

Multi-objective fuzzy optimization for sustainable irrigation planning

Jyotiba B. Gurav^{a,*} and D. G. Regulwar^b

^a Department of Civil Engineering, Amrutvahini College of Engineering, Sangamner 422 608, Maharashtra, India

^b Department of Civil Engineering, Government College of Engineering and Research, Avasari Khurd, Pune 412 405, Maharashtra, India

*Corresponding author. E-mail: jyotiba.gurav@rediffmail.com

Abstract

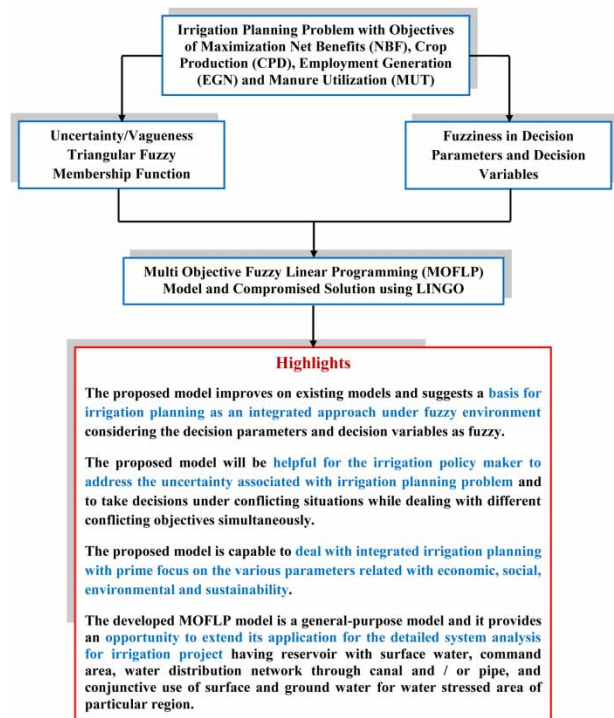
The objective of the present work is to determine an optimal cropping pattern under uncertainty, which maximizes four objectives simultaneously, including net benefits (NBF), crop production (CPD), employment generation (EGN) and manure utilization (MUT). Except the objective of maximizing the NBF, the other objectives are related to sustainability. To deal with uncertainty, a multi-objective fuzzy linear programming (MOFLP) model has developed along with fuzziness in decision parameters (objective function coefficient, cost coefficients, technological coefficients and resources) and decision variables (area to be irrigated under each crop in each season) and applied the same to Jayakwadi Project Stage-I, Maharashtra, India. The present study is in the form of a successful attempt to deal with irrigation planning associated with sustainability and uncertainty.

Key words: decision variables and decision parameters, fuzzy optimization, sustainable irrigation planning, uncertainty

Highlights

- The present study deals with the development of the MOFLP model under uncertainty that maximizes NBF, CPD, EGN and MUT simultaneously and its successful application for the case study of Jayakwadi Project Stage-I, Maharashtra State, India.
- The proposed model suggests a basis for irrigation planning as an integrated approach under a fuzzy environment considering the decision parameters and decision variables as fuzzy.

Graphical Abstract



INTRODUCTION

The randomness of hydrologic variables such as stream flow, rainfall and evapotranspiration, and imprecision in management goals/objectives, constraints, crop response, decision makers' interest, which are the most common things closely associated with uncertainty. The term fuzzy introduced and inducted first time by Zadeh (1965), which allows to develop fuzzy set theory in the form of an improvement over classical set theory. The vagueness and uncertainties associated with various kinds of objectives, decision parameters and decision variables can be well described and addressed by a fuzzy set. The concept of decision-making under a fuzzy environment was presented by Bellman & Zadeh (1970).

Gupta *et al.* (2000) have proposed a multi-objective fuzzy linear programming (MOFLP) area allocation model to analyze the conflicting interests of various decision makers such as the irrigation authority (government) and the individual farmers involved in a particular project and applied the developed model to the case study of the Narmada river basin, India. Gasimov & Yenilmez (2002) have discussed the methodology of fuzzy linear programming (FLP) problems. Jimenez *et al.* (2007) have presented an interactive method to solve linear programming with fuzzy numbers, which informs the decision maker in various stages of the decision process. The fuzzy parametric programming based MOFLP model has been proposed with the illustration of numerical example (Arikan & Gungor 2007).

Fuzziness and randomness associated with objective function and constraints can be dealt with the chance-constrained programming model (Nanda *et al.* 2008). The linear programming problem with the triangular fuzzy number (TFN) has been developed and demonstrated with suitable examples (Nasseri 2008).

The FLP problem in which decision parameters and decision variables, fuzzy in nature can be addressed and solved properly by the method proposed by Allahviranloo *et al.* (2008). The application

of MOFLP in irrigation planning along with fuzziness in objective function only and fuzziness in resources and the objective function is reported in the literature (Raju & Duckstein 2003; Regulwar & Gurav 2010, 2011; Gurav & Regulwar 2012).

Xu *et al.* (2013) have presented a bilevel optimization approach to deal with the regional water resource allocation problem using the technique of interactive fuzzy programming and an entropy-Boltzmann selection-based genetic algorithm under the fuzzy random environment. Guo *et al.* (2014) have presented an inexact fuzzy chance-constrained nonlinear programming (IFCCNP) method for agricultural water resources management to tackle various uncertainties. The model results obtained are helpful for decision makers to deliver insight for tradeoffs related to environmental, economic and system-reliability policy.

