Anisotropic Bayesian linearized stochastic seismic inversion with multi-parameter decoupling

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Abstract

Shale oil reservoir emerges as a significant unconventional energy source, commonly predicted by anisotropic seismic inversion. Considering the intricate nature of shale oil reservoirs, it becomes imperative to consider uncertainties during anisotropic inversion. An effective approach to address this involves stochastic inversion, specifically the anisotropic Bayesian linearized inversion (ABLI), which characterizes statistical and spatial correlations of subsurface parameters through a crucial multivariate correlation matrix constructed through geostatistics. However, an inevitable challenge in stochastic inversion arises from interference during the calibration of statistical and spatial correlations of subsurface parameters. This challenge becomes particularly pronounced in anisotropic
inversion, heightened by the multitude of involved model parameters. Existing decorrelation approaches primarily address statistical correlation, neglecting the impact of spatial correlation. To tackle this issue, a novel multi-parameter decoupling strategy is proposed, formulating decoupling anisotropic Bayesian linearized inversion (D-ABLI). D-ABLI introduces an advanced decorrelation approach, and uses principal component analysis (PCA) to simultaneously eliminate impact of statistical and spatial correlations on ABLI. The decoupling enhances the inversion accuracy of model parameters in ABLI, particularly for density and anisotropic parameters. The theoretical underpinnings of the decoupling strategy are demonstrated to be reasonable, and the effectiveness of D-ABLI is proved through a theoretical data test and a field data test regarding shale oil reservoirs. The D-ABLI results offer the capability to estimate fracture density accurately and unveil the distribution of shale oil.

**Keywords:** Stochastic inversion, shale oil reservoir, correlation decoupling, anisotropic inversion

### 1. Introduction

The depletion of conventional oil and gas reserves has posed challenges to the advancement of human society in recent years. Researchers have turned attention to unconventional reservoirs to addressing this dilemma (Clarkson *et al.* 2018), where shale oil and gas have attracted growing global attention (Davis *et al.* 2018; Liu *et al.* 2018). Numerous oil fields have considerable potential for shale oil development, leading to a significant rise in the demand for shale oil reservoir characterization and sweet spot prediction (Li *et al.* 2017). Geophysical methodologies play crucial roles in characterization of shale-oil reservoir, with seismic inversion methods, in particular, capable of revealing the fine distribution characteristics of underground shale oil reservoirs (Zhu *et al.* 2011; Hu *et al.* 2023; Sun *et al.* 2024).

Typically, shale oil reservoirs demonstrate the development of horizontal thin interbedded structures and horizontal fractures, displaying characteristics of vertical transverse isotropy (VTI)
anisotropic medium (Xue et al. 2017; Rao & Wang 2019; Luo et al. 2022). Consequently, the characterization of anisotropy and fracture density is pivotal in the geophysical exploration of shale oil reservoirs, and these can be delineated by the features of elastic and anisotropic parameters in VTI media. The estimation of these parameters is achieved through anisotropic seismic inversion, wherein the relationship with the amplitude variation of prestack seismograms is leveraged. A key aspect involves constructing distinct reflection coefficient equations for different types of reservoirs (Ursin & Haugen 1996; Rüger 1997; Sayers & Rickett 1997; Rüger 2002). Under the assumptions of weak impedance contrast and weak anisotropy, Ursin & Haugen (1996) derive linearized approximation equations for the reflection and transmission coefficients of VTI equivalent media. Rüger (1997) propose a classical reflection coefficient approximation equation for P-wave at large big incident angles in VTI media, which has found widespread use in anisotropic inversion.

The reflection coefficient equations discussed above serve as the bridge between observed seismic data and parameters associated with shale reservoir characteristics. Seismic inversion serves as the tool that converts seismograms into elastic and rock-physical parameters (Bosch 2010). This process can be broadly categorized into two strategies: stochastic inversion and deterministic inversion (Zhang et al. 2012; Pereira et al. 2019). Deterministic inversion, a strategy that utilizes optimization algorithms, can derive an optimal solution for subsurface parameters from seismic data (Zhi & Gu 2018; Luo et al. 2020; Gao et al. 2023). While deterministic inversion is highly efficient and widely employed in field reservoir characterization, it commonly overlooks inversion uncertainties stemming from various sources, such as ambiguities in seismic data acquisition and processing (Junhwan et al. 2022). In reality, recognizing and addressing inversion uncertainty is crucial for ensuring accuracy and evaluating the risk of seismic exploration (Liu et al. 2022; Ma et al. 2022). Therefore, it is essential to simultaneously estimate inversion results and assess corresponding uncertainties, a task conveniently accomplished through stochastic inversion strategies (Athens & Caers 2022; Hagemann et al. 2022).
The stochastic inversion approach, rooted in statistics, found its initial application in seismic exploration in the late 20th century through a sequential geostatistical inversion (GSI) (Hass & Dubrule 2022). The fundamental concept behind GSI involves exploring a set of potential solutions derived from a probability density function (PDF) constructed by geostatistics using Monte Carlo (MC) methods (Lang & Grana 2017; Nunes et al. 2017). With the evolution of oil field development, GSI has demonstrated the capability to achieve high-resolution inversion results by integrating seismic data with densely distributed drilled wells (Azevedo et al. 2015; Shi et al. 2024). However, GSI encounters a substantial computational burden due to the utilization of MC algorithms (Nunes et al. 2019; Azevedo 2022). Bayesian linearized inversion (BLI) stands as another effective stochastic inversion approach (Buland & Omre 2003a; Yu et al. 2024). It directly estimates the posterior PDF of subsurface parameters without relying on optimization algorithms (Lang & Grana 2018; Yu et al. 2020). BLI has experienced rapid development over the past two decades, giving rise to various linearized stochastic inversion methods (Grana 2014). For instance, Buland & Omre (2003b; 2003c) employ BLI to estimate wavelets, noise level and amplitude-variation-with-offset (AVO) parameters within the BLI framework; Hansen et al. (2006) integrate BLI theorem with geostatistics, proposing a linearized geostatistical inversion. Grana et al. (2017) employ Gaussian mixture distribution in BLI, achieving simultaneous inversion of elastic parameters and lithology. Nevertheless, BLI still suffers from the issue of huge core matrix inverse even it has superiority than GSI in the aspect of efficiency. The challenge can be mitigated to some extent through specific dimension reduction strategies.

