Cross-well 4-D resistivity tomography localizes the oil–water encroachment front during water flooding

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SUMMARY
The early detection of the oil–water encroachment front is of prime interest during the water flooding of an oil reservoir to maximize the production of oil and to avoid the oil–water encroachment front to come too close to production wells. We propose a new 4-D inversion approach based on the Gauss–Newton approach to invert cross-well resistance data. The goal of this study is to image the position of the oil–water encroachment front in a heterogeneous clayey sand reservoir. This approach is based on explicitly connecting the change of resistivity to the petrophysical properties controlling the position of the front (porosity and permeability) and to the saturation of the water phase through a petrophysical resistivity model accounting for bulk and surface conductivity contributions and saturation. The distributions of the permeability and porosity are also inverted using the time-lapse resistivity data in order to better reconstruct the position of the oil water encroachment front. In our synthetic test case, we get a better position of the front with the by-products of porosity and permeability inferences near the flow trajectory and close to the well. The numerical simulations show that the position of the front is recovered well but the distribution of the recovered porosity and permeability is only fair. A comparison with a commercial code based on a classical Gauss–Newton approach with no information provided by the two-phase flow model fails to recover the position of the front. The new approach could be used for the time-lapse monitoring of various processes in both geothermal fields and oil and gas reservoirs using a combination of geophysical methods.

Key words: Inverse theory; Electrical properties; Hydrogeophysics; Permeability and porosity.

1 INTRODUCTION
Water flooding is a common secondary oil recovery process in which water is injected into the reservoir through injection wells in order to push the oil into a set of production wells. Predicting and monitoring the location of the saturation front is an important task to prevent early water breakthrough and to increase vertical displacement efficiency in an intelligent well design (Jackson et al. 2005). Several geophysical methods like seismic and self-potential have been applied to enhance the accuracy and uniqueness of flood front prediction (Lumley 2001; Jaafar et al. 2009). Karaoulis et al. (2012) proposed recently a joint inversion approach of resistivity and seismic data to retrieve the position of the saturation front and various strategies have been developed recently for the joint inversion of geophysical datasets (see for instance Linde et al. 2006, 2008; Moorckamp et al. 2011; Jardani et al. 2013).

In the two last decades, time-lapse resistivity has been applied to a number of problems in Earth geosciences and medical imaging. Examples include the remediation of contaminant plumes (Johnson et al. 2010), hydrothermal processes (Legaz et al. 2009) and salt tracer tests (Müller et al. 2010), just to cite few examples. Traditionally, resistivity and time-lapse resistivity are performed, and the resulting tomograms are then interpreted for properties of interest such as temperature, salinity, or saturation through petrophysical transforms (e.g. Naudet et al. 2004; Nguyen et al. 2009; Robert et al. 2012). Various spatiotemporal regularization approaches have been proposed in the literature (see Johnson et al. 2010 for a discussion of regularization in space and La Brecque & Yang 2001; Miller et al. 2008; Kim et al. 2009; Karaoulis et al. 2011a,b for time-lapse regularization schemes). Pollock & Ciprka (2012) developed recently a fully coupled approach of time-lapse resistivity for salt tracer tests in the realm of hydrogeophysics (see also Kowalsky et al. 2006 and Jardani et al. 2013). Incorporating additional physics of a given process can help to reduce the non-uniqueness of the inverse problem. This requires however to bridge the gap between (reactive) transport modelling and geophysical observables by putting the petrophysics upfront and not merely down front as done too often in geophysics.
We develop below a new approach in which we couple directly the two-phase flow modelling of the water flooding of the reservoir to the cross-well time-lapse resistivity data. The resulting 4-D inversion algorithm is based on the Gauss–Newton algorithm with Tikhonov regularization directly applied on the stochastically generated permeability and porosity distributions using a petrophysical electrical conductivity model valid for shaly sands.

## 2 INVERSION IN A BAYESIAN FRAMEWORK

We consider a 3-D heterogeneous reservoir in which two wells are located. The oil reservoir is initially mostly saturated by oil. Water is then be injected in a well (injector) and oil pumped in a second well (producer). The oil water encroachment front during water flooding is described by a jump in water saturation. Our approach will be based on maximizing the posterior estimation of water saturation based on the apparent resistivity data. Our primary goal is to choose the prior information of water saturation with the help of a streamline two-phase flow model. Given probability distributions for the permeability, we can compute the probability density of its entries: \( p(X) = p(X_1, \ldots, X_N) \), which gives rise to a probability measure on \( \mathbb{R}^N \) with the Borel algebra as a special case of the \( \sigma \)-algebra (the Borel algebra on \( X \) is the smallest \( \sigma \)-algebra containing all open sets, e.g. Weisstein 2013).

