Asymptotic full waveform inversion for arrival separation and post-critical phase correction with application to quasi-vertical fault imaging

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SUMMARY
We propose a new method to separate the incident and reflected arrivals and correct for the post-critical phase shift in wide angle reflection imaging situations. Such a situation arises, for example, in quasi-vertical geological fault imaging using data from small earthquakes. Major faults are often associated with a high degree of seismic activity. There is a contrast in impedance across the fault surface due to the shift of the bounding structures, so that the fault surface can generate reflections of seismic waves visible on seismograms. These two factors make it possible to use reflections of waves from small earthquakes to perform seismic imaging of the fault. Two major challenges arise due to the earthquake sources being very close to the fault: (1) the incident and reflected waves are not well separated on seismograms so that muting of the incident wave is not possible, and (2) most of the waves are reflected post-critically, which causes a distortion in the reflected waveforms. In this paper we present a new technique to simultaneously separate the incident and reflected arrivals, and compute the phase correction for the post-critically reflected waves by formulating these two steps as a single optimization problem. Our implementation of the method is acoustic and 2-D. The method is based on asymptotic representation of the incident and reflected acoustic waves from point sources in two dimensions, and assumes that the source time function of the source is known. The minimization problem is highly non-linear and the objective function is very oscillatory. We propose to solve it by a particle swarm optimization method. We present synthetic numerical examples of fault reconstructions from separated and phase-corrected reflections obtained by our method.

Key words: Body waves; Computational seismology; Wave scattering and diffraction; Wave propagation.

1 INTRODUCTION
The problem of imaging steeply dipping reflectors is of considerable interest. Applications range from imaging of steeply dipping structures in hydrocarbon and mineral exploration to imaging of vertically dipping active faults for earthquake prediction and seismic response modelling.

Major crustal faults often accumulate enough offset so that the structures bounding them have significantly different material properties. Faults are often associated with a high degree of seismic activity. This provides the opportunity to obtain seismic recordings from earthquakes that can be utilized in reflection seismology algorithms. At the same time the high degree of seismic activity also drives the necessity to accurately image the region so that crustal response to earthquakes can be characterized.

Imaging quasi-vertical faults is a complicated problem. Standard surface seismic profiles are not well suited to imaging steeply dipping reflectors. They can be used in layered environments to image faults as breaks in horizontal layers (e.g. Feng & McEvilly 1993), since layered material is readily observed in such profiles; however, this approach is not always applicable in the crystalline rocks, which can have complicated and heterogeneous structure.

Various migration methods with turning waves have been proposed to image steeply dipping reflectors in the subsurface. The idea behind these methods is that the P-wave speed of the Earth increases with the depth, so that seismic waves from surface sources become refracted to horizontal propagation direction and reflect off vertical interfaces. An overview of various migration methods based on turning waves can be found, for example, in the papers by Jia & Wu (2009) and Zheglova et al. (2012). In the last paper applications to imaging reflectors in active fault zones are also given.

Borehole methods are also applicable to imaging steeply dipping faults (e.g. Chavarria et al. 2003). In recent years interferometric imaging has been successfully applied to borehole data to image
steeply dipping reflectors. For an overview and applications the reader is referred to Schuster (2009) and references therein.

Migration is typically performed with active source data recorded at the surface or in boreholes. Migration with turning waves often requires large offsets between the source-receiver pairs and the reflector, especially when the depth-dependent velocity gradient is not very large, which is often the case with crystalline basement rocks (Stradlander & Brown 1997). Therefore the depth to which these methods can penetrate is limited by the wave speed gradient and the offset.

The areas of major faulting, such as the Parkfield area of the San Andreas Fault, are often instrumented with arrays of seismic receivers, and records of local microearthquakes are abundant. Development of travelt ime methods in recent years that can invert simultaneously for the wave speed model and the hypocentre locations (Thurber & Rabinowitz 2000), as well as methods to obtain the source time function from the data, makes it possible to image the fault location using seismic recordings from earthquakes, that is, passive source data.

This is a second paper on quasi-vertical fault imaging. In the first paper by Zhéglova et al. (2012), which we will call Paper 1, a method was proposed to image quasi-vertical geological faults with earthquake data. The authors identified the main difficulties in imaging with earthquake source data, arising from the fact that earthquakes occur very close to the fault surface, as follows: (1) the direct and reflected arrivals are almost simultaneous, and they are not well separated on a seismogram, and (2) most of the reflections occur above the critical angle, that is the angle of specular incidence above which total internal reflection occurs. The angle of specular incidence \( \theta_0 \) is defined as the smallest angle between the wave propagation direction and the normal to the fault at the point of specular incidence, as shown in Fig. 1. Post-critically reflected waves undergo a frequency-dependent phase shift that demonstrates itself as a distortion in the time-dependent waveform (e.g. Brekhovskikh 1960; Pujol 2003; Aki & Richards 2009). The phase of the post-critically reflected waves needs to be corrected before these waves can be used in reflection seismology algorithms. In Paper 1 a two-step seismic fault imaging method was proposed: step 1 included arrival separation and post-critical phase shift correction of the reflected data, and step 2 included an imaging procedure similar to reverse time migration. In the first step the approach is to use a modelled incident wave to cancel the direct arrivals and to compute the post-critical phase correction for the reflected waves. The phase correction algorithm is formulated as a linear least-squares optimization problem and is based on asymptotic representation of the incident and reflected waves.

