Tracing causes for the stress sensitivity of elastic wave velocities in dry Castlegate sandstone

Anna Magdalena Stroisz¹ and Erling Fjær²

¹Norwegian University of Science and Technology, Trondheim, Norway. E-mail: anna.stroisz@ntnu.no
²Pontifical Catholic University of Rio de Janeiro, RJ, Brazil; Norwegian University of Science and Technology, Trondheim, Norway; and SINTEF Petroleum Research, Trondheim, Norway

SUMMARY

The stress sensitivity of elastic wave velocities in dry rock is a resultant of two types of processes—elastic and non-elastic processes. Both processes are activated under stress and both are associated with stress-induced changes in the rock structure. Although they are of the same origin, their stress-dependency may differ.

In this study, a set of tests that separate the elastic and non-elastic processes is used to evaluate the impact of each process on the stress sensitivity of the elastic wave velocities. The tests rely on comparing the stress sensitivity of wave velocities during uniform loading/unloading over a long stress interval (involving both elastic and non-elastic processes) with the stress sensitivity seen in low-amplitude stress oscillations (assumed to be affected mainly by elastic processes). Our study on dry, weak sandstone shows that the impact of elastic and non-elastic processes on the stress sensitivity of the elastic wave velocities is significantly different. This implies that the processes ought to be described separately in order to provide a better foundation for predictive rock physics models.

Observations on artificially fractured samples indicate that large, horizontal fractures reduce the axial wave velocities, whereas they have no notable impact on the stress sensitivity of the velocities. This suggests that the closed macro-fractures contain damaged areas with reduced stiffness which are apparently insensitive to stress changes.

A few basic processes—elastic opening and closure of cracks, friction-controlled shear sliding of closed cracks associated with opening or closure of wing cracks, and crushing of asperities in fractures or grain contacts—may be used to intuitively explain the observations.

Key words: Microstructures; Elasticity and anelasticity; Wave propagation; Acoustic properties.

1 INTRODUCTION

The past several decades have brought an increased interest in the nonlinear elastic behaviour of rocks. Literature provides a large number of theoretical considerations and experimental studies on this subject (e.g. Johnson & Shankland 1989; Johnson & Rasolofosaon 1996; Johnson et al. 2004). The studies confirm significant (compared to other materials) observations of nonlinear elastic behaviour in rocks that is several orders of magnitude higher than that of fluids and mono-crystalline solids, for example (Guyer & Johnson 1999). The large nonlinear elastic behaviour of rocks is associated with their complex structures—the weak features (discontinuities) in a hard rock matrix (Ostrovsky et al. 2000). Discontinuities, due to their sensitivity on the stress changes, are the main sources of nonlinearity.

The nonlinear elastic response of rocks, depending on the test condition, is manifested by many different phenomena. The most commonly investigated are: nonlinear stress–strain relation, hysteresis, stress and strain-dependence of wave velocities, resonance peak shift, pulse distortion, side bands appearance, among others (e.g. Ostrovsky & Johnson 2001).

The stress sensitivity of elastic wave velocities in sedimentary rock is a result of several processes that are activated in the rocks’ structure under variable stress conditions. These processes may be associated with phenomena such as: a mechanism of crack opening and closing, shear sliding (i.e. permanent damage induced by the crack interaction), changes in the grain contacts area (i.e. broadening and multiplying grain contacts) and the breaking of grain bonds. Some of them, like crack mechanisms, are fairly well described (Walsh 1965a,b), while others, such as the effects occurring at the grain contacts, are just briefly mentioned (Liu 1994; Plona & Cook 1995). However, all of these mechanisms give physical explanations for the stress-dependency of both static and dynamic parameters of rock. The phenomena appear in the rock framework simultaneously; thus, it is impossible to distinguish and evaluate them individually by standard wave velocity measurements only. However, a certain type of test enables a separation into two groups of effects, that is, the elastic and non-elastic processes. Here, the term ‘elastic’
refers to a nonlinear elastic response, since linear elasticity does not cause any stress-dependency of wave velocities. Such a separation is useful for tracing the type of structural change, reversible (elastic) or permanent (non-elastic), that is particularly responsible for the stress sensitivity of rock. The separation is essential for understanding how stress path influences the stress-dependency of the velocities.

So far, little has been done to investigate the contribution of the elastic and non-elastic sources separately. The main reason is a difficulty in their separation, since in most of the examined stress conditions, these processes coexist, and thus overlap. In order to investigate them individually, we propose a relevant testing method. The test is a combination of a typical stepwise stress alteration and low-amplitude stress oscillations, resembling dual-frequency acoustic measurements as SURF technique (Másóy et al. 2010). The first part reflects the structural response of the rock at relatively large scale, that is, over a long, uniform and unidirectional (loading/unloading) stress interval, which is a combined effect of both elastic and non-elastic processes. The second part, however, reveals the elastic processes only through a small-scale response as a result of relatively smaller oscillatory stress variations. Proper combination of both parts gives the possibility to estimate the non-elastic processes. This combination is enabled by the use of a model for which we assume TI symmetry and examine two out of five coefficients representing the TI stiffness tensor (i.e. $C_{17}$ and $C_{66}$), since only axial P and S waves were investigated in this study.

The tests are performed on the several samples of Castlegate sandstone at different stress conditions. The impact of pore pressure on the stress sensitivity of the elastic waves was initially tested, and found to have only minor impact, if any, apart from the effect that pore pressure has on the effective stress (Stroisz & Fjær 2011). The main part of the study was therefore focused on dry samples only, enabling an investigation of the stress-induced changes related directly to the rock structure, and to avoid the difficulties connected with fluid interaction. This study includes experiments on intact and artificially fractured specimens. Also, the impact of stress state (three different stress conditions) and stress history (the stress repetition and the direction of stress path) has been studied.