Srivastava & Singh (2017) have proposed fuzzy multi-objective goal programming (FMGP) to find the optimal cropping patterns with objective of maximization of benefit and production along with minimization of investment, fertilizer application and water application to the case study of canal command of Soraon, District Allahabad, Uttar Pradesh (India). Li *et al.* (2016) have proposed the bi-objective programming model with fuzzy inputs such as fuzziness associated with objective functions as well as in constraints, and the developed model has been solved using the fuzzy multi-objective optimization modeling approach for the case study of the Heihe River basin, northwest China. The MOFLP with a two-phase approach can be used for optimal cropping pattern planning, and using this approach, various irrigation planning models have been proposed and presented with its application for different case study of India (Regulwar & Gurav 2013; Mirajkar & Patel 2016).

Zhang *et al.* (2018) have developed an irrigation water allocation model with interval-based fuzzy chance-constrained along with double-sided fuzziness and applied it to the case study of the Heihe River Basin in Northwest China and found the interrelationships to support the decision-making for irrigation water management. Zhang & Guo (2018) have developed a model with the fuzzy linear fractional programming approach with double-sided fuzziness to address and tackle the uncertainty involved for optimal irrigation water allocation in the case study of middle reaches of the Heihe River Basin, in Northwest China. The obtained results help to plan reasonable irrigation water resources management and agricultural production. Banihabib *et al.* (2019) have proposed the fuzzy multi-objective heuristic model to maximize three objective functions along with the uncertainties involved in water resources and economic parameters in a basin and found that optimal operation policies provide better results than the deterministic model. Yue *et al.* (2020) have developed a full fuzzy-interval credibility-constrained nonlinear programming (FFICNP) model to deal with uncertainties in planning irrigation water allocation and applied the same successfully to the case study of the Zhanghe irrigation district in Hubei Province, Central China. The obtained results show lower credibility level with respect to higher net system benefit and system efficiency.

With the above literature review presented, it is found that various models of the LP and MOFLP have been used to tackle the uncertainty and vagueness involved in the irrigation planning scenario. Also, it is found that any model has not been reported in sustainable irrigation planning with fuzzy decision parameters and fuzzy decision variables. Most of the uncertainties associated with irrigation planning are dependent and interrelated with each other in agricultural systems, so it is necessary that in the irrigation planning problem, uncertainties should be considered and addressed properly. Given this, the decision parameters and decision variables are considered fuzzy in nature, which leads in the form of successful attempts in the field of sustainable irrigation planning. The objective of the present study is to develop the MOFLP model and its application to the Jayakwadi Project Stage-I and to find out the optimal cropping pattern for 75% dependable yield for sustainable irrigation planning under the fuzzy environment.

METHODOLOGY AND MODEL DEVELOPMENT

Description of the study area

To represent the applicability of the MOFLP model, a representative agricultural irrigation management problem of the Jayakwadi Project Stage-I is considered. It is an earth type of dam constructed across river Godavari. The reservoir has an active storage capacity of 2,170 Mm³ and a dead storage capacity of 738 Mm³. It has an irrigable command area of 141,640 ha (1,416.40 km²). Also, the project is a hydropower plant, with a pumped storage with a capacity of 12 MW. The main crops grown in the command area are Sugarcane, Banana, Chilies, L S Cotton, Sorghum, Paddy, Wheat, Gram and Groundnut. The location map of the study area is shown in Figure 1.

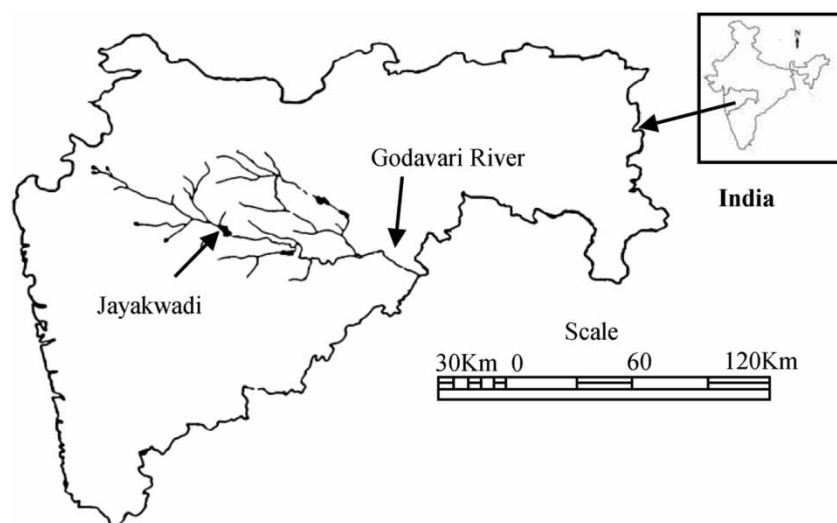


Figure 1 | Jayakwadi Project Stage-I Maharashtra State, India.

Objective function

The various objectives are considered in the formulation of the irrigation planning model depending upon the requirement of the particular region and national importance. In the present study, four objectives of maximization type are considered from an analysis point of view, which are listed as below.