In summary, anisotropic stochastic inversion, which focuses on estimating anisotropic parameters and fracture density while evaluating their uncertainty through stochastic inversion, emerges as a promising tool for characterizing shale-oil reservoirs. Anisotropic inversion methods are widely applied in characterizing reservoirs with diverse anisotropies (Zhang & Li 2016; Zhang et al. 2017; Zhang 2017; Rao & Wang 2019). Most anisotropic inversion cases employ deterministic methods,
emphasizing optimal solutions (Gao et al. 2018; Pan et al. 2018; Gao et al. 2020). By comparison, BLI combined with anisotropic forward seismic model can achieve anisotropy estimation and uncertainty evaluation efficiently, and can be defined as anisotropic Bayesian linearized inversion (ABLI). However, a noteworthy challenge in ABLI is the interference among different parameters, impacting inversion accuracy. This interference largely stems from the statistical correlation between distinct subsurface parameters. Certain existing researches commonly use principal component analysis (PCA) to address this problem in deterministic AVO inversion (Pan et al. 2018). However, stochastic inversion not only needs to consider the statistical correlation between model parameters but also must characterize the spatial correlation of objective parameters at different subsurface locations (Shi et al. 2024). Spatial correlation, typically represented by the variogram in geostatistics, exerts a significant influence on stochastic inversion results whether in BLI or GSI. However, spatial correlation is susceptible to various factors, such as the imprecise calibration of variograms from well-log data (Yu et al. 2021), leading to a loss of inversion accuracy. To address the challenges posed by spatial correlation, Yu et al. (2021) propose a spatial decorrelation method based on the traditional statistical decorrelation in AVO inversion. Cao et al. (2023) apply the spatial decorrelation method in seismic-petrophysics stochastic inversion, achieving satisfactory inversion results for subsurface petrophysical parameters.

While ABLI mentioned above stands as an excellent anisotropic inversion method, there is a notable lack of research addressing statistical and spatial correlation within ABLI. The complexity of model parameters, encompassing five to six types of model parameters, exacerbates the issue of spatial correlation in anisotropic inversion compared to traditional AVO inversion. To tackle this challenge, a novel decoupling strategy for statistical and spatial correlation is introduced in ABLI, giving rise to a decoupling anisotropic Bayesian linearized inversion method (D-ABLI). This innovative approach, integrating the reflection coefficient equation in VTI media, achieves accurate anisotropic inversion through the decoupling of multiple anisotropic parameters. D-ABLI
significantly contributes to the prospecting of shale-oil reservoirs.

The remainder of this paper is structured as follows: In theory part, first, the reflection coefficient and forward seismic model of VTI medium is introduced. Then, the novel decoupling strategy for the model parameters in anisotropic inversion is introduced, and the decoupling prior PDF of model parameters is also derived. Finally, the formulation of D-ABLI is presented by combining the anisotropic forward modelling and the decoupling prior PDF. In the section of data tests, a synthetic case is first used to discuss the effectiveness of D-ABLI, and then a field case from a shale-oil reservoir is used to further validate the impact of D-ABLI.

2. Theory

2.1 Reflection coefficient and forward seismic model of VTI medium

The reflection coefficient formula in VTI media used herein is

$$R_{pp}^{VTI} = R_{pp}^{iso} + R_{pp}^{ani},$$

(1)

where $R_{pp}^{iso}$ and $R_{pp}^{ani}$ represent the isotropic and anisotropic terms in reflection coefficient, with their respective expressions being:

$$R_{pp}^{iso} = \frac{1}{2 \cos^2 \theta} \Delta v_P \frac{\Delta v_P}{v_P} - 4 \gamma^2 \sin^2 \theta \frac{\Delta v_S}{v_S} + \frac{1}{2} \left( 1 - 4 \gamma^2 \sin^2 \theta \right) \frac{\Delta \rho}{\rho},$$

(2)

$$R_{pp}^{ani} = \frac{1}{2} \Delta \delta \sin^2 \theta + \frac{1}{2} \Delta \varepsilon \sin^2 \theta \tan^2 \theta,$$

(3)

where $v_P$, $v_S$ and $\rho$ represent P-wave velocity, S-wave velocity and density; $\theta$ denotes the mean of the longitudinal wave incidence angle and transmission angle; $v_P$, $v_S$ and $\rho$ are the mean values of the three parameters on either side of the reflection layer; $\gamma$ is the ratio of $v_P$ and $v_S$; $\delta$ and $\varepsilon$ denote anisotropy parameters. Especially, $\varepsilon$ has high correlation with the fracture density in shale oil reservoirs. The precise determination of $\varepsilon$ significantly enhances the characterization of shale oil reservoirs. Equation (2) is the Aki-Richards approximation formula of the Zoeppritz equation (Yu et
\( v_p, v_s, \rho, \delta, \epsilon \) are objective parameters of D-ABLI. Under the condition of small incidence angles, the isotropic reflection coefficient, as delineated in Equation (2), can be simplified into the following expression.

\[
R_{vp}^{iso} = \frac{1}{2 \cos^2 \theta} \Delta \ln(v_p^2) - 4 \gamma^2 \sin^2 \theta \Delta \ln(v_s^2) + \frac{1}{2} (1 - 4 \gamma^2 \sin^2 \theta) \Delta \ln(\rho),
\]