2 EXPERIMENTAL SETUP

The experiments were conducted on several cylindrical core plugs (diameter $\sim$1.5” and length $\sim$3”) of dry Castlegate sandstone. Castlegate sandstone is a weak outcrop rock with 26 per cent porosity and low clay content, composed of about 70 per cent quartz, 30 per cent feldspar and other rock fragments. It is characterized by an apparently homogeneous structure, with no distinct lamination, making it a relatively isotropic rock. The unconfining compressive strength (UCS) of this rock is about 16.5 MPa and Young’s modulus at unconfined conditions is about 4.4 GPa (Fjær et al. 2005). All examined samples were taken from the same block of Castlegate sandstone and prepared in the same way. The samples were dried by heating at 80 °C for about 48 hr.

The tests were a combination of quasi-static and dynamic measurements. Ultrasonic wave velocities were measured by transmission of compressional (P) and shear (S) waves in the axial direction (i.e. along the axis of the major external stress). Single sine pulses at the frequency 500 kHz were generated with a function generator (Agilent 33220A, Agilent Technologies, CA, USA) and amplified by 50 dB power amplifier (EIN 2100L RF, NY, USA). A switch unit (HP34970A, Agilent Technologies, CA, USA) provided an alternative excitation of P- and S-wave piezo-elements. In an in-house-created transducer, P- and S-wave piezo-elements were glued together to the front buffer made of PEEK or Titanium. The role of the front buffer was to provide an optimal acoustic matching between the transducer and sample (to maximize the energy transfer) and to protect the piezo-elements against the load applied on the sample (the buffer was bolted to the steel housing). Acoustic data, that is, waveforms, were displayed and recorded with a digital oscilloscope (TDS3012B, Tektronix, OR, USA).

The amplifier belongs to an old generation of amplifiers that may produce distorted output. However, this study investigates the change of the wave velocities due to stress variation (stress oscillations) rather than distortion of the waveforms. In addition, zero-crossing was chosen as picking points for travel time estimation; thus, we believe that a possible pulse distortion should have only a negligible effect on our data. The limited dynamic range of the oscilloscope is not an issue in this study, since the amplitude of the waveform is not taken into consideration.

The static part of the experiments was performed in one of two loading systems—a uniaxial load frame (10 kN, MTS, MN, USA) or a triaxial load cell (MessTek, Messtek Prüfsysteme GmbH, Germany). The axial stress ($\sigma_x$) was exerted on the sample by a movement of axial pistons, being in contact with the transducer housings, and pressurized oil, that is, a confining fluid. Radial stress ($\sigma_r$) was provided by the confining fluid alone. Rubber sleeves wrapped around the samples were used to separate them from the confining oil. Axial displacement was measured by three linear variable differential transformers (LVDTs), while the radial strain, measured only in the triaxial tests, was detected with a chain or a cantilever system.

The samples were subjected to one of three stress conditions listed in Table 1. In all tests, the stress was subsequently loaded and/or unloaded in a stepwise manner; however, the stress range and the rate of change differ depending on the test type. In the uniaxial compression test (Fig. 1a), two samples were examined—sample 1A, tested by a triple repetition of stepwise unloading stress path, and sample 1B, tested by both loading and unloading stepwise stress path repeated four times. In the uniaxial compaction test (Fig. 1b), one sample was examined—sample 2, tested by loading and unloading stepwise stress path repeated twice. In the combined test (Fig. 1c), sample 3 was tested with a single loading stepwise stress path.

The standard stepwise path was modified by the periods of low-amplitude axial stress oscillations added at each step (see Fig. 2). These oscillatory periods represent small-scale stress changes of the rock structure, which is assumed to induce a predominantly elastic response. The oscillations take the form of sinusoidal or square-like shape and their number varies from 10 to 20 cycles, depending on the test type. The frequency of these sine and square cycles was about 0.042 and 0.006 Hz, respectively. The time interval between acoustic pulses (P or S) was about 8 and 20 s (giving approximately three and eight P- and S-wave pulses per one axial stress cycle) for sine and square cycles, respectively. In all cases, however, the peak-to-peak amplitude of the cycles was 1 MPa. Each oscillatory period was preceded by the short period of stabilization, which was also used to analyse the change in velocities due to large-scale stress changes, assumed to induce both elastic and non-elastic processes.

For all test types, we extended our studies by introducing artificial fractures on the samples. The fracture(s) were created in the samples that were previously tested as intact samples, and the same test procedure has been used for examination. Each fracture was created as a through-going lateral crack by carefully splitting the core using
Table 1. Test details.

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Description</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial compression (uniaxial conditions)</td>
<td>Unloading: $\sigma_z = 0.5$–8.5 MPa, $\sigma_r = 0$ MPa</td>
<td>1A Intact, one and two fractures</td>
</tr>
<tr>
<td></td>
<td>Loading/unloading: $\sigma_z = 0.5$–7.5 MPa, $\sigma_r = 0$ MPa</td>
<td>1B Intact and one fracture</td>
</tr>
<tr>
<td>Uniaxial compaction (triaxial conditions)</td>
<td>Loading/unloading: $\sigma_z = 4$–78 MPa, $\sigma_r = 3$–17 MPa, ($\sigma_r$ change due to $K_0$ mode)</td>
<td>2 Intact, one and two fractures</td>
</tr>
<tr>
<td>Combined* (triaxial conditions)</td>
<td>Loading: $\sigma_z = 7$–63 MPa, $\sigma_r = 5$–61 MPa, ($\sigma_r = \sigma_z - 2$ MPa)</td>
<td>3 Intact and one fracture</td>
</tr>
</tbody>
</table>

*Combined test denotes the hydrostatic stepwise loading with $K_0$ mode oscillations. Axial stress ($\sigma_z$) and radial stress ($\sigma_r$).