Net benefits

Most of the time, farmers want to maximize the net benefits (NBF) with the cultivation of particular crops for economic prosperity, due to which the decision maker has to incorporate this objective as part of the irrigation planning policy. Gross benefits are calculated by multiplying the average yield of a particular crop per ha and current market price of that crop. In the present study, the net benefits per ha for each crop, which are calculated by subtracting the input cost of the gross benefits. The input cost considered in the present model, which may vary according to the market conditions and different parts of the region, such variations in the input cost can be

incorporated in the model.

$$\begin{aligned} \text{Maximize } NBF = & \left[\left(\sum_{i=1}^2 J_i^K F_i^K + \sum_{i=1}^3 J_i^R F_i^R + \sum_{i=1}^2 J_i^{TS} F_i^{TS} + \sum_{i=1}^2 J_i^P F_i^P + \sum_{i=1}^1 J_i^{HW} F_i^{HW} \right) \right. \\ & \left. - \left(\sum_{i=1}^2 J_i^K G_i^K + \sum_{i=1}^3 J_i^R G_i^R + \sum_{i=1}^2 J_i^{TS} G_i^{TS} + \sum_{i=1}^2 J_i^P G_i^P + \sum_{i=1}^1 J_i^{HW} G_i^{HW} \right) \right] \end{aligned} \quad (1)$$

[*I* = crop index, which includes various crops in different seasons such as Sugarcane (*P*), Banana (*P*), Chilies (*TS*), L, S Cotton (*TS*), Sorghum (*K*), Paddy (*K*), Sorghum (*R*), Wheat (*R*), Gram (*R*) and Groundnut (*HW*). *P* = Perennial, *K* = Kharif, *R* = Rabi, *TS* = Two Seasonal and *HW* = Hot Weather]

J_i^K = area in ha for *i*th crop (*K*);

J_i^R = area in ha for *i*th crop (*R*);

J_i^{HW} = area in ha for *i*th crop (*HW*);

J_i^P = area in ha for *i*th crop (*P*);

J_i^{TS} = area in ha for *i*th crop (*TS*);

F_i = benefit coefficient for *i*th crop;

G_i = input cost for *i*th crop.

Crop production

To expect maximum production of a particular crop is the natural tendency of every farmer, which needs to be taken into account by the decision maker for the optimal cropping pattern planning with the objective of maximization of crop production (CPD).

The average yield of a crop per ha is taken as CPD coefficient ([Commissionerate of Agriculture, Maharashtra State 2006](#)).

$$\text{Maximize } (CPD) = \left[\left(\sum_{i=1}^2 J_i^K U_i^K + \sum_{i=1}^3 J_i^R U_i^R + \sum_{i=1}^2 J_i^{TS} U_i^{TS} + \sum_{i=1}^2 J_i^P U_i^P + \sum_{i=1}^1 J_i^{HW} U_i^{HW} \right) \right] \quad (2)$$

U_i = average yield of *i*th crop (Tons per ha).

The sufficient food availability in the region can be considered with the help of maximization of CPD, which mainly focuses on the survival of people with satisfaction of food demand of the particular region, and given this the second objective can be thought of sustainability-related.

Employment generation

The socio-economic development of a region cannot be possible without providing employment generation (EGN), which requires the decision maker to include the maximization of EGN in irrigation planning.

$$\text{Maximize } (EGN) = \left[\left(\sum_{i=1}^2 J_i^K M_i^K + \sum_{i=1}^3 J_i^R M_i^R + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{HW} M_i^{HW} \right) \right] \quad (3)$$

M_i = requirement of Man Days for *i*th crop per ha.

The number of Man Days or labor required for a particular crop per ha has been considered based on discussions with farmers and experts from the agricultural field.

The third objective is associated with sustainability with the view of socio-economic development for a developing country like India, where there is uneven distribution of agricultural land due to which many people are available in the form of labor for the agricultural sector.

Manure utilization

$$\text{Maximize } (MUT) = \left[\left(\sum_{i=1}^2 J_i^K M_i^K + \sum_{i=1}^3 J_i^R M_i^R + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{HW} M_i^{HW} \right) \right] \quad (4)$$

M_i = requirement of manure utilization (MUT) for i th crop (Tons per ha).

The discussion with farmers and experts from the agricultural field, which is considered as a basis to work out the requirement of MUT for a crop per ha. The requirement of MUT, which depends upon various parameters such as market condition and its availability, which may vary from place to place. These changes can be incorporated in the model, which may vary the requirement of MUT.

In India, there is a tradition of preparing the green manure by the farmer with the decomposition of waste from various activities of the farming and livestock, which is the richest, recycled and pure source for various ingredients of manure which does not include any harmful chemicals, fertilizers and pesticides. The manure prepared by this procedure helps to retain nutrient sufficiency of soil for various crops in each crop season. So, more and more utilization of green manure relates to sustainability and to be included in the cropping pattern by the decision maker.

Constraints

Total sowing area

The total sowing area constraint can be represented by the following equation:

$$\left(\sum_{i=1}^2 J_i^K + \sum_{i=1}^3 J_i^R + \sum_{i=1}^2 J_i^{TS} + \sum_{i=1}^2 J_i^P + \sum_{i=1}^1 J_i^{HW} \right) \leq TJ \quad (5)$$

TJ = total command area.

Maximum sowing area

The maximum sowing area constraint can be represented by the equation as follows:

Kharif

$$\left(\sum_{i=1}^2 J_i^P + \sum_{i=1}^2 K_i^{TS} + \sum_{i=1}^2 J_i^K \right) \leq TJ_i^P + TJ_i^{TS} + TJ_i^K \quad (6)$$

Rabi

$$\left(\sum_{i=1}^2 J_i^P + \sum_{i=1}^2 K_i^{TS} + \sum_{i=1}^3 K_i^R \right) \leq TJ_i^P + TJ_i^{TS} + TJ_i^R \quad (7)$$

Hot weather and perennial

$$\left(\sum_{i=1}^2 J_i^P + \sum_{i=1}^1 J_i^{HW} \right) \leq TJ_i^P + TJ_i^{HW} \quad (8)$$

TJ_i^K = area for i th crop (K) in ha;

TJ_i^R = area for i th crop (R) in ha;

TJ_i^{HW} = area for i th crop (HW) in ha;

TJ_i^P = area for i th crop (P) in ha.