In this paper we propose an alternative technique to separate the incident and reflected arrivals and compute the post-critical phase shift correction. The main advantage of our new approach is that the arrival separation and phase correction is obtained directly from the data without the use of the modelled incident wave, so that it is unnecessary to know the earthquake source location and the medium parameters to perform arrival separation and phase correction. Earthquake source locations and medium parameters are usually known only approximately in practice. The arrival separation and phase correction step that we developed in Paper 1 appears to be somewhat sensitive to the errors in the source location. In this paper we show that our new method of arrival separation and phase correction is less sensitive to this error. The modelled incident wave is still used for the imaging procedure. We also assume that the source time function is known. In our new method we formulate the arrival separation and post-critical phase correction as a single optimization problem, in which we minimize the misfit between the recorded waveforms and the modelled waveforms in the time domain. The forward modelling is computed using the full waveforms at the receivers using the same asymptotic representations of the incident and reflected waves as in Paper 1. These representations are based on the asymptotic form of the Green’s function for the homogeneous 2-D acoustic medium, and are fully determined by the arrival times of the incident and reflected waves, and the reflection coefficient. We then use the separated phase-corrected reflected wave to image the fault with the imaging technique of Paper 1.

Our method is developed in two dimensions and we assume that the wave propagation is governed by the acoustic wave equation. We also assume that the source is spatially isotropic. Thus we ignore the 3-D elastic properties of the Earth, and the anisotropic radiation pattern of earthquake sources.

Figure 1. Piecewise homogeneous true medium with a straight vertical fault; receivers located on the reflection side of the fault are marked by \( \Delta \). \( u^{inc} \) and \( u^{ref} \) are the incident and reflected waves, \( \theta_0 \) is the angle of specular reflection.

2 THE FAULT IMAGING METHOD WHEN THE REFLECTIONS MAY BE POST-CRITICAL

The set up for the fault imaging problem is presented in Fig. 1. A quasi-vertical fault divides the subsurface into two regions with slowly varying \( P \)-wave speeds \( c_1, c_2 \), and densities \( \rho_1, \rho_2 \), where in general \( c_1(x, z) < c_2(x, z) \) and \( \rho_1(x, z) \leq \rho_2(x, z) \). The earthquake hypocentre \((x_0, z_0)\) is located on the slow side of the fault close to the fault surface. An array of receivers is laid out on the Earth’s surface on the slow side of the surface fault trace. The \( P \)-wave speed and density profiles in the slow region are assumed approximately known, for example, from traveltime and gravity tomography (Roecker et al. 2004). We also assume that we have the following information: (1) an approximate source location, which is necessary for the imaging steps, and (2) the source time function for each source, which can be obtained by means described, for example, by Pratt (1999) and Roecker et al. (2010). In principle, it is possible to incorporate the source time function as an additional set of inversion parameters, however, such an investigation is beyond the scope of this paper and will be the subject of a future study.

The acoustic pressure wave generated by a microearthquake propagates upwards and gets recorded by the receivers. The total recorded wave consists of the incident wave, the reflected wave and the noise, which includes the head waves, multiples and other coherent and incoherent signals:

\[
 u(x, z, t) = u^{inc}(x, z, t) + u^{ref}(x, z, t) + \text{noise}.
\]

Here \((x, z)\) are receiver coordinates.
2.1 Forward model: asymptotic forms of the incident and reflected waves

Our arrival separation and phase correction method is based on the asymptotic high-frequency approximations of the incident and reflected waves from a point source in the frequency domain in two dimensions obtained in Paper 1:

\[ \hat{u}_{\text{inc}} \approx \hat{f}(\omega) \sqrt{\frac{2m c}{\sin^2 \phi}} \exp \left( i \left( \omega t_{\text{inc}} + \frac{\pi}{2} \right) \right), \]  

(1)

\[ \hat{u}_{\text{ref}} \approx V(\theta_0) \hat{f}(\omega) \sqrt{\frac{2m c}{\sin^2 \phi}} \exp \left( i \left( \omega t_{\text{ref}} + \frac{\pi}{2} \right) \right), \]  

(2)

where

\( \hat{f}(\omega) \) is the Fourier transform of the source time function;
\( \omega \) is the angular frequency;
\( t_{\text{inc}} \) and \( t_{\text{ref}} \) are the travel times of the incident and reflected waves;
\( \theta_0 \) is the angle of specular reflection; and
\( V(\theta_0) \) is the reflection coefficient given by

\[ V(\theta_0) = \frac{\rho_\bot / \rho_\parallel \cos \theta_0 - \sqrt{\sin^2 \theta_0 - n^2 \sin^2 \theta_0}}{\rho_\bot / \rho_\parallel \cos \theta_0 + \sqrt{\sin^2 \theta_0 - n^2 \sin^2 \theta_0}}. \]

where \( n \) is the index of refraction. The reflection coefficient has the following values:

\[ 0 < V(\theta_0) < 1, \quad \theta_0 < \theta_{\text{cr}}, \]
\[ V(\theta_0) = 1, \quad \theta_0 = \theta_{\text{cr}}, \]
\[ V(\theta_0) = e^{i \phi} \operatorname{sgn} \omega, \quad \theta_0 > \theta_{\text{cr}}, \]

where the real-valued phase \( \phi \) is given by:

\[ \phi = 2 \arctan \left( \frac{\sqrt{\sin^2 \theta_0 - n^2}}{\rho_\bot / \rho_\parallel \cos \theta_0} \right), \]

and \( \theta_{\text{cr}} = \arcsin \frac{n}{c} \) is the critical angle.