Figure 1. Stress versus time for: (a) uniaxial compression tests, left—sample 1A, right—sample 1B; (b) uniaxial compaction test; (c) combined test. Red line denotes axial stress ($\sigma_z$) and black line denotes radial stress ($\sigma_r$).

hydraulic press and cutting edge. Some specimens were fractured once (sample 1B and sample 3), while others twice (sample 1A and sample 2) (Fig. 3).

3 PROCEDURE FOR DATA ANALYSES

A simple model description was established in order to facilitate the data analysis. The model aims to analyse the two stiffness coefficients ($C_{33}$ and $C_{44}$), derived from the wave velocities, in order to investigate the sources of their stress-dependency. The possible causes of the stress-dependency are ascribed to the elastic and non-elastic processes that coexist during stress application. An estimation of the individual impact of each process is not straightforward; however, it can be done by the appropriate test procedure and a few assumptions. Here, it is done by separating the test into large- and small-scale stress variations, which act differently on the stress.
sensitivity of wave velocities. In the large scale, that is, during uniform loading/unloading over a long stress interval, the response includes both elastic and non-elastic processes. The small scale, that is, low-amplitude stress oscillations, is assumed to be affected by elastic processes only.

In this study, we focus on the dry rock, since the results on saturated samples do not indicate any particular impact on the stress sensitivity of the elastic wave velocities. Furthermore, this restriction allows analysis of the stress sensitivity of the solid framework alone, and thus simplifies the mathematical description. The use of Castlegate sandstone, which has a fairly homogeneous and isotropic structure, implies that the anisotropy in this case is mainly stress-induced, and we therefore assume TI symmetry, with the sample axis as the unique axis. This reduces the number of independent coefficients of the stiffness tensor and gives the relations between the measured velocities $V_{p,z}$ and $V_{s,z}$ (axial $P$- and $S$-wave velocities) and the stiffness coefficients $C_{33}$ and $C_{44}$ as:

$$C_{33} = \rho \cdot V_{p,z}^2,$$

$$C_{44} = \rho \cdot V_{s,z}^2,$$  

where $\rho$ is the density. The stiffness coefficients may change as a consequence of changes in velocities and density with stress. These changes arise from nonlinear elastic and non-elastic deformations of the rock structure. Thus, the stress-dependent stiffness coefficients may be written in a differential form, as a sum of both effects:

$$\Delta C_{33} + \Delta C_{33}^{\text{ne}} = \Delta \rho \cdot V_{p,z}^2 + 2 \rho \cdot V_{p,z} \cdot \Delta V_{p,z},$$  

$$\Delta C_{44} + \Delta C_{44}^{\text{ne}} = \Delta \rho \cdot V_{s,z}^2 + 2 \rho \cdot V_{s,z} \cdot \Delta V_{s,z}. $$

Here, $\Delta C_{ij}$ and $\Delta C_{ij}^{\text{ne}}$ ($i = 3, 4$) are the changes in stiffness due to non-linear elastic effects and non-elastic effects, respectively. In this analysis, we try to evaluate these parameters separately in order to see how they change with stress. The model which aims to do that is based on the one common idea that applies to all three test conditions, namely that the periods of oscillations (labelled $\text{osc}$), representing small-scale changes, are only affected by non-linear elastic effects, and thus can be expressed as

$$\frac{\Delta C_{33}}{\Delta \sigma_z} = 2 \rho \cdot V_{p,z} \cdot \frac{\Delta V_{p,z}}{\Delta \sigma_z} + V_{p,z}^2 \cdot \frac{\Delta \rho}{\Delta \sigma_z}, \quad (5)$$

$$\frac{\Delta C_{44}}{\Delta \sigma_z} = 2 \rho \cdot V_{s,z} \cdot \frac{\Delta V_{s,z}}{\Delta \sigma_z} + V_{s,z}^2 \cdot \frac{\Delta \rho}{\Delta \sigma_z}. \quad (6)$$

This assumption is supported by theoretical studies (Walsh 1965a,b) and experimental studies (Plona & Cook 1995; Fjær et al. 2011). The validity of this assumption is also tested against observations, as shown below, and found to be supported.

The static loading/unloading stepwise path (labelled $\text{slope}$), representing large-scale stress changes, include both the elastic and non-elastic processes. Hence, the equations take the following forms:

$$\frac{\Delta C_{33}}{\Delta \sigma_z} + \frac{\Delta C_{33}^{\text{ne}}}{\Delta \sigma_z} = 2 \rho \cdot V_{p,z} \cdot \frac{\Delta V_{p,z}^{\text{slope}}}{\Delta \sigma_z} + V_{p,z}^2 \cdot \frac{\Delta \rho^{\text{slope}}}{\Delta \sigma_z}, \quad (7)$$

$$\frac{\Delta C_{44}}{\Delta \sigma_z} + \frac{\Delta C_{44}^{\text{ne}}}{\Delta \sigma_z} = 2 \rho \cdot V_{s,z} \cdot \frac{\Delta V_{s,z}^{\text{slope}}}{\Delta \sigma_z} + V_{s,z}^2 \cdot \frac{\Delta \rho^{\text{slope}}}{\Delta \sigma_z}. \quad (8)$$

The slopes of the stress-dependent $P$- and $S$-wave velocity curves $V(\sigma_z)$ are established from measurements prior to the oscillations (i.e. the stabilization periods). These slopes represent the large, unidirectional loading/unloading changes only. This justifies the assumption concerning the influence of both types of effects.

