A scaling law for approximating porous hydrothermal convection by an equivalent thermal conductivity: theory and application to the cooling oceanic lithosphere

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SUMMARY
In geodynamic models of mid-ocean ridges hydrothermal cooling processes are important to control the temperature and thus the rheological behaviour of the crust. However, the characteristic time scale of hydrothermal convection is considerably shorter than that of viscous flow of mantle material or cooling of the oceanic lithosphere and can hardly be addressed in a conjoined model. To overcome this problem we present an approach to mimic hydrothermal cooling by an equivalent, increased thermal conductivity. First the temperature and pressure dependence of crack related porosity and permeability are derived based on composite theory. A characteristic pore closure depth as a function of pressure, temperature and pore aspect ratio is defined. 2-D porous convection models are used to derive scaling laws for parameterized convection including a Rayleigh–Nusselt number relation for a permeability exponentially decreasing with depth. These relations are used to derive an equivalent thermal conductivity to account for consistently evolving hydrothermal heat transport in thermally evolving systems. We test our approach using a 1-D model for cooling of the oceanic lithosphere. Within the context of our modelling parameters we found a pronounced effect for young lithosphere (younger than 10 Ma) down to about 20 km. Significant deviations of the heat flux versus age from the $1/\sqrt{t}$ law may occur due to hydrothermal convection. For the bathymetry versus age curves slopes steeper than $1/\sqrt{t}$ slopes already occur for very young lithosphere. Hydrothermal convection leads to an increase of the total heat flux and heat loss with respect to the classical purely conductive cooling model. Comparison of the total heat flow and its conductive contribution with observations confirm previous suggestions that for young lithosphere heat flow measurements represent only the conductive part, while at older ages the total heat flow is observed. Within their scatter and uncertainties heat flow and bathymetry data are in general agreement with our hydrothermally enforced cooling model suggesting that hydrothermal convection may be important even up to high ages.

Key words: Heat flow; Hydrothermal systems; Permeability and porosity; Mid-ocean ridge processes; Heat generation and transport.

1 INTRODUCTION
Immediately after creation at mid-ocean ridges (MOR) oceanic crust and lithosphere is affected by sudden cooling from above. As it moves away from the ridge this leads to an increase in thickness of the lithosphere with age or distance from the ridge. An increase in lithospheric thickness is accompanied by an increase in seafloor depth and decrease of surface heat flux. This general behaviour is well supported by data both for surface heat flow (e.g. Stein & Stein 1992, 1994) and for bathymetry (e.g. Jaupart & Mareschal 2011 and references therein), although large scatter due to fracture zones, seamounts etc. is found. To account for this general behaviour various half-space and plate cooling models (e.g. McKenzie 1967; Turcotte & Oxburgh 1967; Parsons & Scalter 1977) mostly 1-D have been proposed. In all these models of cooling of the oceanic lithosphere the decrease of heat flow and bathymetry follow at least for younger ocean floor (ages < 80 Ma) a $1/\sqrt{t}$ law (SRTL). However, it has already been found in the early days of ocean heat flow measurements that the data can only approximately be explained by the SRTL (e.g. Sclater & Francheteau 1970; Stein & Stein 1994). If the heat flux of the oceanic lithosphere is deduced from vertical temperature differences in shallow bore holes or needle probes, the
heat flux in very young lithosphere turns out to be less than expected by the SRTL (e.g. Stein & Stein 1992), and for old oceanic lithosphere the decrease of heat flow with age is less than predicted by the SRTL.

Starting with Talwani et al. (1971) and Lister (1972), it has been pointed out, that the large differences between the observed surface heat flow and the SRTL have mainly to be attributed to hydrothermal cooling in the oceanic crust (Fig. 1). Estimates of the fraction of hydrothermally removed heat range between 20 per cent and 40 per cent of the total heat flow of the earth (Selkirk et al. 1988; Stein & Stein 1992; Harris & Chapman 2004; Lowell et al. 2008; Spinelli & Harris 2011). Often such estimates are based on the comparison of observed heat flux measurements to a purely conductive reference model, i.e. a model in absence of hydrothermal convection. Thus, the derived heat flux deficit may be smaller than it would be if related to the total (conductive plus advective) heat flux, because this total heat flux may exceed that of a purely conductive reference model. Quantifying this excess heat flow is one of the aims of this paper. Hydrothermally assisted cooling is most pronounced at the MOR, where also the highest heat flux is expected. While surface heat flux generally decreases away from the ridge, also the efficiency of hydrothermal cooling is reduced due to increasing sedimentary sealing (Stein & Stein 1997; Harris & Chapman 2004; Spinelli et al. 2004).

Beside the cooling behaviour of the lithosphere also the accretion process of oceanic crust at MORs and above oceanic hot spot centres is strongly affected by hydrothermal convection (e.g. Maclellan et al. 2004, 2005, see also Schmeling & Marquart 2008). For the sheeted sill model of crust accretion (Kelemen & Aharonov 1998) hydrothermal convection can cool the entire crust near the ridge axis, producing vertical isotherms in the lower crust (Dunn et al. 2000; Cherkaoui et al. 2003). For the alternative shallow melt lens model, where the crust forms by downbuilding from the melt lens, the depth of the melt lens is controlled by the vigour of hydrothermal convection (Theissen-Krah et al. 2011).

However, a problem arises when studying dynamic processes of the whole lithosphere or mantle lithosphere interaction by numerical modelling. While hydrothermal cooling can well be modelled by porous flow theory, its modelling demands considerably denser spatial and temporal resolution than commonly feasible in large scale dynamic models studying the viscous or viscoelastic behaviour of the lithosphere or lithosphere–mantle interaction. Only on scales of 10 or a few 10 s of kilometres such as in MOR accretion models the physics of hydrothermal convection have been explicitly treated using porous flow models (Cherkaoui et al. 2003; Theissen-Krah et al. 2011). Alternative approaches include parameterizing hydrothermal cooling as an enhanced thermal conductivity (Morgan & Chen 1993; Davis et al. 1997; Chen 2001; Machetel & Garrido 2009; Spinelli & Harris 2011) or by use of heat sources and sinks (Henstock et al. 1993; Maclellan et al. 2004). In most of those approaches the choice of an elevated equivalent thermal conductivity has been arbitrary or has been based on adjusting the depth of the melt lens at the MOR.

From the above discussion it becomes clear that there is a gap between modelling explicitly hydrothermal convection on one side and arbitrarily increasing the thermal conductivity on the other side. It is desirable to link the large scale cooling lithosphere processes more consistently to hydrothermal convective cooling. Therefore, here we present a theory of parameterized hydrothermal cooling (PHC) based on a hydrothermal Rayleigh number to Nusselt number relation (e.g. Otero et al. 2004). This relation can be used to replace PHC by an equivalent thermal conductivity whose value and depth distribution—in contrast to previous approaches—depend on the thermal evolution of the crust and lithosphere. When applying this method to large scale modelling, the layer affected by hydrothermal cooling and its Nusselt number have to be linked to the large scale geodynamic constraints of pressure $P$ and temperature $T$. Here we assume a porosity–permeability relationship for which we develop an expression for $P, T$-dependent porosity closure conditions to define the penetration depth of hydrothermal cooling.

To demonstrate the benefit of our PHC approach we apply it to a simple 1-D cooling model for the oceanic lithosphere and study the evolution of the cooling oceanic lithosphere with age. We discuss our finding in relation to observations of global oceanic heat flow data (Stein & Stein 1992, 1997) and global bathymetry (Jaupart & Mareschal 2011).

2 APPROACH

The approach we follow here is to approximate the effect of hydrothermal cooling by an equivalent thermal conductivity, which, in contrast to previous approaches, depends on the thermal evolution of the system. Our general idea of the parameterization is shown in the strategy flow chart (Fig. 2). A basic assumption is that hydrothermal cooling can be physically described by porous convection which is governed by the rock matrix permeability. Here, we assume that the permeability depends on the ambient

Figure 1. Sketch of a large scale process of rifting at a mid-oceanic ridge with melting, melt extraction and intrusion of melt in a permeable layer. Hydrothermal cooling is controlled by the $P, T$-dependent permeability. The resulting temperature field affects, for example, the macroscopic rheology.
The temperature and pressure dependence of the permeability can be related to the change in porosity due to thermal expansion and isotropic compression. This temperature dependent permeability due to thermoelasticity of a medium containing cracks is described following Germanovich et al. (2000)

\[ k_\varphi = k_{\varphi 0} \cdot \left[ 1 - \gamma \cdot (T - T_0) \right]^n \cdot H \left[ 1 - \gamma \cdot (T - T_0) \right] + k_{res}, \]  

(3)

where \( T_0 \) is a reference temperature, which for our application is equal to the surface temperature and set to 0 °C; \( k_{\varphi 0} \) is the permeability at the surface of the lithosphere containing open cracks at surface temperature \( T_0 \); the parameter \( \gamma \) is defined as \( \gamma = \alpha_{\text{t}}/\alpha_{\varphi 0} \) where \( \alpha_{\text{t}} \) is the thermal expansivity of rock and \( \varphi_0 \) is the porosity at surface temperature; \( H \) is the Heaviside function (which guarantees that with increasing \( T \) the permeability does not become negative), \( k_{res} \) is the residual permeability, present after closure of all major fractures and \( n \) is the power of the permeability–porosity relation (usually 3). Following Germanovich et al. (2000) \( k_{res} \) may be approximated as \( k_{res} \approx 10^{-3} k_{\varphi 0} \). In our approximation we neglect \( k_{res} \), assuming for sufficient high temperature all pores are closed and permeability is zero.