TJ_i^{TS} = area for i th crop (TS) in ha.

The values for TJ (i.e. for particular crop in particular season) have considered according to the existing cropping pattern of the project.

Affinity constraint**Perennial**

$$J_1^P \leq TJ_i^P \quad (9a)$$

J_1^P = area in ha for Sugarcane (P).

$$J_2^P \leq TJ_i^P \quad (9b)$$

J_2^P = area in ha for Banana (P).

Two seasonal

$$J_3^{TS} \leq TJ_i^{TS} \quad (9c)$$

J_3^{TS} = area in ha for Chilies (TS).

$$J_4^{TS} \leq TJ_i^{TS} \quad (9d)$$

J_4^{TS} = area in ha for L S Cotton (TS).

Kharif

$$J_5^K \leq TJ_i^K \quad (9e)$$

J_5^K = area in ha for Sorghum (K).

$$J_6^K \leq TJ_i^K \quad (9f)$$

J_6^K = area in ha for Paddy (K).

Rabi

$$J_7^R \leq TJ_i^R \quad (9g)$$

J_7^R = area in ha for Sorghum (R).

$$J_8^R \leq TJ_i^R \tag{9h}$$

J_8^R = area in ha for Wheat (R).

$$J_9^R \leq TJ_i^R \tag{9i}$$

J_9^R = area in ha for Gram (R).

Hot weather

$$J_{10}^{HW} \leq TJ_i^{HW} \tag{9j}$$

J_{10}^{HW} = area in ha for Groundnut (HW).

Labor availability constraint

With the view of the problem of the unavailability of labor during the farming season, it is suggested that to tackle the problem of uncertainty involved in the availability of labor, the labor requirement should not exceed the total labor availability during that particular crop season.

Kharif

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^2 J_i^K M_i^K \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^2 AM_i^K \tag{10}$$

Rabi

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^3 J_i^R M_i^R \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^3 AM_i^R \tag{11}$$

Perennial and hot weather

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{HW} M_i^{HW} \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^1 AM_i^{HW} \tag{12}$$

AM_i = labor availability for i th crop;
 M_i = requirement of Man Days for i th crop per ha.

Manure availability constraint

With the view of the scarcity of manure and to ensure the fertility of soil, it is considered that in order to maintain the fertility of the soil, the total manure requirement should not exceed the total availability of the manure in that crop season.

Kharif

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^2 J_i^K M_i^K \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^2 AM_i^K \quad (13)$$

Rabi

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^2 J_i^{TS} M_i^{TS} + \sum_{i=1}^3 J_i^R M_i^R \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^2 AM_i^{TS} + \sum_{i=1}^3 AM_i^R \quad (14)$$

Perennial and hot weather

$$\left(\sum_{i=1}^2 J_i^P M_i^P + \sum_{i=1}^1 J_i^{HW} M_i^{HW} \right) \leq \sum_{i=1}^2 AM_i^P + \sum_{i=1}^1 AM_i^{HW} \quad (15)$$

AM_i = manure availability for i th crop;

M_i = requirement of MUT for i th crop per ha.

Water availability constraint

The total water requirement of different crops should not exceed the total water availability in the reservoir,

$$\left(\sum_{i=1}^2 J_i^K W_i^K + \sum_{i=1}^3 J_i^R W_i^R + \sum_{i=1}^2 J_i^{TS} W_i^{TS} + \sum_{i=1}^2 J_i^P W_i^P + \sum_{i=1}^1 J_i^{HW} W_i^{HW} \right) \leq AW_i^j \quad (16)$$

$j = 1, 2, 3, 4, 5$ (No of crop seasons);

W_i = irrigation water requirement for i th crop;

AW_i^j = total water availability for i th crop (all crops) for j th interval (all seasons).

Non negativity constraint

$$J_i^K, J_i^R, J_i^{TS}, J_i^P, J_i^{HW}, U_i, M_i, TJ, TJ_i^K, TJ_i^R, TJ_i^P, TJ_i^{TS}, TJ_i^{HW}, AM_i^P, AM_i^{TS}, AM_i^K, AM_i^R, AM_i^{HW}, M_i^P, M_i^{TS}, M_i^K, M_i^R, M_i^{HW}, W_i^P, W_i^{TS}, W_i^K, W_i^R, W_i^{HW}, AW_i^j \geq 0 \forall i, j \quad (17)$$

Triangular fuzzy number

A fuzzy number \tilde{A} is a convex normalized fuzzy set on the real line R such that:

(a) there exists at least one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$; (b) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Let us assume that the membership function of any fuzzy number \tilde{A} is as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{m^A - x}{\alpha^A}, & m^A - \alpha^A \leq x < m^A \\ 1 - \frac{x - m^A}{\beta^A}, & m^A \leq x \leq m^A + \beta^A \\ 0, & \text{otherwise} \end{cases}$$

where m^A is the mean value of \tilde{A} and α^A and β^A are left and right spreads, respectively, and it is termed as the triangular fuzzy number. The TFN represented as shown in Figure 2.

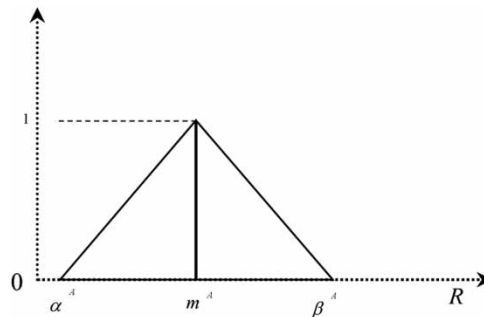


Figure 2 | TFN (\tilde{A}).