Combination of the corresponding eqs (5) with (7) and (6) with (8) enables us to isolate the non-elastic effects and estimate their contribution to the stress-dependency of rock stiffness and wave velocities. The procedure, however, is directly applicable only for the uniaxial compression and uniaxial compaction tests, in which the small- and large-scale stress changes were performed at the same stress conditions. It implies that the non-elastic component of stiffness can simply be estimated by subtracting the elastic component (eqs (5) and (6)) from the total stiffness (eqs (7) and (8)). The case is somewhat more complicated for the combined tests, where the stress conditions of small- and large-scale stress changes are not alike. The oscillatory periods (small-scale) were performed as uniaxial compaction, while the stepwise load/unload (large-scale) path is hydrostatic. This precludes a straightforward use of the eqs (5)–(8) that need to be modified before implementation to create a link between two alterative stress conditions. Here, we converted the hydrostatic loading into the uniaxial compaction ($K_0$) loading, following a procedure as described in Appendix A. This modifies eqs (7) and (8) into the following expressions:

$$\frac{\Delta C_{33}}{\Delta \sigma_z} + \frac{\Delta C_{33}^{\text{ne}}}{\Delta \sigma_z} \approx A \left( 2 \rho \cdot V_{p,z} \cdot \frac{\Delta V_{p,z}^{\text{slope}}}{\Delta \sigma_z} + V_{p,z}^2 \cdot \frac{\Delta \rho^{\text{slope}}}{\Delta \sigma_z} \right),$$

$$A = \left( Q_{33} + 2Q_{11}, \lambda^{K_0}, \frac{\Delta \sigma_z}{\Delta \sigma_z} \right) (Q_{33} + 2Q_{11})^{-1}, \quad (9)$$
The stress sensitivity of wave velocities

The stress sensitivity of wave velocities can be quantified through the relationship between wave velocity and stress. Figure 4 illustrates the dependence of P-wave velocity gradient on pore pressure level, plotted with respect to net axial stress, which is the difference between axial stress and pore pressure ($\sigma_z' = \sigma_z - P_f$). Data refer to the oscillatory periods. Different colors represent different pore pressure levels from 6 to 38 MPa. Data source—combined test conducted in undrained conditions on the intact sample, fully saturated with oil (kerosene).

\[
\frac{\Delta C_{44}}{\Delta \sigma_z} + \frac{\Delta C_{64}}{\Delta \sigma_z} \approx B \left( 2\rho \cdot V_s^2 \cdot \frac{\Delta V_s}{\Delta \sigma_z} \right)
\]

\[
B = \left( Q_{44} + (Q_{44} + Q_{66}) \cdot \frac{K_0}{\Delta \sigma_z} \right) \cdot (Q_{66} + 2Q_{44})^{-1}.
\]  

Here, $Q_{ij}$ are the coefficients representing the impact of the cracks on relevant elements of the stiffness tensor, given by eqs (A13)–(A16) in Appendix A. Combining eqs (9) and (10) that represent the total stiffness (i.e. both elastic and non-elastic components) with a relevant formula, eqs (5) and (6) that represent the elastic component only, it is possible to estimate the contribution of non-elastic effects during the combined test.

4 RESULTS

The combined test conducted on the sample fully saturated with oil under the undrained conditions show that the pore pressure appears to be indifferent to the stress sensitivity of wave velocity (Stroisz & Fjær 2011). For different pore pressure levels, the velocity gradients do not reflect any specific ordering (Fig. 4), and the difference between curves is so insignificant that it may not be related with the pore pressure impact but rather with the uncertainty of measurements. This result convinced us to restrict our later measurements and analyses to a dry rock only.

The P- and S-wave velocities in dry Castlegate sandstone depend strongly on stress (Fig. 5). This stress sensitivity is particularly pronounced at the low-stress levels and appears for both the large- and small-scale stress changes (i.e. stepwise path and oscillatory periods). The stress-induced increase of the velocity and the decrease of the velocity gradient appear to be fairly consistent for both P- and S-wave velocities, and for all tested stress conditions.

Like the velocities, the corresponding stiffness tensors $C_{33}$ and $C_{44}$ (eqs (1) and (2)) increase with increasing stress (Fig. 6a). The rate of this increase depends on the stress level. It is particularly distinctive at low stresses and highly reduced at higher stresses. Therefore, the stiffness gradients decrease most rapidly at low stresses, but remain significant at the highest stress levels (Fig. 6b).

Figure 4. The dependence of P-wave velocity gradient on pore pressure level. The velocity gradient is plotted with respect to net axial stress, which is the difference between axial stress and pore pressure ($\sigma_z' = \sigma_z - P_f$). Data refer to the oscillatory periods. Different colours represent different pore pressure levels from 6 to 38 MPa. Data source—combined test conducted in undrained conditions on the intact sample, fully saturated with oil (kerosene).

Figure 5. The stress sensitivity of P-wave velocity observed in three stress conditions: uniaxial compression (green markers), uniaxial compaction (red markers) and combined test (blue markers). Data refer to the stabilization periods, that is, the large-scale change, but a similar trend is observed in oscillatory periods, that is, the small-scale change.