The derivation of eq. (3) illustrates some further assumptions behind it: First, we derive an expression for the temperature dependence of porosity. Thermal expansion of the solid phase (matrix material) at constant pressure (and in the absence of thermal stresses) is described by

\[ V_i = V_0 \cdot [1 + \alpha(T - T_0)], \]

(4)

where \( V_i \) and \( V_0 \) are the volumes of the solid, nonporous rock at \( T \) and \( T_0 \), respectively. Porosity is defined as

\[ \varphi = \frac{V_p}{V_{tot}} = \frac{V_{tot} - V_i}{V_{tot}} \]

(5)

with \( V_p \) as pore volume and \( V_{tot} = V_i + V_p \) as the total volume of rock matrix and pores. Inserting eq. (4) in eq. (5) yields

\[ \varphi = \varphi_0 \left[ 1 - \frac{V_0}{V_{tot}} \frac{\alpha_{\text{t}}}{\varphi_0} (T - T_0) \right]. \]

(6)

Here \( \varphi_0 \) is the porosity at \( T = T_0 \). For small porosity \( V_0/V_{tot} \) can be approximated by 1, leading to

\[ \varphi \cong \varphi_0 \left[ 1 - \frac{\alpha_{\text{t}}}{\varphi_0} (T - T_0) \right]. \]

(7)

Inserting the porosity (eq. 7) into the permeability eq. (1) finally gives the first part of eq. (3) with \( k_{\varphi 0} \) as permeability at porosity \( \varphi_0 \). This leads to the following implications: our approach is only valid (a) in case of small porosity, (b) under the assumption that \( V_0/V_{tot} = 1 \) and (c) thermal stresses are absent. This means that \( V_{tot} \) remains independent of temperature and any thermal expansion of the solid phase is directly compensated by a decrease in pore space. As we have thermal expansion at constant pressure, this means that our approximation implies that keeping the total volume constant does not lead to a thermal pressure, that is, any thermal stress is relaxed.

At high temperatures water filled pore space will not be kept open by elastic stresses but collapses due to ductile deformation. A simple way to account for this effect is to multiply the right hand side of eq. (3) by the Heaviside function \( H(T_{\text{duc}} - T) \), where \( T_{\text{duc}} \) is the temperature above which the pores close due to ductile deformation. To choose a reasonable value for \( T_{\text{duc}} \) we take a creep law for olivine and require the Maxwell time of the rock to be of the order 10 ka or less, which guarantees that the pores have closed for older lithosphere. This Maxwell time corresponds to an effective
viscosity of O(10^{22} \text{ Pa s}). Assuming that the pore closure stress is of the order of the effective lithostatic stress (O(2 \times 10^9 \text{ Pa}) at 10 km depth), the above viscosity corresponds to a temperature of about 750 ± 25 °C. For other stresses corresponding to 1 or 100 km depth, these temperatures would change to about 850 or 650 °C, respectively. Doing the same exercise with diabase (Kirby & Kronenberg 1984) we arrive at a ductile temperature of 640 °C. For sake of simplicity we will neglect these variations with composition and the depth dependence and keep the value fixed at 750 °C. This is within the experimentally found range of the brittle—ductile transition of 600–800 °C determined for basalt under oceanic crustal conditions (Violay et al. 2010).

We add to eq. (3) (neglecting $K_{\text{dry}}$) the depth-dependent closure of pores due to pressure. If this pore closing is an elastic process, a good approximation is an exponential decrease of porosity with pressure (i.e. depth; Jiang et al. 2009) and we finally have

$$k_{\phi}(T, z) = k_{p0} \cdot [1 - \gamma \cdot (T - T_0)]^\nu \cdot H [1 - \gamma \cdot (T - T_0)] \cdot H(T_{\text{dec}} - T) \cdot \exp \left( -\frac{z}{z_{\text{ch}}} \right),$$

(8)

where $z$ is the depth positive downward and $z_{\text{ch}}$ is the characteristic decay depth, at which the porosity has decreased to 1/e ($e$ is the Euler number) of its surface value. To determine $z_{\text{ch}}$ and to verify the exponential decrease with depth the pressure dependence of porosity has to be derived.

The pressure dependence of the porosity can be described by the pore space stiffness, $K_{\phi}$ (Mavko et al. 1998)

$$\frac{1}{K_{\phi}} = \frac{1}{V_p} \left. \frac{\partial P}{\partial \phi} \right|_{T=\text{const}},$$

(9)

where $P_e$ is the effective pressure.

$$P_e = P - P_t,$$

(10)

with the mean pressure

$$P = (1 - \phi)P_s + \phi P_t,$$

(11)

where $P_s$ and $P_t$ are the pressures in the solid rock and the fluid, respectively.

Using the definition of porosity and the definition of the dry (or drained) effective bulk modulus $K_{\text{dry}}$, that is, the bulk modulus for the rock matrix including pores

$$\frac{1}{K_{\text{dry}}} = \left. \frac{1}{V} \frac{\partial P}{\partial \phi} \right|_{T=\text{const}},$$

(12)

the pore space stiffness (eq. 9) can be written as

$$\frac{1}{K_{\phi}} = \frac{1}{K_{\text{dry}}} \frac{1}{\phi} \left. \frac{\partial \phi}{\partial P} \right|_{T=\text{const}}.$$

(13)

From its definition and the definition of the bulk modulus of the pure matrix material, $K_0$, the pore space stiffness may also be written as

$$\frac{1}{K_{\phi}} = \frac{1}{\phi K_{\text{dry}}} \frac{1 - \phi}{1 - \phi} \approx \frac{1}{\phi K_{\text{dry}}} = \frac{1}{\phi K_0}.$$

(14)

The modulus $K_{\text{dry}}$ can be estimated from composite theory. Based on O’Connell & Budiansky (1977), Schmeling (1985) gave the effective elastic moduli of porous medium. For drained penny-shaped ellipsoidal pores the self-consistent effective bulk modulus is given by

$$\frac{1}{K_{\text{dry}}} = \frac{1}{K_0} + \frac{4}{3\pi} \frac{1 - \nu^2}{K_{\text{dry}}} \frac{\phi}{a_t},$$

(15)

where $\nu$ is the (effective) Poisson ratio of the porous material, $D = 1$ for drained (i.e. perfectly connected) pores, and $a_t$ is the aspect ratio of the pores. The occurrence of the effective quantities $K_{\text{dry}}$ and $\nu$ in the right side of eq. (15) results from the self-consistent approximation, embedding the inclusions into the effective rather than undisturbed material (O’Connell & Budiansky 1977). We abbreviate eq. (15) as

$$\frac{1}{K_{\text{dry}}} = \frac{1}{K_0} + \frac{\phi}{K_{\text{dry}} a_t},$$

(16)

with

$$\frac{\phi}{K_{\text{dry}} a_t} = \frac{3\pi}{4} \left( 1 - 2\nu \right) a_t.$$

(17)

For drained fully connected pores $\nu$ decreases from about 0.3 to 0 for increasing porosity, thus $a_t$ increases from 1.036 $a_t$ to 2.36 $a_t$, respectively. The connected porosity will vary between a maximum value near the earth’s surface and a value close to 0 at the depth of pore closure. The greatest possible value of $\phi_0 \phi_{\text{max}}$ is defined by the asymptotic case of vanishing bulk modulus, $K_{\text{dry}} \rightarrow 0$ and vanishing Poisson ratio, $\nu \rightarrow 0$, thus from eq. (16) it is $\phi_{\text{max}} = \frac{6\pi}{4} (\nu = 0)$. For a given pore aspect ratio and Poisson number, no higher porosity is possible since this would lead to disaggregation of the rock. A first order approximation of eq. (17) taking into account the decrease of the effective Poisson ratio from the pure matrix value, $\nu_0$, to zero for increasing crack density is

$$\frac{\phi}{\phi_0} + c_{\pi} \phi.$$
The alternative assumption is to keep the crack area constant and close the pores by decreasing the crack width, that is, the aspect ratio. In Appendix A, we derive the pressure dependence of porosity following the second assumption and argue in favour of the constant aspect ratio model.

Fig. 3(b) shows the porosity resulting from the depth dependent porosity by directly using eq. (1). Depending on the surface porosity the surface permeabilities are different and may reach maximum values for given aspect ratios of penny shaped cracks as a function of porosity at the surface. Other parameters are as in Fig. 3. The curves terminate at the maximum possible porosity for given aspect ratio corresponding to vanishing dry bulk modulus (disaggregation of the rock).

Fig. 4. Characteristic depth $z_{ch}$ at which the porosity has dropped to $1/e$ of its surface value according to eq. (21) for different aspect ratios of penny shaped cracks as a function of porosity at the surface. Other parameters are as in Fig. 3. The curves terminate at the maximum possible porosity for given aspect ratio corresponding to vanishing dry bulk modulus (disaggregation of the rock).

as it results from eq. (21) for different aspect ratios as a function of surface porosity $\varphi_0$. Except for surface porosities close to their maximum possible values (unconsolidated rock) the characteristic depth is essentially independent of surface porosity for a given aspect ratio. On the other hand, the characteristic depth increases approximately linearly with aspect ratio. However, it has to be noted that our approach assumed the pore fluid at hydrostatic condition, that is, a hydraulic connection to the open pores at the surface. This implies a large pressure difference between the solid and fluid at great depth, leading to high deviatoric stresses around the pores, probably exceeding the elastic strength at depths of a few 10's of kilometres.

