A new versatile method for modelling geomagnetic induction in pipelines

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SUMMARY
Geomagnetic induction drives telluric currents in pipelines and creates fluctuations in pipe-to-soil potentials (PSP) that interfere with pipeline surveys and create conditions where corrosion is more likely to occur. To understand the process of geomagnetic induction and determine the severity and location of troublesome effects requires the ability to model geomagnetic induction in realistic pipeline networks. Previous modelling work, based on transmission line theory, has provided some insights into the process but has to be customized for each situation. This paper presents a new versatile modelling technique that can be easily applied to any pipeline network. The essential part of the new method is the development of an equivalent-pi circuit for geomagnetic induction in a pipeline section. A complex pipeline network can then be represented as a set of equivalent-pi circuits that are combined to form a nodal admittance network comprising connections between nodes and to ground from each node. The nodal admittance matrix method is then used to determine the voltages everywhere in the pipeline system. Sample results are presented for geomagnetic induction in an example pipeline. It is shown how the modelling results can be combined with electric fields calculated from geomagnetic observatory data to determine the PSP variations that occur during geomagnetic disturbances.

Key words: Numerical solutions; Geomagnetic induction.

1 INTRODUCTION
Telluric currents, induced by geomagnetic field variations, have been a concern for pipeline operations for many years. Telluric variations in pipe-to-soil potentials (PSP) disrupt pipeline surveys and interfere with the corrosion-prevention measures for the pipeline (Seager 1991; Gummow 2002). Modelling of geomagnetic induction in pipeline networks can therefore be helpful for understanding where large PSP variations can occur so ameliorative measures can be taken (Edwall & Boteler 2001; Trichtchenko et al. 2001). Pipeline modelling is also being used at the design stage allowing the effects of features such as insulating flanges and placement of ground connections to be examined before the pipeline is constructed (Rix & Boteler 2001).

The first reported observations of telluric effects on pipelines date from the 1950s when Allison & Huddleston (1952) noted ‘the existence of stray currents that could not be accounted for by any of the usual sources’ and considered a possible association with magnetic disturbances. Investigations have since been made in many parts of the world, for example, New Zealand (Proctor 1974), Africa (Barker & Skinner 1980), Australia (Martin 1993) and USA (Peabody 1979). The construction of the Alaska pipeline in the auroral zone region of intense geomagnetic activity prompted more quantitative studies by Campbell (1978) who determined the current flow that could be produced in the pipeline and its dependence on geophysical parameters such as the Earth conductivity structure and frequency spectrum of the geomagnetic disturbances. Subsequent work using observations and modelling (Boteler & Seager 1998) showed that the PSP variations produced by the telluric currents varied considerably along the length of a pipeline. This led to interest in more detailed modelling of the pipeline response to geomagnetic induction.

Modelling of telluric currents in pipelines has used distributed-source transmission line (DSTL) theory. This is based on transmission line theory developed by Schelkunoff (1938) which is widely used for modelling the propagation of signals applied at one end, for example, in phone lines. However, with the addition of voltage sources distributed along the line the theory can also be used to study electromagnetic induction into the line itself. This was applied by Taflove & Dabkowski (1979) to the problem of AC induction in pipelines located close to power transmission lines. Their work was subsequently adapted to the case of geomagnetic induction in pipelines (Boteler & Cookson 1986; Boteler 1997; Pulkkinen et al. 2001). The DSTL modelling has provided insights into how pipeline features affect the telluric PSP variations; however, the modelling technique requires that each pipeline model be set up individually.

This paper presents a more versatile method involving the use of equivalent-pi circuits for modelling telluric currents in pipeline networks. The paper starts by showing how a pipeline can be described
using transmission line theory. It is then shown how an ‘equivalent-pi’ circuit can also be used to represent the pipeline. By deriving the ‘input-output’ relations for the equivalent-pi circuit and comparing this to the same relations obtained from transmission line theory it is possible to determine the parameters of the equivalent-pi network. Finally, it is shown how the equivalent-pi representation facilitates the modelling of a pipeline as a nodal admittance network. This gives the voltages at all ‘nodes’ in the pipeline. The voltage between nodes can then be determined from the transmission line equations, so providing a complete solution for a pipeline network.