These asymptotic formulae were derived for the source–receiver configuration shown in Fig. 1, assuming that \( c_1 < c_2 \), \( \rho_1 \leq \rho_2 \) and \( 0 \leq \theta_0 \leq \pi/2 \). They are true in the high-frequency approximation, that is when the distances travelled by the incident and reflected waves are much larger than the wavelength. For more information see Paper 1 and (Zheligova 2010).

The corresponding representation in the time domain of the reflected wave takes the form:

\[ u_{\text{ref}}(x, z, t) = \begin{cases} \sqrt{\frac{2m c}{\sin^2 \phi}} V(\theta_0) u_{\text{inc}} \left( x, z, t - (t_{\text{ref}} - t_{\text{inc}}) \right) + \text{error}, & \theta_0 \leq \theta_{\text{cr}} \\ \sqrt{\frac{2m c}{\sin^2 \phi}} \left( \cos \phi t_{\text{inc}} \left( x, z, t - (t_{\text{ref}} - t_{\text{inc}}) \right) + \sin \phi \mathcal{H} u_{\text{inc}} \left( x, z, t - (t_{\text{ref}} - t_{\text{inc}}) \right) \right) + \text{error}, & \theta_0 > \theta_{\text{cr}} \end{cases} \]  

(4)

where \( \mathcal{H} \) is the Hilbert transform defined as:

\[ \mathcal{H} f(t) = P.V. \int_{-\infty}^{\infty} \frac{f(s)}{s - t} \, ds. \]

In this integral \( P.V. \) denotes Cauchy principal value.

It is evident from (4) that for the post-critically reflected wave the phase shift \( \phi \) causes a waveform distortion in the time domain. This distortion needs to be removed before the reflected waveform can be used in the fault imaging method.

2.2 Arrival separation and phase shift correction

In this section we formulate the method to separate the incident and reflected arrivals and determine the phase correction. The need for the arrival separation is illustrated in Paper 1, Fig. 2. This figure shows that the incident and reflected waves arrive almost simultaneously and are merged on the seismogram, so that it is not possible to mute the direct arrival without also muting the reflected wave.

To formulate our method of arrival separation and phase shift correction, we first rewrite the reflection coefficient in its complex polar form as follows:

\[ V(\theta_0) = A e^{i\phi}. \]  

(5)

Here

\[ 0 < A \leq 1, \quad \phi = 0, \quad \text{if } \theta_0 \leq \theta_{\text{cr}}, \]
\[ A = 1, \quad 0 < \phi < 2\pi, \quad \text{if } \theta_0 > \theta_{\text{cr}}. \]

We omit the \( \text{sgn} \) factor from the exponent of the reflection coefficient since we only use the positive frequencies in the Fourier synthesis of the time-dependent incident and reflected waves. We can do this because the time domain waveforms are real so that their Fourier transforms are even. Representations (1) and (2) are rewritten using (5) and neglecting the approximation error as follows:

\[ \hat{u}_{\text{inc}} = \hat{f}(\omega) \sqrt{\frac{2m c}{\sin^2 \phi}} e^{\left( i \omega t_{\text{inc}} + \frac{\pi}{2} \right)}, \]  

(7)

\[ \hat{u}_{\text{ref}} = A e^{i\phi} \hat{f}(\omega) \sqrt{\frac{2m c}{\sin^2 \phi}} e^{\left( i \omega t_{\text{ref}} + \frac{\pi}{2} \right)}. \]  

(8)

We formulate the problem of separation of the incident and reflected arrivals and determination of the post-critical phase correction as a minimization problem, in which the square misfit between the recorded and the modelled waveforms in the time domain is minimized. That is, we minimize the functional:

\[ J\left( \tau_{\text{inc}}, \tau_{\text{ref}}, A, \phi \right) = \int_{0}^{\tau} \left( u(x, z, t) - u_{\text{obs}}(x, z, t) \right)^2 \, dt, \]  

(9)

where \( u(x, z, t) \) is the modelled waveform, \( u_{\text{obs}}(x, z, t) \) is the recorded (observed) waveform and \( T \) is the length of the observed time trace in seconds. The modelled waveform \( u(x, z, t) \) is obtained by taking the inverse Fourier transform of:

\[ \tilde{u}(x, z, \omega) = \tilde{u}_{\text{inc}}(x, z, \omega) + \tilde{u}_{\text{ref}}(x, z, \omega), \]  

(10)

where \( \tilde{u}_{\text{inc}}(x, z, \omega) \) and \( \tilde{u}_{\text{ref}}(x, z, \omega) \) are obtained from (7) and (8). Optimization problem (9) needs to be solved for each time trace, that is, for each source–receiver pair.

\( J \) is determined by the four unknown parameters: \( \tau_{\text{inc}}, \tau_{\text{ref}}, A \) and \( \phi \), however, \( A \) and \( \phi \) are not independent, as evidenced by (6). In this paper we test two strategies to minimize \( J \):

(i) Three parameter optimization which proceeds as follows: we first set \( A = 1 \) and solve for \( \phi, \tau_{\text{inc}} \) and \( \tau_{\text{ref}}, \) that is, we assume a post-critical reflection; then we set \( \phi = 0 \) and solve for \( A, \tau_{\text{inc}} \) and \( \tau_{\text{ref}}, \) that is, we assume a subcritical reflection; then out of the two sets of the recovered parameters we choose the one that produces the lowest value of \( J \).

