Source and processing effects on noise correlations

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Accepted 2014 March 11. Received 2014 March 9; in original form 2014 February 6

SUMMARY

We quantify the effects of spatially heterogeneous noise sources and seismic processing on noise correlation measurements and their sensitivity to Earth structure. Using numerical wavefield simulations and adjoint techniques, we calculate interstation correlations and sensitivity kernels for arbitrarily distributed noise sources where—as in the real Earth—different frequencies are generated in different locations. While both heterogeneous noise sources and processing can have profound effects on noise correlation waveforms, narrow-band traveltime measurements are less affected, in accord with previous analytical studies. Sensitivities to Earth structure depend strongly on the source distribution and the processing scheme, and they reveal exotic frequency dependencies that go beyond the well-known frequency scaling of the Fresnel zone width. Our results indicate that modern full waveform inversion applied to noise correlations is not possible unless one of the following measures is taken: (1) properly account for noise source distribution and processing, or (2) limit measurements to phase or time shifts in narrow frequency bands. Failure to do so can lead to erroneous misfits, tomographic artefacts, and reduced resolution.

Key words: Inverse theory; Tomography; Interferometry; Computational seismology.

1 INTRODUCTION

Following contributions of Shapiro et al. (2005) and Sabra et al. (2005), seismic tomography based on ambient noise correlations has become standard. The method, frequently referred to as ‘noise tomography’, rests on the assumed equality of noise correlations and Green’s functions, from which information on Earth structure can be extracted. This assumption, however, only holds under special conditions, including wavefield diffusivity and equipartitioning (e.g. Lobkis & Weaver 2001; Sánchez-Sesma & Campillo 2006), or the isotropic distribution of both monopole and dipole uncorrelated noise sources (e.g. Wapenaar 2004).

Since none of these conditions is satisfied in the Earth, various processing steps are commonly applied to obtain ‘good’ Green’s function estimates. Examples include one-bit normalization, spectral-whitening, averaging causal and anticausal parts and many others (e.g. Bensen et al. 2007). The choice of processing is to some degree subjective; and it depends on the data and the type of information that one wishes to extract (e.g. surface-wave dispersion or body-wave arrival times). Both the heterogeneous distribution of noise sources and the processing affect the details of noise correlations and their sensitivity to Earth structure.

With the advent of full waveform inversion (FWI) methods (e.g. Tape et al. 2010; Fichtner et al. 2013) and the noise-based monitoring of time-dependent Earth structure (e.g. Brenguier et al. 2008), these details have acquired a new level of relevance. Powerful optimization schemes in FWI can map small waveform differences into spurious Earth structure, and the time dependence of correlations related to noise source variations may be misinterpreted as subsurface changes.

While source effects on noise correlations are well understood (e.g. Tsai 2009; Tromp et al. 2010), the purpose of this paper is to go one step beyond and to quantify the effects of both heterogeneous noise sources and seismic processing on noise correlations and their sensitivity to Earth structure. Given the plethora of data- and application-specific scenarios, we have decided to focus on the physics of the problem, and to describe a limited number of illustrative examples that allow us to draw conclusions that are as general as possible.

2 EFFECTS OF PROCESSING AND HETEROGENEOUS NOISE SOURCES ON MEASUREMENTS

We start our analysis with a description of noise correlation modelling with spatially heterogeneous noise sources where different frequencies are generated in different locations. This will be followed, in Section 2.2, by examples where we compute biases in narrow-band traveltime and time-frequency phase measurements, induced by heterogeneous sources and the optional spectral whitening prior to correlation.
2.1 Noise correlation modelling

To emphasize the physics of the problem, we limit ourselves to
acoustics, that is, to wavefields \( u(x, \omega) \) governed by the acoustic
wave equation

\[
\mathcal{L}u(x, \omega) = -c^2 u(x, \omega) - c^2(x) \Delta u(x, \omega) = N(x, \omega). \tag{1}
\]