Finally, the reflection coefficient shown in equation (1) can be further expressed as

\[
R_{vp}^{VTI} = \frac{1}{2 \cos^2 \theta} \Delta \ln(v_p^2) - 4 \gamma^2 \sin^2 \theta \Delta \ln(v_s^2) + \frac{1}{2} (1 - 4 \gamma^2 \sin^2 \theta) \Delta \ln(\rho)
+ \frac{1}{2} \Delta \delta \sin^2 \theta + \frac{1}{2} \Delta \epsilon \sin^2 \theta \tan^2 \theta,
\]

The equations presented above depict the reflection coefficient for a single point in space. When considering one or multiple trace, the model parameter can be expressed as

\[
m = \begin{bmatrix} v_p & v_s & \rho & \delta & \epsilon \end{bmatrix}^T.
\]

Meanwhile, considering different incident angles, the reflection coefficient vector for a single or multiple traces can be expressed as

\[
\mathbf{R}(\mathbf{m}) = \mathbf{K} \mathbf{D} \mathbf{m},
\]

Where \( \mathbf{D} \) is a difference matrix, and the expression of \( \mathbf{K} \) is

\[
\mathbf{K} = \begin{pmatrix}
\mathbf{A}(\theta_1) & \mathbf{B}(\theta_1) & \mathbf{E}(\theta_1) & \mathbf{X}(\theta_1) & \mathbf{Y}(\theta_1) \\
\mathbf{A}(\theta_2) & \mathbf{B}(\theta_2) & \mathbf{E}(\theta_2) & \mathbf{X}(\theta_2) & \mathbf{Y}(\theta_2) \\
... & ... & ... & ... & ... \\
\mathbf{A}(\theta_M) & \mathbf{B}(\theta_M) & \mathbf{E}(\theta_M) & \mathbf{X}(\theta_M) & \mathbf{Y}(\theta_M)
\end{pmatrix},
\]

where the submatrices \( \mathbf{A}, \mathbf{B}, \mathbf{E}, \mathbf{X}, \) and \( \mathbf{Y} \) are all diagonal matrices with similar expressions. \( \mathbf{A} \) is taken as an example, and its expression can be given by

\[
\mathbf{A}(\theta_i) = \begin{pmatrix}
A(\theta_i, t_1) & ... & 0 \\
... & ... & ... \\
0 & ... & A(\theta_i, t_N)
\end{pmatrix},
\]

where \( \theta_i \) is the \( i^{th} \) incident angle of the prestack seismic data. \( N \) is the sampling point number of the model parameters. The expressions of \( \mathbf{B}, \mathbf{E}, \mathbf{X}, \) and \( \mathbf{Y} \) can also be expressed as the manner shown in equation (8). Taking the example of the \( i^{th} \) incidence angle and \( j^{th} \) time sample point, the elements of
each submatrix can be expressed as:

\[ A(\theta_i, t_j) = \frac{1}{2 \cos^2 \theta_i}, \]  
(9)

\[ B(\theta_i, t_j) = 4 \gamma_j^2 \sin^2 \theta_i, \]  
(10)

\[ E(\theta_i, t_j) = \frac{1}{2} \left( 1 - 4 \gamma_j^2 \sin^2 \theta_i \right), \]  
(11)

\[ X(\theta_i, t_j) = \frac{1}{2} \sin^2 \theta_i, \]  
(12)

\[ Y(\theta_i, t_j) = \frac{1}{2} \sin^2 \theta_i \tan^2 \theta_i. \]  
(13)

Finally, the seismogram of the P-P wave can be calculated through the convolution between the wavelet \( W \) and the reflection coefficient \( R \) in VTI media

\[ d = WR + e = WKDm + e = Gm + e, \]  
(14)

where \( G = WKD \) is the forward operator.

**2.2 Decoupled multivariate prior probability distribution of model parameter**

Generally, seismic inversion methods operate under the assumption that model parameters adhere to a specific probability distribution. In conventional deterministic AVO inversion, it is typical to assume that model parameters conform to either a trivariate Gaussian or Cauchy distribution. A trivariate covariance matrix is commonly employed to express the statistical correlation among various AVO parameters. However, the spatial correlation of parameters among different sampling points is non-negligible in multi-parameter stochastic inversions, such as AVO inversion and anisotropic inversion. For stochastic inversion, a multivariate Gaussian PDF is commonly used to simultaneously characterize the statistical and spatial correlation of model parameters

\[ m \sim N(\mu_m, \Sigma_m), \]  
(15)

where \( \mu_m \) is the prior mean of parameter \( m \), and \( \Sigma_m \) represents the multivariate correlation matrix, depicting both statistical and spatial correlation of \( m \). For anisotropic inversion in VTI media
\(\Sigma_m\) can be expressed as

\[
\Sigma_m = \begin{bmatrix}
\Sigma_{pp} & \Sigma_{ps} & \Sigma_{ps} & \Sigma_{pc} \\
\Sigma_{ps} & \Sigma_{ss} & \Sigma_{ps} & \Sigma_{sc} \\
\Sigma_{ps} & \Sigma_{ps} & \Sigma_{ss} & \Sigma_{pc} \\
\Sigma_{pc} & \Sigma_{sc} & \Sigma_{pe} & \Sigma_{ee}
\end{bmatrix}_{5N \times 5N},
\]

whose size is \(5N \times 5N\). \(\Sigma_m\) includes 25 submatrices of size \(N \times N\), and each submatrix represents the statistical and spatial correlation of the parameters denoted by its subscript. These submatrices in traditional BLI are estimated by geostatistics. \(\Sigma_{ps}\) is taken as an example, and its expression is

\[
\Sigma_{ps} = \begin{bmatrix}
\gamma_{ps} & \gamma_{ps} - \gamma_{12} & \ldots & \gamma_{ps} - \gamma_{ij} \\
\gamma_{ps} - \gamma_{21} & \gamma_{ps} & \ldots & \gamma_{ps} - \gamma_{2j} \\
\ldots & \ldots & \ldots & \ldots \\
\gamma_{ps} - \gamma_{ij} & \gamma_{ps} - \gamma_{12} & \ldots & \gamma_{ps}
\end{bmatrix}_{N \times N},
\]

where \(\gamma_{ps}\) represents the sill value of the cross-variogram between \(p\) and \(s\), and it’s also the maximum of matrix \(\Sigma_{ps}\). \(\gamma_{ij}\) is the cross-variogram value with the point distance of \(|i - j|\). As is known that variogram in geostatistics can characterize the spatial correlation of subsurface parameters. Since \(\Sigma_{ps}\) is constructed by a cross-variogram, it can depict the statistical correlation for various model parameters and spatial correlation of parameters across different inversion nodes. In geostatistical inversion and simulation methods, spatial correlation holds substantial importance as it governs the resolution and distribution of the inverted and simulated models.