(ii) Four parameter optimization: we solve the optimization problem for all four parameters simultaneously, without putting constraints on the relationship between \( A \) and \( \phi \).
Asymptotic full waveform inversion

Once the optimum parameter values have been found, the phase-corrected reflected wave is obtained as:

\[ \hat{u}_{\text{ref}} = \hat{f}(\omega) \sqrt{\frac{2\pi}{\omega \tau_{\text{ref}}}} e^{i(\omega \tau_{\text{ref}} + \frac{\pi}{4})}, \]  

(11)

where \( \tau_{\text{ref}} \) is optimal. The time domain version of this wave is computed by the Fourier transform. This phase-corrected reflected wave is used in the fault imaging step.

2.3 Optimization method: Particle Swarm Optimization (PSO)

Due to the complexity of the optimization problem presented in the previous section we use a global optimization approach to solve it. The complexity arises primarily from the oscillatory dependence of the objective \( J \) on \( \tau_{\text{inc}} \) and \( \tau_{\text{ref}} \), as can be seen from (7) and (8). This makes this problem unsuitable for a local gradient-based optimization method, since a very good initial guess would be required to ensure convergence to the global minimum. Such an initial guess may be difficult to obtain if the source time function is complicated.

We choose PSO which is a multi-agent stochastic heuristic (Kennedy & Eberhart 1995). Introduction of stochastic element to optimization strategy allows to limit influence of typical problems such as local minima, non-linearity or general ill-conditioning of the target function. PSO was chosen because of some important advantages of this method over other similar method (e.g. Genetic Algorithms—GA). First of all, PSO does not require careful algorithm parameter tuning. This advantage is relevant not only for other multi-agent methods but also for classic stochastic techniques like Simulated Annealing. The other advantages include relatively small number of agents, very straightforward way of introducing restrictions to certain dimensions of the solution space and relatively low likelihood of premature convergence. Additionally, PSO implementation is simple and its computational efficiency is high. Of course for limited number of parameters (here 3 or 4) the difference between computational efficiency of GA and PSO is small (see e.g. Donelli et al. 2006). In general, PSO is a stochastic technique that simulates social behaviour of birds or fish searching for food. Particles are defined as points in the \( N \)-dimensional solution space, where \( N \) is the number of the model parameters used in the inversion (in our case \( N = 3 \) or 4). In the case of the canonical PSO the ‘awareness’ of the \( n \)th particle is usually limited to its current position and velocity, \( y_i \) and \( v_i \); its previous best position \( p_i \); and the global (when ‘global best’ static topology is applied (see e.g. Poli et al. 2007) or group best position \( p_g \). Here \( i = 1 \ldots P \) with \( P \) being the number of particles.

When \( p_g \) is tested against all the updated \( p_i \)'s, new positions \( y_i \) and velocities \( v_i \) are calculated for all particles. In the canonical version of PSO the following update strategy is used to obtain these values (Clerc & Kennedy 2002):

\[ v_i \leftarrow \chi \left[ v_i + U(0, \Phi) \otimes (p_i - y_i) + U(0, \Phi) \otimes (p_g - y_i) \right] \]

\[ y_i \leftarrow y_i + v_i \]

\[ \chi = \frac{1}{\Phi - 1 + \sqrt{\Phi^2 - 2\Phi}}. \]

where \( U \) represents a vector of \( P \) random numbers uniformly distributed between 0 and \( \Phi \), and \( \otimes \) is component-wise multiplication. In regular applications \( \Phi \) is commonly set to 2.05, which means that constant velocity multiplier \( \chi \) is approximately 0.7298. This scheme guarantees convergence if no particle speed limitations are applied, although, to make computations faster, various restrictions...
limitations are put on $v_i$ in practice to ensure that it is on the same order of magnitude as the interval of admissible $y_i$’s (Eberhart & Shi 2000).

The main disadvantage of this approach is the fact that when particles are close to the final solution a lot of unnecessary particle movements are observed, as discussed by Kennedy (2003). In the same paper, the fact that random particle positions can be approximated by fixed probability distributions similar to a Gaussian or Cauchy distribution with a maximum between $p_i$ and $p_j$ was used to formulate a new, velocity-less PSO update strategy called ‘bare bones PSO’ (BBPSO):

$$y_i \leftarrow G\left(\frac{1}{2}(p_i + p_j), |p_i - p_j|\right).$$

Here $G$ represents a Gaussian distribution with mean $\frac{1}{2}(p_i + p_j)$ and standard deviation (SD) $|p_i - p_j|$. The advantages of BBPSO include fast convergence, precision and the limited number of unnecessary, time-consuming position updates in the vicinity of the final solution.

In this study we use the canonical PSO with a 64-particle swarm to optimize for $\tau^{\text{inc}}$, $\tau^{\text{ref}}$ and $\phi$ with $A$ fixed in the three-parameter optimization. The BBPSO is used for all other optimization runs. To reduce the risk of accepting local minima as final solutions in the BBPSO runs, we use five independent swarms of 64 particles. The intraswarm calculations are performed in parallel using OpenMP application programming interface. The use of different PSO methods is due to the fact that we first performed the $(\tau^{\text{inc}}, \tau^{\text{ref}}, \phi)$ optimization with the canonical PSO and then switched to BBPSO for the rest of computation, since it produces very similar results with less computational time.