In eq. (1), \( \mathcal{L} \), \( c \) and \( N \) denote the wave equation operator, the acoust-
cic wave speed and the noise source distribution, respectively. The
modelling of ambient noise correlations is most easily derived in
the frequency domain, where the representation theorem is given
by

\[
u(x) = \int G(x, \xi) N(\xi) d\xi . \tag{2}
\]

with \( G(x, \xi) \) being the Green’s function with source at position \( \xi \).
 Dependencies of \( u \), \( G \) and \( N \) on the circular frequency \( \omega \)
 are omitted in the interest of a condensed notation. Using (2), the correlation
of recordings at positions \( x_1 \) and \( x_2 \) takes the form

\[
C(x_1, x_2) = u(x_1) u^*(x_2)
= \int \int G(x_1, \xi_1) G^*(x_2, \xi_2) N(\xi_1) N^*(\xi_2) d\xi_1 d\xi_2 , \tag{3}
\]

where \( * \) denotes complex conjugation. Following Woodard (1997),
we compute the expectation \( \langle \cdot \rangle \) of (3), and assume that noise
sources are spatially uncorrelated in the sense that \( \langle N(\xi_1) N^*(\xi_2) \rangle = S(\xi) \delta(\xi_1 - \xi_2) \),
with the power-spectral density (psd) \( S \). Taking into account that Green’s functions do not change over time, the
expected noise correlation now takes the form of a single integral
over space:

\[
C(x_1, x_2) = \langle C(x_1, x_2) \rangle = \int G(x_1, \xi) G^*(x_2, \xi) S(\xi) d\xi . \tag{4}
\]

Comparing (2) and (4), we note that \( C(x_1, x_2) \) can be understood as
a ‘correlation wavefield’

\[
C(x, \xi) = \int G(x, \xi) [G^*(x_2, \xi) S(\xi)] d\xi . \tag{5}
\]
evaluated at position \( x = x_1 \), and excited by the deterministic source
\( G^*(x_2, \xi) S(\xi) \). Eq. (5) provides a recipe for the calculation of expected
correlations at any position \( x \) and a reference station at position \( x_1 \). Illustrations of the correlation wavefield can be found in
the Supporting Information.

2.2 Measurement biases in traveltimes and time–frequency
phase differences

To compute correlations for heterogeneous media and source distri-
butions, we use 2-D finite-differences. As illustrated in Fig. 1,
the computational domain is split into two parts, with velocities
approximating the phase velocity of Rayleigh waves at 10 s pe-
period in oceanic \( (c = 3.5 \text{ km s}^{-1}) \) and continental \( (c = 3.0 \text{ km s}^{-1}) \)
regions, respectively. Seven receivers at equal distance from a ref-
ence station are placed on the continental side. All boundaries are
absorbing.

We perform three types of simulations: (1) To establish a refer-
nce, we compute correlations for a homogeneous source dis-
tribution and a psd with peaks at 0.07 Hz and 0.13 Hz (Fig. 1,
left-hand panel). The correlations are shown in black in Fig. 1(right-
hand panel). (2) To mimic the one-sided illumination typical for
coastal regions, we place two source regions on the oceanic side.
Their psd’s peak at 0.07 and 0.13 Hz, roughly representing pri-
mary and secondary microseisms, respectively. Superimposed is
a weak homogeneous source with a psd as in the previous sce-
nario. The correlations are shown in red in Fig. 1(right-hand panel).
(3) Finally, correlations for spectrally whitened noise are com-
puted by replacing the original psd \( S(\xi) \) by the new effective psd
\( S(\xi)/|\mu(u(x_1))/|u(x_2))| \). Examples are shown in Fig. 2(right-hand panel).

The correlations computed for these scenarios reveal large differ-
ences. To assess their impact on a hypothetical traveltome tomogra-
phy, we compute frequency-dependent cross-correlation traveltim-
e shifts (Luo & Schuster 1991) between the reference correlations and
the correlations for geographically heterogeneous sources. These
traveltim shifts, which should ideally be zero, are a proxy for trav-
eltim biases induced by heterogeneous noise sources. Results for
the frequency bands 0.06–0.08 and 0.14–0.16 Hz are summarized in
Fig. 2 (left-hand panel). Traveltim biases are on the order of \( \pm 1 \) s,
and strongly position- and frequency-dependent. However, the ef-
fect of subjective choices (measuring/averaging causal or anticausal
parts, spectral whitening) is less than 0.1 s for a given station and
frequency band.