FLP problem

The FLP model can be represented in the following manner:

$$\begin{aligned}
 \max \tilde{z} &= \tilde{c}_1 \otimes \tilde{x}_1 \oplus \dots \oplus \tilde{c}_n \otimes \tilde{x}_n \\
 \text{s.t.} \\
 \tilde{a}_{11} \otimes \tilde{x}_{11} \oplus \dots \oplus \tilde{a}_{1n} \otimes \tilde{x}_n &\leq \tilde{b}_1 \\
 \cdot \\
 \cdot \\
 \cdot \\
 \tilde{a}_{m1} \otimes \tilde{x}_1 \oplus \dots \oplus \tilde{a}_{mn} \otimes \tilde{x}_n &\leq \tilde{b}_m \\
 \tilde{x}_1 \geq 0, \tilde{x}_2 \geq 0, \dots, \tilde{x}_n &\geq 0.
 \end{aligned}
 \tag{I}$$

The matrix form of the above equation is:

$$\begin{aligned}
 \max \tilde{z} &= \tilde{c} \otimes \tilde{x} \\
 \text{s.t.} \\
 \tilde{A} \otimes \tilde{x} &\leq \tilde{b} \\
 \tilde{x} &\geq 0.
 \end{aligned}
 \tag{II}$$

The coefficient matrix $\tilde{A} = [\tilde{a}_{ij}]_{m \times n}$, $1 \leq i, j \leq n$ is $m \times n$ fuzzy matrix where $\forall i, j$, $\tilde{a}_{ij} > 0$ or $\tilde{a}_{ij} < 0$ and $\tilde{x}_i, \tilde{b}_j \in F(\mathbb{R})$.

If matrix \tilde{A} denoted by $\tilde{A} = (A, A', A'')$ that $A = [a_{ij}]$, $A' = [a'_{ij}]$, $A'' = [a''_{ij}]$, $\tilde{x} = (x, x', x'')$, $\tilde{b} = (b, b', b'')$. Then we have:

$$\begin{aligned}
 \max \tilde{z} &= (c, c', c'') \otimes (x, x', x'') \\
 \text{s.t.} \\
 (A, A', A'') \otimes (x, x', x'') &\leq (b, b', b'') \\
 (x, x', x'') &\geq 0.
 \end{aligned}
 \tag{III}$$

Algorithm for MOFLP

The detailed solution process for the MOFLP model can be described as follows:

1. Formulate the irrigation planning problem with the help of TFN for decision parameters and decision variables of objective functions and constraints.
2. Solve the model as an FLP problem, considering only one objective individually.
3. Work out the corresponding values of each objective with reference to the procedure adopted in step 2.
4. Pick up the best value (Z_U) and worst value (Z_L) for each objective function comparing the values obtained steps 2 and 3.
5. Formulate the linear membership function with reference to (Z_U) and (Z_L) values for each objective.
6. Introduce a new dummy variable as level of satisfaction (λ), which is subject to the new constraints incorporating vagueness involved in the objective function values and original constraints and maximize it.
7. Identify significant parameters and interactions and formulate the equivalent FLP model as a MOFLP model and solve the formulated model.
8. Analyze the results and find out the compromised solution with the level of satisfaction (λ) under the fuzzy environment.
9. Stop.

The structure of the fuzzy approach for a compromised solution of the MOFLP model is as depicted in [Figure 3](#).

RESULTS AND DISCUSSION

The present study explores the development of the MOFLP model with a TFN of linear membership functions and its applicability for sustainable irrigation planning to the Jayakwadi Project Stage-I, India. Maximization of NBF, CPD, EGN and MUT, these are the objectives for which the irrigation policy maker has to focus with the same priority with available limited resources of the project under uncertainty.

The MOFLP problem considers fuzzy input data by fuzzy membership functions. The objectives (Equations (1)–(4)) and constraints ((5)–(17)), which are imprecise and uncertain, are represented by fuzzy sets in the form of TFN. The fuzziness associated with stipulations (resources) can be well described by considering the tolerance range of membership functions. Various decision parameters (cost coefficients, technological coefficients and resources) and decision variables (area to be irrigated under each crop in each season) are treated as imprecise (fuzzy) in the form of constraints for the existing cropping pattern of the project. Similarly, for objective functions, various decision parameters (objective function coefficients, cost coefficients) and decision variables (area to be irrigated under each crop in each season) considered as fuzzy. The fuzzy objective functions (Equations (1)–(4)) are being maximized individually subject to fuzzy constraints (Equations (5)–(17)) using LINGO 13.0 (Language for INteractive General Optimization). The MOFLP model which gives compromised solution involves total 35 variables and 51 constraints.