Figure 6. The dependency of stiffness coefficients (a) and stiffness gradients (b) on stress change at three stress conditions: uniaxial compression (green markers), uniaxial compaction (red markers) and combined test (blue markers). Data refer to the stabilization periods, thus including both the elastic and non-elastic processes. Note that curves in combined test refer to gradients after conversion from hydrostatic to $K_0$ mode.
According to eqs (3) and (4), the stiffness gradient depends on the stress-induced changes of both velocity and density. Thus, considering the equations with respect to the terms including the velocity gradient \((2\rho \cdot V \cdot \Delta V)\) and density gradient \((V^2 \cdot \Delta \rho)\) separately, we are able to investigate the impact of these two parameters on the stiffness gradient. Our study shows that this impact differs significantly, and the stress sensitivity of the stiffness is predominantly associated with the velocity gradient. The influence of the density gradient appears to be negligible in particular at the low stresses at which the density variation constitutes only 0.5 per cent of the stiffness gradient. This impact increases with increasing stress and at the highest stress level, it reaches about 16 and 21 per cent of the stiffness gradient components \(C_{33}\) and \(C_{44}\), respectively, for the initial loading path. For reloading and unloading stress paths, the impact of the density gradient does not increase with stress as much as for the first loading path, and at the highest stresses, it reaches about 7 per cent for reloading and 3 per cent for unloading.

Our test method and analysis enable us to distinguish between elastic and non-elastic processes in the "total" stiffness gradient. Both effects are activated in the rock structure under monotonic, large amplitude stress increase, and as Fig. 7 shows both are highly stress-dependent and diminish with rising stress. The magnitude of the non-elastic part drops close to zero or takes a negative value for high stresses, while the elastic part remains significant even at the highest stress levels.

Parameters for the regression lines representing the stiffness gradients, presented in Fig. 7, are given in Table 2. A power type of regression line has been used to obtain the most satisfactory fit according to the least squares method. Note that two different mathematical formulations are used to retrace the results. The elastic part of the stiffness gradient is represented by the pure power function that excludes negative values, which is according to our observations. Whereas the total and the non-elastic part of the stiffness gradients are given as the power functions with constant factors, which allow taking negative values.

The ratio of the elastic part to the total stiffness gradient may be estimated as

\[
R_{33} = \frac{\Delta C_{33}}{\Delta \sigma_2} \quad \text{and} \quad R_{44} = \frac{\Delta C_{44}}{\Delta \sigma_2}
\]

The ratio, for both the \(C_{33}\) and the \(C_{44}\) coefficients, is in the range from about 50 per cent (at low stress levels) to 100 per cent (at high stress levels). The rate of change depends on the test conditions; for uniaxial tests (uniaxial compression test), it is much more rapid than for the triaxial tests (uniaxial compaction and combined test) (Fig. 8).

The stress sensitivity of the stiffness coefficients \(C_{33}\) and \(C_{44}\) appears to be different, in all test conditions. This relates to the direction of wave propagation and wave polarity with respect to the direction of applied stress. The value of the total stiffness gradient of \(C_{33}\) is approximately three to five times higher than the gradient of \(C_{44}\) (Fig. 9a). The difference between the two stiffness gradients is seen both in the elastic and non-elastic parts (Fig. 9b).

Effects of stress history on the stiffness gradients were tested by running several loading/unloading/reloading cycles. Fig. 10 shows the results for \(C_{33}\), but comparable behaviour is revealed by \(C_{44}\) component as well. Several subsequent loading sequences give nearly identical results; however, these are different from the results of the initial loading path. Subsequent unloading sequences are also nearly identical; however, loading and unloading stress paths give

![Figure 7](https://academic.oup.com/gji/article-abstract/192/1/137/596394/137)}
The stress sensitivity of wave velocities

Table 2. Fitted power function for the gradient of $C_{33}$ and $C_{44}$ stiffness components.

<table>
<thead>
<tr>
<th>Test</th>
<th>Stiffness gradient</th>
<th>Elastic part</th>
<th>Non-elastic part</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td><strong>Total</strong></td>
<td><strong>$y_{33}$</strong></td>
<td><strong>$y_{44}$</strong></td>
</tr>
<tr>
<td>Uniaxial compression</td>
<td>$y_{33} = 3640x^{-0.60} - 827$</td>
<td>$y_{33} = 1811x^{-0.99}$</td>
<td>$y_{44} = 379x^{-0.91}$</td>
</tr>
<tr>
<td></td>
<td>$y_{44} = 1009x^{-0.40} - 414$</td>
<td>$y_{44} = 2893x^{-0.04} - 2671$</td>
<td></td>
</tr>
<tr>
<td>Uniaxial compaction</td>
<td>$y_{33} = 1315x^{-0.34} - 281$</td>
<td>$y_{33} = 1267x^{-0.85}$</td>
<td>$y_{44} = 294x^{-0.77}$</td>
</tr>
<tr>
<td></td>
<td>$y_{44} = 365x^{-0.23} - 129$</td>
<td>$y_{44} = 660x^{-0.25} - 222$</td>
<td></td>
</tr>
<tr>
<td>Combined test</td>
<td>$y_{33} = 1572x^{-0.63} - 82$</td>
<td>$y_{33} = 1402x^{-0.85}$</td>
<td>$y_{44} = 294x^{-0.79}$</td>
</tr>
<tr>
<td></td>
<td>$y_{44} = 469x^{-0.80} - 11$</td>
<td>$y_{44} = 910x^{-0.85} - 29$</td>
<td></td>
</tr>
</tbody>
</table>

$^*$Data in the table refer to intact samples (sample 1A, sample 2 and sample 3) during loading path. Note that the curves in combined test represent gradients after conversion from hydrostatic to $K_0$ mode. The parameters denote: $x$—axial stress ($\sigma_z$) [MPa]; $y_{33}$—stiffness gradient for $P$ wave ($\Delta C_{33}/\Delta \sigma_z$) [no unit]; $y_{44}$—stiffness gradient for $S$ wave ($\Delta C_{44}/\Delta \sigma_z$) [no unit].

somewhat different results. Thus, the direction of the stress path apparently has an impact on the stiffness.

A closer investigation shows that the stress path repetition does not have any particular effect on the elastic or the non-elastic effect (Fig. 11). In both cases, the subsequent loading or subsequent unloading (not presented here) has comparable characteristics. The only difference appears in the initial loading path and this difference is seen in both elastic and non-elastic processes.