Having derived the characteristic depth $z_{ch}$ (eq. 21), we compare the simple exponential depth dependence of permeability suggested in eq. (8) (dashed curves in Fig. 3b) with the full solution (bold curves, based on eq. 20). As we aim to use the simplified exponential solution in the following, we adjust $c_{fit}$ in eq. (21) to 0.6 by trial and error, which gives a best fit to the full nonexponential solutions. Thus, in our further simplifying approach when applying eq. (8) we use $z_{ch}$ as defined in eq. (21) with $c_{fit} = 0.6$.

We emphasize the importance of considering reasonable assumptions of pore geometry when dealing with pore closure with depth. In our theoretical case, $\alpha_i$ is a good parameter to control the penetration depth of the hydrothermally convecting layer. Using reasonable values for $v_0$ (in the range of 0.25–0.3), this dependence simply reduces to $z_{ch} \approx 1.3 \cdot \alpha_i \cdot K_0 / [(\tilde{\rho} - \rho_i) \cdot g]$. Next we discuss the temperature effect on porosity and permeability according to eq. (7) and eq. (8), the results are shown in Fig. 5. From Fig. 5(a) we can deduce that closure of pore space is linear with temperature given by the thermal expansivity of the rock. To close the pores from a starting porosity of about 2 per cent, a temperature increase of about 800 °C is necessary. Due to the power law of the porosity–permeability relation (eq. 1 with $n = 3$, $k_0$ drops faster, so that a starting permeability of $10^{-13}$ m$^2$ drops to 10 per cent already at crustal temperatures of 600 °C. For higher permeabilities, the
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Figure 5. Temperature dependent porosity (a) and permeability (b) for different starting porosities at the surface, an initial temperature of $T_0 = 0 \degree C$, $n = 3$, and a thermal expansivity of the rock of $\alpha_r = 2.4 \times 10^{-5} K^{-1}$. No pressure effect is considered. For temperature above $T_{duc} = 750 \degree C$ ductile behaviour of the rock matrix is assumed, leading to complete closure of the pores and vanishing permeability.

2.2 Thickness and bottom temperature of hydrothermally convecting layers

The present study is done to consider hydrothermal cooling in the crust, when dynamic processes in the lithosphere which involve heat transport processes, are investigated. The effect of hydrothermal convection on the lithosphere geotherm will be a decrease of the geotherm towards lower temperature in the uppermost crust, as indicated schematically in Fig. 6.

The depth down to which hydrothermal convection takes place is given by the lower boundary of the layer for which the depth-dependent permeability is larger than a threshold permeability $c_{thr} \cdot k_{\varphi 0}$, where $k_{\varphi 0}$ is the surface permeability and $c_{thr}$ is a parameter <<1 depending on the vigour of convection. We denote this depth by $z_{hy}$. The actual temperature at that depth, defined as $T_{hy}$, can be equated to $\Delta T_{hy}$ (assuming zero temperature at the top), see Fig. 6. $\Delta T_{hy}$ will be used as the temperature difference driving hydrothermal convection.

2.3 Parameterized hydrothermal convection

Assuming hydrothermal convection in the upper layer of the lithosphere as shown in Fig. 6, we will now derive an expression to account for the hydrothermal heat transport by an equivalent thermal conductivity for this layer. The only length scale in our problem is the characteristic depth $z_{ch}$ at which the permeability drops to 1/e of its surface value. As we are faced with a Neuman boundary condition the convection is uniquely described by the bottom heat flux $q_0$. Then the vigour of convection is described by a nominal heat flow Rayleigh number, $Ra_q$:

$$Ra_q = \frac{\alpha_f g \rho_f^2 c_{pf} k_{\varphi 0} q_0 z_{ch}^2}{\eta_f \lambda_m z_{ch}}.$$  \hspace{1cm} (22)

Since hydrothermal convection usually penetrates deeper than $z_{ch}$ it is convenient to define a hydrothermal Rayleigh number $Ra_{hy}$ based on the penetration depth $z_{hy}$ and the temperature difference $\Delta T_{hy}$ between the top and the depth at $z_{hy}$, both quantities being functions of $Ra_q$:

$$Ra_{hy} = \frac{\alpha_f g \rho_f^2 c_{pf} k_{\varphi 0} \Delta T_{hy} z_{hy}}{\eta_f \lambda_m} = c_{Ra} \Delta T_{hy} z_{hy}$$ \hspace{1cm} (23)

with

$$c_{Ra} = \frac{\alpha_f g \rho_f^2 c_{pf} k_{\varphi 0}}{\eta_f \lambda_m},$$ \hspace{1cm} (24)

where the various rock and fluid parameters are specified in Table 1.
Table 1. Symbols, their definitions, numerical values, and physical units used in this study.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Geometrical factor of permeability–porosity relation</td>
<td>1/162</td>
<td></td>
</tr>
<tr>
<td>$a_t$</td>
<td>Aspect ratio of all pores</td>
<td>varied</td>
<td></td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>Defined in eq. (17)</td>
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<td></td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat capacity</td>
<td>1200</td>
<td>J kg$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$c_{pf}$</td>
<td>Specific heat capacity of water</td>
<td>$4.185 \times 10^3$</td>
<td>J kg$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$c_{No}$</td>
<td>Pre-factor of $Nu$–$Ra$ relation</td>
<td>0.5617</td>
<td>Non-dimensional</td>
</tr>
<tr>
<td>$c_{Ra}$</td>
<td>Independent part of hydrothermal Rayleigh number</td>
<td>See definition eq. (24)</td>
<td>K$^{-1}$ m$^{-1}$</td>
</tr>
<tr>
<td>$d$</td>
<td>Grain size</td>
<td>$10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$D$</td>
<td>Pore connection factor</td>
<td>1 (fully connected)</td>
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</tr>
<tr>
<td>$g$</td>
<td>Gravity acceleration</td>
<td>10</td>
<td>m s$^{-2}$</td>
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<tr>
<td>$H$</td>
<td>Heaviside function</td>
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<td>$K_0$</td>
<td>Bulk modulus of uncracked rock</td>
<td>$0.4 \times 10^{11}$</td>
<td>Pa</td>
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<tr>
<td>$K_{BY}$</td>
<td>Effective bulk modulus</td>
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<tr>
<td>$K_p$</td>
<td>Pore space stiffness</td>
<td>Pa</td>
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<td>$k_0$</td>
<td>Permeability</td>
<td>m$^2$</td>
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<tr>
<td>$k_{0s}$</td>
<td>Permeability at $T_0$ and surface pressure (denoted as $k_0$)</td>
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<td>m$^2$</td>
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<tr>
<td>$N_{Res}$</td>
<td>Residual permeability after pore closure</td>
<td>0</td>
<td>m$^2$</td>
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<tr>
<td>$n$</td>
<td>Mean mid-cell temperature gradient of hydrothermal convection layer</td>
<td>See definition eq. (34)</td>
<td>Non-dimensional</td>
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<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
<td>3</td>
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<tr>
<td>$P_c$</td>
<td>Power of permeability–porosity relation</td>
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<td>$P_f$</td>
<td>Effective pressure</td>
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<td>$P_t$</td>
<td>Fluid pressure</td>
<td>Pa</td>
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<tr>
<td>$P_s$</td>
<td>Solid (rock) pressure</td>
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<tr>
<td>$\bar{P}$</td>
<td>Mean Pressure</td>
<td>$\bar{P} = (1 - \psi)P_f + \psi P_t$</td>
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<td>$q_{eq}$</td>
<td>Surface heat flow with hydrothermal convection</td>
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<td>$Ra_{hy}$</td>
<td>Hydrothermal Rayleigh number</td>
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<td>$Ra_d$</td>
<td>Critical hydrothermal Rayleigh number</td>
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<tr>
<td>$T_0$</td>
<td>Temperature at surface defining initial permeability $k_{0s}$</td>
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<td>C</td>
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<td>$T_{base}$</td>
<td>Temperature for pore closure due to ductile deformation</td>
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<td>C</td>
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<tr>
<td>$V_f$</td>
<td>Rock volume</td>
<td>m$^3$</td>
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<tr>
<td>$V_0$</td>
<td>Rock volume at surface temperature</td>
<td>m$^3$</td>
<td></td>
</tr>
<tr>
<td>$V_p$</td>
<td>Pore volume</td>
<td>m$^3$</td>
<td></td>
</tr>
<tr>
<td>$V_{tot}$</td>
<td>Total volume</td>
<td>$V_f + V_p$</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$z_{by}$</td>
<td>Thickness (depth) of the equivalent hydrothermal convection layer, positive downward</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>$z_{ch}$</td>
<td>Characteristic depth (depth of e-fold porosity decrease)</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal expansivity water</td>
<td>$10^{-3}$</td>
<td>K$^{-1}$</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>Thermal expansivity rock</td>
<td>$2.4 \times 10^{-5}$</td>
<td>K$^{-1}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Factor of $T$-dependence of permeability</td>
<td>$\alpha_t/\alpha_0$</td>
<td>K$^{-1}$</td>
</tr>
<tr>
<td>$\Delta T_{by}$</td>
<td>Temperature difference across the equivalent hydrothermal convection layer. Equal $T_{by} = T(z_{by})$</td>
<td>Obtained from $z_{by}$</td>
<td>K</td>
</tr>
<tr>
<td>$\delta_1, \delta_2$</td>
<td>Upper and lower thermal boundary layer thickness</td>
<td>$\delta_1 = 0, \delta_2$ from eq. (37)</td>
<td>m</td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>Thermal conductivity of rock matrix</td>
<td>Equal $\lambda_0$</td>
<td>W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Thermal conductivity of the crust (lithosphere)</td>
<td>$=\kappa_0 \rho_0 c_p$ from scaling parameters</td>
<td>W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{eq}$</td>
<td>Scaling thermal conductivity</td>
<td>$\lambda_{eq}$</td>
<td>W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>Thermal diffusivity</td>
<td>$10^{-6}$</td>
<td>m$^2$ s</td>
</tr>
<tr>
<td>$\eta_t$</td>
<td>Viscosity of water</td>
<td>$10^{-3}$</td>
<td>Pa s</td>
</tr>
<tr>
<td>$\nu_0, \nu$</td>
<td>Poisson ratio of pure matrix, effective Poisson’s ratio</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Density of water</td>
<td>1000</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Mean density of fluid-filled crustal rock</td>
<td>Approximated by $\rho_0$</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Porosity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>Porosity at $T_0$ and at surface</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A measure for the intensity of convective heat transport is the Nusselt number

$$Nu = \frac{q_{eq} z_{by}}{\lambda_0 \Delta T_{by}}$$  \hspace{1cm} (25)
be small we can substitute it by \( \lambda_m \), the thermal conductivity of the rock matrix.