2 TRANSMISSION LINE EQUATIONS FOR INDUCTION IN A PIPELINE

The electrical properties of a pipeline are determined by the series impedance, \( Z \), along the pipeline, determined by the cross-sectional area of the pipe and the resistivity of the pipeline steel, and the parallel admittance \( Y \) determined by the surface area per unit distance of the pipeline and the conductivity of the pipeline coating. The \( Z \) and \( Y \) components determine the propagation constant \( \gamma \) and the characteristic impedance \( Z_0 \) of the pipeline.

\[
\gamma = \sqrt{ZY} \quad Z_0 = \frac{|Z|}{\sqrt{Y}}. \quad (1)
\]

An electric field induced in a pipeline can be represented in the transmission line model by a voltage source, \( E \), in each distributed element (Fig. 1).

The voltage and current along the line are given by

\[
\frac{dV}{dx} + ZI = E, \quad (2)
\]

and

\[
\frac{dI}{dx} + YV = 0. \quad (3)
\]

From which differentiation and substitution lead to the equations

\[
\frac{d^2V}{dx^2} - \gamma^2 V = \frac{dE}{dx}, \quad (4)
\]

\[
\frac{d^2I}{dx^2} - \gamma^2 I = -YE. \quad (5)
\]

There are two approaches to use these expressions to obtain solutions for the voltages and currents produced in a pipeline by an arbitrary electric field which varies with position along the pipeline. The first, used by Taflove & Dabkowski (1979), is to include the variation of the electric field, \( dE/dx \), in eq. (4) and derive expressions involving integration of the electric field along the pipe. The second approach, used by Boteler (1997), is to break the pipeline into sections short enough that the electric field can be considered uniform within each section. The variation of the electric field along the pipeline is then taken into account by changes in the electric field from section to section.

It is this second approach that will be used here.

On the assumption that the electric field is uniform within a section of pipeline, \( dE/dx = 0 \) and eq. (4) becomes

\[
\frac{d^2V}{dx^2} - \gamma^2 V = 0. \quad (6)
\]

This has solutions of the form

\[
V = Ae^{\gamma x} - Be^{-\gamma x}. \quad (7)
\]

Therefore

\[
\frac{dV}{dx} = \gamma Ae^{\gamma x} - \gamma Be^{-\gamma x}. \quad (8)
\]

Substituting this into eq. (2) and substituting \( \gamma/Z = 1/Z_0 \) gives

\[
I = \frac{E}{\gamma Z_0} - \frac{A}{Z_0} e^{\gamma x} + \frac{B}{Z_0} e^{-\gamma x}. \quad (9)
\]

where \( A \) and \( B \) can be found from conditions at the ends of the line. Boteler (1997) derived \( A \) and \( B \) in terms of the Thevenin equivalent circuits for the terminations at the end of the lines. However, for the network solutions it is more useful to express \( A \) and \( B \) in terms of the voltages at the ends of the line (Grainger & Stevenson 1994).

To derive expressions for \( A \) and \( B \) consider Fig. 1, where \( V_i \) and \( V_k \) and \( I_i \) and \( I_k \) are the voltage and currents at the start and end of the pipeline section respectively.

At the start, \( x = 0 \), eqs (7) and (9) give

\[
V_i = A + B, \quad (10)
\]

\[
I_i = \frac{E}{\gamma Z_0} - \frac{A}{Z_0} + \frac{B}{Z_0}. \quad (11)
\]

At the end of the section, \( x = L \), eqs (7) and (9) reduce to

\[
V_k = Ae^{\gamma L} + Be^{-\gamma L}. \quad (12)
\]

and

\[
I_k = -\frac{A}{Z_0} e^{\gamma L} + \frac{B}{Z_0} e^{-\gamma L} + \frac{E}{\gamma Z_0}. \quad (13)
\]

Eqs (10) and (12) can be combined to solve for \( A \) and \( B \)

\[
A = \frac{V_i - V_k e^{-\gamma L}}{e^{\gamma L} - e^{-\gamma L}}, \quad (14)
\]

\[
B = \frac{V_i e^{\gamma L} - V_k}{e^{\gamma L} - e^{-\gamma L}}. \quad (15)
\]

Substituting for \( A \) and \( B \) in eq. (7) for voltage gives

\[
V = \left( \frac{V_i e^{\gamma L} - V_k}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma(L-x)} + \left( \frac{V_i e^{\gamma L} - V_k}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma x}. \quad (16)
\]

