Frequency of gaps observed in a structurally aligned protein pair database suggests a simple gap penalty function

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ABSTRACT

Gap penalty is an important component of the scoring scheme that is needed when searching for homologous proteins and for accurate alignment of protein sequences. Most homology search and sequence alignment algorithms employ a heuristic ‘affine gap penalty’ scheme \( q + r \times n \), in which \( q \) is the penalty for opening a gap, \( r \) the penalty for extending it and \( n \) the gap length. In order to devise a more rational scoring scheme, we examined the pattern of gaps that occur in a database of structurally aligned protein domain pairs. We find that the logarithm of the frequency of gaps varies linearly with the length of the gap, but with a break at a gap of length 3, and is well approximated by two linear regression lines with \( R^2 \) values of 1.0 and 0.99. The bilinear behavior is retained when gaps are categorized by secondary structures of the two residues flanking the gap. Similar results were obtained when another, totally independent, structurally aligned protein pair database was used. These results suggest a modification of the affine gap penalty function.

INTRODUCTION

The gap penalty is also important for the sensitivity (ability to find remotely related sequences) of the search tool and the accuracy of the alignment. Most sequence homology detection algorithms employ the ‘affine gap penalty’ scheme of the form \( (q + r \times n) \), where \( n \) is the length of the gap and \( q \) and \( r \) are empirically chosen parameters representing the cost of opening and extending a gap, respectively. The affine gap penalty scheme recognizes that it costs to open a gap as well as to extend an existing one and has proved to be superior to length proportional gap costs, which often produce a large number of short insertions or deletions (7). However, no firm theoretical or experimental support has been presented for such a gap penalty scheme. It has also been criticized for over-penalizing long gaps (8). Several methods for improving this gap penalty function have been proposed (7–10).

There have been several attempts to deduce a gap penalty function by examining patterns of insertions–deletions (indels) in aligned sequences (11–13). The assumption used in these studies is that a linear relationship exists between the gap penalty and the logarithm of the probability of gap lengths. Under such an assumption, an exponential distribution of gap lengths will give rise to an affine gap penalty scheme. Benner et al. (11) examined a database of aligned protein sequence pairs that they constructed and concluded that the frequency distribution of gap lengths observed in this database was best described by a power law. However, they aligned protein sequences using dynamic programming methods with explicit gap penalty functions, thus introducing a certain circularity to the analysis. More recently, Qian and Goldstein (13) examined gaps that occur in structurally aligned proteins. Using the FSSP database (14), they fitted the probability distribution of gap lengths to a complex quadruple exponential function. There have been several other efforts to derive gap penalty functions by examining insertions and deletions that occur in human gene–pseudogene pairs (12,15). However, it is likely that the constraints placed on the formation of gaps in functional proteins are different from neutral pseudogenes, and thus gap penalty functions derived from the latter may not be applicable for detecting homologous proteins.

Here, we report the results of an examination of the pattern of gaps that occur in a database of structurally aligned protein domain pairs constructed from an all-against-all pairwise structural alignment of 3992 SCOP (16) protein domains of

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low sequence homology. We find that the distributions of the logarithm of the probability of gaps, as a function of gap length, exhibit a bilinear behavior with a break at a gap of length 3. The slope of the regression line at long gap lengths (>3) is much smaller than that for the short gap lengths (≤3). The statistical support for the bilinear regression is very strong, with $R^2$ values of 1.0 and 0.99 for the first and the second linear segments, respectively. Assuming a linear relation between the gap penalty function and the logarithm of the probability of observing a gap of given length, these results suggest a modification of the affine gap penalty scheme.

**MATERIALS AND METHODS**

**Generation of a structurally homologous protein domain pair (SHoPP) database**

*Pairwise structural superposition of protein domains.* A non-redundant set of 3992 protein domains with <40% sequence identity to each other, which excluded structures determined by NMR, were selected from the ASTRAL SCOP v1.59 database (17). These domains were subjected to an all-against-all pairwise structural superposition using the structure comparison program SHEBA (18) in order to generate a structurally superposed domain pair set. The computations were performed on the NIH Biowulf cluster (a Beowulf parallel processing system), and a total of 7 966 036 domain pairs were structurally superposed by this method.