### 2.1 MAP model

We assume that the \( L \times W \times H \) sized oil reservoir can be described by a 3-D grid composed of \( N_{\text{cell}} = N_L \times N_W \times N_H \) cells. A total of \( L \) bipole–bipole dc electrical resistivity measurements, over \( N_L \) electrodes located in each of the Producer and the Injector wells, are simulated. These measurements are repeated for a set of \( N_T \) snapshots (\( T \) time length). We use below the maximum a posteriori (MAP) estimation process to determine the form of the objective function to minimize by maximizing the posterior probability density function related to the permeability and porosity distributions given the electrical potential measurements in the two wells. In other words, we want to maximize the following probability density:

\[
p(k|V_1, \ldots, V_{N_T}) \propto \prod_{i=1}^{N_T} [p(V_i|\sigma_i)p(\sigma_i|S_i)]p(S_1, \ldots, S_{N_T}|k)p(k).
\]

where \( k \) (in \( m^2 \)) denote the heterogeneous intrinsic permeability distribution of the reservoir \( (N_{\text{cell}} \) sized random vector), \( S_i \) (dimensionless) denotes the saturation of the wetting phase namely water at time step \( t \) \( (N_{\text{cell}} \) sized random vector for each snapshot), \( \sigma_i \) (in \( S \ m^{-1} \)) denotes the electrical conductivity distribution at the time step \( t \) \( (N_{\text{cell}} \) sized random vector), \( V_i \) denotes the electrical potential measurements at time step \( t \) \( (L \) sized random vector expressed in Volts, V). In the following, we will note \( \tau \) (in seconds) as the arrival time of water front \( (N_{\text{cell}} \) sized random vector). The electrical conductivity model will be related to the permeability in Section 4.

Note that previous work on time-lapse electrical resistivity tomography (e.g. La Brecque & Yang 2001; Karaoulis et al. 2011a,b) could also be formulated using a MAP estimator by maximizing the probability density associated with saturation given a set of electrical potential measurements. This approach yields

\[
p(S_1, \ldots, S_{N_T}|V_1, \ldots, V_{N_T}) \propto \prod_{i=1}^{N_T} [p(V_i|\sigma_i)p(\sigma_i|S_i)]p(S_1, \ldots, S_{N_T}) \propto \prod_{i=1}^{N_T} [p(V_i|\sigma_i)p(\sigma_i|S_i)] \frac{1}{(2\pi)^{N_{\text{cell}}/2}\sqrt{|\Sigma|}}
\]

\[
\times \exp \left[ -\frac{1}{2} \begin{bmatrix} S_1 \\ \vdots \\ S_{N_T} \end{bmatrix} \Sigma^{-1} \begin{bmatrix} S_1 \\ \vdots \\ S_{N_T} \end{bmatrix} \right].
\]

The main difference between the approach underlined by eq. (2) and the approach developed below is that \( p(S_1, \ldots, S_{N_T}) \) is changed into \( p(S_1, \ldots, S_{N_T}|k)p(k) \) in the approach we plan to follow below. The classical approach simply assumes \( p(S_1, \ldots, S_{N_T}) \) is described by a Gaussian distribution with a simple spatial-temporal covariance structure \( \Sigma \). Our method, on the other hand, computes \( p(S_1, \ldots, S_{N_T}) \) with the help of two-phase flow model \( [p(S_1, \ldots, S_{N_T}|k)] \) and some petrophysical models for the permeability prior distribution [i.e. the probabilities \( p(k) \) associated with the permeability models]. Therefore, our approach incorporates more information into the inversion of the apparent resistivity data. This additional information is used to reduce the non-uniqueness of the inverse problem, especially far from the electrodes.

Another difference is that our approach reduces the unknown variables by \( N_T \) as only the porosity and permeability are inverted. Indeed, the \( N_T \) time steps of water saturations are generated from the inverted distributions of the porosity and the permeability using the two-phase flow model. Given probability distributions for the permeability, we can compute the probability density associated with the saturation. Using the probability density for the saturation, we can build the probability density for the electrical conductivity. Finally, we can compute the probability density for the electrical potential measurements. This sequential Bayesian technique is at the heart of our approach.
2.2 Permeability prior distributions

In order to define a heterogeneous siliciclastic reservoir, we use the petrophysical model of Revil & Cathles (1999), which is valid for clay sand mixtures. This model is a mixture model that can be used to compute the porosity and the permeability given a random distribution for the clay content. We define the clay volume fraction \( \phi_c \) (dimensionless, \( 0 \leq \phi_c \leq 1 \)). The permeability is described by

\[
\kappa = \kappa_{sd} \left[ \phi_{sd} - \phi_c(1 - \phi_{sd}) \right]^\frac{2}{3},
\]

where \( \phi_{sd} \) denotes the porosity of the clean sand end-member (dimensionless) and \( \phi_{sh} \) denotes the porosity of the shale end-member (dimensionless). \( \kappa_{sd} \) denotes the permeability of the clean-sand end-member (taken equal to 1 mD = \( 10^{-15} \) m\(^2\)). The spatial distribution of the volumetric clay content of the sand clay mixture, \( 0 \leq \phi_c \leq 1 \), is assumed to be obtained through a stationary Gaussian process with an anisotropic exponential semi-variogram model:

\[
g(\mathbf{h}) = 1 - \exp \left( - \| \text{Diag}(r)^{-1} \mathbf{h} \| \right),
\]

where \( \mathbf{h} \) denotes the distance between locations and range vector \( r \) usually estimated from well log data. The derived \( N \times N \) covariance matrix given by,

\[
\Sigma_{ij} = (1 - \phi_{sh})^2 \exp \left( - \| \text{Diag}(r)^{-1} (\mathbf{x}_i - \mathbf{x}_j) \| \right).
\]