The relation between the nominal Rayleigh number \( Ra_{\text{hy}} \), the Nusselt number \( Nu \), and the effective hydrothermal Rayleigh number \( Ra_{\text{hy}} \) is given by

\[
Ra_{\text{hy}} = \frac{Ra_{\text{hy}}}{Nu \ ch} \quad (26)
\]

To parameterize hydrothermal convection with depth-dependent permeability it is thus sufficient to carry out one series of experiments with varying nominal Rayleigh number \( Ra_{\text{hy}} \) and determine functional relationships for the various quantities \( Nu, z_{\text{hy}}, \Delta T_{\text{hy}}, \) and \( Ra_{\text{hy}} \). In the following we will write these relationships in terms of functions of the temperature difference-based Rayleigh number \( Ra_{\text{hy}} \) by using eq. (26).

For high \( Ra \)-convection the non-linear relationship is given by

\[
Nu = c_{Nu} \ Ra_{\text{hy}}^{\beta_{Nu}}, \quad (27)
\]

where the scaling parameters \( c_{Nu} \) and \( \beta_{Nu} \) are now to be determined. A multitude of numerical and laboratory experiments have been carried out to determine \( Nu-Ra \) relationships (see e.g. summaries by Davis et al. 1997; Wang 2004; and references therein). As those are valid mostly for constant permeability we carried out numerical 2-D depth-dependent permeability experiments on hydrothermal convection with permeable (constant pressure), constant temperature boundary conditions and impermeable, constant temperature bottom boundary conditions using the commercial FE-code COMSOL (Fig. 7). The following equations for conservation of energy, mass and momentum (Darcy equation, where \( \tilde{u} \) is the Darcy velocity) for porous convection including the equation of state for water are solved:

\[
\frac{\partial T}{\partial t} + \tilde{\nabla} \cdot \tilde{u} T = \nabla^2 T, \quad (28)
\]

\[
\tilde{\nabla} \cdot \tilde{\rho} \tilde{u} = 0, \quad (29)
\]

\[
\tilde{u} = -\frac{k_w}{\eta} \left( \tilde{\nabla} p_f - \tilde{\rho} \tilde{g} c_e \right), \quad (30)
\]

\[
\tilde{\rho} = \rho_l \left[ 1 - \alpha \left(T - T_0\right) \right], \quad (31)
\]

where \( c_e \) is the unit vector pointing downward. The permeability was assumed to decrease exponentially with depth (eq. 8). Interestingly, near-steady state convection with constant spacing between the plumes was found even up to high Rayleigh numbers. This is in contrast to constant permeability and constant bottom temperature calculations (cf. Cherkaoui & Wilcock 1999): a decreasing permeability with depths leads to a conductive basal layer of finite thickness beneath the effectively convecting layer and stabilizes the flow. Care was taken to reach steady state or statistical steady state and to guarantee that \( z_{\text{hy}} \) was sufficiently far above the bottom of the numerical box. The depth \( z_{\text{hy}} \) was determined by evaluating the depth at which the horizontally averaged advected heat flow became smaller than the conductive heat flow. This depth and the horizontally averaged temperature at that depth were taken to define the Rayleigh number. From these experiments, for the permeable surface case we obtained the parameters \( c_{Nu} = 0.56 \) and \( \beta_{Nu} = 0.2 \pm 0.07 \) by fitting the numerical results with straight lines in the log-log plots (Fig. 8). It is noteworthy that this exponent is considerably smaller than that found for constant bottom temperature and constant permeability conditions (Cherkaoui & Wilcock 1999). Even in case that \( Nu \) is referred to the nominal Rayleigh number \( Ra_{\text{hy}} \) (eq. 22) and using eq. (26) the exponent is as small as 0.23. The permeability threshold parameter \( c_{tu} \), defined in chapter 2.2 (Fig. 6) decreases with increasing \( Ra_{\text{hy}} \) following

\[
c_{tu} = c_k \ Ra_{\text{hy}}^{\beta_k} \quad (32)
\]

with \( c_k = 3.376 \) and \( \beta_k = -0.84 \pm 0.05 \) according to Fig. 8(b). Eqs (27) and (32) link the permeability of the hydrothermally convecting layer given in the Rayleigh number to its heat transfer characteristic which will be used to define an equivalent thermal conductivity.

For the purpose of comparing model results for hydrothermal cooling with observed heat flux data, another quantity needs to be derived. The thermal gradient at the top of a hydrothermally convecting layer with a permeable upper boundary is smaller than the thermal gradient within the lower thermal boundary layer (cf. Fig. 6) and thus, smaller than a corresponding conductive thermal gradient describing the entire heat flux across the layer. Using the reduced gradient to infer surface heat flux from observations underestimates the total heat flux \( q_{\text{tot}} \) by a factor \( q_{\text{cond}} / q_{\text{tot}} \), where \( q_{\text{cond}} \) is the conductive part of the surface heat flux. From the 2-D numerical results we determined this part (horizontally averaged) as a function of \( Ra \)
Parameterized porous convection

Figure 8. (a) \( \text{Nu–Ra}_h \) relation for 2-D hydrothermal convection with an exponentially decreasing permeability with depth derived from a series of models some of which are shown in Fig. 7. \( \text{Ra}_h \) is based on the surface permeability and on the horizontally averaged depth \( (z_h) \) and corresponding temperature \( \Delta T_h \) at which advective heat transport becomes smaller than conductive transport. Boundary conditions at the surface are constant fluid pressure and temperature. The Nusselt number has been determined by using eq. (27), but taking the horizontally and time averaged conductive bottom heat flow for sufficiently long time. This was done because the surface heat flow consists of conductive and advective parts, whose numerical determination is less accurate. The dashed line shows the approximated scaling law. (b) Threshold permeability at the depth \( z_{hy} \) and approximated scaling law (dashed). (c) Penetration depth \( z_{hy} \) scaled by characteristic depth as a function of \( \text{Ra}_h \) and approximated scaling law (dashed). (d) Horizontally averaged conductive part of the surface heat flux relative to the total heat flux; dashed line according to fitted scaling law (eq. 33).

(33)  
\[ \frac{q_{\text{cond}}}{q_{\text{tot}}} = \begin{cases} 1 & \text{Ra}_h < \text{Ra}_c, \\ \left( \frac{\text{Ra}_h}{\text{Ra}_c} \right)^{-\beta_{q_{\text{cond}}}} & \text{else} \end{cases} \]

where the exponent \( \beta_{q_{\text{cond}}} \) was determined by fitting as 0.75 (dashed line in Fig. 8d). Interestingly, the conductive contribution of the horizontally averaged heat flux decreases strongly with increasing Rayleigh number.

### 2.4 Approximating hydrothermal cooling by an equivalent thermal conductivity

We now replace the heat transfer characteristics of the hydrothermally convecting layer by an equivalent thermal conductivity based on the parameterized \( \text{Nu–Ra} \) relationship derived above. It should be noted that strictly speaking this is only valid for a permeability exponentially decreasing with depth, while our permeability also depends on temperature (eq. 8). Thus, as long as high temperatures dominate the system and effectively reduce the permeability, our approach will overestimate the Nusselt number, as will be discussed in chapter 3.3. To apply our scaling law let us recall the Nusselt number (eq. 25) and reconsider the temperature distribution through a convection cell (Fig. 9). While for very high \( Ra \) numbers the cell interior of hydrothermal convection will be isothermal, for practical reasons we introduce a finite non-dimensional slope or temperature gradient \( m = (dT/dz)_{\text{cell}} z_h / \Delta T_h \) of the cell interior. For lower Rayleigh number even physically \( m > 0 \) is found because in the central part of the convection layer both advection and conduction work. If the Rayleigh number is critical \( (\text{Ra}_h = \text{Ra}_c) \), just at the onset of convection, the purely conductive case is valid and \( m = 1 \). We define \( m \) in a heuristic way by

\[ m = \begin{cases} 1 & \text{Ra}_h < \text{Ra}_c, \\ (1 - m_{\text{min}}) \cdot \frac{\text{Ra}_h}{\text{Ra}_c} + m_{\text{min}} & \text{Ra}_h \geq \text{Ra}_c \end{cases} \]

where \( m_{\text{min}} \) is an artificial minimum value for numerical purpose (chosen on the order of 0.2) which guarantees numerical convergence for small \( m \) when the temperature gradient in the mid-cell is very small and the scaling thermal conductivity becomes large. Tests with a cooling lithosphere with variations of \( m_{\text{min}} \) by \( \pm 50 \) per cent show that at high Rayleigh numbers both the slopes and scaling conductivities vary by a similar amount but in opposite direction, so that the resulting heat flow and bathymetry deviate only by a few per cent from the values obtained with \( m_{\text{min}} = 0.2 \). Only for times around 0.1 Ma after onset of cooling the deviations reach 20 per cent. The critical Rayleigh number is taken using eq. (26) and setting \( \text{Nu} = 1 \).