As is observed in equation (16) that the calculation of the 25 submatrices is necessary to construct \(\Sigma_m\). Due to the symmetry of the matrix, in reality, it is only necessary to construct 15 matrices from the variogram and cross-variogram of the model parameters. However, the calibration of variogram, typically relying on well-log data, is a rough process that usually results in noticeable errors. The calibration error leads to inaccurate statistical and spatial correlation between the five model parameters, further reducing the accuracy of Bayesian stochastic inversion.
An effective method to address the aforementioned issue is the correlation decoupling of these parameters, which can circumvent the estimation of statistical and spatial correlation. Similar to the existing decorrelation method in traditional AVO inversion, the decoupling strategy herein is also based on the PCA. The first step is calculating the covariance for the above five parameters,

$$
C_e = \begin{bmatrix}
c_{pp} & c_{ps} & c_{ps} & c_{p\delta} & c_{pc} \\
c_{ps} & c_{ss} & c_{ps} & c_{p\delta} & c_{sc} \\
c_{ps} & c_{ps} & c_{ss} & c_{p\delta} & c_{ps} \\
c_{p\delta} & c_{ps} & c_{p\delta} & c_{ss} & c_{p\delta} \\
c_{pc} & c_{sc} & c_{ps} & c_{p\delta} & c_{sc} 
\end{bmatrix}_{5 \times 5}
$$

(18)

where the diagonal values of matrix $C_e$ represent their variances, while the off-diagonal elements are the covariances, indicating statistical correlations between different parameters. Singular value decomposition (SVD) is conducted on $C_e$

$$
C_e = u \begin{bmatrix}
\lambda_1 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & 0 \\
0 & 0 & 0 & 0 & \lambda_5 
\end{bmatrix} u^T, \quad u = \begin{bmatrix}
u_{11} & u_{12} & u_{13} & u_{14} & u_{15} \\
u_{21} & u_{22} & u_{23} & u_{24} & u_{25} \\
u_{31} & u_{32} & u_{33} & u_{34} & u_{35} \\
u_{41} & u_{42} & u_{43} & u_{44} & u_{45} \\
u_{51} & u_{52} & u_{53} & u_{54} & u_{55} 
\end{bmatrix},
$$

(19)

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and $\lambda_5$ are singular value of this covariance matrix. A big transform operator is obtained by $u$

$$
U = \begin{bmatrix}
U_{11} & U_{12} & U_{13} & U_{14} & U_{15} \\
U_{21} & U_{22} & U_{23} & U_{24} & U_{25} \\
U_{31} & U_{32} & U_{33} & U_{34} & U_{35} \\
U_{41} & U_{42} & U_{43} & U_{44} & U_{45} \\
U_{51} & U_{52} & U_{53} & U_{54} & U_{55}
\end{bmatrix}_{5N \times 5N}, \quad U_y = \begin{bmatrix}
u_{ij} & 0 & \ldots & 0 \\
0 & \nu_{ij} & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & \nu_{ij}
\end{bmatrix}_{N \times N},
$$

(20)

where $i, j \in [1, 5]$. $UU^T = I$, and $I$ is an unit matrix, namely $U^T = U^{-1}$. Then, the decoupled subsurface parameters should be expressed as

$$
m' = U^T m = (v'_p, v'_s, p', \delta', \varepsilon').
$$

(21)

According to statistical theory, the covariance matrix of $m'$ should be
i.e., the covariances of $v_p$, $v_i$, $\rho$, $\delta$, $\varepsilon$ are zeroed, and the corresponding correlation regarding these parameters is eliminated. The process mentioned above is actually the traditional decorrelation method used in AVO inversion, and it only addresses the correlation illustrated by $C_s$. However, Bayesian linearized stochastic inversion must consider both the statistical and spatial correlation. Therefore, a multivariate decoupling strategy is further proposed based on the traditional decorrelation, formulating the following multivariate correlation matrix

$$
C' = \begin{bmatrix}
    c'_{pp} & c'_{ps} & c'_{ps} & c'_{ps} & c'_{ps} \\
    c'_{ps} & c'_{ss} & c'_{sp} & c'_{sp} & c'_{sp} \\
    c'_{ps} & c'_{sp} & c'_{pp} & c'_{pp} & c'_{pp} \\
    c'_{p\delta} & c'_{p\delta} & c'_{p\delta} & c'_{p\delta} & c'_{p\delta} \\
    c'_{p\delta} & c'_{p\delta} & c'_{p\delta} & c'_{p\delta} & c'_{p\delta}
\end{bmatrix} = \begin{bmatrix}
    \lambda_1 & 0 & 0 & 0 & 0 \\
    0 & \lambda_2 & 0 & 0 & 0 \\
    0 & 0 & \lambda_3 & 0 & 0 \\
    0 & 0 & 0 & \lambda_4 & 0 \\
    0 & 0 & 0 & 0 & \lambda_5
\end{bmatrix},
$$

(22)