In all production runs the stopping criterion was defined by the number of particles gathered in a vicinity of the global minimum. We defined this vicinity as the hypersphere in the solution space with radius equal to 1 and the number particles as half of the swarm, namely 32. Parameter constraints were set to prevent algorithm to accept non-physical values (such as $A < 0$), and to avoid multivaluedness (e.g. $\phi$ is restricted to the interval $[0, 2\pi]$). Starting models were randomly chosen within physically possible parameters.

2.4 Imaging method

We use the same imaging method as in Paper 1. To make our discussion in this paper self-contained, we briefly reproduce the imaging procedure below. It consists of the following steps:

(i) The incident wave is modelled from the approximate source location and source time function. The modelling is performed by solving the acoustic wave equation in a background medium constructed by extending the wave speed and density profiles from the reflection side of the fault to the whole computational region. The incident wave is recorded at the ‘virtual’ receivers, placed symmetrically with respect to the fault trace on the transmission side of the fault.

(ii) The time reversed phase-corrected reflected wave $\tilde{u}^{\text{ref}}$ is backpropagated from the receivers on the reflection side of the fault into the background medium.

(iii) The time reversed incident wave is backpropagated from the virtual receivers into the background medium.

(iv) The zero-lag correlation in time of the two backpropagated fields is computed at each point $(x, z)$ of the computational domain.

(v) For each depth $z$, the position of the fault is determined by the horizontal maximum of the zero-lag correlation

$$C(x^*(z), z) = \max_x C(x, z).$$

The set $\{(x^*(z), z)|z_{\min} \leq z \leq z_{\max}\}$ is the recovered fault location. The zero-lag correlations from several sources are summed to increase the accuracy of the reconstruction. Then step (v) is applied to the zero-lag correlation stack.

3 NUMERICAL RESULTS

We apply our arrival separation and phase shift correction method to a series of noise-free, synthetic 2-D data examples of quasi-vertical fault reconstruction. We use the same numerical examples as in Paper 1 to directly compare the results. We refer the reader to this paper for the details on the numerical methods used in calculations.

In all examples, the computational domain is 11.8 km wide and 8.8 km deep. The range $x$ varies from $-6.8$ km to 5 km; the depth $z$ varies from $-1.6$ km to 7.2 km, where depth is directed downwards. $z = 0$ corresponds to the sea level and the Earth’s surface is assumed to be located at the depth $z = -0.6$ km, that is, 600 m above the sea level. The surface trace of the fault is at the range $x = -1.2$ km. 10 receivers spaced by 500 m are placed at the surface on the reflection side of the fault between $x = -1.1$ (receiver 1) and 3.4 km (receiver 10). 10 virtual receivers are placed symmetrically with respect to the fault trace on the surface between $x = -1.3$ and $x = -5.8$ km. The observed data were synthesized by solving the acoustic wave equation in the true media. The Ricker wavelet source time function with the peak frequency of 9 Hz was used in the generation of the observed data in all examples, and the length of the time traces in all examples is 3 s.

During the PSO optimization procedure, both the observed and the modelled traces are scaled before evaluation of the objective function, so that maximum amplitude of each trace in the time domain is 1. The search space of the optimization parameters is constrained as follows: $0.5 \text{ s} \leq \tau^{\text{inc}} \leq \tau^{\text{ref}} \leq 2.5 \text{ s}$, $0.01 \leq A \leq 1$, $0 \leq \phi \leq 2\pi$. Thus, we assume that the reflected wave arrives after the incident wave. It is possible for some fault geometries to generate refracted reflected arrivals that arrive before the incident wave, however, it is not clear how to handle such reflections by our fault imaging procedure, so we discard them. Both the phase-corrected reflected wave and the modelled incident wave traces are scaled to unit amplitude before the time reversal and backpropagation.

3.1 Benchmark example: straight vertical fault

We first test our method on an example of a straight vertical fault separating two regions of constant wave speeds: $c_1 = 4 \text{ km s}^{-1}$ and $c_2 = 6 \text{ km s}^{-1}$, as shown in Fig. 2(a). The reconstruction is performed with a single source located at $(-1.09, 3.5)$.

Fig. 3 demonstrates arrival separation and phase correction for traces at receivers 4 and 10. The top row shows traces separated by the three parameter optimization; the bottom row shows traces separated by the four parameter optimization. The observed waveforms are plotted with asterisks, the total modelled waveform is a solid line, the incident wave is plotted in a dashed line, the reflected wave is plotted in a dotted line and the phase-corrected reflected wave is plotted in a dash-dot line.

For this example, it is possible to calculate the theoretical values of the parameters from geometric optics and compare them to the values obtained by our optimization procedures. Table 1 shows the
Figure 3. Traces obtained by arrival separation and phase correction for numerical Example 1: left-hand column—traces at receiver 4, \((x, z) = (0.4, -0.6)\); right-hand column—traces at receiver 10, \((x, z) = (3.4, -0.6)\). Notation: asterisks denote the synthetic observed waveforms, solid, dashed and dotted lines denote the full, incident and reflected waveforms computed by formulae (7), (8) and (10), where \(A\), \(\phi\), \(\tau_{inc}\) and \(\tau_{ref}\) are obtained by minimizing (9). The dash-dot line represents the phase-corrected reflected wave computed by formula (11). The top row shows traces computed with parameters obtained by the three parameter optimization. The bottom row shows traces computed with parameters obtained by the four parameter optimization. In image (b) the dotted line coincides with the dash-dot line.

Table 1. Specular angle of reflection and the values of \(\tau_{inc}\), \(\tau_{ref}\), \(A\) and \(\phi\) predicted by the geometric optics for Example 1.