[Table 1](#) shows the results of four different objectives for its individual maximization, with the help of which, the linear membership function has constructed for each objective by choosing the best value (z^+) and worst value (z^-), which are indicated in bold figures. The objective functions rise linearly from zero to one for membership functions $\mu_1(X)$, $\mu_2(X)$, $\mu_3(X)$ and $\mu_4(X)$ of the fuzzy sets. The

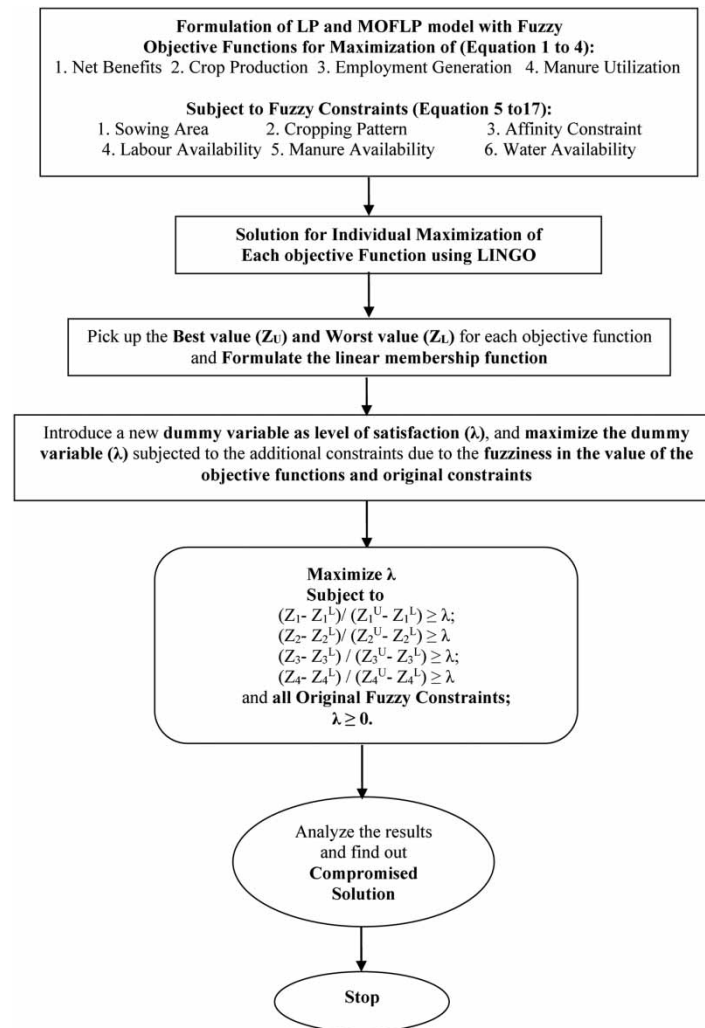


Figure 3 | Structure of the fuzzy approach for the compromised solution of the MOFLP model.

highest values obtained are $Z_1 = 1,261.99$ Million Rs, $Z_2 = 355,352.60$ Tons, $Z_3 = 25.83$ Million Man Days and $Z_4 = 137,038.10$ Tons, respectively.

In case of NBF, $Z_1 = 941.77$ Million Rs or less if level of satisfaction (λ) rises from zero and when it becomes one, $Z_1 = 1,261.99$ Million Rs or more. Similarly, for CPD, $Z_2 = 80,250.94$ Tons or less if λ rises from zero and when it becomes to one, $Z_2 = 355,352.60$ Tons or more. Also, $Z_3 = 17.44$ Million Man Days or less if λ rises from zero and when it becomes to one, $Z_3 = 25.83$ Million Man Days or more for EGN. Similarly, for MUT, $Z_4 = 52,932.98$ Tons or less if λ rises from zero and when it becomes to one, $Z_4 = 137,038.10$ Tons or more.

The MOFLP model has been formulated for the irrigation planning problem on the basis of FLP formulation and applied successfully to the case study under consideration. The optimal fuzzy solution for the first objective is: $X_1 = (4,249.20, 4,249.20, 0)$, $X_2 = (2,124.60, 2,124.60, 0)$, $X_3 = (4,249.20, 4,249.20, 0)$, $X_4 = (0, 0, 0)$, $X_5 = (16,996.23, 16,996.23, 0)$, $X_6 = (6,445.84, 6,445.84, 0)$, $X_7 = (0, 0, 0)$, $X_8 = (35,410, 35,410, 0)$, $X_9 = (7,082, 7,082, 0)$, $X_{10} = (0, 0, 0)$ and the optimal value of the NBF = 1,261.99 Million Rs.

Similarly, the optimal fuzzy solution for each individual objective and optimal values of the other objectives are represented in Table 1. From the results presented in Table 1, the solution of decision variables for various crops is in the form of the TFN. The TFN has values in the form of left spread, mean value and right spread. When the demand or constraint is satisfied, the membership grade is 1, it

