize the initial holding period value function, and we need to choose the forest land use which maximizes the value of the land thereafter. This approach differs significantly from the basic forest land-use options model and the conventional Faustmann formula and provides a new application of the Faustmann timber production model.

There are several important extensions to our analyses that need to be considered. First, we have taken account of timber benefits only, and the inclusion of nontimber forest benefits is likely to be highly significant (see Hartman 1976, Parks et al. 1998, Burgess 2000, p. 104–125). Second, we assume that the forestland manager starts with a tract of bare land, whereas in many situations a commercially valuable forest already exists (see Faustmann 1849, Barbier and Burgess 1997). Third, we have not included the costs of conversion between alternative forest land uses, whereas these are likely to be considerable, especially for the re-establishment of “natural” forestland (see Barbier and Burgess 1997).

Although the key insights derived from our analysis may still pertain to more complicated versions of this model, extensions to our analysis to include these considerations would make the model more applicable to real world situations. For example, one would expect that even if the current value of maintaining an old-growth forest is less than that from converting the land to plantation timber or agricultural production, it may be optimal to maintain the existing forest if the timber and nontimber benefits derived from this use are expected to increase at a relatively high rate over time and the costs of re-establishing natural forestland are prohibitively high.

Literature Cited


is the factor which discounts given by:

$$Q_i = \frac{K^i (K^i - Q_0^i) \alpha_i e^{-\alpha_i T^i}}{1 + \left(\frac{K^i - Q_0^i}{Q_0^i}ight) e^{-\alpha_i T^i}} = Q'(T^i) \alpha_i e^{-\alpha_i T^i} \left[ \frac{1}{Q_0^i} - \frac{1}{K^i} \right]$$  \hspace{1cm} (A2)

We assume that the length of rotation of relevance to the forest manager is chosen such that the standard assumption of increasing timber biomass with rotation age but at a decreasing rate are applicable:

$$Q^i_T > 0, \quad Q^i_{T-T} < 0$$  \hspace{1cm} (A3)

We assume that it costs a fixed amount, $S_i$, for silvicultural effort to undertake seedling establishment in the initial year. After this, no further silvicultural effort is required, and the trees are left to grow naturally over time. The costs of harvesting, $C_i$, and the stumpage prices of timber, $P_i$, are assumed to be in real value terms, and the market rate of interest is given by $r$, where $e^{-rT}$ is the factor which discounts net benefits at age $T^i$ to their present value.

As in the main text, let $N^i(T^i)$ be the net revenue function if the forest land manager allocates the bare land to type $i$ forestland use:

$$N^i(T^i) = (P^i - C^i) Q^i(T^i) e^{-rT^i} - S^i \quad i = 1, ..., n$$  \hspace{1cm} (A4)

Let $V^i(T^i)$ represent the net present value of the bare land:

$$V^i(T^i) = (P^i - C^i) Q^i(T^i) e^{-rT^i} - S^i + V^i(T^i) e^{-rT^i}$$  \hspace{1cm} (A5)

This implies:

$$V^i(T^i) = \frac{(P^i - C^i) Q^i(T^i) e^{-rT^i} - S^i}{1 - e^{-rT^i}} = \frac{N^i(T^i)}{1 - e^{-rT^i}}$$  \hspace{1cm} (A6)

The first-order condition for the optimal rotation period is found by differentiating (A6) with respect to $T^i$ and then setting this equal to zero: This yields the necessary condition for an efficient rotation age:

$$V^i_T = \left(1 - e^{-rT^i}\right) [(P^i - C^i) Q^i_T e^{-rT^i} - re^{-rT^i} (P^i - C^i) Q^i(T^i)]$$

$$- \frac{re^{-rT^i} [(P^i - C^i) Q^i(T^i) e^{-rT^i} - S^i]}{(1 - e^{-rT^i})^2} = 0$$  \hspace{1cm} (A7)

which can be rewritten as:

$$(P^i - C^i) Q^i_T e^{-rT^i} = re^{-rT^i} (P^i - C^i) Q^i(T^i) + re^{-rT^i} V^i(T^i)$$  \hspace{1cm} (A8)

**Numerical Illustration**

We develop a numerical example for tropical hardwood timber production, such as mahogany, grown in a “natural” forest setting and softwood timber production grown in a plantation setting to illustrate the theoretical model. The data