In eq. (5), \( \mathbf{x}_i \) denotes the spatial coordinate vector for the \( i \)th cell. The prior distribution of the permeability \( \kappa \) is based on the change of variables,

\[
p(\kappa) = \frac{-\phi_{sd}}{(2\pi)^{3/2}\sqrt{\Sigma}} \exp \left\{ -\frac{1}{2} \left[ \frac{\phi_{sd} (\kappa/\kappa_{sd})^{2}}{(\phi_{sd} + \phi_{sh})/2} - \frac{(\phi_{sd} + \phi_{sh})}{2} \right] \Sigma^{-1} \left[ \phi_{sd} (\kappa/\kappa_{sd})^{2} - \frac{(\phi_{sd} + \phi_{sh})}{2} \right] \right\}.
\]

We approximate eq. (6) by the following probability density,

\[
p(\kappa) \approx \frac{1}{(2\pi)^{3/2}\sqrt{\Sigma}} \exp \left\{ -\frac{1}{2} [\kappa - \bar{\kappa}]^T \Sigma^{-1} [\kappa - \bar{\kappa}] \right\},
\]

where \( \bar{\kappa} = \kappa_{sd} [(\phi_{sd} + \phi_{sh})/(2\phi_{sh})]^3 \). Note that eq. (6) has the same mode as eq. (7), and the covariance structures of the two probability densities are very close to each other.

3 STREAMLINE TWO-PHASE FLOW MODEL

The water encroachment is governed by the two-phase flow model which is highly non-linear with respect to saturation. In addition, the flooding time is usually long in duration. There are efforts to approximate this system by means of a streamline asymptotic technique as done for instance by Vasco (2011) and Vasco et al. (2014). In the inversion process, we use the full non-linear two-phase flow model to calibrate the streamline approximation parameters at each Gauss–Newton update step. The two-phase flow governing equations are described for instance in Peaceman (1977). The explicit streamline approximation can be found in Vasco et al. (2014) and is used to compute the Jacobian matrix.

3.1 Incompressible two-phase flow model

We describe now the modelling of water flooding in the heterogeneous aquifer realized by the mixture model developed in the previous section. We consider clayey sand/sandstone (Fig. 2) with oil being the non-wetting pore fluid phase and water being the wetting pore fluid phase. Such type of mixes is known to accurately describe a number of siliciclastic formations as discussed by Revil et al. (2002).

The incompressible two-phase flow field equations ignoring gravity effect are (Peaceman 1977):

\[
\frac{\nabla \cdot \left[ \frac{\kappa(\mathbf{x})}{\mu_w} S(\mathbf{x}) \right]}{\mu_w} \nabla p = \frac{\partial (\rho_w S_w)}{\partial t},
\]

\[
\frac{\nabla \cdot \left[ \frac{\kappa(\mathbf{x})}{\mu_n} S(\mathbf{x}) \right]}{\mu_n} \nabla p = -\frac{\partial (\rho_n S_n)}{\partial t},
\]

where \( S \) denotes the wetting phase saturation (dimensionless), \( p \) the fluid pressure (in Pa), \( \rho_w \) and \( \rho_n \) denote the densities (in kg m\(^{-3}\)) of the wetting and non-wetting phases, \( \mu_w \) and \( \mu_n \) denote the two dynamic fluid viscosities (in Pa s) of the wetting and non-wetting phases, \( \phi \) (dimensionless) denotes the porosity of the reservoir. We use two vertical wells, one water injection well with controlled flow rate \( q_{w, \text{inj}} \) (m\(^3\) s\(^{-1}\)) and one production well with flow controlled by the bottom-hole pressure \( p_{w, \text{prod}} \) (Pa).
Table 1. Definition and value of the parameters used for the simulation.