Similarly substituting for \( A \) and \( B \) in eq. (9) for current gives

\[
I = -\frac{1}{Z_0} \left( \frac{V_i e^{\gamma L} - V_k}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma(L-x)} + \frac{1}{Z_0} \left( \frac{V_i e^{\gamma L} - V_k}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma x} + \frac{E}{\gamma Z_0}. \quad (17)
\]

The relation between the voltages and currents, \( V_i, I_i \) and \( V_k, I_k \), at the ends of the section is found by combining eqs (10) and (11) to give the expressions for \( A \) and \( B \) in terms of \( V_i \) and \( I_i \)

\[
A = \frac{V_i}{2} - \frac{1}{2} \left( Z_0 I_i - \frac{E}{\gamma} \right), \quad (18)
\]

\[
B = \frac{V_k}{2} - \frac{1}{2} \left( Z_0 I_i - \frac{E}{\gamma} \right). \quad (19)
\]
\[ B = \frac{V_i}{2} + \frac{1}{2} \left( Z_0 i_i - \frac{E}{\gamma} \right). \]  

(19)

Substituting these into the expressions for \( V_k \) and \( I_k \) gives

\[ V_k = V_i \cosh \gamma L - i_i Z_0 \sinh \gamma L + \frac{E}{\gamma} \sinh \gamma L, \]  

(20)

\[ I_k = -\frac{V_i}{Z_0} \sinh \gamma L + i_i \cosh \gamma L - \frac{E}{\gamma Z_0} (\cosh \gamma L - 1). \]  

(21)

3 EQUIVALENT-PI CIRCUIT FOR INDUCTION IN A PIPELINE

Equivalent-pi circuits have been found to be useful in AC power system studies (Gross 1986; Grainger & Stevenson 1994; Machowski et al. 1997) and for modelling AC induction in pipelines (Dommell 1979; Haubrich et al. 1994).

3.1 Using a voltage source

To develop an equivalent-pi circuit for geomagnetic induction in a pipeline start with the transmission line element representation shown in Fig. 2.

Using Kirchhoff’s voltage law around the loop gives

\[ V_k = V_i - i Z' + E'. \]  

(22)

Kirchhoff’s current law at nodes \( A \) and \( B \) give

\[ I_i = I + V_i \frac{Y'}{2}, \]  

(23)

\[ I_k = I - V_k \frac{Y'}{2}, \]  

(24)

which gives an expression for \( I \)

\[ I = I_i - V_k \frac{Y'}{2}. \]  

(25)

Substitute for \( I \) in eq. (22) for \( V_k \) and collecting terms gives

\[ V_k = \left( 1 + \frac{Y'}{2} \right) V_i - Z' I_i + E'. \]  

(26)

Relating these equations to the transmission line solution, eq. (20), gives

\[ Z' = Z_0 \sinh \gamma L. \]  

(27)

Figure 2. Pipeline and its equivalent-Pi circuit.

\[ \frac{Y'}{2} = (\cosh \gamma L - 1) \frac{1}{Z_0 \sinh \gamma L}. \]  

(28)

\[ E' = \frac{E}{\gamma} \sinh \gamma L. \]  

(29)

Thus, we now have definitions for all the parameters of the equivalent-pi circuit.

3.2 Using an equivalent current source

An alternative way of representing the induced voltage \( E' \) in series with the impedance \( Z' \) is by an equivalent current source \( I_E \) in parallel with an admittance \( Y_E \) as shown in Fig. 3. This is a convenient representation for use in a nodal admittance matrix representation of a pipeline network.

To derive the parameters for the equivalent-pi circuit in Fig. 3 we start with Kirchhoff’s voltage law

\[ V_k = V_i + Y_E I_E - Y_E I. \]  

(30)

Again using Kirchhoff’s current law at nodes \( A \) and \( B \) gives the expression for \( I \) as shown in eq. (25). Substituting in expression for \( V_k \) gives

\[ V_k = \left( 1 + \frac{Y'}{2 Y_E} \right) V_i - I_i \frac{Y_E}{Y_E} + I_E \frac{Y_E}{Y_E}. \]  

(31)

Comparing to the transmission line eq. (20) shows that

\[ 1 + \frac{Y'}{2 Y_E} = \cosh \gamma L, \]  

(32)

\[ \frac{1}{Y_E} = Z_0 \sinh \gamma L, \]  

(33)

\[ \frac{1}{Y_E} I_E = \frac{E}{\gamma} \sinh \gamma L, \]  

(34)

which give the equivalent-pi circuit components

\[ Y_E = \frac{1}{Z_0 \sinh \gamma L}, \]  

(35)

\[ \frac{Y'}{2} = (\cosh \gamma L - 1) \frac{1}{Z_0 \sinh \gamma L}. \]  

(36)

\[ I_E = \frac{E}{Z}. \]  

(37)

The equivalent-pi circuits provide a suitable representation of a pipeline section that can be combined in a network to represent a complete pipeline system with any configuration including branches.