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Coordinates</th>
<th>(\theta_0)</th>
<th>(\tau_{inc})</th>
<th>(\tau_{ref})</th>
<th>(A)</th>
<th>(\phi)</th>
<th>Reflection type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((-1.1, -0.6))</td>
<td>87.055°</td>
<td>1.0250</td>
<td>1.0263</td>
<td>1</td>
<td>3.0036</td>
<td>Post-critical</td>
</tr>
<tr>
<td>2</td>
<td>((-0.6, -0.6))</td>
<td>80.163°</td>
<td>1.0323</td>
<td>1.0403</td>
<td>1</td>
<td>2.6790</td>
<td>Post-critical</td>
</tr>
<tr>
<td>3</td>
<td>((-0.1, -0.6))</td>
<td>73.545°</td>
<td>1.0545</td>
<td>1.0687</td>
<td>1</td>
<td>2.3619</td>
<td>Post-critical</td>
</tr>
<tr>
<td>4</td>
<td>((0.4, -0.6))</td>
<td>67.351°</td>
<td>1.0906</td>
<td>1.1106</td>
<td>1</td>
<td>2.0558</td>
<td>Post-critical</td>
</tr>
<tr>
<td>5</td>
<td>((0.9, -0.6))</td>
<td>61.662°</td>
<td>1.1394</td>
<td>1.1644</td>
<td>1</td>
<td>1.7608</td>
<td>Post-critical</td>
</tr>
<tr>
<td>6</td>
<td>((1.4, -0.6))</td>
<td>56.527°</td>
<td>1.1992</td>
<td>1.2287</td>
<td>1</td>
<td>1.4756</td>
<td>Post-critical</td>
</tr>
<tr>
<td>7</td>
<td>((1.9, -0.6))</td>
<td>51.933°</td>
<td>1.2686</td>
<td>1.3018</td>
<td>1</td>
<td>1.1933</td>
<td>Post-critical</td>
</tr>
<tr>
<td>8</td>
<td>((2.4, -0.6))</td>
<td>47.852°</td>
<td>1.3461</td>
<td>1.3823</td>
<td>1</td>
<td>0.90062</td>
<td>Post-critical</td>
</tr>
<tr>
<td>9</td>
<td>((2.9, -0.6))</td>
<td>44.235°</td>
<td>1.4303</td>
<td>1.4691</td>
<td>1</td>
<td>0.55851</td>
<td>Post-critical</td>
</tr>
<tr>
<td>10</td>
<td>((3.4, -0.6))</td>
<td>41.034°</td>
<td>1.5201</td>
<td>1.5611</td>
<td>0.73347</td>
<td>0</td>
<td>Subcritical</td>
</tr>
</tbody>
</table>

The values of \(\tau_{inc}\), \(\tau_{ref}\), \(A\), \(\phi\) and the objective function \(J\) produced by the three and four parameter optimizations are shown in Tables 2 and 3, respectively. These tables also show the relative errors in the computed parameters with respect to the theoretical values. Overall, the traveltimes are recovered very accurately by both optimization methods: the error is less than 1 per cent for all but one of the
traveltimes, and the majority of the traveltimes are recovered with less than 0.5 per cent error. These results are good, especially considering that the search intervals for the traveltimes were wide. The values of $\phi$ and $A$ are not recovered as accurately. For the three-parameter optimization the reflection type is recovered correctly for all traces except the trace at receiver 9. This trace is interpreted by our three-parameter optimization method as subcritical, whereas it is post-critical. On average, the value of $\phi$ recovered by the three-parameter optimization becomes less accurate as reflection angle becomes closer to the critical angle, which is consistent with the accuracy estimates presented in Paper 1. The errors in the values of $A$ are quite high at receivers 9 and 10, which are close to the critical angle. This is also consistent with the theory. For the four-parameter optimization, the value of $A$ becomes less accurate as the critical angle is approached; however, this trend is not consistent. There is no clearly defined trend in the accuracy of $\phi$ recovered by this method. We note that the value of the objective function is lower for the four parameter optimization method in most cases. This is expected, since this method is looking for an optimum over a larger set. The three parameter optimization is essentially looking for an optimum in the union of two lower dimensional subsets of this set by constraining either $A$ or $\phi$. The lower optimum that the four parameter method recovers is not necessarily in this union. At receiver 1, the value of the objective is higher for the four parameter method. This is unexpected and most likely signifies that the PSO has not identified the global minimum and has converged to a local minimum.

Fig. 4 shows the zero-lag correlation images and the fault reconstructions from reflected wave traces separated and phase-corrected by the three parameter optimization (top row) and the four parameter optimization (bottom row). Although the results of trace separation in Figure 3 differ somewhat in details, the zero-lag correlations and the fault recoveries are very similar for the two methods and are as good as the corresponding results in Paper 1.

Fig. 5 shows plots of the objective function for receiver 4. Fig. 5(a) shows a plot of $J$ as a function of $\tau^{inc}$ and $\tau^{ref}$ when $A$ and $\phi$ are fixed at their theoretical values, and $J$ is plotted over the region $1 \leq \tau^{inc} \leq \tau^{ref} \leq 1.2$. Fig. 5(b) shows a plot of $J$ as a function of $A$ and $\phi$, when $\tau^{inc}$ and $\tau^{ref}$ are fixed at their theoretical values, and $J$ is plotted over the region $0.8 \leq A \leq 1, 2.0308 \leq \phi \leq 2.1308$. It is clear from these figures that the minimum in $\tau^{inc}$ and $\tau^{ref}$ is much better defined than the minimum in $A$ and $\phi$, so that the optimization problem is much more sensitive in recovery of the traveltimes. This explains the relative precision of the recovered traveltimes and the variability in the accuracy of $A$ and $\phi$.