<table>
<thead>
<tr>
<th>Name</th>
<th>Variable (unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>$L \times W \times H$ (m)</td>
<td>$150 \times 150 \times 200$</td>
</tr>
<tr>
<td>Reservoir cell size</td>
<td>$\Delta x \times \Delta y \times \Delta z$ (m)</td>
<td>$10 \times 10 \times 10$</td>
</tr>
<tr>
<td>Number of reservoir cells</td>
<td>$N_L \times N_M \times N_H$ (–)</td>
<td>$15 \times 15 \times 20$</td>
</tr>
<tr>
<td>Total number of cells</td>
<td>$N_{cell}$</td>
<td>4500</td>
</tr>
<tr>
<td>Time length</td>
<td>$T$ (d)</td>
<td>170</td>
</tr>
<tr>
<td>Time step</td>
<td>$\Delta t$ (d)</td>
<td>11</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>$N_T$</td>
<td>15</td>
</tr>
<tr>
<td>Semi-variogram ranges</td>
<td>$r$ (m)</td>
<td>$[30 \ 60 \ 30]$</td>
</tr>
<tr>
<td>Oil viscosity</td>
<td>$\mu_n$ (Pa s)</td>
<td>$10 \times 10^{-3}$</td>
</tr>
<tr>
<td>Water viscosity</td>
<td>$\mu_w$ (Pa s)</td>
<td>$1 \times 10^{-3}$</td>
</tr>
<tr>
<td>Oil density</td>
<td>$\rho_n$ (kg m$^{-3}$)</td>
<td>700</td>
</tr>
<tr>
<td>Water density</td>
<td>$\rho_w$ (kg m$^{-3}$)</td>
<td>1000</td>
</tr>
<tr>
<td>Grain density</td>
<td>$\rho_g$ (kg m$^{-3}$)</td>
<td>2650</td>
</tr>
<tr>
<td>Water residual saturation</td>
<td>$S_{wr}$ (–)</td>
<td>0.25</td>
</tr>
<tr>
<td>Oil residual saturation after water flooding</td>
<td>$S_{or}$ (–)</td>
<td>0.3</td>
</tr>
<tr>
<td>End points water relative permeability</td>
<td>$k_{rw}$ (–)</td>
<td>1</td>
</tr>
<tr>
<td>End points oil relative permeability</td>
<td>$k_{rn}$ (–)</td>
<td>1</td>
</tr>
<tr>
<td>Pressure producing wells</td>
<td>$p_{p,prd}$ (Pa)</td>
<td>$1 \times 10^5$</td>
</tr>
<tr>
<td>Rate water injection well</td>
<td>$q_{w,inj}$ (m$^3$ s$^{-1}$)</td>
<td>$1.1574 \times 10^{-5}$</td>
</tr>
<tr>
<td>Clean sand end-member porosity</td>
<td>$\phi_{sd}$ (–)</td>
<td>0.4</td>
</tr>
<tr>
<td>Shale end-member porosity</td>
<td>$\phi_{sh}$ (–)</td>
<td>0.6</td>
</tr>
<tr>
<td>Clean sand end-member permeability</td>
<td>$k_{sd}$ (mD)</td>
<td>2000</td>
</tr>
<tr>
<td>Cementation exponent</td>
<td>$m$ (–)</td>
<td>1.83</td>
</tr>
<tr>
<td>Conductivity of the pore water</td>
<td>$\sigma_w$ (S m$^{-1}$)</td>
<td>1.0</td>
</tr>
<tr>
<td>Mobility of the cations</td>
<td>$\beta_S$ (m$^2$ V$^{-1}$ s$^{-1}$)</td>
<td>$5.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>Excess of charge per unit pore volume</td>
<td>CEC (C kg$^{-1}$)</td>
<td>4000</td>
</tr>
<tr>
<td>Saturation exponent</td>
<td>$n$ (–)</td>
<td>2.00</td>
</tr>
<tr>
<td>Logistic shape coefficient</td>
<td>$\alpha$ (–)</td>
<td>$7.9578 \times 10^{-7}$</td>
</tr>
<tr>
<td>Arrival time adjustment coefficient</td>
<td>$a$ (–)</td>
<td>0.7961</td>
</tr>
<tr>
<td>Potential measurement error variance</td>
<td>$\sigma_V$ (V)</td>
<td>0.001</td>
</tr>
</tbody>
</table>

We use a Corey type model for the relative permeabilities (Corey 1954). Therefore, the dimensionless expressions of the relative permeabilities $k_{rw}(S)$ and $k_{rn}(S)$ for the wetting and non-wetting phases are:

$$k_{rw}(S) = \begin{cases} 0 & S \leq S_{wr} \\ k_{rw}^* \left( \frac{S - S_{wr}}{1 - S_{or} - S_{wr}} \right)^2 & S_{wr} < S \leq 1 - S_{or} \\ k_{rw}^* & S > 1 - S_{or} \end{cases}$$

$$k_{rn}(S) = \begin{cases} 0 & 1 - S \leq S_{nr} \\ k_{rn}^* \left( \frac{1 - S - S_{nr}}{1 - S_{or} - S_{nr}} \right)^2 & S_{nr} < 1 - S \leq 1 - S_{or}, \\ k_{rn}^* & 1 - S > 1 - S_{or} \end{cases}$$

where $S_{wr}$ and $S_{or}$ are irreducible/residual saturation of the wetting and non-wetting phases, and $k_{rw}^*$ and $k_{rn}^*$ denote the end points relative permeability (dimensionless) of the wetting and non-wetting phases. All the petrophysical properties that are used for the simulations are reported in Table 1. We simulate this type of two-phase flow model by MATLAB Reservoir Simulation Toolbox (Lie et al. 2012).

3.2 Streamline two-phase flow model approximations

The set of equations given in the previous section is non-linear in the two unknown temporal-spatial function $p(x, t)$ and $S(x, t)$ (fluid pressure and saturation). Intuitively, we think saturation is changing according to a logistic function based on the arrival time of water front,

$$S = S_{wr} + \frac{1 - S_{nr} - S_{or}}{1 + \exp[-\alpha(t - \tau)]},$$

where the front shape parameter $\alpha$ is estimated by comparing with MATLAB Reservoir Simulation Toolbox simulation results as briefly discussed above.
Using an asymptotic technique, we follow the streamline-based approach developed recently by Vasco et al. (2014) to approximate this system of non-linear equations. Arrival time solutions $\tau(x)$ to eqs (8) and (9) can be approximated by,

$$
\tau(x) = \int_{x(s)} \frac{\phi \mu_n}{\kappa(x) \kappa_n(S)} \, dr,
$$

(13)

where $x(s)$ is geodesic connecting point $x$ to injection well locations. We get $x(s)$ by the performing the shortest path algorithm (Dijkstra 1959) on a weighted graph spanned by the natural 6-connected neighbourhood 3-D edges with slowness $\phi \mu_n / [\kappa(x) \kappa_n(S) | \mathcal{P}]$ as weightings. Mean saturation and mean fluid pressure are calculated as

$$
\overline{S}(x) = \frac{1}{N_T} \sum_{t=1}^{N_T} S(x, t),
$$

(14)

$$
\overline{p}(x) = \frac{1}{N_T} \sum_{t=1}^{N_T} p(x, t),
$$

(15)

where $S(x, t)$ and $p(x, t)$ are obtained from the incompressible two-phase flow standard reservoir simulator based on current permeability estimate.