Figure 3. Equivalent-pi circuit with an equivalent current source.
FORMING A NODAL NETWORK

The first step in obtaining a solution for a complete pipeline network is to combine the equivalent-pi circuits for each pipeline section into a nodal admittance network.

Consider a pipeline comprising three sections, A, B, and C with equivalent-pi circuits as shown in Fig. 4(a). At junctions between sections there are the admittance to ground \( y' \) from adjacent sections in parallel which combine to give the total admittance to ground at that point. For example, in Fig. 4, the junction between sections A and B becomes node 2 and the junction between sections B and C becomes node 3, in the admittance network shown in Fig. 4(b). The admittance to ground at these nodes is then given by combining the admittances from the adjacent sections

\[
y_2 = \frac{y'_A}{2} + \frac{y'_B}{2} \quad y_3 = \frac{y'_B}{2} + \frac{y'_C}{2}.
\]  

At the ends of the pipeline there is only the admittance to ground of one section, so:

\[
y_1 = \frac{y'_A}{2} \quad \text{and} \quad y_4 = \frac{y'_C}{2}.
\]  

The series admittance of the equivalent-pi sections becomes the series admittance of the corresponding section in the nodal network

\[
y_{12} = y_A \quad y_{23} = y_B \quad y_{34} = y_C
\]  

Similarly, the current sources from the equivalent-pi sections become the current sources in the nodal network

\[
j_{12} = j_A \quad j_{23} = j_B \quad j_{34} = j_C.
\]

We can now proceed to find a solution for the voltages of the nodes.

Applying Kirchoff’s current law that the algebraic sum of the currents entering any node is zero, that is the sum of the currents entering the nodes from the lines equals the current flowing to ground, we can write equations for each node:

\[
-\left( v_1 - v_2 \right) j_{12} = -i_{12} = i_1, \quad (42)
\]

\[
i_{12} - i_{23} = i_2, \quad (43)
\]

\[
i_{23} - i_{34} = i_3, \quad (44)
\]

\[
i_{34} = i_4. \quad (45)
\]

The current in each line is determined by the potential difference between the nodes at the ends of the line, the admittance of the line and the equivalent current source for the line

\[
\left( v_1 - v_2 \right) j_{12} + i_{12} = \left( v_2 - v_3 \right) j_{23} + i_{23}, \quad (46)
\]

\[
\left( v_3 - v_4 \right) j_{34} + i_{34} = i_4. \quad (47)
\]

The current to ground at each node, \( i_1, i_2, i_3, i_4 \) are given by the node voltage times the admittance to ground, \( i_i = v_i y_i \) where \( i = 1, 2, 3 \) or 4. Making these substitutions in eqs (42)-(45) above gives for each node:

\[
-\left( v_1 - v_2 \right) j_{12} - j_{12} = v_1 y_1, \quad (49)
\]
The currents on the left-hand side represent the total of the equivalent current sources directed into each node, $J_i$. Thus we can rewrite the equations in the form

$$J_i = (y_{1i} + y_{12})v_1 - y_{12}v_2, \quad (57)$$

$$J_2 = -y_{12}v_1 - (y_{12} + y_{23})v_2 - y_{23}v_3, \quad (58)$$

$$J_3 = -y_{23}v_2 + (y_{23} + y_{34})v_3 - y_{34}v_4, \quad (59)$$

$$J_4 = -y_{34}v_3 + (y_{34} + y_4)v_4. \quad (60)$$

This can be written in matrix form

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} y_{11} + y_{12} & -y_{12} & 0 & 0 \\ -y_{12} & y_{12} + y_{23} & y_{23} & -y_{23} \\ 0 & y_{23} & y_{34} + y_{34} & -y_{34} \\ 0 & 0 & -y_{34} & y_{34} + y_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}. \quad (61)$$

The first matrix on the right-hand side of eq. (61) is termed the admittance matrix. Taking the inverse of the matrix and multiplying by the nodal current source then gives the voltages at the nodes.