### 3.2 Examples with a shifting fault

In this section we show two examples in which we reconstruct the fault that shifts towards the slow region as it dips using data from 6 sources. The true media for Examples 2 and 3 are shown in Figs 2(b) and (c). The true medium in Example 2 is piecewise constant with the wave speed $c_1 = 4 \text{ km s}^{-1}$ and $c_2 = 6 \text{ km s}^{-1}$ in the low and high speed region respectively. In Example 3 the high and low wave speed profiles are depth dependent and are shown in Fig. 2(d).

The shape of the fault is described by the following smooth piecewise polynomial function:

$$x(z) = \begin{cases} 
-1.2, & -1.6 \leq z \leq 2.5 \\
33.35 - 36.28z + 12.44z^2 - 1.38z^3, & 2.5 \leq z \leq 3.5 \\
-0.51, & 3.5 \leq z \leq 7.2.
\end{cases}$$

The source locations for Examples 2 and 3 are listed in Table 4. In these examples the zero-lag correlations from the six sources are stacked and the reconstruction is obtained from the stack, as described in item (vi) of Section 2.4.

Fig. 6 shows the images of zero-lag correlation stack and the fault reconstruction for Example 2. The top row shows recoveries
Figure 4. Zero-lag correlations (a) and (c) and fault reconstructions (b) and (d) for Example 1: top row images are obtained from the traces separated by the three-parameter optimization, bottom row images are obtained from the traces separated by the four-parameter optimization.

Figure 5. Plots of the objective function for receiver 4, Example 1: (a) $J(\tau_{\text{inc}}, \tau_{\text{ref}})$ with $A$ and $\phi$ fixed at their theoretical values; (b) $J(A, \phi)$ with $\tau_{\text{inc}}, \tau_{\text{ref}}$ fixed at their theoretical values. In panel (a) the plot of the objective function is only shown for $\tau_{\text{inc}} \leq \tau_{\text{ref}}$.

from the reflected wave traces separated and phase-corrected by the three parameter optimization. The bottom row shows the recoveries from the traces obtained by the four parameter optimization. The zero-lag correlations and the fault recoveries are very similar for the two methods. The straight parts of the fault are recovered very well. The shifting part is slightly better defined on the reconstruction corresponding to the three parameter method. The two optimization methods produced very close values of both the
Table 4. Source positions in numerical Examples 2 and 3.

<table>
<thead>
<tr>
<th>Source</th>
<th>Location ((x_0, z_0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((-0.4, 2.6))</td>
</tr>
<tr>
<td>2</td>
<td>((-0.4, 3.5))</td>
</tr>
<tr>
<td>3</td>
<td>((-0.4, 4))</td>
</tr>
<tr>
<td>4</td>
<td>((-0.4, 4.5))</td>
</tr>
<tr>
<td>5</td>
<td>((-0.4, 5))</td>
</tr>
<tr>
<td>6</td>
<td>((-0.4, 6.5))</td>
</tr>
</tbody>
</table>

parameters and the objective function for approximately 1/3 of the 60 traces.

Fig. 7 shows the images of zero-lag correlation stack and the fault reconstruction for Example 3. The top row shows recoveries from the reflected data separated and phase-corrected by the three parameter optimization, and the bottom row shows the recoveries from the data obtained by the four parameter optimization. In the top row the lower part of the fault is recovered better, whereas in the bottom row the top part of the fault is recovered slightly better.

In both Examples 2 and 3, the vertical parts of the fault are well imaged, and the location of the shift is identified correctly, but the shape of the shifting part is not well recovered. This is consistent with our expectations, since our method is geared to recovering vertical faults with singly reflected waves. We also note that our new arrival separation and phase correction method only picks up one incident arrival and one dominant reflected arrival, as opposed to the method presented in Paper 1, where the shifting part is recovered slightly better. In most cases the dominant reflection comes from the straight parts of the fault. The remaining energy in the time trace, which may contain multiples and additional reflections from the shifting parts of the fault, is lost from the reconstituted phase-corrected reflected wave. It may be possible to enhance the recovery of the shifting part, if more than one reflected arrival is separated by including more terms in formula (10). Investigation of this possibility remains a future goal.

3.3 Performance of the method when source locations are uncertain

In practice, the earthquake source locations are seldom known with high accuracy. In this section we show an additional Example 4 that demonstrates that the new method of arrival separation and phase correction makes the fault imaging method more robust to the errors in the source location. Again we choose an example from Paper 1 to compare the results. In this example the fault configuration is as in
Asymptotic full waveform inversion

Example 1, Fig. 2(a). We use 20 sources located along two straight vertical lines: sources 1–10 are located at the range $x = -1.09$ km and sources 11–20 are located at the range $x = -0.8$. The depths of the sources range from 2 to 6.5 km with a spacing of 500 m. These are exact source locations, used to generate the synthetic data. To mimic the inexact source locations available to us, we perturb the exact source locations. The perturbation is as follows: the distance between the true and the perturbed source location is normally distributed with a SD of 500 m; the direction of the perturbation has a uniform distribution. The resulting rms deviation of the perturbed sources from the true sources is 500 m. In this example we only use the three-parameter optimization to separate the arrivals and to compute the post-critical phase correction.