The likelihood of saturation for the $T$ snapshots is defined as:

$$
P[S_1, \ldots, S_N | \kappa(x)] \propto \prod_{t=1}^{N_T} \delta[S_t - \hat{S}_t(k)],
$$

(16)

$$
\hat{S}_t(k) = S_{\text{sat}} + \frac{1 - S_{\text{sat}} - S_{\text{eff}}}{1 + \exp \left(-\alpha \left(t \Delta t - \int_{x(s)} \frac{\phi \mu_n}{\kappa(x) \kappa_n(S)} \, dr \right) \right)},
$$

(17)

where $\delta$ denotes the Dirac distribution. Note that we may have other choices of the likelihood function (e.g. Gaussian distribution with spatial prior covariance) but the use of the Dirac distribution is numerically simpler to compute.

4 DC RESISTIVITY

4.1 Petrophysical conductivity transform

In isothermal conditions, electrical conductivity depends on saturation, salinity, clay content and clay mineralogy, and porosity (Waxman & Smits 1968; Revil et al. 1998). In this section, we consider clayey sands or clayey sandstones that are water-wet. The electrical conductivity $\sigma$ (in S m$^{-1}$) of the porous material as a function of the water saturation can be written using the following petrophysical transform based on the work of Waxman & Smits (1968) (see also Jougnot et al. 2010; Revil 2013),

$$
\sigma = f(S) = \frac{1}{F} S^t \left[ \sigma_w + \rho_n \left( \frac{1 - \phi}{\phi} \right) \beta_S \frac{\text{CEC}}{S} \right],
$$

(18)

where $S$ denotes water saturation, $n$ is called the saturation exponent (Archie 1942), $F$ (dimensionless) denotes the formation factor $F = \phi^{-m}$ (Archie 1942), $m (>1$, dimensionless) is called the porosity exponent, $\sigma_w$ denotes the conductivity of the pore water (in S m$^{-1}$), $\beta_S$ denotes the apparent mobility of the cations within the electrical double layer that are responsible for surface conductivity (m$^3$V$^{-1}$s$^{-1}$), and CEC (C kg$^{-1}$) denotes the Cation Exchange Capacity of the material. The CEC denotes the total amount of exchangeable charge located on the mineral surface and that need to be compensated in then Stern and diffuse layers around the grains. This quantity serves as a proxy to describe the properties of the electrical double layer (see Leroy & Revil 2009). The saturation dependence in eq. (18) is consistent with the volume averaging approach developed by Revil (2013) and discussed in details in Revil et al. (2014a,b). The normalized CEC per unit pore volume (in C m$^{-3}$) is written as $\bar{Q}_v = \rho_n (1 - \phi) \text{CEC}/\phi$.

The likelihood of the electrical conductivity is therefore defined as:

$$
P(\sigma_1 | S_1) \propto \delta \left[ \sigma_1 - f(S) \right],
$$

(19)

where $\delta$ denotes the Dirac (delta) function and $\hat{f}(S_t)$ is defined by eq. (18) above. Here again, other choices of the likelihood function are possible (e.g. Gaussian distribution with spatial prior covariance) but the use of the Dirac function is numerically simpler to compute and this is why we use this choice in the inverse modelling approach developed below.

In the following, we also assume that the injected water is at the same salinity as the connate water so changes in electrical conductivity are driven by changes in saturation only. In reality, a change in resistivity could be driven both by changes in salinity and saturation and the approach developed below will need to be extended.
4.2 Electrical potential model

The 3-D potential field due to a known dc current injection \( I \) (in Ampères) is related to the conductivity structure via a 3-D Poisson equation for the electric potential (e.g. Johnson et al. 2010),

\[
- \nabla \cdot [\sigma(x) \nabla V(x)] = I \delta(x - x_s),
\]

(20)

where the point \( x_s \) denotes a source current injection point where a current of magnitude \( I \) (in A) is injected (\( I > 0 \)) or retrieved (\( I < 0 \)). In eq. (20), the electric potential \( V \) (in V) is the electrical potential field in the space domain (\( E = -\nabla V \) represents the quasi-static electrical field in V m\(^{-1}\), \( (\nabla \times E) = 0 \) in the low-frequency limit of the Maxwell equations), \( \sigma(x) = 1/\rho(x) \) denotes the electrical conductivity of the porous sediment (in S m\(^{-1}\)), \( \rho(x) \) denotes its resistivity (in Ohm m) and \( \delta \) represents the delta function. By expressing eq. (20) using the finite volume approach, we have,

\[
[D \cdot S(\sigma_i) \cdot \mathbf{G}] \cdot \mathbf{u}_i = \mathbf{q}_i,
\]