The impact of the rock framework was examined by introducing artificial fracture(s) to the intact samples. Although a fracture is potentially a weak plane with highly stress-sensitive stiffness, their presence appears to have no impact on the stress sensitivity of the elastic and non-elastic parts of stiffness coefficients $C_{33}$ and $C_{44}$ (Fig. 12). The stiffness gradients remain fairly comparable for intact and fractured specimens, and the minor differences are comparable to the fluctuations between different runs. This suggests that the differences reveal the test uncertainty, rather than the actual difference in the structure.

The wave velocities reveal a clear difference between the three types of specimens: intact, with one fracture, and with two fractures (Fig. 13). This difference is mainly seen in the magnitude of the velocity and far less in the velocity gradient (rate of velocity change with stress).

5 DISCUSSION

Stress changes may induce several processes in the rock structure. A part of these processes, which may intuitively be symbolized by the opening and closing of cracks with face normal to the applied stress (Fig. 14a), is elastic and reversible. For this part, an increasing stress causes stiffening of the rock framework due to crack closure, while stress removal causes softening due to opening of cracks and recovery of the initial structure and stiffness. Consequently, the
stiffness gradients associated with the elastic processes are always positive, as confirmed by our observations and reflected in the fitted power functions (Table 2).

Another part may involve friction controlled shear sliding along internal surfaces (Figs 14b and c). This process is usually accompanied by opening (Fig. 14b) or closure (Fig. 14c) of nearby cracks (so-called “wing cracks”). The wing cracks will affect the velocities like any other cracks, and therefore the process is important for the stress-dependency of the wave velocities. This process is controlled by friction; thus, a certain magnitude of shear stress is required to overcome the static friction and activate the shear sliding. In the reversal process, the level of stress reduction needs to be high enough to overcome the static friction and activate the shear sliding in the opposite direction. For the oscillatory periods, we have tested herein, the stress amplitude is so small that this type of process is not expected to be activated (see Appendix B), and the process will therefore be classified as ‘non-elastic’ in our analysis. Depending on the unloading conditions, the structural changes induced by this process may be completely or partly recovered upon total stress relaxation. Hence, the process is, strictly speaking, ‘non-perfectly-elastic’. The crushing of asperities at grain contacts (Fig. 14d) is an example of a purely non-elastic process, since it changes the rock structure permanently. We assume, however, that this process is restricted to loading, and mainly to the initial loading path, since destroyed asperities remain permanently in this condition. This process may explain why the initial loading path differs from the reloading paths (Figs 10 and 11). The initial loading represents stress that sample experiences for a first time. Hence, the initial stress introduces the most extensive changes to the rock structure. This is confirmed by the large density change, about 30 per cent higher than during reloading. This means that during reloading, the rock structure remains fairly unchanged, as long as the stress does not exceed the maximum value of the initial loading. Contrary to the large density change, the change of $P$- and $S$-wave velocity was smaller for initial loading than for reloading (the difference of about 40 and 45 per cent for $P$ and $S$ waves was observed). As a result, the $P$- and $S$-wave stiffness gradients increase of about 10 and 25 per cent, respectively, during reloading.

The direction of applied stress, that is, loading versus unloading stress path, has an impact on the stress sensitivity. The total stiffness gradients for the loading paths differ from the gradients for the unloading paths. For the uniaxial compression test (Fig. 10a),

Figure 10. The effect of stress history on the total stiffness gradient $C_{33}$. Data source—uniaxial compression test, intact sample 1B (a) and uniaxial compaction test, intact sample 2 (b). Note that the stress history was not investigated in the combined test. The lines in the figure are guides for the eye only.

Figure 11. The effect of the stress path repetition on the elastic (solid line) and non-elastic (dashed line) stiffness gradient $C_{33}$. Data source—uniaxial compaction test. The lines in the figure are guides for the eye only.

Figure 12. The effect of through-going horizontal fractures on the elastic (solid line) and non-elastic (dashed line) stiffness gradient $C_{33}$. Data source—uniaxial compaction test. The lines in the figure are guides for the eye only.
The stress sensitivity of wave velocities

1.45

1.45

Figure 13. The effect of through-going horizontal fractures on the P-wave velocity (a) and the P-wave velocity gradient (b). Data source—uniaxial compaction test. The lines in the figure are guides for the eye only.

The gradients are significantly larger for the loading than unloading paths. This is not as obvious for the uniaxial compaction test (Fig. 10b).

Our experiments reveal that both elastic and non-elastic effects have significant contribution to the stress sensitivity of stiffness, and as follows to the stress-dependence of the elastic waves. Their impact depends on the stress level—at low stresses they have comparable impact, while at high stress levels, the elastic process dominates (Fig. 8). However, it does not imply that at high stress levels, the non-elastic processes disappear or are less active than the elastic processes. In fact, they can be very active, since among the non-elastic processes are those which increase as well as reduce the stiffness of the rock framework. The stiffness increment, due to crack closure (Fig. 14c), decreases in intensity with increasing stress since the rising stress decreases the number of crack possible to be closed. The stiffness reduction is associated with crack opening (Fig. 14b), which increases in intensity when the failure envelope is approached. This process, in contrast to crack closure, is basically unlimited by the stress level; hence, the new cracks appear until the failure. The total effect that these opposite processes have on the stiffness gradient is thus suppressed due to cancellation. This most likely explains why the stress levels where the elastic processes dominate are different for the different tests (Fig. 8), as the tests approach the failure envelope along different paths.