A set of novel in-equations is employed herein to derive a multivariate decoupling strategy, which can be expressed as,

$$
c_{ps} \geq \gamma_{ps}, \quad c_{ps} \geq \gamma_{ps}, \\
c_{pp} \geq \gamma_{pp}, \quad c_{ic} \geq \gamma_{ic}, \\
c_{p\delta} \geq \gamma_{p\delta}, \quad c_{p\delta} \geq \gamma_{p\delta}, \\
c_{pi} \geq \gamma_{pi}, \quad c_{pi} \geq \gamma_{pi}, \\
c_{sp} \geq \gamma_{sp}, \quad c_{ic} \geq \gamma_{ic}
$$

(24)

namely the covariance is no less than the sill of cross-variogram for all the model parameters. The above relationship is the basic of D-ABLI, and its detailed derivation can be found in Yu et al (2021). Actually, the relationship is proposed under two assumptions. Firstly, the range values of these cross-variograms are significantly smaller than the number of sampling nodes. Secondly, there is a consistent positive correlation for $v_p$, $v_i$, $\rho$, $\delta$, $\varepsilon$. Universally, one inversion trace includes hundreds even thousands of sampling nodes, while the length of the variogram range in ABLI is commonly 10–30. From this point of view, the first assumption is reasonable. However, the second assumption may not always be reasonable in anisotropic inversion. To address the potential issue in anisotropic inversion where the correlation between any two model parameters is not initially
positive, certain specific transformations can be employed to ensure a positive correlation. As mentioned after equation (17), the \( \gamma'_{ps} \) should be the maximum of \( \Sigma'_{ps} \), and this relationship are apparently suitable for the other parameters, not only \( \nu_p \) and \( \nu' \). Therefore, as per equation (20), the values of submatrices away from the diagonal in equation (23) should be all zero. Therefore, the final expression of \( \Sigma'_m \) should be

\[
\Sigma'_m = \begin{bmatrix}
\Sigma'_{pp} & 0 & 0 & 0 & 0 \\
0 & \Sigma'_{ss} & 0 & 0 & 0 \\
0 & 0 & \Sigma'_{sp} & 0 & 0 \\
0 & 0 & 0 & \Sigma'_{ss} & 0 \\
0 & 0 & 0 & 0 & \Sigma'_{ss}
\end{bmatrix},
\]

(25)

and the corresponding prior PDF of the model parameter \( m' \) is

\[
m' \sim N(\mu'_m, \Sigma'_m) = N(U\mu'_m, \Sigma'_m).
\]

(26)

It’s mentioned that the non-diagonal submatrices in \( \Sigma'_m \) present the statistical and spatial correlation of the five objective parameters. Apparently, there is no correlation between \( v'_p, v'_s, \rho', \delta', \varepsilon' \), and the calibrations of the cross-variograms between the five parameters are avoided, further diminishing the corresponding effect of correlation interference on anisotropic stochastic inversion precision. In addition, according to equation (25), the decoupling stochastic inversion only need to consider variograms for \( v'_p, v'_s, \rho', \delta', \varepsilon' \), and can ignore their cross-variograms. Therefore, the multivariate decoupling significantly simplifies the inversion process.

2.3 Anisotropy Bayesian linearized inversion with decoupling prior information

The seismic forward model for the decoupling parameter \( m' \) can be derived from equation (14) and (21)

\[
d = Gm + e = GUU^Tm + e = G'm' + e,
\]

(27)

where the PDF of \( e \) is also a Gaussian one

\[
e \sim N(0, \Sigma_e),
\]

(28)

\( \Sigma_e \) presents the correlation matrix for \( e \). According to above parameters, the seismogram distribution should be

\[
d \sim N(G'm'_m, G'S'_mG'^T + \Sigma_e).
\]

(29)
Consider a second observed datasets like borehole dataset, and construct a forward relationship between this kind of data and \( m' \)

\[
q = Qm = QU'U^Tm = Q'm',
\]

where \( Q' \) is the forward operator for the decoupled parameters and \( Q \) is the operator for the coupled parameters. Like equation (29), the PDF of the dataset \( q \) can also be expressed as

\[
q \sim N\left( Q'\mu'_m, Q'S'_mQ'^T \right).
\]

Following the BLI framework, the posterior PDF of the decoupled \( m \), constrained by the aforementioned two types of datasets, is given by

\[
m' \mid (d, q) \sim N\left( \mu'_{m(d,q)}, \Sigma'_{m(d,q)} \right),
\]

where the expressions of the final inversion result and the corresponding multivariate correlation matrix for uncertainty assessment are

\[
\begin{align*}
\mu'_{m(d,q)} &= \mu'_m + \left[ \Sigma'_m Q'^T \Sigma'_m G'^T \right] C_{(d,b)}^{-1} \left[ q - Q'\mu'_m \right], \\
\Sigma'_{m(d,q)} &= \Sigma'_m - \left[ \Sigma'_m Q'^T \Sigma'_m G'^T \right] C_{(d,b)}^{-1} \left[ Q'\Sigma'_m \right].
\end{align*}
\]

where

\[
C_{(d,b)} = \begin{bmatrix}
B\Sigma'_m B'^T & B\Sigma'_m G'^T \\
G\Sigma'_m B'^T & G\Sigma'_m G'^T + \Sigma_e
\end{bmatrix}.
\]

However, equation (33) represents only the PDF of the decoupling parameter, not the PDF of the real objective parameter. Therefore, an inverse decoupling transform must be conducted herein to obtain the posterior PDF of \( m \)

\[
m \mid (d, q) \sim N\left( \mu_{m(d,q)}, \Sigma_{m(d,q)} \right),
\]

and the posterior mean and correlation matrix can be expressed as,

\[
\begin{align*}
\mu_{m(d,q)} &= U\mu'_{m(d,q)} = U\mu'_m + U\left[ \Sigma'_m Q'^T \Sigma'_m G'^T \right] C_{(d,b)}^{-1} \left[ q - Q'\mu'_m \right], \\
\Sigma_{m(d,q)} &= U\Sigma'_{m(d,q)} U^T = U\Sigma'_m U^T - U\left[ \Sigma'_m Q'^T \Sigma'_m G'^T \right] C_{(d,b)}^{-1} \left[ Q'\Sigma'_m \right] U^T,
\end{align*}
\]

where the posterior mean \( \mu_{m(d,q)} \) represents the D-ABLI results of the objective parameters, and the posterior matrix \( \Sigma_{m(d,q)} \) characterizes the inversion uncertainty.
In summary, the D-ABLI can be divided into the following five steps, and the roadmap of D-ABLI is displayed in Figure 1.