Depending on the Rayleigh number and the boundary conditions, thermal boundary layers of thickness \( \delta_1 \) and \( \delta_2 \) develop at the top and bottom boundary.
With these values we define the equivalent thermal conductivity \( \lambda_{eq} \) as

\[
\lambda_{eq} = \begin{cases} 
\lambda_{sc} & \delta_1 < z < z_{by} - \delta_2 \\
\lambda_0 & \text{else}
\end{cases}
\]  

(39)

This formulation involves still a sharp transition for thermal conductivity at the depth \( z_{by} - \delta_2 \). To smooth this transition and to account only for a lower thermal boundary layer (i.e. \( \delta_1 = 0 \)), since we assume a permeable surface, we finally define

\[
\lambda_{eq} = \begin{cases} 
\frac{\delta_1 z + \delta_2}{\delta_2} (\lambda_0 - \lambda_{sc}) + \lambda_{sc} & 0 \leq z < \delta_1 \\
\frac{z - z_{by} + \delta_2}{\delta_2} (\lambda_0 - \lambda_{sc}) + \lambda_{sc} & \delta_1 \leq z < z_{by} - \delta_2 \\
\lambda_0 & \text{else}
\end{cases}
\]  

(40)

Eqs (40) should produce a thermal boundary layer at the depth ranges \( 0 \leq z < \delta_1 \) and \( z_{by} - \delta_2 < z < z_{by} \) and an almost constant temperature within the hydrothermal convection layer.

The concept behind the use of eqs (39) or (40) in numerical modelling is the following: At any time at each position \( (x, y) \) at the surface, the temperature field \( T(z) \) below is checked to determine the depth where \( \lambda_0 \) is equal to \( c_{eq} \cdot k_{by} \) using eq. (8). This can be done, for example, by inserting \( c_{eq} \cdot k_{by} \) for the left-hand side of eq. (8) and solving it for the temperature, which we might call \( T_{ctl} \) (pore closure temperature for pressure \( P = 0 \)). Determining the point of intersection of this \( T_{ctl}(z) \) curve with the evolving \( T(z) \) curve gives a time-dependent depth which can be identified with \( z_{by} \). Because \( c_{eq} \) is not known \( a \ priori \) but has to be determined using the scaling law eq. (32), a starting value of \( c_{eq} = 0.1 \) can be taken which is of the order of that value at the critical Rayleigh number. The temperature at \( z_{by} \) gives the temperature \( \Delta T_{by} \). Based on these values and other parameters in eq. (37) the equivalent thermal conductivity is determined based on eq. (40) in a region between depth \( \delta_1 \) and \( z_{by} - \delta_2 \). This changes the temperature field during the following time steps, so that a perturbed \( T \) field is obtained (shallow dashed curve in Fig. 6). As the temperature drops in that region due to the enhanced heat conduction, deeper and deeper parts will become cooler and the permeability increases according to eq. (8), thus the simulated hydrothermal convection layer thickens (deeper dashed curve in Fig. 6) down to a depth where all pores are closed due to pressure. In this formulation it is assumed that the time scale of adjustment of the depth of the hydrothermal convection layer, that is, \( z_{by} \), is much larger than the time scale of hydrothermal convective overturns, so that the hydrothermal convection can adjust immediately to any change of \( z_{by} \), that is, of \( R_{th} \). In fact this is justified for high \( R_{th} \), as the Rayleigh number give exactly the ratio of these time scales.

As already mentioned, in case of constant pressure at the top (permeable boundary) no thermal boundary layer exists at the top (see Fig. 6) and \( \delta_1 \) has to be chosen as zero in eq. (39) or (40), while the scaling thermal conductivity \( \lambda_{sc} \) depends on the presence or absence of the top thermal boundary layer only indirectly via the Nusselt number.

At this point it should be emphasized that although we achieve a high degree of physical consistency, at every point along the way the formulation derived above rests on broad assumptions and on strongly simplified descriptions of possibly complicated geologic formations. When applying it to real nature conclusions should be taken with care.
Figure 10. (a) Vertical temperature profiles at successive times $t_0$, $2t_0$, $4t_0$, $8t_0$, . . . with $t_0 = 0.05$ Ma (i.e. the last time is 102.4 Ma; for simplicity, only the approximate times are indicated near the curves) for a 1-D cooling model with parameterized hydrothermal convection in the upper part. Initial temperature is 1300 °C. At $t = 0$ the surface is cooled to 0 °C, the bottom temperature is kept at 1300 °C. (b) Zoomed section showing the equivalent thermal conductivity $\lambda_{eq}$ simulating the parameterized hydrothermal convection for the same times. Crack aspect ratio is 0.01, surface porosity and permeability are 2.4 per cent and $8 \times 10^{-14}$ m$^2$, respectively.

3 APPLICATION TO 1-D LITHOSPHERIC COOLING

To demonstrate how our approach works we choose as an example the simple 1-D case of plate cooling, a model often used for the cooling of the oceanic lithosphere after being created at the mid-ocean ridges (Turcotte & Schubert 2002).

We use the 1-D heat conduction equation

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial}{\partial z} \left[ \lambda_{eq}(T, z) \frac{\partial T}{\partial z} \right], \quad (41)$$

which we solve forward in time and fourth-order central in space with a finite difference scheme on a grid with 601 gridpoints. We address a 100-km thick layer with a constant temperature at the lower boundary of 1300 °C. As initial condition we set the temperature in the entire layer to 1300 °C and assume a sudden drop to 0 °C at the surface which is kept as a constant temperature boundary for further times. Hydrothermal cooling is accounted for according to the equivalent thermal conductivity theory and elastic pore space closure formulation described in the previous sections. In this example a crack aspect ratio of 0.01 and an associated maximum surface porosity $\phi_0$ corresponding to a surface permeability of $8 \times 10^{-14}$ m$^2$ was chosen. This aspect ratio corresponds to a characteristic depth $z_{ch}$ (porosity drop to $1/e$ of surface value) of 13.6 km. Hydrothermal convection is mimicked by using the equivalent thermal conductivity formulation using eqs (38) and (40) with $\delta_1 = 0$ and the $Nu-Ra_{hy}$ relation derived as eq. (27).

3.1 Temporal evolution

Fig. 10(a) shows the evolving temperature profiles for the times $t_0$, $2t_0$, $4t_0$, $8t_0$, . . . with $t_0 = 0.05$ Ma (i.e. the last time is 102.4 Ma) and Fig. 10(b) shows depth profiles down to 20 km of the corresponding equivalent thermal conductivity $\lambda_{eq}$ evolving from the dynamics of the system. As cooling starts the simulated hydrothermal convection layer is thin because almost everywhere pore space (cracks) is absent because of $T$ being higher than $T_{duc}$. Yet effective cooling commences near the surface as the pore space opens, and deeper and deeper parts are incorporated in the hydrothermal cooling layer, which thickens with time. This type of shallow temperature drop followed by a steep temperature gradient has been observed at drilling sites at the mid-Atlantic Ridge and the Costa Rica Rift flank, though at shallower depths (see Davis et al. 2004 and references therein). The simulated hydrothermal Rayleigh number increases due to increasing thickness of the convecting layer, $z_{hy}$, which can be seen by the increasing maximum value of the effective heat conductivity. After the permeable layer reaches a depth of about 20 km, the pressure effect closing the pore space dominates and no deeper penetration of hydrothermal convection is possible. Further cooling decreases the temperature at the bottom of the convecting layer, that is, at $z_{hy}$, and thus the Rayleigh number starts to drop. The equivalent thermal conductivity starts decreasing during the later phase of cooling. At the end of the evolution a steady state solution is approached in which still hydrothermal convection effectivelly reduces the mean temperature gradient within the upper 20 km of the model.
To understand the cooling process in more detail we study here the time dependence of a number of key parameters. In Fig. 11 the evolution is shown for the hydrothermal Rayleigh number (a), the Nusselt number (b), the penetration depth \( z_{hy} \) of the hydrothermal circulation, that is, the layer of high effective thermal conductivity (c), the bottom temperature \( \Delta T_{bottom} \) of this layer (d), the effective thermal conductivity (e) and the surface heat flux (f). Taking into consideration the logarithmic time scale in Fig. 11, we observe the following: in the first phase, during the first Mio years, rapid thickening of the permeable layer occurs (Fig. 11c) since heat is effectively transported upwards, rapidly cooling the top layer of the lithosphere. New pore space opens within this cooling layer increasing permeability and further enhancing hydrothermal cooling. The bottom temperature of this layer is limited by the ductile pore closure temperature \( T_{duc} = 750 \, ^\circ\text{C} \) (Fig. 11d). The Rayleigh number (Fig. 11a) and Nusselt number (Fig. 11b) strongly increase during the thickening phase of the hydrothermal layer. This increasing convective vigour is modelled by the equivalent thermal conductivity according to eq. (40) (Fig. 11e). The second phase (1–10 Ma) starts when the penetration of hydrothermal cooling reaches its maximum depth (Fig. 11c); the bottom temperature of the convecting layer already starts to decrease a few 10^5 years earlier (Fig. 11d) because the pressure effect takes over determining the depth of the permeable layer. As a result also the Rayleigh and Nusselt numbers decrease. In the third phase (10–100 Ma) steady state conditions are going to be approached, characterized by weak ongoing hydrothermal convection within the upper 20 km and a linear conductive thermal gradient below down to the bottom of the lithosphere (cf. Fig. 10a).