(21)

where \( \mathbf{u}_i \) and \( \mathbf{q}_i \) are each \( N_{\text{cell}} \times 1 \) and represent an electrical potential vector and source vector at \( i \)th measurement and time \( t \), respectively. The matrices \( \mathbf{D} \) and \( \mathbf{G} \) denote the divergence and gradient operator matrix written in finite difference, respectively. The matrix \( \mathbf{S} \) denotes the diagonal matrix with conductivity at flow time \( t \) as the diagonal elements. The electrical potential at the \( i \)th measurement at snapshot \( T \) at cell \( i \), \( V_{i/T} \), can be expressed as:

\[
\hat{V}_{i/T} = \mathbf{I}_i \cdot \mathbf{u}_i = \mathbf{I}_i \cdot (D \cdot S(\sigma_i) \cdot \mathbf{G})^{-1} \cdot \mathbf{q}_i,
\]

(22)

where \( \mathbf{I}_i \) is a \( 1 \times N_{\text{cell}} \) vector with one in the location of \( i \)th electrodes and zero elsewhere. Here we use RESINVM3D as the implementation of both forward and inverse electrical potential model. We also considered boundary condition corrections already implemented in RESINVM3D for better modelling effects outside region of interest.

The likelihood of electrical potential is therefore defined as:

\[
p(V_i|\sigma_i) = \frac{1}{(2\pi)^{L/2} \sigma_{\text{cell}}^{L \times M}} \exp \left(-\frac{1}{2} \frac{V_i - \hat{V}_i(\sigma_i)}{\sigma_V} \right)^2,
\]

(23)

where the \( L \times 1 \) vector \( \hat{V}_i(\sigma_i) \) is defined by eq. (22), and \( \sigma_V \) denotes the uniform measurement error variance estimated from the recorded data.

5 GAUSS–NEWTON METHOD FOR THE MAP ESTIMATOR

5.1 Posterior probability density

Based on eqs (7), (16), (19) and (23), we can express the probability density of \( p(\kappa|V_1, \ldots, V_T) \) as,

\[
p(\kappa|V_1, \ldots, V_T) \propto \prod_{t=0}^{T} \iint \left[ \exp \left(-\frac{\sigma_V}{2} \left\| V_t - \hat{V}_t(\sigma_t) \right\|^2 \right] \delta \left[ \sigma_t - \hat{\sigma}(S_t) \right] \delta \left[ S_t - \hat{S}(\kappa) \right] \right. \text{d}\sigma_t \text{d}S_t \exp \left(-\frac{1}{2} [\kappa - \hat{\kappa}]^T \Sigma^{-1} [\kappa - \hat{\kappa}] \right)
\]

\[
\exp \left\{ -\frac{\sigma_V}{2} \left\| V_t - \hat{V}_t(\hat{\sigma}(\hat{S}(\kappa))) \right\|^2 - \frac{1}{2} [\kappa - \hat{\kappa}]^T \Sigma^{-1} [\kappa - \hat{\kappa}] \right\}. \]

(24)

5.2 Gauss–Newton method

Maximizing eq. (24) is equivalent to minimizing

\[
G(\kappa) = \sum_{t=1}^{N_T} \left\| V_t - \hat{V}_t(\hat{\sigma}(\hat{S}(\kappa))) \right\|^2 + [\kappa - \hat{\kappa}]^T \Sigma^{-1} [\kappa - \hat{\kappa}].
\]

(25)

The first term of \( G \) is the data misfit and the latter two terms are spatial regularization functions for \( \kappa \). We consider \( \hat{G}_t(\kappa) = \hat{V}_t(\hat{\sigma}(\hat{S}(\kappa))) \). According to the Gauss–Newton algorithm, the variables could be updated as:

\[
\hat{\kappa}^{k+1} = \hat{\kappa}^k + (J^T J + \Sigma^{-1})^{-1} J^T \begin{bmatrix} V_1 - \hat{G}_1(\kappa) \\ \vdots \\ V_{N_T} - \hat{G}_{N_T}(\kappa) \end{bmatrix},
\]

(26)
where $J$ denotes the Jacobian matrix for the electrical conductivity problem with respect to the porosity and permeability. The matrix $J$ is defined as:

$$J = \begin{bmatrix}
\frac{\partial \hat{G}_{ij}}{\partial \tau} \\
\frac{\partial \hat{f}_{ij}}{\partial \tau} \\
\vdots \\
\frac{\partial \hat{S}_{ij}}{\partial \tau}
\end{bmatrix}. \quad (27)$$

The sensitivity of the $j$th electrode-node in the $l$th measured at flow time $t$ corresponding to the change of permeability at $j$th cell $\left(\frac{\partial \hat{G}_{ij}}{\partial k_j}\right)$ could be expressed as:

$$\frac{\partial \hat{G}_{ij}}{\partial k_j} = \sum_k \frac{\partial \hat{V}_{li}}{\partial \sigma_{ti}} \frac{\partial \hat{f}_{ij}}{\partial \sigma_{ti}} \frac{\partial \hat{S}_{ij}}{\partial k_j}, \quad (28)$$

respectively. According to eq. (18), the derivative $\frac{\partial \hat{f}_{ij}}{\partial \sigma_{ti}}$ is explicitly given by,

$$\frac{\partial \hat{f}_{ij}}{\partial \sigma_{ti}} = \frac{n}{\phi_{ui} S_{li}} [\sigma_u + n\beta_S \left(\frac{\sigma}{S_{li}}\right) - \frac{n}{\phi_{ui}} S_{li} \left(\beta_S \left(\frac{\sigma}{S_{li}}\right)\right)], \quad (29)$$