1. Conduct the PCA-based decoupling strategy on the prior models of the objective parameters, and obtain decoupled prior models;
2. Conduct geostatistical analysis on decoupled prior models, and build the decoupled multivariate correlation matrix for decoupled parameters;
3. Construct PDF of prestack seismic data based on VTI forward model;
4. Estimate the inversion results of the spatial decoupled anisotropy model parameters in equation (33) based on Bayesian theorem;
5. Obtain the final D-ABLI results for $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ in VTI media using an inverse decoupled transformation.

**Figure 1.** The roadmap of D-ABLI.

### 3. Numerical examples

#### 3.1 Synthesized data test

To showcase the effectiveness of D-ABLI, the Overthrust model is employed for testing purposes. Figure 2 illustrates the tested models of $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$. The dataset consists of five models,
comprising 801 traces (spanning 8 km horizontally), and each trace comprises 187 nodes in the time direction.

The synthetic seismogram is generated based on the tested models depicted in Figure 2. Figures 3a–3c illustrate the noise-free synthesized angle gathers with incident angle of 9, 15, and 21 degrees. Figures 3d–3f present the seismic data with added noise, featuring a signal-to-noise ratio (SNR) of 6.

Figure 2. Real models of (a) $v_p$, (b) $v_s$, (c) $\rho$, (d) $\delta$, and (e) $\epsilon$. 
Figure 3. (a), (b), and (c) display the prestack seismic data without noise, corresponding to incident angles of 9, 15, and 21 degrees. (d), (e), and (f) showcase the angle gathers with added noise for angles of 9, 15, and 21 degrees.

1D tests of D-ABLI and ABLI are both conducted to verify the effectiveness of the decoupling strategy. The test data is derived from the first trace of the 2D Overthrust model. Statistical analyse is first conducted to verify the theorem expressed in equations (23), (24) and (25). As mentioned above, the correlation matrix illustrated in equation (16) is constructed through geostatistics, and the maximum of each submatrix in equation (23) depends on the sill value of the variogram and cross-variogram. The sill values of cross-variograms between the five objective variables without decoupling are displayed in Table 1, and the corresponding sills estimated from the decoupled parameters are shown in Table 2. Apparently, the sills in Table 2 are much smaller than those in...
Table 1, indicating a significant reduction in magnitude. Therefore, compared to the sills in Table 1, the sills in Table 2 can be regarded as zero, leading to the expressions of the two multivariate correlation matrices without and with decoupling in equations (23) and (25).

### Table 1

The sill values of cross-variograms for $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ before decoupling.

<table>
<thead>
<tr>
<th>Cross-variogram</th>
<th>$\gamma_{ps}$</th>
<th>$\gamma_{pp}$</th>
<th>$\gamma_{pc}$</th>
<th>$\gamma_{p\delta}$</th>
<th>$\gamma_{sp}$</th>
<th>$\gamma_{sc}$</th>
<th>$\gamma_{s\delta}$</th>
<th>$\gamma_{pc}$</th>
<th>$\gamma_{p\delta}$</th>
<th>$\gamma_{sc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sill</td>
<td>1.5×10^{-3}</td>
<td>3.0×10^{-2}</td>
<td>3.0×10^{-3}</td>
<td>5.0×10^{-4}</td>
<td>2.5×10^{-2}</td>
<td>1.5×10^{-3}</td>
<td>2.0×10^{-4}</td>
<td>3.5×10^{-4}</td>
<td>5.0×10^{-3}</td>
<td>4.5×10^{-6}</td>
</tr>
</tbody>
</table>

### Table 2

The sill values of cross-variograms for $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ after decoupling.

<table>
<thead>
<tr>
<th>Cross-variogram</th>
<th>$\gamma_{ps}$</th>
<th>$\gamma_{pp}$</th>
<th>$\gamma_{pc}$</th>
<th>$\gamma_{p\delta}$</th>
<th>$\gamma_{sp}$</th>
<th>$\gamma_{sc}$</th>
<th>$\gamma_{s\delta}$</th>
<th>$\gamma_{pc}$</th>
<th>$\gamma_{p\delta}$</th>
<th>$\gamma_{sc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sill</td>
<td>7.0×10^{-3}</td>
<td>6×10^{-4}</td>
<td>0</td>
<td>0</td>
<td>1×10^{-5}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure 4.** Results of (a) D-ABLI and (b) ABLI estimated from noise-free seismic data. $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ are listed from left to right in each subfigure. The black dashed line, along with the red and blue lines, symbolizes...
the actual models, inversion outcomes, and prior models for the five objective parameters. The green dashed lines are the boundaries of 95% confidence interval of the inversion results.