Fig. 8(a) shows the zero-lag correlation stack for the 20 sources, and Fig. 8(b) shows the fault reconstruction. The fault is recovered very well. The recovery is notably better than for the corresponding example in Paper 1. The results of this example confirm that our new method of arrival separation and phase correction is indeed robust with respect to the errors in the source location.

The improvement in performance of the proposed method is achieved by excluding the modelled incident wave from the arrival separation and phase correction steps, and formulating these steps as a non-linear optimization problem. The advantage of using the modelled incident wave, as in Paper 1, is that the phase correction can be formulated as a linear optimization problem. However, when the error in the source location is large, then the incident wave modelled from the approximately located source can have a significantly different arrival time than the true direct wave in the recorded data. In this case, an incomplete cancellation of the direct arrival may occur, causing the phase correction $\phi$ also to be computed incorrectly. This is illustrated in Fig. 9 where the phase-corrected reflected waves for three sources with significantly different location inaccuracy, obtained by the method in Paper 1 and our new method are shown. The columns correspond to sources 2 (left), 3 (middle) and 5 (right) that have the absolute spatial location error of 3 m, 138 m and 932 m, respectively, and the rows correspond to the method of Paper 1 (top) and our new method (bottom). The method in Paper 1 produces a good phase correction for sources 2 and 3, images (a) and (b), with the smaller location error. For source 5, image (c) with a large location error, the modelled incident wave from the approximately located source arrives earlier than the true incident wave in the data, so it is not cancelled well from the data. Instead, the subtracted modelled incident wave appears as the earlier arrival with a reversed polarity. Our new method produces a well-separated reflected wave with a correct phase for all three sources.

In all examples considered so far the origin time of the source is assumed known and fixed. In practice the origin time is known only approximately. The error in the origin time of the source has
Figure 8. Zero-lag correlation stack (a) and fault reconstruction (b) for Example 4. The source locations shown in the figures are the perturbed source locations. The true source locations are not shown.

Figure 9. The phase-corrected reflected waves for three sources obtained by the method in Paper 1 and our new method. The columns correspond to sources 2 (left panel), 3 (middle panel) and 5 (right panel) with absolute spatial source location error of 3 m, 138 m and 932 m, respectively. The true source locations are (2.5, −0.79), (3.0, −0.79) and (4.0, −0.79). The approximate source locations are (2.50, −0.79), (2.87, −0.74), (4.10, 0.14). The top row shows the phase-corrected reflected wave obtained by the method of Paper 1, and the bottom row shows the same wave obtained by our new method. Note the effect of the incorrect arrival time of the modelled incident wave from the approximate source 5.

a similar effect on the arrival separation and phase correction as the error in the spatial location of the source. For the method in Paper 1, the source origin error can cause incorrect direct arrival cancellation and phase correction. Our new method will correctly separate the arrivals and compute the phase correction, but the arrival time of both arrivals will be incorrect. Indeed, assuming the spatial source location is fixed, changing the origin time of the source by −Δτ results in shifting the full recorded time trace by Δτ, so the traveltimes will also change by Δτ. It can be shown from formulae (1) and (2) that the phase shift $\phi$ of the reflected wave
does not depend on the source origin time, and will be recovered correctly by our new method if the source origin is wrong. Also, it follows from formulae (1) and (2) that the amplitude of the reflected wave will change by the multiplicative factor \( \sqrt{\frac{\tau_{\text{inc}} + \Delta \tau}{\tau_{\text{inc}}}} \). In our case this factor is \( \approx 1 \) since the incident and reflected arrival times are similar, that is, \( \frac{\tau_{\text{inc}}}{\tau_{\text{ref}}} \approx 1 \). For example, for \( \tau_{\text{inc}} = 1 \) s, \( \tau_{\text{ref}} = 1.05 \) s, and \( \Delta \tau = 0.2 \) s this factor is 1.004. Therefore, the amplitude of the reflection coefficient recovered by our method is also essentially independent of the error in the source origin. These conclusions are confirmed by numerical tests.

For local events, when all the receivers are located on the surface, the depth of the hypocentre is usually more uncertain than the epicentre. It is often observed in practice that the uncertainties in the source depth and origin have a negative covariance, so that errors in these parameters tend to offset each other. Both the proposed method and the method in Paper 1 are equally affected by errors in the source location and origin at the imaging step, but in the proposed method the effect of these errors is removed at the arrival separation and phase correction step compared to the method in Paper 1. This is achieved at the expense of formulating the arrival separation and phase correction as a non-linear instead of linear optimization problem.

## 4 CONCLUSIONS AND FUTURE WORK

In this paper, we presented a new technique to separate the incident and reflected arrivals and to perform the post-critical phase correction of the reflected waves from an earthquake source located close to the fault. Our new method is based on asymptotic representations of the 2-D incident and reflected waves from a point source and uses full recorded waveforms as input data. The method requires only the knowledge of the source time function, and does not require the knowledge of the medium parameters or the source location. We showed that our new technique, combined with the existing imaging method, produces good recoveries of the quasi-vertical faults in piecewise constant and depth-dependent media. We also demonstrated the robustness of our method with respect to the uncertainty in the earthquake source locations. Possible future work directions include experimenting with incorporation of the source time function as a set of parameters in the optimization, and using the separated incident wave to obtain a more accurate estimate of the source location. A more long-term goal is the extension of this method to the 3-D linear elastic case.

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## REFERENCES