While the bias in narrow-band traveltimes is relatively small,
measurements with a higher time resolution—frequently used in
FWI—suffer substantially. These include time-domain waveform
mists (e.g. Brossier et al. 2009) and time–frequency phase differ-
ences (Fichtner et al. 2008, 2013), for which examples are shown in
Furthermore, these biases strongly depend on whether spectral whitening is applied or not. Animations of the correlation wavefields for the previously discussed scenarios can be found in the Supporting Information.

3 EFFECTS OF PROCESSING AND HETEROGENEOUS NOISE SOURCES ON STRUCTURAL SENSITIVITY

To quantify source and processing effects on sensitivity to Earth structure, we compute sensitivity kernels for noise correlation measurements. In Section 3.1, we propose a derivation of noise correlation sensitivities that is more compact than the one provided by Tromp et al. (2010). This will be followed in Section 3.2 by examples of source- and processing-dependent sensitivities for finite-frequency traveltime measurements.

3.1 Computing sensitivity kernels

As a result of the chain rule, we can write the first variation of a measurement \( \chi(C) \) with respect to Earth model parameters \( \mathbf{m}(x) \) as

\[
\delta \chi = \text{Re} \int \delta C(x_1, x_2, \omega) f(\omega) d\omega,
\]

where the specific form of \( f \) depends on the definition of \( \chi \) (see Section 3.2 for examples). Introducing into (6) the first variation of \( C(x_1, x_2, \omega) \) from eq. (4), gives the variation of the measurement \( \chi \) in terms of the variation of Green’s functions:

\[
\delta \chi = \text{Re} \int G(x_1, \xi) \delta G^S(x_2, \xi) S(\xi) f d\xi d\omega
+ \text{Re} \int \delta G(x_1, \xi) G^S(x_2, \xi) S(\xi) f d\xi d\omega. \tag{7}
\]

Denoting by \( G^* \) the adjoint Green’s function, we invoke Green’s theorem (e.g. Hanasoge et al. 2011)

\[
\delta G(x_1, \xi) = - \int G^*(x_1, x_2) \delta C(x_2, \xi) dx_2, \tag{8}
\]

to transform (7) into a sequence of products of forward and adjoint Green’s functions:

\[
\delta \chi = -\text{Re} \int \int \left[ G^*(x, x_2) G(x_1, \xi) \delta C^* G^S(x, \xi) S(\xi) \right] f dx d\xi d\omega.
\]

\[
+ G^*(\xi, x_2) G^S(x_1, \xi) \delta C G(\xi, x) f(\omega) \tag{9}
\]

Isolating integrals over \( \xi \), we identify the correlation fields \( C^*(x, x_1) \) and \( C(x, x_2) \) in (9), allowing us to condense the expression for \( \delta \chi \) to

\[
\delta \chi = -\text{Re} \int u^*(x, x_2) [\delta C(x_1, \xi)] dx d\omega
\]

\[
-\text{Re} \int u^S(x_1, \xi) [\delta C(x_2, \xi)] dx d\omega, \tag{10}
\]

with the adjoint fields \( u^*(x, x_2) \) and \( u^S(x_1, \xi) \) defined as

\[
u^*(x, x_2) = G^*(x_2, x) f^* \quad \text{and} \quad u^S(x_1, \xi) = G^S(x_1, \xi) f. \tag{11}
\]

From eq. (1) we see that the variation of \( \mathcal{L} \) with respect to velocity \( c \) is \( \delta \mathcal{L} = -2c \delta c \Delta \). Substituting this result into (10), gives

\[
\delta \chi = \int K(x) \delta c(x) dx, \tag{12}
\]

with the sensitivity kernel

\[
K(x) = 2c(x) \text{Re} \int \left[ u^S(x, \xi) \Delta C(x, x_1) + u^*(x_1, \xi) \Delta C(x_2, x) \right] dx d\omega. \tag{13}
\]

The calculation of \( K(x) \) requires 4 forward simulations for the Green’s functions \( G(x, x_1) \) and \( G(x, x_2) \), and the correlation fields \( C(x, x_1) \) and \( C(x, x_2) \), plus two adjoint simulations for \( u^S(x_1, \xi) \) and \( u^*(x_2, \xi) \). Examples are given in the following section.