Interestingly the vigour of hydrothermal convection peaks at 1 Ma. This behaviour is observed qualitatively only for slow spreading ridges. In our model this delayed peak is a consequence of the initial temperature condition where the hot temperature reached to the very top.

It should be noted that our time-dependent cooling model is based on a steady state \( Nu-Ra \) relationship and does not include transient hydrothermal convection effects. From our parameterization experiments we infer that the non-dimensional transient times for onset of convection is roughly given by \( 100 \times Ra^{-1/2} \). If scaled to the model shown in Fig. 10, this corresponds to O(0.1 Ma) or less for a 1-km thick convection layer, that is, at early times steady state is quickly reached. For thicker convection layer at later times the transient time is of similar order as the cooling age.

### 3.2 Effect on surface heat flow

Fig. 11(f) shows the total heat flux at the surface (red curve), being the sum of convective and conductive contributions in comparison to the reference case of no hydrothermal cooling (black curve). This reference case represents the \( 1/\sqrt{T} \) law (SRTL) of a cooling 100-km thick plate up to about 100 Ma. During the first phase (first Mio years) the total heat flux including hydrothermal convection is higher by a factor 3 than the reference case. A linear decrease in the log-log plot is found with a slope in agreement with a SRTL. This is surprising in view of the time dependence of the vigour and depth extent of hydrothermal convection. From some numerical tests, this apparent SRTL can be explained by the counteracting effects of an increasing \( z_{hy} \) with time leading to a steeper slope than SRTL, and an increasing \( \lambda_{eq} \) with time leading to a shallower slope than SRTL. It should be noted that this apparent SRTL during phase 1 is not a...
3.3 Comparison with a full 2-D convection solution

In order to test our parameterized approach we carried out full hydrothermal convection calculations (eqs 28–31) with the FE-code COMSOL assuming a $P, z$-dependent permeability according to eq. (8) with $z_{ch}$ according to eq. (21). All relevant parameters, boundary conditions and dimensions have been chosen as in the model shown in Figs 10 and 11, the width of the model was 10 km, as initial temperatures distribution an error function profile with 1-km thick thermal boundary layer was chosen to avoid numerical problems. Comparing the full convection solution in 2-D with the parameterized 1-D solution shows the following points: (1) during the first Ma transient effects of time-dependent hydrothermal convection are visible in the heat flux (Fig. 12) but are absent in the parameterized approach. (2) As expected the horizontally averaged vertical temperature gradient within the hydrothermal convection region is very small because no artificial slope $m$ is assumed to mimic high heat transport. (3) Despite this difference the temporal evolution of the horizontally averaged surface heat flow is similar to the parameterized model (Fig. 12). The parameterized model overestimates the full solution during the first few Ma by 10–20 per cent. This stems from assuming a $P$-dependent permeability in the $Nu$-$Ra$ parameterization while the numerical model additionally assumes a $T$-dependence. (4) This overestimation is associated with an overestimation of the effective total diffusion time resulting in an earlier flattening of the heat flux resulting from the approximation compared to the full solution. (5) The temporal evolution of the hydrothermal penetration depth agrees well with that of the parameterized model. All in all we regard the agreement between the approaches sufficiently good and refrain from fine tuning of parameters such as $c_{fit}$, $m_{min}$, $c_{lim}$ at the onset, or other than linear variations of $\lambda_{eq}$ within the thermal boundary layers, or scaling laws (eqs 27, 32 and 37). (6) Since 2-D hydrothermal convection is not explicitly computed, the reduction to 1-D saves computing time: While the full 2-D calculation takes about 40 min on an 8 core PC (using parallelized COMSOL solvers), the parametric 1-D run takes only 50 s for comparable vertical resolution with Matlab without optimization. Furthermore, numerical instabilities are frequently encountered in the COMSOL formulation for various mesh distributions.

3.4 Varying permeability parameters

So far our model assumed a particular aspect ratio and the related maximum possible surface porosity and permeability. However, it is interesting to study the effect of different crack porosity properties controlling the permeability distribution, where the most critical parameters are the crack aspect ratio and the surface permeability. We performed plate cooling models similar to the case shown in Figs 10 and 11 for aspect ratios ranging from $a_s = 0.003$ to 0.03 and surface permeabilities $k_{vo}$ varying from $2 \times 10^{-15}$ m$^2$ to almost $10^{-13}$ m$^2$. The resulting maximum hydrothermal Rayleigh number, maximum penetration depth $z_{hy}$, and total heat loss increase due to hydrothermal convection are shown in Fig. 13. The model discussed in the previous section shows up as crosses with the highest surface permeability in Figs 13(a), (b) and (c), respectively. Models with decreasing $k_{vo}$ result in decreasing Rayleigh numbers, the critical Rayleigh number being reached for $k_{vo}$ around $2 \times 10^{-15}$ m$^2$, and decreasing penetration depths. The characteristic depth $z_{ch}$ (depth of e-fold porosity decrease) is defined for constant temperature (cf. eq. 21) and is essentially independent of the surface porosity (and permeability; cf. Fig. 4). However, the convective penetration depth $z_{hy}$ of the fully hydrothermal approach strongly decreases with decreasing $k_{vo}$ (Fig. 13b). This can be explained by the scaling law for hydrothermal convection exponentially decaying permeability with depth relating the convective penetration depth with the Rayleigh number (Fig. 8c): An increase of Rayleigh number by one order of magnitude increases the convective penetration depth $z_{hy}$ by the amount of two characteristic depths ($2 z_{ch}$). Models, which have been carried out with an aspect ratio 0.03 (and 0.1, not shown), result in a strong increase of penetration depth with $k_{vo}$, probably
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Figure 13. (a) Maximum hydrothermal Rayleigh number, (b) maximum penetration depth of hydrothermal convection and (c) total extra heat loss due to hydrothermal convection, \( Q_{\text{tot}} - Q_{\text{noHC}} \), normalized by the total heat loss without hydrothermal convection, \( Q_{\text{noHC}} \), for a cooling lithosphere for crack porosities with different crack aspect ratios and different surface permeabilities. For cracks with an aspect ratio of 0.003 and 0.01, the maximum possible surface permeabilities are given by the points most to the right, for cracks with an aspect ratio of 0.03, higher surface permeabilities than shown are theoretically possible but lead to unrealistic deep penetration depths. The dashed line in (a) indicates the critical Rayleigh number, which marks the onset of hydrothermal convection.

above realistic values. This stems from the fact that closing higher aspect ratio pores by purely elastic compression probably exceeds the elastic limit, which is not accounted for here.

It is interesting to estimate the total heat loss of the plate due to hydrothermal convection compared to the reference case of no hydrothermal convection (Fig. 13c). Integrating the total surface heat flux over time (up to 100 Ma) and subtracting the corresponding total heat loss of the reference model without hydrothermal convection shows that hydrothermal convection not only shifts the cooling to earlier ages (cf. Fig. 11f), but also increases the total heat loss of the plate considerably. In other words, it might be misleading to state (as is done in the introduction following the usual argumentation in literature; e.g. Harris & Chapman 2004) that a certain fraction of the heat loss of the classical, conductive \( 1/\sqrt{t} \) heat loss is removed by hydrothermal convection. Instead, hydrothermal convection adds an additional amount of heat loss to the classical conductive \( 1/\sqrt{t} \) law which may not be easily detectable due to the variability of observed heat flow measurements. Thus, previous estimates of hydrothermal oceanic heat loss based on the deficit of measured heat flux with respect to the classical \( 1/\sqrt{t} \) law may underestimate the real heat loss by a considerable amount. Again, we discard the model results with aspect ratio 0.03 and surface permeabilities larger than \( 10^{-14} \) m\(^2\) for the same reasons as stated above.

To investigate the effect of different rock permeability parameters on the age-dependent distribution of the heat flux, Fig. 14 compiles models with a crack aspect ratio of 0.01 and 0.03, and different surface permeabilities \( k_{\phi0} \). As expected, high \( k_{\phi0} \) cases and consequently strong hydrothermal convection (red curve) show a larger increase of heat flux relative to the reference case of no hydrothermal cooling (bold black curve) compared to cases with smaller \( k_{\phi0} \) (black, green and blue curve). Interestingly, the maximum deviation from the reference SRTL is progressively delayed for decreasing \( k_{\phi0} \). For example, for the case with \( k_{\phi0} = 2.2 \times 10^{-15} \) m\(^2\) the maximum (but small) deviation occurs as late as at about 60 Ma. The reason is the decrease of the vigour of hydrothermal convection and the associated increase of the apparent lithospheric diffusion time (which scales with \( 1/\lambda_{eq} \), see Section 3.2).

A typical behaviour of plate cooling models is a deviation from the SRTL at old ages >80 Ma, visible as flattening of the reference curve (Fig. 14a, bold black curve). As discussed above, we
observe that intense hydrothermal cooling such as shown by the red curve shifts this starting point of flattening to earlier ages (around 60 Ma) due to a decrease of the apparent lithospheric diffusion time for cooling. In Fig. 14, we see that this effect becomes smaller for smaller surface permeabilities, which can be explained by the smaller decrease of the apparent lithospheric diffusion times.

The dashed curves in Fig. 14(a) show the conductive contribution of the cooling according to eq. (33). For considerably high permeabilities only a small fraction of the total heat flux is conductively observable as long as the surface is permeable for hydrothermal flow. With decreasing $k_{\phi 0}$ the deviation from the reference model decreases.