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**APPENDIX 1**

**Timber Production**

Based on an approach developed by Hartwick and Oleviler (1998, p. 345–350), we assume logistic growth for timber production. The quantity of timber produced is determined by the initial stock of commercial timber biomass, $Q_0^i$ (i.e., the value of $Q^i(T^i)$ when $T^i = 0$) that is planted on the “bare” land (i.e., land that is clear of any commercially valuable timber), the intrinsic rate of increase of commercial timber biomass, $\alpha_i$, and the carrying capacity of the land, $K^i$ for forest land use type $i = 1, ..., n$. The initial stock of commercial timber biomass depends on the quantity and size of the tree seedlings that are planted at the start of the rotation. These tree seedlings may be artificially re-generated and established as a plantation, or naturally regenerated from existing seed trees and grown within the ongoing forest. The intrinsic rate of timber biomass growth depends on the type of tree species; for example, softwoods tend to be fast growing and hardwoods much slower growing. The carrying capacity of the land, which is the maximum volume of commercial timber biomass that the land can sustain, depends on the type and quantity of commercially valuable tree species grown and the quality of the land (e.g., soil quality, temperate, rainfall).

The volume of commercially valuable timber biomass is given by:

$$Q^i(T^i) = \frac{K^i}{1 + \left(\frac{K^i - Q_0^i}{Q_0^i}\right) e^{-\alpha_i T^i}} = \frac{K^i}{1 + m^i e^{-\alpha_i T^i}}$$  \hspace{1cm} (A1)

where $m^i = (K^i - Q_0^i) / Q_0^i$. Differentiating (A1) with respect to $T^i$ and rearranging gives the logistic growth equation for the commercially valuable timber stock:
for the numerical analysis are based on a number of case studies in the tropics and should be considered as a representative examples rather than specific case studies (see for example, Browder et al. 1996, Wunder 1996, Pearce and Warford 1993, Vincent 1990, Boscolo et al. 1997, Ross 1984).

For natural forest management, we assume that the initial stock of commercially valuable timber, $Q_i^0$, is 5 m$^3$/ha, the intrinsic rate of tree growth, $a_i$, is 0.1, and the carrying capacity of the land in terms of commercially valuable timber, $K_i$, is 275 m$^3$/ha. Using (A2) we can derive the total volume of commercially valuable timber in the natural forest over time, which is about 100 m$^3$/ha after 35 yr and at 70–80 yr reaches maturity with a total volume of timber of 270 m$^3$/ha. For natural forest management the initial establishment cost, $S_i$, is $100/ha, the harvesting cost, $C_i$, is $25/m^3, the stumpage price of timber, $P_i$, is $100/m^3, and the market rate of discount, $r$, is 0.05.

We assume that for timber production from plantation forestry the total carrying capacity of the bare land in terms of commercially valuable timber, $K_i$, is 700 m$^3$/ha. This is considerably higher than that under natural forest management. The initial stock of commercially viable timber at the start of the rotation, $Q_i^0$, is 5 m$^3$, which is the same as in the case of natural forest management. The intrinsic rate of growth of plantation tree species, $a_i$, is 0.15, slightly higher than those under natural forest management, as softwood tree species tend to be faster growing. Using (A2) we find that the total volume of plantation timber reaches approximately 400 m$^3$/ha after 35 yr, and the trees reach maturity at around 50–60 yr of age, with a total timber volume of 650–690 m$^3$/ha. As in the case of natural forest management, we assume that it costs a fixed amount, $S_i = $100/ha, to undertake seedling planting in the initial year of the rotation and that the cost of timber harvesting is $C_i = $25/m$^3$. The stumpage price of softwood timber from plantation forests is $P_i = $75/m$^3$, which is lower than the price of quality hardwood tree species from natural tropical forests.

Using (A8), we can establish the numerical value of the optimal rotation length and the maximum bare land value for both types of forest land-use options. For natural forest management, the optimal rotation length is 36 yr, and the maximum bare land value is $1,530/ha. For plantation forestry, the optimal rotation length is also 36 yr, and the maximum bare land value is $4,138/ha. The fact that the optimal rotation length of both types of timber is 36 yr is purely a coincidence and a result of the key parameter values chosen. In particular, the slower growth but higher value of the hardwood trees on the natural forest stand offsets the faster growth but lower value of the softwood plantation timber. Given these comparative bare land values, we can see that plantation timber production yields the highest returns, and the forestland manager will allocate the bare land to this forest land-use option.