5 MATRIX SOLUTION FOR A GENERAL PIPELINE NETWORK

The equivalent-pi circuits and nodal admittance network approach can be used for any general pipeline network. Consider a pipeline network with $n$ nodes connected by straight pipeline sections where the pipeline characteristics and induced electric field are uniform within each pipeline section. For each pipeline section between two nodes $i$ and $k$, the section length, pipeline characteristics, and induced electric field can be used to determine an equivalent-pi circuit model with parameters $Y_{ik}$, $Y_{ik}^{1/2}$, $j_k$.

These can be used to construct a nodal admittance network as shown in Fig. 5, where $y_{ik}$ is the admittance of the line between nodes $i$ and $k$, and $y_i$ is the admittance to ground from node $i$ which is the sum of the admittances to ground of the equivalent-pi circuits for all of the branches going to that node plus the admittance $Y_G^i$ of any ground bed added at that node.

![Figure 5. Nodal admittance network for modelling geomagnetic induction in a pipeline system.](https://academic.oup.com/gji/article-abstract/193/1/98/741245)
where $[Y]$ is the admittance matrix in which the diagonal elements are the sum of the admittances of all paths connected to node $i$, and the off-diagonal elements are the negatives of the admittances between nodes $i$ and $k$, that is

$$Y_{ii} = y_i + \sum_{k=1}^{N} y_{ki} \quad k \neq i,$$

(71)

$$Y_{ki} = -y_{ki}.$$ 

(72)

The voltages of the nodes are then found by taking the inverse of the admittance matrix and multiplying by the nodal current sources:

$$[V] = [Y]^{-1} [J].$$ 

(73)

These voltages at the ‘nodes’ in the network derived from eq. (73) represent the values for $V_i$ and $V_k$ at the ends of the pipeline sections which can be used in eqs (16) and (17) to determine the voltage $V(x)$ and current $I(x)$ in each pipeline section.

### 6 Examples of Modelling Pipeline Networks

To validate the modelling technique it is useful to make comparison with observations. As part of the design of the corrosion-prevention systems for the Norman Wells–Zama pipeline in northern Canada a series of measurements were made by Seager to determine the relation between PSP at different points along the pipeline (Boteler & Seager 1998). This showed that the PSP variations were largest at the ends of the pipeline, but oppositely directed, and with small PSP in the middle section of the pipeline (see left-hand curve in Fig. 6a). For these measurements an insulating flange at kilometre 335 was by-passed with a ‘jumper’ cable so that the pipeline was electrically continuous from end to end. Seager also made a second set of measurements with the jumper cable disconnected so that the pipeline was broken into two sections. These measurements also showed the same pattern with the largest, and oppositely directed, PSP variations at opposite ends of each section (see right-hand curve in Fig. 6a).

The new pipeline model has been used to calculate the PSP that would be expected on the Norman Wells–Zama pipeline. The model was set up with a pipeline with series impedance,
Z = 0.028 ohm km⁻¹ and parallel admittance Y = 0.01 S km⁻¹ and an electric field parallel to the pipeline, E = 1 V km⁻¹. Two sets of calculations were made, without and with the insulating flange. The first set of results, without the insulating flange, are shown in the left-hand curve of Fig. 6(b) and show the characteristic shape with oppositely directed PSP as seen in the observations (Fig. 6a). The model results with the insulating flange are shown in the right-hand curve of Fig. 6(b) and again closely resemble the observations.

To illustrate the application of the modelling technique where there are junctions we consider the simple pipeline network shown in Fig. 7. Every junction or bend in the pipeline system is specified as a node for modelling purposes. The main pipeline extends from node 3 to node 8 with branch lines from node 3 to node 1 and to node 2 and branch line from node 7 to node 9.

The pipeline parameters for the main pipeline and for the branch lines are given in the Appendix. Then, taking account of the length of each pipeline section, the equivalent-pi circuit parameters are calculated for each section. These are used to determine the elements of the nodal admittance matrix for the pipeline system. The equivalent current sources are then calculated for the specified electric field. Solution of the matrix eq. (73) gives the voltages for all the nodes which are then used to give the variation with voltage along each section using eq. (16). The pipeline voltages produced by a northward electric field of 1 V km⁻¹ are shown in Fig. 8. Repeating the procedure for an eastward electric field of 1 V km⁻¹ gives the pipeline voltages shown in Fig. 9. This shows the capabilities of the modelling method.