3.5. Comparison with observations

It is interesting to compare our model results with observations of the oceanic surface heat flux and bathymetry. We choose a typical model for the comparison and it should be noted that we did not try to optimize our model parameters such as plate thickness or lithospheric thermal conductivity $\lambda_0$ to fit the data. Furthermore this typical model rests on numerous assumptions made in chapter 2, which may be needed to be adjusted if the model is to be applied to particular regions.

3.5.1. Heat flow

In Fig. 15 the observed heat flow data are taken from Stein & Stein (1992) for ages above 2 Ma, and estimated from data given by Stein & Stein (1997) for younger ages. We used these data for comparison with our prediction of the cooling of the oceanic lithosphere including the effect of hydrothermal flow (thin solid and dashed black curves, partly enhanced as bold green curves). Crack aspect ratio and surface permeability are free parameters in our approach, but for the comparison we use $a_t = 0.01$ and $k_{\phi 0} = 8.1 \times 10^{-15}$ m$^2$ for which a reasonable fit is found. A nice overview of the problem with heat flow measurements in the oceans and what the likely effect of hydrothermal cooling might be is given in Pollack et al. (1993) and Hamza et al. (2008).

For better visualization the data and model results are shown both in a log–log and a linear-linear scale (Figs 15a and b, respectively). Their mean values together with their upper and lower standard deviations are indicated as circles, and red and blue curves, respectively. The green curve indicates the heat flux as it can be observed in shallow holes in the ocean floor. As already discussed by Stein & Stein (1994), the data lie below the SRTL, which is clearly observed also in Fig. 15(b) when comparing the circles for ages $< 40$ Ma with our reference curve (bold black curve). However, for ages below 5–10 Ma, the conductive contribution of a hydrothermally cooling lithosphere fits the data better (left segment of the bold green curve enhancing the young part of the black dashed curve). From our model, the total heat loss including the advective contribution (black thin curve) is up to a factor 4 higher than the conductive contribution. For ages $> 5$ Ma it might be more appropriate to assume a sediment cover sealing the hydrothermal convection layer from the ocean (see e.g. Harris & Chapman 2004). In this case the top thermal boundary layer of the convecting layer represents the total heat
flow, and thus the full heat flow curve will be a better approximation (right section of bold green curve). Strictly speaking a different parameterized hydrothermal convection model would have to be used for that case, whose Nusselt number and thus total heat flow will be slightly smaller compared to the model presented here. The transition from the observable conductive to the sediment covered full heat flow curve may lead to an apparent heat flow curve increasing with age over a certain age interval. Such a characteristic increase has been observed, for example, for the Juan de Fuca ridge for ages between 0.8 and 1.6 Ma (Davis et al. 1999; see also discussion by Jaupart & Mareschal (2011)).

3.5.2 Bathymetry

Due to progressive cooling of the oceanic lithosphere as it moves away from the ridge the bathymetry shows a significant dependence on age. Assuming constant rock properties within the lithosphere the subsidence of the seafloor relative to the depth of the ocean ridge can be determined by accounting for thermal contraction (e.g. Turcotte & Schubert 2002) and opening of pore space due to cooling

\[
w = \frac{\rho_w \alpha_l}{\rho_l - \rho_w} \left( T_m h - \int_0^h T(z) \, dz \right) - \frac{\psi}{\rho_m} \int_0^h \psi \, dz,
\]

(42)

where \(\rho_{w}\) and \(\rho_{l}\) are the asthenospheric mantle density and the water density, respectively, \(\alpha_{l}\) is the thermal expansivity of the lithosphere, \(T_{m}\) is the asthenospheric mantle temperature, \(h\) is the thickness of the cooling layer and \(\psi\) is the porosity.

Fig. 16 shows the bathymetry of our models with various crack aspect ratios as it decreases with age relative to the depth of the oceanic ridge. The reference case without hydrothermal cooling shows a subsidence proportional to \(1/\sqrt{t}\) up to ages of 60–80 Ma (cf. Jaupart & Mareschal 2011), while enhanced cooling by hydrothermal convection increases the subsidence considerably and leads to a deviation from the classical \(1/\sqrt{t}\) law. This deviation is qualitatively similar to the deviation discussed for the heat flux.

The model can be compared with bathymetry observations determined for different oceans at different ages. For this purpose we took the ocean depth data as summarized by Jaupart & Mareschal (2011) and subtracted a mean mid-ocean ridge depth of 2.5 km from these data. The resulting subsidence depths below the ridge compare well with the models with some moderate hydrothermal cooling, although the scatter does not allow preferring a specific hydrothermal model.

It should be noted that in the model shown in Fig. 16 we used \(\alpha_l = 3.1 \times 10^{-5} \text{ K}^{-1}\) (Jaupart et al. 2007) as representative of the whole, mostly peridotitic, cooling lithosphere. However, the thermal expansivity \(\alpha_l\) of the shallow part of the hydrothermally convecting lithosphere has been kept at a value typical for gabbroic rocks \((2.4 \times 10^{-5} \text{ K}^{-1})\) as used in the previous chapters; e.g. Richter & Simmons 1974). We think this different choice is reasonable, as \(\alpha_l\) essentially controls the temperature dependence of the permeability (eq. (3)) in the shallow part, while \(\alpha_l\) controls the subsidence due to cooling of the whole lithosphere. For test purpose we ran a model with \(\alpha_l\) increased to the value of \(\alpha_l = 3.1 \times 10^{-5} \text{ K}^{-1}\). This test showed that the temperature dependent closure of crack porosity is slightly increased, and thus the vigour of hydrothermal convection and its effect on enhanced lithospheric cooling is reduced by about roughly 20 per cent.

It might be noteworthy that the commonly used equation for bathymetry does not include the porosity dependent term (eq. (42)). This term stems from the physically consistent opening of the crack pores space upon cooling. In our simple model approach we assumed isotropic porosity. In case of anisotropic crack porosity with predominantly vertical cracks (‘basalt column’—type cracking), the contribution to subsidence will be smaller, possibly approaching the end member case of no horizontal cracks, that is, the last term in eq. (42) would have to be dropped. The corresponding bathymetry curves look similar to those shown in Fig. 16, but are shifted downwards by a few 100 m (not shown). In other words, the differences to the reference case without hydrothermal convection are enhanced in this case.

4 SUMMARY AND DISCUSSION

4.1 Method and its limitations

In this study we first developed a method based on composite theory to describe the pressure and temperature dependence of porosity and related permeability in an elastic medium containing cracks. These cracks are approximated by ellipsoidal pores of uniform aspect ratio. We investigated the pore closure with pressure or depth depending on the porosity at the surface and the aspect ratio of the pores. Although a principal agreement with laboratory experiments is found, natural crack and fracture geometries might be different. We found the sphericity of the pores (or width-to-length ratio of cracks) to be the decisive parameter to control pore closure. For an aspect ratio of 0.1 pores will remain open (in our elastic approach) down to more than 100 km, while for an aspect ratio of 0.01 or less pores are essentially closed at 20 km or shallower depth, respectively. The temperature dependence of porosity is considerably less than the pressure dependence, but at temperatures above a value of about 750 °C complete ductile pore closure is expected and assumed in our approach.

In a second step we derived a scaling law between Nusselt and Rayleigh number for hydrothermal convection based on the results of numerical experiments for a series of Rayleigh numbers.
We found in these experiments that the depth of penetration of hydrothermal convection strongly depends on the vigour of convection (i.e. Rayleigh number) for identical exponential decrease of permeability with depth.

In a third step, based on the pressure and temperature dependence of porosity and the Rayleigh–Nusselt number scaling law, we derived an expression to approximate hydrothermal cooling by an equivalent thermal conductivity, where the depth over which this equivalent thermal conductivity has to be applied is determined by the evolving Rayleigh number. The variations of this equivalent thermal conductivity are much larger than the possible range of rock thermal conductivity depending on composition, temperature and pressure which highlights the greater importance of hydrothermal heat transport compared to possible variations in conductive heat transport with variable rock properties (McKenzie et al. 2005).

It should be emphasized that the approach and the definition of an equivalent thermal conductivity not only consistently models the heat flux due to (implicitly considered) hydrothermal convection, but also predicts the thermal boundary layers within the layer supposed to be effected by hydrothermal convection. However, the thermal gradient within the convection layer is somewhat overestimated, while the average vertical heat flux is captured correctly. We tested our approach quantitatively using a 1-D cooling problem typically for the cooling of the oceanic crust/lithosphere away from the mid-oceanic ridge The strong point of our model is its ability to reproduce the principal temperature evolution in the modelled scenario and to quantitatively assess the effect of hydrothermal cooling in large scale lithospheric models to a high degree of physical consistency, though subject to several simplifying assumptions.

While this approach has a wide range in applicability for the oceanic lithosphere and for crustal accretion by magma intrusion in volcanic centres, it involves a number of limitations. Our approach uses an incompressible Boussinesq fluid. However, pore fluids are neither incompressible nor consist of pure water and large variations in density, specific heat, and viscosity which leads to differences in the convective pattern and the Nu–Ra relationship (e.g. Coumou 1990). A crucial assumption is a porosity–permeability relation. The porosity assumed in our study is generally small, below 0.1. However, as mentioned above, only interconnected pores are addressed in this study. Porosity might be higher due to non-connected pores, but these pores are not related to permeability. The pores considered here have a small aspect ratio (less than 0.1), thus they can be considered as an interconnected crack network.