*Selection of structurally homologous domain pairs.* Members of the structurally homologous protein domain pair database (SHoPP) were selected from the superposed domain pair set using the following criteria: (i) $m > 40$; (ii) $m \geq 0.6 \times$ number of residues in the larger domain of the domain pair; (iii) the root-mean-square deviation of superposed residues $\leq$2.0 Å. These criteria are based on the number of superposed residue pairs $m$. Residue pairs were considered to be superposed if the distance between the respective alpha carbons was $<3.5$ Å after superposition of the domains by SHEBA. These criteria were developed, in part, by manual examination of a sample of superposed domain pairs.

The structurally homologous protein domain pair database (SHoPP) consists of 9806 domain pairs that satisfied the above criteria. Domain pairs selected in this manner also had $z$ scores $>3$ with respect to each domain, when the $z$ score was defined as

$$z = \frac{m - \langle m \rangle}{\sigma}$$

where $m$ is divided by the number of residues in the larger domain, $\langle m \rangle$ is the mean of $m_i$ over all pairs involving the given domain and $\sigma$ is the standard deviation of $m_i$ over all pairs involving the given domain.

**Classification of gaps**

The structural alignment of two protein domains also yields a concomitant sequence alignment containing regions of aligned residues separated by gaps. The gaps that occurred in the SHoPP database were classified in terms of the secondary structure of the residues flanking the gap as follows.

**Definitions of secondary structure.** Secondary structure assignments were made using the program DSSP (19). H, I and G were classified as helical (H), B and E as strand (S) and S and T as coil (C). The secondary structure of the residues immediately before (ginit) and immediately after (gend) the gap. Using the simplified three-state classification of secondary structure, gaps were divided into five types: (i) gaps occurring within a helix (ginit = gend = H); (ii) gaps occurring within a strand (ginit = gend = S); (iii) gaps occurring in a coil region (ginit = gend = C); (iv) gaps at the edge of a helix (ginit = H, gend = C or ginit = C, gend = H); (v) gaps at the edge of a strand (ginit = S, gend = C or ginit = C, gend = S).

Gaps that occur at the junction of a helix and a strand (ginit = H, gend = S or ginit = S, gend = H) were ignored because there were too few cases where a helix and a sheet were directly adjacent to each other.

**Probability of a gap**

All gaps were classified in terms of gap type (as described above) and the length of the gap. The probabilities were computed by converting raw frequencies into normalized frequencies. For example, the probability of a gap of length $n$ occurring within a helix (where the secondary structure of the residues at positions $g_{init} = H$ and $g_{end} = H$) was computed as follows:

$$Pr(H[-]_nH|HH) = \frac{\sum_{\text{domain} \_ \text{pairs}} H[-]_nH}{\sum_{x=0,1,2} \sum_{\text{domain} \_ \text{pairs}} H[-]_xH}$$

where

$$\sum_{\text{domain} \_ \text{pairs}} H[-]_nH$$

refers to the number of gaps of length $n$ between residues of secondary structure type $H$ and $H$ in the SHoPP database. Probabilities for other types of gaps were computed in an analogous manner.

**RESULTS**

**Structurally homologous protein domain pair database (SHoPP)**

The ASTRAL SCOP v1.59 (17) database contained 3992 protein domains with <40% sequence identity among them. An all-against-all structure comparison of these protein domains, using the structure comparison program SHEBA (18), resulted in 9806 domain pairs that were judged to be structurally highly homologous to each other according to the criteria detailed in Materials and Methods. The SHoPP database consists of these protein domain pairs.
Analysis of probability distribution curves

The numbers of gaps observed in the SHoPP database, categorized by length and the secondary structure of the residues flanking the gap, are given in Table 1. The total numbers of residue pairs in the coil, helix and strand regions were roughly the same (last row of Table 1). However, as expected, more gaps occurred between coil residues than between residues of any other secondary structural types. For gaps of length >2, the frequency of gaps within a coil region is >10 times that of gaps of similar length within a helical or strand region. This result is consistent with the expectation that the relatively unstructured coil regions are more likely to tolerate insertions than regions of regular secondary structure such as helices and strands.