The modelling described above provides the techniques for making a comprehensive examination of the effects of geomagnetic disturbances on pipelines. The PSP results obtained for ‘reference’ northward and eastward electric fields can be used to determine the PSP for any size and direction of the electric field:

\[ V(x, A, \theta) = \alpha(x)\hat{E}\cos\theta + \beta(x)\hat{E}\sin\theta, \]  

(74)

where

\[ \alpha(x) = \frac{V_N(x)}{E_N(ref)}, \quad \beta(x) = \frac{V_E(x)}{E_E(ref)}. \]  

(75)

\( \hat{E} \) is the amplitude of the electric field (V km⁻¹) and \( \theta \) is the angle of the electric field measured clockwise from the north direction, and \( V_N(x) \) and \( V_E(x) \) are the PSP at point \( x \) produced by reference northward and eastward electric fields \( E_N(ref) \) and \( E_E(ref) \), respectively. The \( \alpha \) and \( \beta \) values can also be used with time-series of \( E(north) \) and \( E(east) \) values to determine the PSP as a function of time at any point on the pipeline:

\[ V(x, t) = \alpha(x)E_N(t) + \beta(x)E_E(t). \]  

(76)

The electric fields can easily be calculated from geomagnetic observatory data by assuming a layered model to represent the change in Earth conductivity with depth. Fig. 10 shows the magnetic disturbance recorded at Ottawa on 2004 November 8 (Trichtchenko et al. 2007) and the northward and eastward components of the electric fields calculated using a layered-Earth conductivity model of the region (Trichtchenko & Boteler 2002). Combining these with the model results for the sample pipeline in Fig. 7 gives the PSP variations shown in Fig. 11. These show that the pipeline potential would reach 70 volts at times during the disturbance. Such a pipeline potential would create the electrochemical conditions where pipeline corrosion could occur. The short intervals of high potentials shown in Fig. 11 would...
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Figure 8. Pipe-to-soil voltage produced in the pipeline network, shown in Fig. 7, by a northward electric field of 1 V km$^{-1}$. Top: for the main pipeline. Bottom: for each branch line.

Figure 9. Pipe-to-soil voltage produced in the pipeline network, shown in Fig. 7, by an eastward electric field of 1 V km$^{-1}$. Top: for the main pipeline. Bottom: for each branch line.

not, on their own, cause any pipeline problems; however, if these conditions occurred repeatedly they could be a cause for concern. An assessment of the long-term geomagnetic hazard to a pipeline can be made using the many years of archived magnetic data to calculate the electric fields and, with the pipeline modelling, determine the accumulated time that the pipeline potential can be expected to reach certain levels and determine if this poses a problem for pipeline integrity.

7 DISCUSSION

The modelling results above show the characteristic features of the pipeline response to geomagnetic induction. The most distinctive feature is the ‘S’ shape shown in Fig. 6 with the peak PSP occurring at the ends of the pipeline and falling off exponentially with distance from the end. This fall-off is characterized by an ‘adjustment distance’ equal to the inverse of the propagation constant. In the
middle of the pipeline the PSP is zero and there is no flow of telluric currents on or off the pipeline. In this region the telluric currents flow along the pipeline driven purely by the induced electric field. At each end of the pipeline the telluric currents produce a build up of charge resulting in a potential difference between the pipe and the soil. The resulting effect is that the telluric current flows onto the pipeline where the PSP is negative, flows along the pipeline driven by the induced electric field to the other end of the pipeline where the positive PSP drives the telluric current off the pipeline. Because of the high resistance of the coating the telluric current cannot flow on or off the pipeline just at the ends of the pipeline and these regions of current inflow and outflow spread out away from the ends of the pipeline with the current density proportional to the exponential decrease of the PSP shown in Fig. 6. On shorter pipelines these regions of exponential decrease overlap to give an approximately linear variation in PSP from one end of the pipeline to the other as shown in Fig. 9. These characteristics of long and short pipelines are consistent with the modelling results shown by Boteler et al. (1998).