3.2 Source- and processing-dependent sensitivity of finite-frequency traveltimes

We compute sensitivity kernels under the assumption that the noise source is known and that processing—in our examples in the form of spectral whitening—is correctly taken into account. Our measurements are frequency-dependent traveltimes determined by

Figure 2. Left-hand panel: traveltime bias measured by cross-correlation in the frequency bands 0.06–0.08 and 0.14–0.16 Hz for non-whitened (black) and whitened (red) noise. Measurements were made on the causal (circle), anti-causal (star), and the averaged causal/anti-causal (triangle) correlations. Right-hand panel: comparison at stations 2 and 4 of the causal parts of the reference (black), non-whitened (red) and whitened (blue) correlations in terms of time-domain waveforms (top) and time–frequency phase differences (below).
cross-correlation, for which $f$ in eq. (6) is equal to $\text{iooC} / \int \omega^2 |C|^2 \omega d\omega$ (e.g. Luo & Schuster 1991; Dahlen et al. 2000). Fig. 3 shows sensitivity kernels for measurements at frequencies around 0.07 and 0.12 Hz on the causal part of the correlations between the reference station and station 4. Kernels for the heterogeneous source distribution are displayed to the left, and kernels for the heterogeneous source distribution from Fig. 1 are shown in the centre (‘without’ spectral whitening) and to the right (‘with’ spectral whitening).

In addition to the classical Fresnel zone connecting the two receivers, high sensitivity extends far left of the reference station. This feature does not exist for standard source-receiver geometries. A comparison of the different scenarios in Fig. 3 reveals that the shape of the sensitivity kernels is strongly affected by the noise source distribution, as noted already by Tromp et al. (2010). Spectral whitening effectively changes the noise source distribution, and therefore has a strong effect on the sensitivities as well.

4 DISCUSSION AND CONCLUSIONS

We study the effects of heterogeneous noise sources and processing on noise correlation measurements and their sensitivity to Earth structure. In accord with analytical approaches (e.g. Tsai 2009; Froment et al. 2010), we find that traveltimes measured in narrow frequency bands are weakly affected by noise source heterogeneity, with errors of $\sim 1$ s over 500 km distance. The effect of processing (spectral whitening, measuring at positive or negative time lags) on narrow-band traveltimes is practically negligible. This explains the similarity of earthquake and noise tomographies based on similar measurements (e.g. Lin et al. 2008; Shen et al. 2013). Nevertheless, these measurement biases are relevant in monitoring applications.

While narrow-band traveltimes are robust, correlation waveforms are strongly affected by the source distribution and the processing that modifies the effective sources. This is consistent with the work of Halliday & Curtis (2008), Kimman & Trampert (2010) and Cobden et al. (2010). It follows that misfit measures with higher time resolution such as time-domain waveform misfits or time–frequency phase difference (Fig. 2) are hardly meaningful when noise sources and processing are not accounted for correctly. This implies that FWI schemes (e.g. Tape et al. 2010; Fichtner et al. 2013), when applied to noise correlations, should either reduce the exploited information to narrow-band traveltimes, or properly account for all factors contributing to the details of noise correlation waveforms.

As illustrated in Fig. 3, noise source characteristics strongly affect sensitivity to Earth structure. Since processing modifies the effective noise source, it leaves an imprint on structural sensitivity as well, and we illustrated this effect using the example of spectral whitening. Seismic noise at different frequencies is generated in different locations, with considerable variation existing already within the narrow secondary microseism band (Ardhuin et al. 2011). This complexity introduces an additional frequency dependence of sensitivity kernels that goes beyond the well-know frequency scaling of the Fresnel zone width.

Ignoring source and processing effects, will—as in earthquake tomography—slow down the convergence of optimization schemes, and potentially lead to tomographic artefacts (Hanasoge 2014). Correctly modelling the fine structure of sensitivity kernels has the potential to significantly improve tomographic resolution. This potential, which motivates the calculation of finite-frequency kernels, should in the future also be used for seismic tomography based on interstation ambient noise correlations.

ACKNOWLEDGEMENTS

I would like to thank Michael Afanasiev, Moritz Bernauer, Laura Cobden, Laura Ermert, Lucia Gualtieri, Shravan Hanasoge and Victor Tsai for comments and discussions that helped me to improve this manuscript.

REFERENCES


**SUPPORTING INFORMATION**

Additional Supporting Information may be found in the online version of this article:

**Movie S1.** correlation_field_whitened.mp4

**Movie S2.** correlation_field_homogeneous.mp4


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