The probability of a gap of given length occurring in a given secondary structural region was obtained by dividing the number of gaps of that length and secondary structural type by the total number of residue pairs of the same secondary structural type (equation 1). The negative logarithms of these probabilities, plotted as a function of the gap length for each of the five secondary structural types, are given in Figure 1. Interestingly, each type exhibits a similar trend: a rapid linear increase for gaps of length <3, followed by another linear, but more gradual, increase. Each plot is well approximated by two linear regression lines given by

\[
g(n) = q_1 + r_1 \times n \quad (n < 3) \\
q_2 + r_2 \times n \quad (n \geq 3)
\]

where \(g(n)\) is the negative of the logarithm to base 2 of the probability of a gap of length \(n\). The best-fitting values for the parameters \(q\) and \(r\) are given in Table 2. The values for the square of the correlation coefficient \((R^2)\) are given in Table 3.

For all types, the gradient \(r_1\) of the first regression line is larger than the gradient \(r_2\) of the second regression line by a factor of 3–4 (Table 3). The combined data (Fig. 2A) exhibited similar behavior, and could also be well approximated by two linear regression lines with \(R^2\) values of 1.0 and 0.99, indicating almost perfect linear regression.

Table 1. Frequency of gaps found in SHoPP database by length and by gap type

<table>
<thead>
<tr>
<th>Gap length</th>
<th>Gap type</th>
<th>Within coil</th>
<th>Within strand</th>
<th>Within helix</th>
<th>Edge of strand</th>
<th>Edge of helix</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>493,602</td>
<td>576,334</td>
<td>499,274</td>
<td>189,739</td>
<td>96,990</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>25,587</td>
<td>79,788</td>
<td>439,600</td>
<td>650,000</td>
<td>452,900</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>909,392</td>
<td>1,166,615</td>
<td>1,515,400</td>
<td>1,250,000</td>
<td>1,292,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>322,424</td>
<td>314,332</td>
<td>332,451</td>
<td>335,335</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2,290</td>
<td>162,191</td>
<td>175,136</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1,721</td>
<td>70,109</td>
<td>196,106</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>976</td>
<td>54,72</td>
<td>112,46</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>617</td>
<td>33,47</td>
<td>67,40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>417</td>
<td>34,54</td>
<td>38,46</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>394</td>
<td>14,32</td>
<td>34,23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>255</td>
<td>8,31</td>
<td>30,19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>538,176</td>
<td>586,167</td>
<td>506,053</td>
<td>198,592</td>
<td>103,562</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)Gaps of length >10 were not considered for analysis, as they were judged to be too few in number for some gap types.

\(^{b}\)This is the number of residue pairs without a gap, for each gap type.

The shape of the probability distribution curve for gaps is independent of structural database

In order to verify that the observations made above are not a property of the particular database used, we computed the gap
probability distributions using another database, DAPS, which was constructed independently by Mallick et al. (20). The SHoPP and DAPS databases differed from each other in the definition of protein domains (17,21), computer programs used for the structural alignment of domains (18,22) and criteria used for selection of domain pairs to the database (see Materials and Methods). SHoPP, with 9806 domain pairs, was considerably smaller than DAPS, which had 34 778 pairs. All SHoPP entries had a sequence identity \( \leq 40\% \) and a root-mean-square deviation (RMSD) \( \leq 2.0 \) Å. By contrast, DAPS had 1592 entries with a sequence identity of 100\%, and 1717 domain pairs with RMSD \( > 5 \) Å. Nevertheless, the probability distribution curves obtained for the two databases are very similar (Fig. 2); each exhibits a strong bilinear character, with a break at a gap of length 3 and a ratio of 3–4 between the gradients of the first and second regression lines. For a subset of DAPS satisfying some of the selection criteria of pairwise sequence identity \( < 40\% \) and RMSD \( < 2.0 \) Å, the differences in the values of the coefficients for the two regression lines in Figure 2 were significantly reduced such that the two curves were nearly superimposed (inset to Fig. 2B).