The derivative $\frac{\partial \hat{S}_{ij}}{\partial k_j}$ is calculated by differentiating eq. (17) as

$$\frac{\partial \hat{S}_{ij}}{\partial k_j} = \begin{cases} 
\sigma (1 - S_{ui} - S_{wi}) \exp \left(-\alpha \left(t \Delta t - \sum_{k \in PT_i} \frac{\phi_{u_k}}{\sigma \cdot \phi_{u_k}} [\sigma \cdot \phi_{u_k}] \right) \right) \left(\frac{\phi_{u_k}}{\sigma} \right), & j \in PT_i, \\
1 + \exp \left(-\alpha \left(t \Delta t - \sum_{k \in PT_i} \frac{\phi_{u_k}}{\sigma \cdot \phi_{u_k}} [\sigma \cdot \phi_{u_k}] \right) \right), & j \notin PT_i,
\end{cases} \quad (30)$$

where the geodesics $PT_i$ is shortest path connecting the injection wells $Well_{inj}$ and cell $i$.

The derivative $\frac{\partial \hat{V}_{li}}{\partial \sigma_{ti}}$ is calculated by assuming electrical potential $u_{iT}$ of eq. (21) is also a function of the conductivity model, and taking the derivatives of eq. (21) with respect to $\sigma_{iT}$ we obtain:

$$\frac{\partial [(D \cdot S(\sigma_T) \cdot G) \cdot u_{iT}(\sigma_T)]}{\partial \sigma_{iT}} = 0. \quad (31)$$

So the derivative of the potential vector could be expressed as:

$$\frac{\partial u_{iT}}{\partial \sigma_{iT}} = [D \cdot S(\sigma_T) \cdot G]^{-1} D \cdot \frac{\partial S(\sigma_T)}{\partial \sigma_{iT}} \cdot G \cdot u_{iT}. \quad (32)$$

Finally, the sensitivity of the $j$th electrode-node in the $l$th measurement at time $t$ corresponding to the change of conductivity at $j$th cell, $\frac{\partial \hat{V}_{li}}{\partial \sigma_{jt}}$, can be expressed as:

$$\frac{\partial \hat{V}_{li}}{\partial \sigma_{jt}} = I_t \cdot \frac{\partial u_{lj}(\sigma_t)}{\partial \sigma_{jt}} = I_t \cdot (D \cdot S(\sigma_t) \cdot G)^{-1} D \cdot \frac{\partial S(\sigma_t)}{\partial \sigma_{jt}} \cdot G \cdot (D \cdot S(\sigma_t) \cdot G)^{-1} \cdot q_{lj}, \quad (33)$$

where $I_t$ denotes a $1 \times N$ vector with one in the location of $l$th electrodes and zero elsewhere.

Rather than explicitly calculating the inverse of the eqs (26) and (33), we use the preconditioned conjugate gradient algorithm to solve it as a linear equation for each data mismatch vector and current source vector. The flow chart for the inversion of the electrical potential data is shown in Fig. 1.

### 6 APPLICATION TO A SYNTHETIC EXAMPLE

We implemented a MATLAB-based Gauss–Newton solver with conjugate gradient algorithm to update the resistivity model. A Bayesian prior with more weighting emphasis placed on the cross-borehole plane is used for the spatial regularization function. We assume that borehole logs are also used to constrain permeability (see for instance Rabaute et al. 2003) and so these two parameters are perfectly constrained along the two wells. We first test our algorithm on a synthetic fixed impulse variation dataset to ensure that everything works, and we then apply the time-lapse algorithm to a water-flood and secondary oil recovery experiment in which water is injected in one well and the oil is produced in a second well (dipole test configuration for which the external boundary conditions have a much smaller effect on the flow field than for a single pumping test).

We generated the reservoir porosity and permeability using the petrophysical model of Revil & Cathles (1999) for clay sand mixes (see Fig. 2). The heterogeneous and isotropic spatial distribution of the volumetric clay content of the sand clay mixture, $\phi_{uc}$, is generated with
Figure 1. Flow chart used for the inversion process. This chart shows how the porosity, the permeability and the saturation are used to update the electrical conductivity model.

Figure 2. Sand clay mixture model used to build the heterogeneous reservoir. The clayey sand domain is characterized by clay contents ranging from zero to the porosity of the clean-sand end-member. The clayey sand domain is used to determine the porosity and the permeability of the heterogeneous reservoir in our simulations.

the SGeMS library (see Stanford University, Stanford Geostatistical Earth Modelling Software, http://sgems.sourceforge.net/). We used the following semi-variogram:

\[ \gamma(x, y, z) = 1 - \exp \left( -3 \left( \frac{x^2}{30^2} + \frac{y^2}{60^2} + \frac{z^2}{30^2} \right) \right) \]  

(34)

We create a 150 m × 150 m × 200 m reservoir (grid size 10 m × 10 m × 10 m) based on this method. The true permeability distribution as well as the inverted permeability distribution are shown in Fig. 3. The producer and two injector wells are located on each corner of the block shown in Fig. 3.

Our forward modelling approach is based on TOUGH2 (Pruess et al. 1999). It is used to the two-phase flow problem over 170 d and to model the saturation front propagation during this time interval. Then, we pick \( T = 15 \) snapshots (11 days between snapshots) for which resistivity synthetic acquisitions are simulated. Note that the length of simulation is kept before breakthrough to make the two-phase flow equation valid through the entire process. We consider 18 electrodes with a 10 m electrode spacing located in both the injection well and two production wells (54 electrodes in total). We simulate 1420 bipole–bipole measurements for each of the 15 time steps. The true saturation and resistivity distributions for the 5 out of 15 snapshots are shown in Figs 4 and 5, respectively.