Table 1 | Solution of the LP model and compromised solution of the MOFLP model

Fuzzy variable	Crop and season	Solution for individual maximization of each objective function				Compromised solution $\lambda = 0.58$
		NBF (Z ₁) (ha)	CPD (Z ₂) (ha)	EGN (Z ₃) (ha)	MUT (Z ₄) (ha)	
\tilde{X}_1	Sugarcane (P)	(4,249.20, 4,249.20, 0)	(4,249.20, 4,249.20, 0)	(0, 0, 0)	(4,247.93, 4,247.93, 0)	(0, 0, 6,498.64)
\tilde{X}_2	Banana (P)	(2,124.60, 2,124.60, 0)	(2,124.60, 2,124.60, 0)	(0, 0, 0)	(2,124.60, 2,124.60, 0)	(2,124.60, 2,124.60, 0)
\tilde{X}_3	Chilies (TS)	(4,249.20, 4,249.20, 0)	(4,249.20, 4,249.20, 0)	(4,249.20, 4,249.20, 0)	(4,249.20, 4,249.20, 0)	(4,249.20, 4,249.20, 0)
\tilde{X}_4	L S Cotton (TS)	(0, 0, 0)	(0, 0, 0)	(35,410, 35,410, 0)	(35,410, 35,410, 0)	(21,403.70, 0, 0)
\tilde{X}_5	Sorghum (K)	(16,996.23, 16,996.23, 0)	(16,996.23, 16,996.23, 0)	(16,996.23, 16,996.23, 0)	(16,996.23, 16,996.23, 0)	(0, 0, 50,988.68)
\tilde{X}_6	Paddy (K)	(6,445.84, 6,445.84, 0)	(0, 0, 0)	(14,164, 14,164, 0)	(14,164, 14,164, 0)	(14,164, 14,164, 0)
\tilde{X}_7	Sorghum (R)	(0, 0, 0)	(14,683.87, 14,683.87, 0)	(0, 0, 0)	(20,276.34, 20,276.34, 0)	(0, 0, 0)
\tilde{X}_8	Wheat (R)	(35,410, 35,410, 0)	(35,410, 35,410, 0)	(35,410, 35,410, 0)	(0, 0, 0)	(23,846.35, 23,846.35, 0)
\tilde{X}_9	Gram (R)	(7,082, 7,082, 0)	(0, 0, 0)	(6,437.84, 6,437.84, 0)	(0, 0, 0)	(7,082, 7,082, 0)
\tilde{X}_{10}	Groundnut (HW)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(4,249.20, 4,249.20, 0)	(0, 0, 0)
NBF (Million Rs)		1,261.99 (Z₁⁺)	1,240.78	941.77 (Z₁⁻)	1,094.60	1,127.60
CPD (Tons)		354,319.70	355,352.60 (Z₂⁺)	80,250.94 (Z₂⁻)	334,781.70	239,897.50
EGN (Million Man Days)		18.54	17.44 (Z₃⁻)	25.83 (Z₃⁺)	19.63	22.31
MUT (Tons)		57,480.78	52,932.98 (Z₄⁻)	119,802.80	137,038.10 (Z₄⁺)	115,969.60

means that level of satisfaction is highest (i.e. at apex) or it is having no spread on the left and right of the TFN. When the demand or constraint is not satisfied, the membership grade is lowest, the level of satisfaction is minimum, and corresponding spread on the left and right is observed for a TFN. The situation in which the constraint or demand is not satisfied at all, which can be represented as a horizontal line, i.e. no spread on left and right and zero mean value of the TFN. These features of TFN allow the irrigation policy maker to address and tackle the vagueness and/or uncertainty involved in the various parameters of irrigation planning, and given these, to formulate the irrigation planning problem, the TFN is used for the present study.

From Table 1, when the NBF is maximized, it is observed that the benefit coefficient is low for various crops such as Groundnut (*HW*), Sorghum (*R*) and LS Cotton (*TS*); because of this, the area to be allocated for irrigation under these crops is zero. Similarly, when the CPD is maximized, as the CPD coefficients are low for various crops such as Groundnut (*HW*), Gram (*R*), Paddy (*K*) and LS Cotton (*TS*); because of this, the area to be allocated for irrigation under these crops is zero. Also, in case of maximization of EGN, the area under irrigation is zero for Groundnut (*HW*), Sorghum (*R*), Banana (*P*) and Sugarcane (*P*), which is because the labor requirement per ha for these crops is low and also, the limited area allocation of these crops under the existing cropping pattern of the project. If MUT is maximized, then for crops such as Gram (*R*) and Wheat (*R*), the area under irrigation is zero because the manure requirement per ha is low and also due to the limited area under the existing cropping pattern of the project. If four conflicting objectives are considered simultaneously under the fuzzy environment, then crops such as Groundnut (*HW*) and Sorghum (*R*), the area under irrigation is zero.

The results are shown graphically for Sugarcane (*P*) for the fuzzy optimization of individual objectives in the form of TFN in Figure 4. The maximum level of satisfaction (λ) from the membership functions of four participating/conflicting objectives has been designated as the 'best' achieved/compromised solution. The final modified form of the MOFLP model with adding a new dummy variable $\lambda = \min [\mu_1(X), \mu_2(X), \mu_3(X), \mu_4(X)]$ such that:

Maximize λ Subject to:

$$(Z_1 - 941.77 \times 10^6)/(1261.99 \times 10^6 - 941.77 \times 10^6) \geq \lambda; (Z_2 - 80250.94)/(355352.60 - 80250.94) \geq \lambda$$

$$(Z_3 - 17.44 \times 10^6)/(25.83 \times 10^6 - 17.44 \times 10^6) \geq \lambda; (Z_4 - 52932.98)/(137038.10 - 52932.98) \geq \lambda$$

And all other original fuzzy constraints given in the model;

$$\lambda \geq 0.$$

The obtained value of the level of satisfaction (λ) = 0.58 is in the form of compromised solution, and it provides NBF = 1,127.60 Million Rs, CPD = 239,897.50 Tons, EGN = 22.31 Million Man Days and MUT = 115,969.60 Tons, respectively.

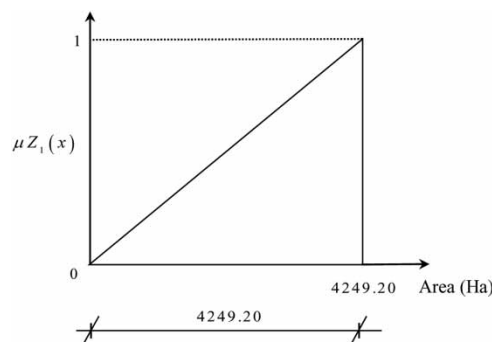


Figure 4 | Optimal solution of crop 1 in the form of TFN for maximization of NBF.