The foundation of D-ABLI has been substantiated through the preceding analysis. Subsequently, a noise-free test of D-ABLI is performed. The inverted models of these objective parameters are showcased in Figure 4. Figure 4a shows the inverted models estimated by D-ABLI, and Figure 4b shows the inversion results obtained by ABLI without parameter decoupling. In both figures, the black dashed line, along with the red and blue lines, represents the actual models, inversion results, and initial models for the five objective parameters, respectively. The green dashed lines indicate the boundaries of the 95% confidence interval of the inversion results. The inverted curves maintain excellent accordance with the real models, even though the prior models are quite smooth. The root-mean-square errors (RMSEs) between real and inverted models in Figure 4a are 0.055 km/s, 0.036 km/s, 0.008 g/cc, 0.0009, and 0.0001 for $v_p$, $v_s$, $\rho$, $\delta$, $\varepsilon$, respectively. In comparison, RMSEs for the five objective parameters in Figure 4b are 0.078 km/s, 0.051 km/s, 0.026 g/cc, 0.0021, and 0.0003, respectively. The inversion accuracy of D-ABLI is apparently higher than that of ABLI, especially for density, epsilon, and delta. The precision of inverted $v_p$ and $v_s$ are also acceptable. Therefore, the calibrations of spatial and statistical correlations may have a more serious effect on the inversion of density and the two types of anisotropy parameters.
Figure 5. Results of (a) D-ABLI and (b) ABLI estimated from noisy seismic data. $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ are listed from left to right in each subfigure. The black dashed line, along with the red and blue lines, symbolizes the actual models, inversion outcomes, and prior models for the five objective parameters. The green dashed lines are the boundaries of 95% confidence interval of the inversion results.

Then, 1D D-ABLI and ABLI are also tested using noisy seismic data to verify the effectiveness of the proposed decoupling strategy with noisy data. Figure 5a presents the inversion results of D-ABLI, while Figure 5b showcases the inversion results of ABLI. The meanings of different curves in Figure 5 are the same as those in Figure 4. The RMSEs between the real models and inversion results are 0.11 km/s, 0.08 km/s, 0.02 g/cc, 0.001, and 0.0002 for the five objective parameters in Figure 5a, and 0.15 km/s, 0.13 km/s, 0.05 g/cc, 0.0024, and 0.0006 in Figure 5b. Compared with the noise-free results of D-ABLI in Figure 4a, the noisy D-ABLI results in Figure 5a apparently have lower accuracy. However, the inversion results are still acceptable for identifying thin layers in the models, illustrating the satisfying noise-immunity of D-ABLI. Comparing Figures 4b with 5b, the ABLI without decoupling seems more susceptible to noise. The noisy ABLI results shown in Figure 5b lack accuracy and deviate from the real model, especially for density, epsilon and delta. Therefore, it is apparent that the correlation calibration can influence the accuracy of Bayesian linearized
stochastic inversion method.

Figure 6. Inverted models for (a) $v_p$, (b) $v_s$, (c) $\rho$, (d) $\delta$, and (e) $\varepsilon$ from noise-free seismic data.
Finally, a 2D D-ABLI is performed using both 2D noise-contaminated and noise-free seismogram. Figures 6a-6e depict the noise-free inverted models for $v_p$, $v_s$, $\rho$, $\delta$, and $\varepsilon$, respectively, and Figures 7a-7e display the noisy inversion results. In Figure 6, the inverted models align well with real models and also have high resolution. The layers, faults, and certain subtle geological bodies are clearly illustrated. In Figure 7, although the accuracy and lateral continuity are affected by noise, the small-scale geological bodies can still be identified apparently. Therefore, D-ABLI demonstrates satisfying performance in both 1D and 2D synthetic data tests, thereby validating the contribution of the proposed decoupling strategy to the accuracy of Bayesian stochastic inversion.

3.2 Field data test

After verifying the effectiveness in the synthetic data test, D-ABLI is applied in a field case to calculate anisotropy parameters and further to calculate the subsurface fracture density. The tested field data comes from a shale-oil reservoir characterized by mainly horizontal fractures. Hence, the forward model in VTI media is suitable for this field data test.
Figure 8. Angle gathers from an oil field with a shale oil reservoir, corresponding to incident angles of (a) 9°, (b) 15°, and (c) 21°.

The tested field prestack seismic data are showcased in Figure 8, with the corresponding incidence angle of 9, 15, and 21 degrees in Figures 8a–8c. The 2D dataset comprises 197 traces in horizontal direction and 700 sampling points in the time direction. The time range of the tested seismic data varies from 1250 milliseconds (ms) to 1950 ms, with the target layer producing shale oil spanning from 1700 ms and 1890 ms (marked by black dashed line in Figure 8a). A drilled well located at the 175th trace of the 2D seismic profile is used as the blind well to verify the effectiveness of D-ABLI. Then, the D-ABLI is conducted to characterise the distribution of the shale-oil reservoir. Reasonable prior models are crucial for D-ABLI to ensure the precision of inverted models. The prior models of $v_p$, $v_s$, $\rho$, $\delta$, $\varepsilon$ are provided by the oil field with this shale oil reservoir, and displayed in Figures 9a-9e.
Using the above datasets, 1D inversion is first conducted to verify the effect of D-ABLI, and 1D ABLI without decoupling is also conducted for comparison. Figures 10a and 10b illustrate the inverted model of $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ obtained by D-ABLI and ABLI at the blind well, where $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ are shown from left to right. The well-log curves are represented by the black dashed line, while the inversion results and prior models for the five objective parameters are illustrated by the red and blue lines, respectively. The green dashed lines are the boundaries of 95% confidence interval of the inversion results. In Figure 10a, the D-ABLI results in red lines agree well with the well-log curves in black lines for all the five objective parameters. In contrast, the inverted P-wave velocity from ABLI is similar with that calculated by D-ABLI. However, the ABLI results apparently lack accuracy for S-wave velocity, density, epsilon, and delta especially at the positions indicated by the blue arrows. The accuracy of density, epsilon, and delta in Figure 10b is much lower than that in Figure 10a. Therefore, as can be observed explicitly in the 1D field data test, the decoupling strategy can improve the inversion results, especially for density, epsilon, and delta.
Figure 10. 1D inverted models calculated by (a) D-ABLI and (b) ABLI at the blind well. $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ are listed from left to right in each subfigure. The well-log curves are depicted by the black dashed line, while the inversion results and prior models for the five objective parameters are represented by the red and blue lines, respectively. The green dashed lines are the boundaries of 95% confidence interval of the inverted models.