In the calculations shown above the induced electric field is assumed to be the same everywhere along the pipeline. This would be a reasonable approximation for a short pipeline in a geologically uniform area where a 1D Earth conductivity model could be used. For accurate modelling of extensive pipeline networks it is necessary to include the spatial variations in the electric fields. These variations can arise because of both spatial variations in the magnetic disturbance and changes in the Earth conductivity structure. Spatial variations in the magnetic disturbance could be included by using data from several magnetic observatories and interpolating between these measurements to give the magnetic disturbance at pipeline locations between the observatories. Changes in geology along the pipeline route can be included by dividing the pipeline route into a number of regions each with its own 1D Earth conductivity model. Then, for each section of pipeline, the interpolated magnetic field would be used with the local 1D Earth conductivity model to calculate the electric field. The electric fields are then used as inputs to the network pipeline modelling to calculate the pipeline potentials throughout the pipeline network.

The above analysis is adequate for modelling gradual changes in the Earth conductivity structure. When there is a sharp change in the Earth conductivity, for example, at a coastline, local enhancements of the electric fields are produced. These could be included in the pipeline network modelling by breaking the pipeline into short sections each with a different electric field value. The challenge is to determine the electric fields in the vicinity of conductivity boundaries. Previous studies have examined the effect of the coast on electric fields (Beamish et al. 2002; Gilbert 2005) but more work needs to be done to accurately model the effect of 3D conductivity structures on surface electric fields. There is evidence of enhanced PSP variations on pipelines crossing or adjacent to Earth conductivity boundaries (Fernberg et al. 2007) so this is an important topic for future research to understand geomagnetic effects on pipelines.

Geomagnetic hazards are just one aspect of managing the integrity of a pipeline system. The principal concern is corrosion because of electrochemical processes at the steel–soil interface. To prevent this pipelines are usually covered with a coating intended to stop the steel being in direct contact with the soil. This protects much of the pipeline surface but it is difficult to install a pipeline without the coating having some holes where corrosion could occur. To provide a second form of corrosion prevention, electric current sources are connected to the pipeline to make the pipeline approximately 1 volt negative with respect to the surrounding soil. This produces a potential difference between the pipeline steel and the soil that inhibits the electrochemical corrosion processes. In this arrangement the pipeline is the cathode of the circuit so the technique is called “cathodic protection” (Peabody 2001). Over the last 50 years there have been a number of changes in the types of pipeline coatings used but a general trend is that the newer coatings have higher electrical resistivity. This has the advantage that smaller current sources can be used to produce the potential difference required for cathodic protection. However, a disadvantage is that when the geomagnetically induced currents produced in the pipeline flow to
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Figure 11. Pipe-to-soil potential variations calculated for the pipeline network in Fig. 7 using the electric fields in Fig. 10(b) and eq. (76).

Ground through the coating a larger potential drop is produced than on earlier pipelines. Thus geomagnetic disturbances are producing larger variations in PSP on newer pipelines than on older ones. This has resulted in more attention being paid to the geomagnetic aspect of pipeline protection.

Several ways of minimizing geomagnetic variations in PSP have been suggested. Early calculations (Campbell 1978; Pirjola & Lehtinen 1985; Osella et al. 1998) were made of the induced currents flowing along a pipeline; and pipeline protection was focussed on blocking this current flow by inserting insulating flanges into the pipeline (Peabody 1979). However, modelling and observations by Boteler & Seager (1998) showed that a flange created a site where the induced currents were forced to flow between the pipe and the soil, thereby creating large swings in PSP. Subsequent modelling work done as part of the design work for new pipelines showed the same effect and led to a change of approach to one where every effort was made to make the pipeline electrically continuous to minimize the places where the induced currents flow on and off the pipeline (Rix & Boteler 2001). There are, however, places where this cannot be avoided: the ends of the pipeline, bends, and pipeline junctions. In these cases the pipeline modelling can be used to determine how good a connection to ground (i.e. low grounding resistance) is needed to drain the induced currents off the pipeline without creating significant PSP variations. The ability to include all these pipeline features in the modelling described in this paper makes the technique a valuable tool for not only evaluating the possible geomagnetic hazard to a pipeline, but also for planning mitigation measures during the design of the pipeline. To aid in this use the new modelling technique has been built into a Telluric Current Simulator that is available under an Open Source licence (Boteler et al. 2013).