We invert the 3-D distribution of the permeability and porosity based on the 1420 × 15 = 21 300 electrical potential measurements. These measurements have been contaminated with an additive white Gaussian noise of homogeneous variance equal to 1 mV. The inverted permeability distribution between the two wells is shown in Fig. 3. The inverted distribution of the porosity and permeability can be used to compute the distribution of the saturation for each snapshot and the distribution of the resistivity can then be inferred from the calculated saturation and porosity distributions. A comparison between the true and inverted saturation distributions can be found in Fig. 4 while a comparison between the true and inverted resistivity distributions can be found in Fig. 5.
Encroachment front localization with ERT

Figure 3. Permeability distribution used for the reservoir modelling and inverted permeability distribution. The producing wells are located on two left-hand side corners of the block while the injector is located on the right-hand side of the block. The computation is based on the algorithm described in Fig. 1 and the petrophysical model for clay–sand mixtures described in Fig. 2.

Figure 4. Comparison between the true saturation distribution and the inverted saturation distribution. Note that the position of the front is pretty well-recovered through the inversion of the voltages.

The data misfit is calculated by,

\[
\text{Data misfit} = \frac{\sum_{i,j,t} [V_{i,j,t} - \hat{G}_{i,j}^t(\kappa)]^2}{\sum_{i,j,t} V_{i,j,t}^2},
\]

where \( V_{i,j,t} \) is electrical potential measured at \( i \)th measurement, \( j \)th electrode and time \( t \), and where \( \kappa \) denote the inverted permeability and porosity distributions for the recovered model. Fig. 6 shows the eight iterations where our algorithm has converged. The decrease in the root mean square error is smaller than 88 per cent.

From these results, we can see that the position of the saturation front is well recovered. Its dynamics can therefore be monitored using cross-well resistivity tomography and the approach developed above. That said, the permeability distributions are not very well recovered, showing that the sensitivity of resistivity to saturation is higher than to porosity and permeability.

In Fig. 5, we compare the results from the true resistivity distribution with the results of a commercial software (RES3DINV) and our inversion algorithm. The classical approach based on smoothness-constrained least-squares inversion technique (Sasaki 1994; Loke & Barker 1996) and has been recently used for various hydrogeophysical problems (Dahlin et al. 2002; Rucker et al. 2010). RES3DINV is able to see where the lowest and highest resistivity structures are located but no more. For a cross-well tomography, this approach is not able to locate
Figure 5. Comparison between the true resistivity model, the result from RES3DINV, and the approach developed in this paper. (a) Time-series of the true resistivity distribution for the five snapshots. (b) Inverted resistivity from the approach derived in the main text. (c) Time-series of the inverted resistivity distribution for the five snapshots from the commercial software RES3DINV.

Figure 6. Data model misfit error versus the iteration number (see eq. 35). The algorithm stops when the difference between two iterations is below a specified value.

the oil water encroachment front. This is showing how incorporating additional information in the inverse problem regarding the two phase flow problem strongly reduces the non-uniqueness of the inverse problem and offers a breakthrough with respect to conventional methods such as the classical Gauss–Newton inversion with isotropic smoothness.

7 DISCUSSION

The approach developed above can easily be adapted to other geophysical methods including time-lapse seismic, gravity, seismoelectric and self-potential data. All these methods have shown some sensitivity to the detection of saturation fronts (e.g. Jackson et al. 2005; Karaoulis et al. 2014; Revil et al. 2014a). Since all these methods have distinct sensitivity maps with respect to the distribution of the heterogeneities, they would probably increase a lot the resolution of the final permeability tomogram. In addition, we can use a geophysical method that is directly sensitive to the permeability distribution, namely spectral induced polarization. Indeed, it has been shown recently that spectral
induced polarization can be used to determine the pore size distribution (Revil et al. 2014b) and permeability (Binley et al. 2005; Revil & Florsch 2010; Revil 2012, 2013; Revil et al. 2012). This approach could be also combined with the image-guided inversion technique recently introduced by Zhou et al. (2014), which consists in introducing structural constraints in the inverse problem, in order to improve the resolution of the method with respect to permeability inversion.

8 CONCLUSION

We have developed a novel 4-D inversion approach to better image the position of the oil/water encroachment front in a heterogeneous siliciclastic reservoir by combining an approximated version of two-phase flow model with the electric resistivity tomography. The two-phase flow model gives us a better weight of the time-lapse electric resistivity tomography data over the time and spatial domains. The new approach predicts the position of the encroachment front, and also gives an inference, to some degree, of the permeability of the reservoir. This work could be further improved by combining different types of geophysical measurements (especially electromagnetic, seismolectric, self-potential and gravity data) to perform a joint inversion of these data with reservoir data inside the Bayesian framework introduced in this paper. The same type of approach could be used in geothermal systems to monitor change in saturation and temperature or for the sequestration of CO$_2$ in sedimentary or basaltic formations. Our model currently works for (1) shallow reservoir or light oil, (2) stable injection influx, (3) small reservoir pressure and temperature change case and (4) negligible capillary pressure. We plan to relax these assumptions and to apply our model to field data in future works.

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