Finally, the 2D field D-ABLI is conducted to determine the distribution of the shale-oil reservoir. Figures 11a-11e exhibit the 2D inverted models of $v_p$, $v_s$, $\rho$, $\delta$, $\epsilon$ respectively. These 2D inverted models intuitively have high resolution and satisfying lateral continuity, illustrating the effectiveness of D-ABLI. The well-log curves pasted on the 2D models further demonstrate the inversion accuracy.
The solid and dashed lines correspond to the borehole data and the position at this blind well. The inverted profiles exhibit satisfying agreement with the well-log curves. Besides, fracture density is essentially the final parameters of the anisotropic inversion herein, and it is related to epsilon. Hence, the D-ABLI result of epsilon is displayed again in Figure 11f, where the embedded curve is the fracture density, rather the epsilon curve. As is observed in Figure 11f, the fracture density is high between 1700~1870 ms, where the value of epsilon is big and a shale-oil reservoir is developed. Therefore, according to this field test, D-ABLI can predict anisotropic accurately, and further estimate fracture density for the characterization of shale oil reservoirs.

![Figure 11](https://academic.oup.com/jge/advance-article/doi/10.1093/jge/gxae049/7654000)

Figure 11. D-ABLI results of (a) $v_p$, (b) $v_s$, (c) $\rho$, (d) $\delta$, and (e) $\varepsilon$. The black dashed line denotes the location of blind well, and the black solid lines represent the well-log curves for various parameters. (f) also displays the inversion result of epsilon, while the black full line in (f) is the curve of fracture density.
4. Discussion

Anisotropic parameters play a crucial role in representing the distribution of fractures and, consequently, contribute to shale-reservoir characterization. Anisotropic inversion, essentially a multi-parameter inversion method, typically involves simultaneously inverting five or six parameters. Certain research also simplifies anisotropic forward model as a form including three objective parameters (Zhang et al. 2020). The interference among different parameters presents a significant challenge in multi-parameter inversion, and this problem becomes more pronounced as the number of model parameters increases. Consequently, the interference issue in anisotropic inversion is more pronounced than in traditional AVO inversion. D-ABLI presents a novel solution to the interference issue in stochastic anisotropic inversion.

The existing decorrelation strategy is employed to mitigate interference arising from statistical correlation in both deterministic AVO inversion and anisotropic inversion. However, there has been a lack of research specifically addressing interference in stochastic anisotropic inversion, which necessitates the consideration of both statistical and spatial correlation of model parameters. The key point of decoupling strategy outlined here is conducting PCA-based decorrelation on the crucial multivariate correlation matrix in stochastic inversion. This approach is extended to spatial decorrelation, which further leads to the formulation of D-ABLI. The effectiveness of D-ABLI is evident in both synthetic and field data tests, where D-ABLI can achieve satisfactory inversion accuracy and systematic uncertainty evaluation simultaneously. These outcomes align with the theoretical derivation. The core of D-ABLI, spatial correlation decoupling, is proposed based on the relationship between covariance and cross-variogram among different model parameters. While this relationship is demonstrated approximately and relies on certain assumptions, it remains suitable for practical cases, as validated by the datasets presented in Tables 1 and 2.

As evidenced in the data tests, parameters with small magnitudes are more vulnerable to interference caused by correlation. The inversion results of density and anisotropic parameters
obtained through D-ABLI significantly outperform those from regular ABLI, whereas the differences in inverted $v_p$, and $v_s$ between D-ABLI and ABLI are not distinctly pronounced. It is well-known that the inversion of density and anisotropic parameters poses a challenge in most commercial inversion software. Therefore, D-ABLI holds promise in addressing this issue and enhancing the inversion accuracy of parameters with small magnitudes.

Spatial correlation decoupling has been overlooked in both GSI and BLI until now. However, a critical impact of stochastic inversion is to characterize the spatial correlation of subsurface parameters, typically presented by a multivariate correlation matrix and achieved through geostatistics. Consequently, correlation decoupling becomes an inevitable challenge in both GSI and BLI. Additionally, the correlation estimation is universally affected by the variogram calibration in field application, regardless of the amount of known data (e.g., the well-log data), which exacerbates the coupling problem. Therefore, the correlation decoupling should be considered an essential process in the field applications of stochastic inversion. While the proposed decoupling method is currently applied in BLI, its potential value in GSI is noteworthy. Moreover, the decoupling of statistical and spatial correlation is a critical subject in mathematical geosciences, not only in seismic inversion. Therefore, the decoupling method introduced herein holds a broad and promising application prospect.

An inevitable drawback of D-ABLI lies in the inversion of a large core matrix, limiting its practical applications. However, various dimension reduction approaches are proposed to tackle this issue, effectively reducing the size of the core matrix. Therefore, D-ABLI remains a promising tool for shale-oil reservoir characterization, provided that the matrix inversion problem is appropriately addressed.

5. Conclusion

D-ABLI is presented herein as an efficient and accurate method for achieving stochastic anisotropic inversion in the characterization of shale oil reservoirs. This approach leverages the
Bayesian linearized stochastic inversion framework to circumvent the heavy iterative calculation burden associated with GSI. A novel decoupling strategy, utilizing a PCA-based decorrelation method, is employed to alleviate the impact of spatial and statistical correlation interference on anisotropic inversion precision. The reasonability of this decoupling strategy is demonstrated by both the theoretical derivation and numerical examples. Results from synthetic and field data tests highlight that D-ABLI can accurately estimate subsurface elastic and anisotropic parameters while simultaneously evaluating their uncertainty. Particularly noteworthy is the contribution of D-ABLI to enhancing the inversion accuracy of parameters with small magnitudes, such as density and anisotropic parameters, which may represent a potential advantage compared to existing prestack inversion methods.

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