8 CONCLUSIONS

A new method of modelling geomagnetic induction in pipeline systems has been developed. Pipeline sections are represented by equivalent-pi circuits that are then combined to give a nodal admittance network. A matrix solution is used to give the voltages at the nodes. Transmission line equations then give the variation of voltage with distance within each pipeline section.

The equivalent-pi circuit for a pipeline section of length \( L \) with series impedance \( Z \) (ohms per unit length) and parallel conductance \( Y \) (Siemens per unit length), subjected to an induced electric field \( E \), is given by an equivalent current source \( I_E = \frac{E}{Z} \)

in parallel with a series admittance \( Y_E = \frac{1}{Z_0 \sinh \gamma L} \)

and admittances to ground \( Y/2 \) at each end of section

\[
\frac{Y}{2} = \frac{1}{Z_0 \sinh \gamma L},
\]

where \( \gamma \) is the propagation constant and and \( Z_0 \) is the characteristic impedance of the pipeline given by

\[
\gamma = \sqrt{ZY}, \quad Z_0 = \sqrt{\frac{Z}{Y}}.
\]

The pipeline sections can be combined into a nodal admittance network defined by an admittance matrix \([Y]\) with diagonal elements equal to the sum of the admittances of each branch from the node, including the admittance to ground which is the sum of the admittances to ground of the equivalent-pi circuits for all of the branches going to that node plus the admittance \( Y^G \) of any ground bed added at that node.

\[
Y_{ii} = \sum_{j \neq i} Y_{ij} + \sum_{j \neq i} \frac{Y_j^G}{2} + Y_{iG},
\]

and off-diagonal elements which are the negatives of the admittances between nodes

\[
Y_{ki} = -y_{ki}.
\]

A nodal current vector is defined as the sum of the equivalent current sources directed into each node

\[
J_i = \sum_{k=1}^{N} J_{ki}.
\]

The voltages of the nodes are then found by taking the inverse of the admittance matrix and multiplying by the nodal current vector

\[
[V] = [Y]^{-1} [J].
\]
The voltages and currents along each pipeline section between nodes $i$ and $k$ are then given by

$$V = \left( \frac{V_i e^{\gamma L}}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma x} + \left( \frac{V_k e^{\gamma L}}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma x},$$

and

$$I = \frac{1}{Z_0} \left( \frac{V_i e^{\gamma L}}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma x} + \frac{1}{Z_0} \left( \frac{V_k e^{\gamma L}}{e^{\gamma L} - e^{-\gamma L}} \right) e^{-\gamma x} E \gamma Z_0,$$

where $V_i$ and $V_k$ are the voltages at nodes $i$ and $k$.

The new method provides a versatile way of modelling geomagnetic induction in realistic pipeline systems, with features such as branches and ground connections. This can be used to assess the geomagnetic hazard to pipelines and aid in the design of pipeline cathodic protection systems.

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**REFERENCES**


**APPENDIX: CALCULATION OF PIPELINE PARAMETERS**

The pipeline response is defined by the series impedance of the pipeline, $Z$, determined from the steel resistivity $\rho$ and the pipe diameter and wall thickness

$$Z = \frac{\rho}{\pi (r_o^2 - r_i^2)} \Omega \text{ km}^{-1},$$

where $r_o$ and $r_i$ are the outer radius and inner radius of the pipe, respectively. The parallel admittance, $Y$, determined from the coating conductance, $C$, and the surface area per unit distance of the pipeline

$$Y = C 2\pi r_o \text{ S km}^{-1}.$$
Combining these gives the characteristic impedance $Z_0$, and the propagation constant, $\gamma$.

For the sample pipeline shown in Fig. 7, the main pipeline has a pipe diameter of 30 inches (762 mm) and wall thickness of 15.6 mm, and the branch lines have a diameter of 15 inches (381 mm) and wall thickness of 10 mm. For the pipeline steel we use the standard resistivity value of 0.18 $10^{-3}$ $\Omega$-m. Different coatings have different conductance values. For the example pipeline we assume a value of $5.10^{-6}$ S m$^{-2}$ which is a typical value for a modern coating. These give the pipeline parameters shown in Table A1.

The equivalent-$\pi$ circuit for a pipeline section of length $L$ with series impedance $Z$ and parallel conductance $Y$ has the parameters shown in Table A2.

Electric fields of 1 V km$^{-1}$ northward or eastward produce the nodal voltages shown in Table A3.