The problem of irrigation planning can be analyzed to deal with uncertainty in various strategies such as (i) fuzzy objective functions (\tilde{Z}_k), (ii) fuzzy cost coefficients (\tilde{C}_j), (iii) fuzzy resources (\tilde{b}_i), (iv) fuzzy technological coefficients (\tilde{A}_{ij}) and fuzzy resources (\tilde{b}_i) simultaneously and (v) fuzzy resources (\tilde{b}_i) and fuzzy decision variable (\tilde{X}_j) simultaneously (Regulwar & Gurav 2010, 2011, 2012; Gurav & Regulwar 2012). The uncertainty/vagueness arises in the values of the objective function because of sharing the same resources for more than one objective(s) (\tilde{Z}_k); which can be tackled successfully by trading off between the values of objective functions with fuzzy sets in the first strategy.

Also, the cost coefficients including the fluctuations in market price, input cost and monetary benefits associated with uncertainty/vagueness can be dealt with the second strategy. When the resources (b_i) are limited but with variation to certain extent due the practical condition/difficulties (e.g. migration of labor from one region to the other) which leads to uncertainty/vagueness in resources, that can be tackled by implementing the third strategy. Sometimes, the uncertainty associated with technological coefficients (A_{ij}), and resources (b_i) can be dealt with the fourth strategy.

Similarly, depending upon the field situation the strategy sixth (developed and proposed model for the present study) can be used by irrigation policy makers for irrigation planning. In the present study, the triangular membership function has been used to represent the fuzzy sets. Along with the triangular membership function, the other, such as linear membership functions-monotonously increasing and/or decreasing; linear membership functions; convex and/or concave, exponential membership functions and trapezoidal membership functions, can also be feasible for such problems.

The results which are represented in the form of Table 1 (present study) are improved qualitatively and quantitatively over the results presented in Table 2 (Regulwar & Gurav 2011). The uniqueness of the present manuscript is to deal the uncertainty/vagueness involved in the problem of sustainable irrigation planning issues in developing countries like India. The present paper deals with the uncertainty included in the objective functions (\tilde{Z}_k), cost coefficients (C_j), technological coefficients (A_{ij}), resources (b_i) and decision variable (X_j) simultaneously, by treating these as fuzzy. This seems to be probably closer to the real-world problem of irrigation planning associated with uncertainty.

Table 2 | Salient parameters of the optimal cropping pattern planning for \tilde{Z} , \tilde{b}_i , \tilde{a}_{ij} and \tilde{b}_i (Regulwar & Gurav 2011)

Sr no.	Crop and season	Compromised solution for \tilde{Z} ($\lambda = 0.580$) Area (ha)	Compromised solution for \tilde{b}_i ($\lambda = 0.503$) Area (ha)	Compromised solution for \tilde{a}_{ij} ($\lambda = 0.501$) Area (ha)	Compromised solution for \tilde{a}_{ij} and \tilde{b}_i ($\lambda = 0.287$) Area (ha)
1	Sugarcane (P)	2,166.18	1,871.80	1,872.20	1,839.55
2	Banana (P)	2,124.60	1,857.26	2,146.01	2,000.64
3	Chilies (TS)	4,249.20	3,714.52	4,357.91	4,090.02
4	LS Cotton (TS)	28,567.80	24,725.47	30,124.79	19,102.49
5	Sorghum (K)	16,996.80	14,858.08	17,889.34	16,988.02
6	Paddy (K)	14,164.00	12,381.73	14,907.79	14,156.69
7	Sorghum (R)	0	0	0	0
8	Wheat (R)	23,832.78	21,017.54	32,603.18	38,326.00
9	Gram (R)	7,082.00	6,190.86	7,263.18	6,816.70
10	Groundnut (HW)	0	0	0	0
Net Cropped Area (ha)		99,183.36	86,617.26	111,164.42	103,320.11
NBF (Million Rs)		1,503.73	1,314.87	1,617.66	1,602.43
CPD (Tons)		319,563.50	278,042.5	312,941.30	308,066.20
EGN (Million Man Days)		29.74	25.99	34.03	32.60
MUT (Tons)		154,506.50	134,365.3	163,647.90	131,783.00
Irrigation Intensity (%)		70.02	61.15	78.48	72.94

CONCLUSION

The present study deals with the development of the MOFLP model under uncertainty that maximizes NBF, CPD, EGN and MUT simultaneously and its successful application for the case study of Jayakwadi Project Stage-I, Maharashtra State, India.

The MOFLP model compromised solution has worked out with $\lambda = 0.58$, which suggests NBF = 1,127.60 Million Rs, CPD = 239,897.50 Tons, EGN = 22.31 Million Man Days and MUT = 115,969.60 Tons respectively. These obtained results of the present model are promising as it considers the uncertainty involved in irrigation planning problem which is closer to the scenario of the real world.

The proposed model improves on existing models and suggests a basis for irrigation planning as an integrated approach under the fuzzy environment considering the decision parameters and decision variables as fuzzy.

The proposed model will be helpful for the irrigation policy maker to address the uncertainty associated with irrigation planning problems and to take decisions under conflicting situations while dealing with different conflicting objectives simultaneously.

The proposed model is capable of dealing with integrated irrigation planning with prime focus on the various parameters related to economic, social, environmental and sustainability.

The MOFLP model developed in this paper is a general-purpose model, and it provides an opportunity to extend its application for the detailed system analysis for irrigation project having reservoir with surface water, command area, water distribution network through canal and/or pipe, and conjunctive use of surface and ground water for water-stressed area of the particular region.

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DATA AVAILABILITY STATEMENT

Data cannot be made publicly available; readers should contact the corresponding author for details.

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