Our analysis shows that the stress-dependence of the wave velocities is mostly controlled by changes in the rock stiffness, not changes in the density. This indicates that the stress sensitivity of the rock largely relates to micro-cracks, since opening and closure of cracks does not cause large porosity changes and thus density variations, but have a significant impact on the rock stiffness (Guéguen & Schubnel 2003). The orientation of these cracks relative to the applied stress (as indicated in Fig. 14) and the sensitivity of P and S waves to open cracks (see, for instance, Hudson 1981; Schubnel & Guéguen 2003) explains the larger stress sensitivity of the axial P wave than the axial S wave, as seen in our study.

The tests on the fractured samples indicate that the large-scale fracture(s) have an impact on the magnitude of wave velocities. The velocities decrease with each introduced fracture (Fig. 13), reaching a difference of about 70 m s$^{-1}$ (corresponding to about 1.5 per cent) between the intact and doubly fractured sample. It appears, however, that the fractures have no measurable effect on the gradients. The observations indicate that these closed macro-fractures contain damaged areas with reduced stiffness which is apparently insensitive to stress changes. The fact that the difference in velocity between fractured and intact rock is almost the same for all stress levels (Fig. 13) supports this interpretation.

6 CONCLUSIONS

Our study on dry, weak sandstone indicates that the stress-induced changes of wave velocities are dominated by changes in the rock stiffness, while density fluctuations are insignificant. The stress sensitivity of the stiffness can be ascribed to two types of processes—elastic and non-elastic processes—activated in the solid framework under stress. Both processes are highly stress-dependent; however, their stress-dependencies are different. At low stress levels, both processes have comparable contributions to the stiffness gradient, while at the higher stress levels explored in our study, the elastic processes seem to dominate. The elastic part of the stiffness gradient takes non-negative values only, and may be represented by a power

Figure 14. Idealized examples of processes activated in the rock framework under stress. Elastic process (a), non-perfectly-elastic process (b) and (c), and non-elastic process (d).
function, whereas the non-elastic part needs a constant component to be added to the power function to allow also for negative values. These findings show that the elastic and non-elastic processes have to be handled separately in rock physics models.

Although a number of elastic and non-elastic processes are activated in the solid framework under stress, we found that only a few basic processes are sufficient for an intuitive explanation of all observations in our study. These are: elastic opening and closure of cracks, friction-controlled shear sliding of closed cracks associated with opening or closure of wing cracks, and crushing of asperities in fractures or grain contacts.

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REFERENCES


APPENDIX A

This appendix describes the method used to convert data recorded during hydrostatic loading into parameters describing uniaxial compaction. The general expression for the stiffness gradient under hydrostatic conditions is given as

$$\frac{\Delta C_{ij}^{\text{Hyd}}}{\Delta \sigma} = \frac{\partial C_{ij}}{\partial \sigma_z} + \frac{\partial C_{ij}}{\partial \sigma_x} + \frac{\partial C_{ij}}{\partial \sigma_y},$$

and the corresponding expression for uniaxial compaction conditions as

$$\frac{\Delta C_{ij}^{\text{Uniax}}}{\Delta \sigma_z} = \frac{\partial C_{ij}}{\partial \sigma_z} + \frac{\partial C_{ij}}{\partial \sigma_x} = \frac{\partial C_{ij}}{\partial \sigma_y} = \frac{\partial C_{ij}}{\partial \sigma_z}.$$  

If we consider that the stiffness tensor responds to the stress changes primarily because of opening and closure of cracks, and that the crack density is only a function of the stress normal to the crack (assuming that the rock possesses three orthogonal crack sets), we find (Fjær 2006b) that

$$\frac{\partial C_{33}}{\partial \sigma_x} \propto Q_{33},$$

$$\frac{\partial C_{11}}{\partial \sigma_x} \propto Q_{11},$$

$$\frac{\partial C_{44}}{\partial \sigma_x} \propto Q_{44},$$

$$\frac{\partial C_{66}}{\partial \sigma_x} \propto Q_{66},$$

where $Q_x$ are the coefficients representing the impact of the cracks on relevant elements of the stiffness tensor (see, for instance, Fjær et al. 2008). Combining these expressions, we find that the stiffness gradients for hydrostatic test take the form:

$$\frac{\Delta C_{33}^{\text{Hyd}}}{\Delta \sigma} = \frac{\partial C_{33}}{\partial \sigma_z} + \frac{\partial C_{33}}{\partial \sigma_x} + \frac{\partial C_{33}}{\partial \sigma_y} \propto Q_{33} + 2Q_{11},$$

$$\frac{\Delta C_{44}^{\text{Hyd}}}{\Delta \sigma} = \frac{\partial C_{44}}{\partial \sigma_z} + \frac{\partial C_{44}}{\partial \sigma_x} + \frac{\partial C_{44}}{\partial \sigma_y} \propto Q_{66} + 2Q_{44}.$$
while for the uniaxial compaction test, and with $\Delta \sigma_\lambda = \Delta \sigma_\nu = \Delta \sigma_z$, take the form:

\[
\frac{\Delta C_{33}}{\Delta \sigma} = \frac{\partial C_{33}}{\partial \sigma} + \frac{\partial C_{33}}{\partial \sigma_\lambda} \cdot \frac{\Delta \sigma_\lambda}{\Delta \sigma} + \frac{\partial C_{33}}{\partial \sigma_\nu} \cdot \frac{\Delta \sigma_\nu}{\Delta \sigma} + \frac{\partial C_{33}}{\partial \sigma_z} \cdot \frac{\Delta \sigma_z}{\Delta \sigma} \cdot Q_{33} + 2Q_{11} \cdot \frac{\Delta \sigma_\lambda}{\Delta \sigma} .
\]  