### DISCUSSION

We have found that the distribution of the negative logarithm of the probability of gaps, by length, observed in the SHoPP database could be well represented by a bilinear function with the breakpoint at the gap of length 3 (equation 2 and Fig. 2). The values of the parameters of equation 2 are given in Table 2. The bilinear equation is an excellent representation of the data as can be seen from the values of the square of the correlation coefficient, \( R^2 \), which are 1.0 and 0.99, respectively, for the first and second regression lines (Table 3). Remarkably, similar bilinear behavior, with a break at a gap of length 3, is observed for each gap type when gaps are categorized by the secondary structural environment (within coil, within helix, within strand etc.) (Fig. 1).

It is not clear why the gap-length distribution is bilinear with a break at 3. This is not a property of the SHoPP database alone, since the same basic behavior is observed in the DAPS and apparently also in the FSSP (see below) databases, which are all entirely independent of each other. Since the bilinear behavior is observed for all secondary structural environments, the mechanism that produces this behavior is operating outside the constraints of the secondary structure of the protein. There are many mechanisms that will cause an insertion and/or a deletion in a protein sequence (23,24). A linear regression with gap length is expected if each gap (of unit length) is introduced independently (12). However, the reason for the break in the regression line at length 3 is unclear at present.

The gap-length distribution in aligned protein sequence databases has been studied before (11,13). Benner et al. (11) used protein pairs that were aligned by sequence homology using dynamic programming and a gap penalty function. They concluded that the gap-length distribution followed a power law rather than the exponential behavior that we observe. The difference may arise from the different methods used to obtain the alignments. Qian and Goldstein (13) studied gaps that

### Table 3. Correlation coefficients for the first (\( R_1 \), gap lengths 1–3) and the second (\( R_2 \), gap lengths \( \geq 3 \)) regression lines and the ratio of slopes (\( r_1/r_2 \)) of the two regression lines for the logarithm of the probability versus length for gaps found in SHoPP and DAPS databases

<table>
<thead>
<tr>
<th>Gap type</th>
<th>SHoPP</th>
<th>DAPS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_1^2 )</td>
<td>( R_2^2 )</td>
</tr>
<tr>
<td>Within coil</td>
<td>1.0</td>
<td>0.98</td>
</tr>
<tr>
<td>Within strand</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>Within helix</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>Edge of strand</td>
<td>1.0</td>
<td>0.92</td>
</tr>
<tr>
<td>Edge of helix</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>Combined</td>
<td>1.0</td>
<td>0.99(^d)</td>
</tr>
</tbody>
</table>

\(^a\)For the regression lines in Figures 1 and 2A.
\(^b\)Corresponding regression lines are not shown except for the combined category, which is shown in Figure 2B.
\(^c\)Pooled data in which categorization of gaps by secondary structure is ignored.
\(^d\)When gaps of length \( \leq 15 \) were considered, the \( R_2^2 \) values (gaps of length 3–15) for the combined data were 0.98 and 0.99 for the SHoPP and DAPS databases, respectively.
Although the logarithm of the probability distribution of gap lengths for all gap types exhibits the same bilinear behavior as noted above, the parameters describing the regression lines are clearly a function of the secondary structure of residues flanking the gap (Table 2). This division of gaps into just two categories, based on the presence/absence of a coil residue flanking the gap (26), is an oversimplification when modeling gap penalty functions where the structure of the query sequence is known (or predicted). Such secondary structure-dependent gap penalty functions could be useful in cases wherein protein sequences are aligned to a known protein structure.

When considering a non-affine gap penalty scheme, computational cost can be an issue (8). The affine gap penalty, as well as the bilinear gap penalty described in this work, belong to the class of convex (also called concave) gap penalties, where \( g(k+1) - g(k) \leq g(k) - g(k-1) \) and \( K \) is the gap length. Waterman and Beyer (27) showed that the computational complexity for aligning two sequences of lengths \( n \) and \( m \) using a convex gap penalty was, at most, \( O(nm(m+n)) \). This was later improved to \( O(nm[\log(m)]) \) by Miller and Myers (25), who also showed that, for a piecewise linear convex gap penalty made from \( K \) straight lines, the computational complexity is reduced to \( O[nm\log(K)] \). Since the bilinear gap penalty presented here is a special case for \( K = 2 \), the computational complexity is \( O(nm) \), the same as for the affine gap penalty. This is a major advantage of the bilinear form of the gap penalty, which is not shared by a multi-exponential form like that suggested by the studies of Qian and Goldstein (13).

### References