Crack densities (cf. eq. A1) in natural systems have been inverted from seismic tomography distributions by Adelinet et al. (2011). They found values up to 0.1–0.6 within the upper 6 km of oceanic crust of the Reykjanes ridge, corresponding to a crack porosity of 0.3–1.7 per cent for an aspect ratio of 0.007. If compared to Fig. 3 these values are in good agreement with the experimental data by Fortin et al. (2011) and strengthen our constant aspect ratio model discussed in Section 2.1. It should be emphasized that pore aspect ratio is an important parameter in our study, since narrow cracks close much faster under increasing pressure as wider ones.

As our model implies pore space in the form of fully connected cracks, it is equivalent to a Kozeny–Carman relation with \( n = 3 \) (tortuosity = 1). The Kozeny–Carman approach for porosity–permeability relation is valid for packed spherical particles and can be applied to both, consolidated sediments and crystalline rocks containing cracks (e.g. Sidborn 2007). However, for natural fracture systems such as a layered, highly heterogeneous oceanic crust where volcanic and tectonic structure, off-axis stresses, and alteration exert the real controls on permeability, the Kozeny–Carman relation and thus also our model approach is of approximate validity. The assumptions fail, if grains are strongly non-spherical, if the rock type has a broad grain size distribution or if the material is not consolidated, in this case fluid flow is governed by conduit flow on the grain scale. Also, for very high porosity which is not considered in this study, the model assumption breaks down.

In our analysis we assumed elastic behaviour of the pore walls. This might not be justified for depth greater than 15 km since increasing pressure will lead to stress concentration at pore tips and thus, nonlinear behaviour not considered here. Depending on the depth, brittle fracturing at pore tips may increase/decrease permeability if the pore fluid has a higher/lower pressure than the ambient rock. Ductile deformation of the pore walls will enhance pore closure if pore fluids can escape. To account for ductile behaviour, we considered a brittle/ductile transition temperature of 750 °C which restricted our approach to a depth above 20 km. In case the permeability is dominated by brittle behaviour of fissure or fracture networks different closure conditions have to be formulated. However, our general approach to determine an equivalent thermal conductivity could still be applied if the hydrothermal circulation can be described by porous flow.

When investigating hydrothermal flow two conditions at the surface are possible: open pores (equivalent with an unconfined aquifer) or an impermeable upper boundary (confined aquifer). In the second case a thermal boundary layer will develop at the upper bound. While our approach (eq. 40) addresses both cases, the example shown in Fig. 10 addresses only the case of open pores at the surface. While this is certainly correct for a young oceanic lithosphere it is well known to be questionable for an older lithosphere of age much less than 100 Ma covered by sediments which might act as sealing layers and delay further cooling (e.g. Stein & Stein 1994; Harris & Chapman 2004). A second effect of impermeable surface might also be important. In the presence of a sealing upper boundary, or in general, at greater depth, fluids are no more in a drained condition, and the pore pressure will rise to values between hydrostatic and lithostatic pressure. This is commonly observed in sedimentary basins (e.g. Mann & Mackenzie 1990). In our model such an increase of pore pressure will decrease the effective pressure used to elastically close the pore space and thus impede pore closure with depth. In natural systems this effect may be counterbalanced by dissolution and precipitation in the convecting layer, and cementation and compaction will lead to closure of pore space (Walderhaug et al. 2001). In any case, for an impermeable surface the pressure conditions within the pores are not only controlled by hydrostatic pressure and dynamic pressure of the hydrothermal convection but also affected by additional pressure sources such as surface loads or tectonic stresses. Since these effects have to be specified case by case we did not consider a test example for an impermeable surface.

The applicability of our approach is not restricted to porous convection only, but may also be formulated for solid–liquid systems such as mantle convection containing magma bodies. While solving for liquid convection flow within magma bodies would be prohibitive in a mantle convection code, replacing the magma bodies by regions with appropriately defined high equivalent thermal conductivities based on thermal convection scaling laws derived for magma volumes may consistently predict temperature distributions within and around the magma bodies.

Our approach may be applicable to a multitude of geothermal scenarios as long as one is only interested in the laterally averaged temperature field within the region affected by hydrothermal convection. Such cases include geothermally anomalous regions, temperature fields above magmatic intrusions (e.g. above sills or
within the crust subjected to magmatic underplating), crustal accretion regions such as mid-ocean ridges or continental rift zones, or cooling of the oceanic lithosphere as it moves away from the MOR.

While our predictions on penetration depth and vigour of hydrothermal convection can be applied to all these cases, they are only valid within the limitations discussed above and major controlling parameters such as the chemical composition of the pore fluid are not considered. Therefore, one carefully has to keep in mind the limitations and pitfalls discussed above.

4.2 Applicability to the cooling plate problem

In the application of our method we demonstrate how cooling of the lithosphere can consistently be coupled with parameterized hydrothermal convection. This is an important step forward for understanding the physics of this process, as long as the limitations are kept in mind.

The parameterized hydrothermal convection includes the effect of pore closure with increasing temperature and pressure and addresses in a self-consistent way the penetration depth and vigour of hydrothermal flow. We applied the method to a plate model of oceanic cooling, however, it can easily also be incorporated in any other physical concept of oceanic plate cooling. Possible applications include approaches with more sophisticated thermal boundary conditions such as the CHABLIS model (Doin & Fleitout 1996), with or without small scale convection below the oceanic lithosphere (e.g. Korenaga 2009), mantle convection models with plate like behaviour (e.g. Grigné et al. 2005), or cooling models with variable thermal conductivity (McKenzie et al. 2005).

We compared the results of our improved, but still strongly simplified cooling model to observations of oceanic heat flux and bathymetry and found a confirmative agreement within the scatter of the data. However, the age dependence of the fit or misfit (see e.g. Fig. 16) suggests that a more complex age dependence of hydrothermal cooling may be important. Such age dependence may include changing surface boundary conditions due to sedimentary sealing, lateral variation of permeability due to chemical precipitation–dissolution effects (see e.g. Harris & Chapman 2004; Spinelli et al. 2004), or age dependent variations of transition depth between the shallow region characterized by drained open pore space to the deeper, pressurized pore space region (which has been neglected in the present models altogether).

It is interesting to note that the heat flux—age and the bathymetry—age relations of the hydrothermally assisted lithospheric cooling models show characteristic different deviations from the classical SRTL: while the heat flux curves depict typically bell-shaped curves in a log–log plot, the bathymetry curves do not, at young ages they are already steeper than the SRTL and start flattening at considerably earlier ages.

While our approach has several limitations as discussed in detail above, we emphasize that cooling of the oceanic lithosphere is significantly influenced by hydrothermal flow, and now may be modelled in a consistently coupled way using an equivalent thermal conductivity based on parameterized hydrothermal convection. With this approach several important new effects have been found and are compared with observations:

1) Hydrothermal convection not only leads to a deviation of the total heat flux from the purely conductive lithospheric cooling model (e.g. Stein & Stein 1994) but also to an increase of the total heat loss, with respect to the purely conductive cooling model. Thus, previous estimates of the fraction of hydrothermally removed heat might be too low.

2) Significant deviations from the $1/\sqrt{t}$ law may occur due to the effect of hydrothermal convection: for young to moderately old lithosphere steeper than the $1/\sqrt{t}$ slopes may occur for the heat flow, up to 1/s slopes have been found in our models. For the bathymetry, steeper slopes already occur for very young lithosphere.

3) Due to higher equivalent thermal conductivity in the presence of hydrothermal convection the plate-cooling related flattening of the heat flux and bathymetry curve (typically observed after 80–100 Ma) is shifted to somewhat younger lithosphere.

4) Taken together, in regard to heat flow a hydrothermally assisted cooling plate may be characterized by a logarithmically bell-shaped rather than a flattening $1/\sqrt{t}$ law; for bathymetry flattening is shifted to even younger ages.

5) Comparison of the total heat flow and its conductive contribution with observations confirm previous suggestions (Stein & Stein 1994) that for young lithosphere heat flow measurements represent only the conductive part, while at older ages the total heat flow is observed. Within their scatter and uncertainties the heat flow and bathymetry data are in general agreement with our hydrothermally enforced cooling model suggesting that hydrothermal convection may be important even up to high ages.

The MALAB-code for parameterized hydrothermal cooling of an oceanic lithosphere used to generate Figs 10 and 11 and related figures can be downloaded from the site:

http://user.uni-frankfurt.de/~schmelin/hydroth_cooling_lith

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Here we derive the pressure dependence of porosity under the assumption of a constant crack area, this means the closure of the pores leads to a decreasing crack width, that is, aspect ratio. This is equivalent to keeping the crack density constant. For ellipsoidal cracks the crack density $\varepsilon$ is defined as (e.g. Schmeling 1985)

$$\varepsilon = \frac{3}{4\pi} \frac{\varphi}{a_i}. \quad (A1)$$

If we substitute the aspect ratio in eq. (18) using eq. (A1) and keep $\varepsilon$ constant when integrating eq. (19), the porosity becomes

$$\varphi = \varphi_0 - 1 - \frac{v_0^2}{2} \frac{\bar{\rho} - \rho_f}{1 - \frac{1}{2} v_0} \left[ \frac{4}{3} \cdot \frac{4\pi}{\varepsilon a_i} - 1 \right] P. \quad (A2)$$

The porosity decrease with depth assuming constant crack density is also shown in Fig. 3(a) and the effect on the permeability in Fig. 3(b). It has the same slope at the surface, but drops to zero at finite depths. Comparison with the experimental data (even when using other aspect ratios) suggests that the constant aspect ratio model better describes the porosity decrease with depth, but at greater depth may overestimate the porosity.