(A9)

\[
\frac{\Delta C_{44}}{\Delta \sigma} = \frac{\partial C_{44}}{\partial \sigma} + \frac{\partial C_{44}}{\partial \sigma_\lambda} \cdot \frac{\Delta \sigma_\lambda}{\Delta \sigma} + \frac{\partial C_{44}}{\partial \sigma_\nu} \cdot \frac{\Delta \sigma_\nu}{\Delta \sigma} + \frac{\partial C_{44}}{\partial \sigma_z} \cdot \frac{\Delta \sigma_z}{\Delta \sigma} \cdot Q_{44} + (Q_{44} + Q_{66}) \cdot \frac{\Delta \sigma_\lambda}{\Delta \sigma} .
\]  

(A10)

Combining the eq. (A7) with (A9) and eq. (A8) with (A10), we find a link between the hydrostatic and $K_0$ mode conditions as

\[
\frac{\Delta C_{33}}{\Delta \sigma} \approx \frac{Q_{33} + 2Q_{11} \cdot \frac{\Delta \sigma_\lambda}{\Delta \sigma}}{Q_{33} + 2Q_{11}} \cdot \frac{\Delta C_{33}^{\text{Hyd}}}{\Delta \sigma} .
\]  

(A11)

\[
\frac{\Delta C_{44}}{\Delta \sigma} \approx \frac{Q_{44} + (Q_{44} + Q_{66}) \cdot \frac{\Delta \sigma_\lambda}{\Delta \sigma}}{Q_{44} + 2Q_{66}} \cdot \frac{\Delta C_{44}^{\text{Hyd}}}{\Delta \sigma} .
\]  

(A12)

The coefficients $Q$, are given as (Fjær 2006b)

\[
Q_{11} = \frac{16}{3} \cdot \frac{\nu^2}{1 - 2 \cdot \nu} \cdot D,
\]  

(A13)

\[
Q_{33} = \frac{16}{3} \cdot \frac{(1 - \nu^2)}{1 - 2 \cdot \nu} \cdot D,
\]  

(A14)

\[
Q_{44} = \frac{16}{3} \cdot \frac{1}{2 - \nu},
\]  

(A15)

\[
Q_{66} = 0,
\]  

(A16)

where $\nu$ is the Poisson’s ratio. In this study, we have used a fixed value ($\nu \approx 0.2$) for Poisson’s ratio in these expressions. $D$ is the drainage parameter ($D = 1$ for the dry rocks). Note that with reference to Sayers & Kachanov (1995), the stress-dependency of rock properties is ascribed to displacement discontinuities, which do not necessarily obey the predictions of the open, penny-shaped cracks model ($B_s/B_T = 1 - v/2 \leq 1$) with respect to the ratio between normal ($B_N$) and shear ($B_T$) compliance (Fjær et al. 2008). This can be compensated for by giving the drainage parameter $D$ a wider definition, and accepting a wider range of values for this parameter.

Assuming that the hydrostatic part, as a representation of large-scale unidirectional changes, reveals both elastic and non-elastic processes, we have

\[
\frac{\Delta C_{33}}{\Delta \sigma}^{\text{Hyd}} = \frac{\Delta C_{33}^{\text{e}}}{\Delta \sigma} + \frac{\Delta C_{33}^{\text{ne}}}{\Delta \sigma} = 2\rho \cdot \frac{V_{p,z}}{\Delta \sigma} \cdot \frac{\Delta V_{p,z}}{\Delta \sigma} ,
\]  

(A17)

\[
\frac{\Delta C_{44}}{\Delta \sigma}^{\text{Hyd}} = \frac{\Delta C_{44}^{\text{e}}}{\Delta \sigma} + \frac{\Delta C_{44}^{\text{ne}}}{\Delta \sigma} = 2\rho \cdot \frac{V_{s,z}}{\Delta \sigma} \cdot \frac{\Delta V_{s,z}}{\Delta \sigma} .
\]  

(A18)

Combining the eq. (A11) with (A17) and (A12) with (A18), we are able to represent the total stiffness gradients (i.e. both elastic and non-elastic components), derived from hydrostatic test, in a uniaxial compaction manner.

**APPENDIX B**

In this appendix, we present the estimation of the non-elastic contribution to stiffness during small amplitude (1 MPa) stress cycling. Assuming that the non-elastic impact on stiffness during unloading/reloading follows the linear trend described by Fjær et al. (2011):

\[
S_{H} = \frac{1}{H} - \frac{1}{H_e} = a \cdot \Delta \sigma .
\]  

(B1)

Here, $H$ and $H_e$ represent the static and dynamic stiffness component, respectively; $S_{H}$ denotes a non-elastic compliance component of the rock under uniaxial compaction conditions; $a$ is a slope of trend line in non-elastic compliance versus axial stress characteristics; and $\Delta \sigma$ is a stress increment. The ratio of non-elastic contribution, relative to the elastic stiffness, for a stress increment $\Delta \sigma$ is then

\[
\frac{H_e - H}{H_e} = H \cdot S_{H} = H \cdot a \cdot \Delta \sigma .
\]  

(B2)

From the uniaxial compaction data, we find $a \approx 0.5$ GPa $^{-2}$ while $H \approx 20$ GPa for the unloading sequence at 80 MPa. Inserting these numbers into eq. (B2), we find that the non-elastic ratio, for the stress amplitude of 1 MPa, is about 1 per cent.

Since the relation given in eq. (B1) has only been demonstrated for larger stress amplitudes, we cannot claim that this is also valid for amplitudes as small as 1 MPa. However, it seems intuitively reasonable that if the actual relation deviates from eq. (B1) for small amplitudes, it will be lower rather than higher since we expect purely elastic behaviour for sufficiently small amplitudes. Hence, the 1 per cent estimate may be considered as an upper limit.